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DYNAMICAL EFFECTS DOMINATE THE EVOLUTION OF CATACLYSMIC VARIABLES IN DENSE STAR CLUSTERS

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ABSTRACT

Strong interactions between the single and binary stars in the cores of dense clusters often cause the binaries' semimajor axes to contract. These "hardened" binaries are potent dynamical energy sources. Once significant physical interaction between a binary's components begins (e.g., mass transfer), the stellar evolution of that binary is intimately linked to the dynamical evolution of all the stars in the cluster. We self-consistently simulate the stellar dynamics, and binary and single-star evolution of a 100,000 star cluster with 5000 primordial binaries. The production of very close binaries containing a white dwarf is enhanced over that in the field; we focus on their formation, evolution, and fate. We report on a class of utterly novel CVs that never undergo a common envelope phase but are instead formed in exchange reactions. Exchange interactions are more likely to make CVs in which the main-sequence star mass is greater than 0.7 $M_{\odot}$, as opposed to CVs with low-mass donors. These dynamically produced CVs are more likely to be short-lived than their field counterparts. We find that the shorter lives of CVs in the harsh cluster environment decrease the expected number of CVs, at any given time, by a factor of 3. Finally, we provide the first self-consistent simulation of the period distribution of dynamics-dominated cataclysmic variables. We predict that there will be no 2–3 hour period gap for cluster CVs; the gap is smeared out by dynamical interactions of CVs with cluster stars.

Subject headings: globular clusters: general — methods: n-body simulations — novae, cataclysmic variables — open clusters and associations: general — stellar dynamics

1. INTRODUCTION

Stellar dynamicists have long known that binary stars are prodigious sources and sinks of energy in open and globular clusters (Aarseth 1971; Hut 1983). Three- and four-body encounters in crowded cluster cores provide dynamical support against cluster collapse, thereby stabilizing cluster evolution on timescales of Myr to many Gyr. These same interactions sometimes lead to the high-speed ejection from the cluster of one or more of the interacting stars. This evaporative process carries away both energy and mass from the cluster, driving the remaining stars into an ever denser and hotter configuration. In such dense environments, stars occasionally approach to within a few stellar radii of each other, with dramatic results. Grazing to head-on physical collisions begins to occur. Stellar partners are swapped into and out of binaries ("stellar promiscuity": Hurley & Shara 2002). The orbits of single and multiple stars are violently changed.

Physical collisions in open and globular clusters between stars with similar core densities (e.g., two main-sequence stars or two neutron stars) are inevitably amalgamative (Shara 2002). Very different stellar densities—e.g., a white dwarf (WD) striking a red dwarf (RD), or a neutron star (NS) running into a red giant (RG)—result in supersonic shocks, often accompanied by a large and sudden nuclear energy release. Dispersal and the consequent destruction of the less dense star follow on a dynamical timescale.

The results of stellar promiscuity are somewhat less spectacular but no less profound for a cluster's stellar populations (Hurley & Shara 2002). The lightest stars in three-body or four-body interactions are often expelled at speeds high enough to escape the cluster, leaving behind successively more massive binaries. The end product of this process is often a close double white dwarf (DWD). Successive interactions with passing single and double stars harden such binaries further. Type Ia supernovae are probably produced by such dynamically forced mergers of double degenerates. Shara & Hurley (2002) demonstrated an order-of-magnitude enhancement of mergers of double degenerates (with system mass in excess of the Chandrasekhar mass) relative to the field in a moderately dense cluster. Dense clusters may well be important factories for Type Ia supernovae, ultimately driven by N-body stellar dynamics.

In this paper we simulate the formation and dynamically driven evolution of cataclysmic variables (CVs) in star clusters. Double degenerates must approach to within a few WD radii before they begin to exchange mass. In contrast, the RD in a CV starts to transfer mass to its degenerate companion when they are much farther apart—a few RD radii. The eruptive behavior of a CV is critically dependent on the long- and short-term mass-transfer rates of hydrogen-rich matter to the WD. These rates, in turn, are controlled by the degree to which the RD overfills its Roche lobe, a sensitive function of the separation between the CV stellar components. The semimajor axis of a CV is changed during successive interactions with cluster stars. The most likely outcome is "hardening," which leads to an increased mass-transfer rate. In contrast, a "softening" interaction decreases this rate or, in the extreme, may even lead to cessation of mass transfer or to the CV becoming unbound. Thus, the evolution of

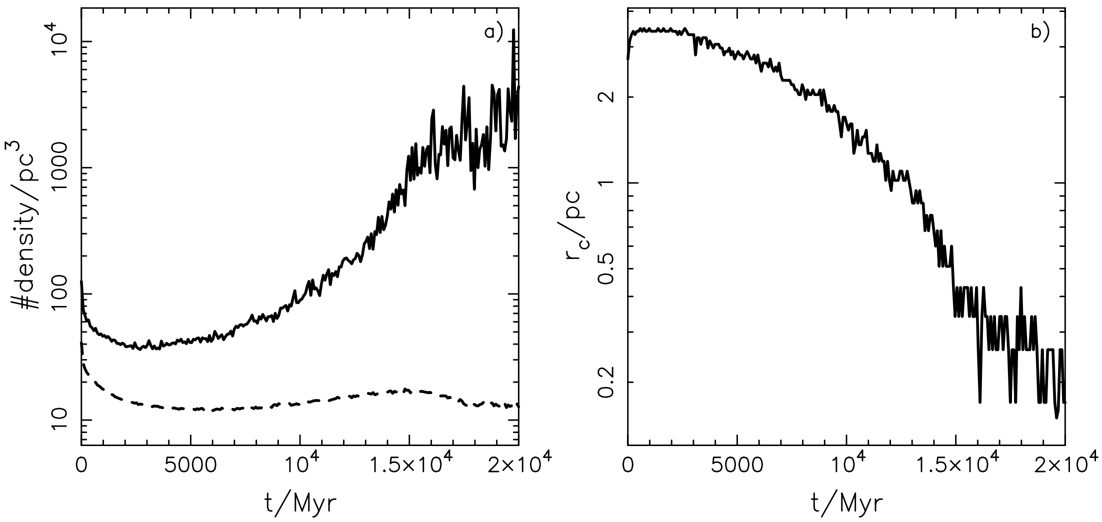

FIG. 1.—Parameters of the evolution for the 100,000 star model: (*a*) number density of stars in the core (*solid line*) and within the cluster half-mass radius (*dashed line*), and (*b*) core radius (*solid line*).

a CV is just as tightly coupled to its host cluster *N*-body dynamics as is the evolution of a double degenerate.

Our simulation method and initial conditions are detailed in § 2. In § 3 we briefly discuss the overall evolution of the star clusters. In § 4 we focus on the core results of this paper: the evolution of cataclysmic variables immersed in those clusters. Section 5 is a discussion of the broader implications of our results for binary evolution in clusters. We briefly summarize our results in § 6.

2. INITIAL MODEL

All simulations are performed on the GRAPE-6 boards (Makino 2002) located at the American Museum of Natural History. We use the NBODY4 code (Aarseth 1999) to self-consistently model the dynamical evolution of a star cluster as well as the stellar and binary evolution of the cluster stars (Hurley et al. 2001).

The focus of this paper is an *N*-body simulation that started with $N = 100,000$ objects, where 5% of these are primordial binaries and the remainder are single stars. Masses for the single stars were drawn from the initial mass function (IMF) of Kroupa et al. (1993) between the mass limits of 0.1 and 50 $M_{\odot}$. Each binary mass was chosen from the IMF of Kroupa et al. (1993), as this had not been corrected for the effect of binaries. The component masses were set by choosing a mass ratio from a uniform distribution. This is supported by the measured halo binary mass ratio distribution of Goldberg et al. (2003). Their definitive data set suggest, at most, modest q variations for their halo binary subsample (a significant excess of low-q systems is apparent in their disk subsample). We assume that all stars are on the zero-age main sequence when the simulation begins and that any residual gas from the star formation process has been removed from the cluster. We took $Z = 0.001$ for the metallicity of the stars. The orbital separations of the 5000 primordial binaries were drawn from the lognormal distribution suggested by Eggleton et al. (1989), with a peak at 30 AU. A maximum separation of 100 AU was imposed; the hard/soft binary limit of the initial model is about 30 AU. Orbital eccentricities were assumed to follow a thermal distribution (Heggie 1975). There is some suggestion that short-period binaries follow an eccentricity distribution with a mean of about 0.3 (Duquennoy & Mayor 1991). We find that artificially setting all initial binary eccentricities to 0.3 yields essentially no changes in the production of CVs from binary evolution. Thus, the results on CVs in clusters should not change with different initial eccentricities.

We used a Plummer density profile (Aarseth et al. 1974) and assumed that the stars are in virial equilibrium when assigning the initial positions and velocities. There was no mass segregation built into the initial model, and the binaries followed the overall mass distribution of the cluster stars. The cluster is subject to a standard Galactic tidal field: a circular orbit with a speed of 220 km s^{-1} at a distance of 8.5 kpc from the Galactic center. Stars are removed from the simulation when their distance from the density center exceeds twice that of the tidal radius of the cluster.

The initial model had a total mass of 51,770 $M_{\odot}$ contained within a tidal radius of 51 pc. The core and half-mass radii were 2.7 and 6.7 pc, respectively. This gave an initial half-mass relaxation timescale of 1400 Myr. The initial velocity dispersion of the model was 3.6 km s^{-1}.

3. 100,000 STAR CLUSTER EVOLUTION

The 100,000 star cluster was evolved to an age of 20 Gyr, which required about 6 months of dedicated time on a 32 chip GRAPE-6 board. Baumgardt & Makino (2003) have evolved clusters of a similar size, but this is the first simulation reported in the literature with such a large number of stars *and* binaries. While it might seem excessive to evolve a simulation beyond a Hubble time, there are good reasons for doing so. First, the behavior at 20 Gyr gives us some insight into the likely behavior, at 13 Gyr, of an initially denser cluster. Second, the evolution of classes of binaries over the entire lifetime of the cluster is important to study in order to see if long-term trends develop. Finally, predicting the end states of clusters and their populations can only be done by allowing simulations to run until the clusters are nearly exhausted. In particular, we want to know if the fraction and central concentration of binaries increases monotonically throughout the history of a cluster, or if some not-yet-recognized effect can saturate either or both values.

At 20 Gyr there were 8,080 single stars and 465 binaries remaining in the cluster. The *ratio* of binaries to single stars hardly changed over the cluster lifetime: 5% at time $t = 0$ and 5.8% at time $t = 20$ Gyr (see § 4 for further discussion). During this time the total mass of the cluster decreased to 4894 $M_{\odot}$, or 9% of the initial cluster mass. The half-mass radius at 20 Gyr was 4.1 pc.

Figure 1*a* shows the behavior of the number density of stars within the core and the half-mass radius as the 100,000 star model evolves. We see that after an initial drop (due to cluster expansion

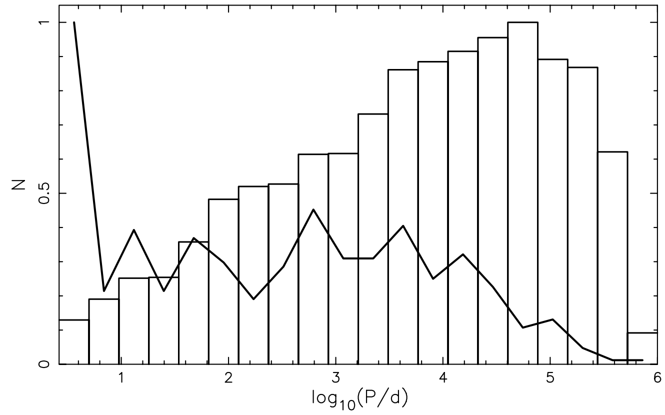

FIG. 2.—Normalized period distribution for the 5000 primordial binaries (*histogram*) and for the 465 binaries remaining after 20 Gyr.

driven by stellar evolution mass loss), the core density steadily increases to a local maximum at about an age of 16 Gyr. The cluster at this age was 15 half-mass relaxation times old: we identify this point with the cluster having reached core collapse. In Figure 1*b* we show the density-weighted core radius (Casertano & Hut 1985) of the model, which clearly reaches a minimum at 16 Gyr. Hurley et al. (2005) have noted that clusters with a significant primordial binary population do not show deep core collapse, and we find the same result here. The mass of the cluster at this point was 10,020 $M_\odot$, about 20% of the initial mass contained within a tidal radius of 30 pc. The core radius was 0.17 pc, and the half-mass relaxation timescale had reduced to 500 Myr. Just before the termination of the model, the core density became as high as 10^4 stars pc^{-3}.

Figure 2 shows the period distribution of the 5000 primordial binaries and compares this with the distribution for the 465 binaries remaining after 20 Gyr (note that both distributions are normalized). We see clearly that short-period binaries, or so-called *hard* (Heggie 1975) binaries, are strongly favored at late times in the evolution. This is as expected for a dynamically old cluster in which the wide binaries have been broken up in encounters with other stars.

It has been shown previously that exchange interactions (Heggie et al. 1996) resulting from three- and four-body encounters are important for determining the properties of stellar populations, such as blue stragglers (Hurley et al. 2005) in clusters. In our $N = 100,000$ simulation, a total of 1260 of the stars in the cluster were involved in at least one exchange interaction while resident in the cluster. The majority experienced only one such event (899), while the record for promiscuity was claimed by a neutron star that had eight distinct partners. This 2.0 $M_\odot$ NS was involved in seven different binaries between the ages of 12 and 17 Gyr, with periods ranging from 120 days up to 6×10^4 days. The period distribution of exchange binaries formed in the simulation is shown in Figure 3, and we see that, in general, the exchange process does *not* tend to produce close binaries. Not surprisingly, we also find that the incidence of exchange binaries, as shown in Figure 4, increases as the central density increases (compare Fig. 4 with Fig. 1). The histogram of final/initial binding energy for the exchange binaries is shown in Figure 5. The most likely outcome of an exchange interaction (the highest probability bin in Fig. 5) is a binary with binding energy similar to that of the initial binary, as indicated by Heggie et al. (1996) in their work on binary-single star interactions of arbitrary mass ratios. However, the data show that 83% of the exchanges actually lead to an increase in the binding energy of the binary. If we look at only exchanges that involved an initially

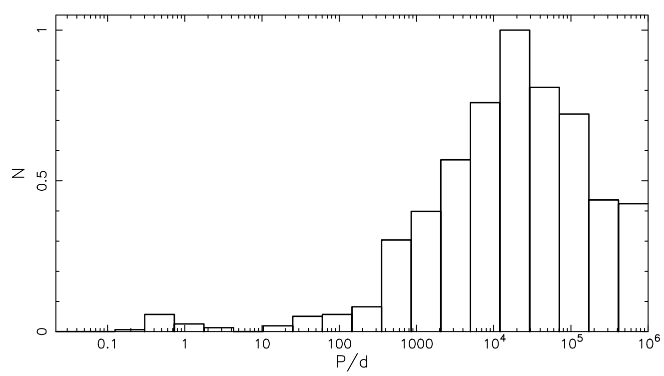

FIG. 3.—Period distribution for exchange binaries formed during the simulation.

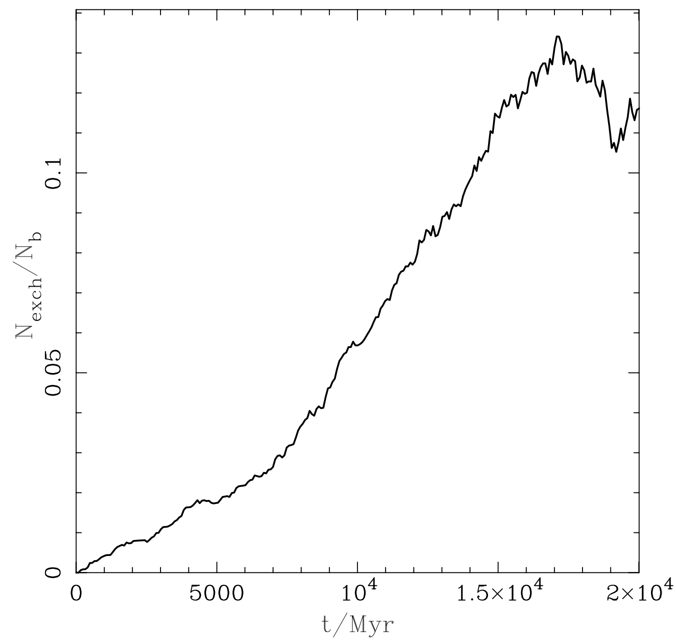

FIG. 4.—Fraction of binaries that formed as a result of an exchange interaction as a function of cluster age.

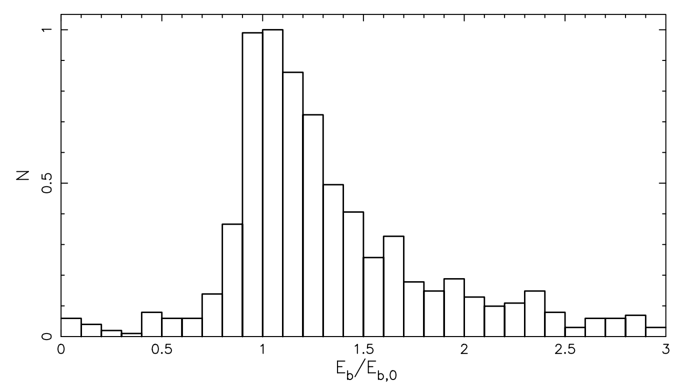

FIG. 5.—Normalized histogram of final/initial binding energy for the exchange binaries. The most likely outcome is a binary with a binding energy similar to that of the initial binary. However, 83% of all exchanges result in harder binaries, and 90% of exchanges involving initially hard binaries produce an increase in binding energy. Note that a few percent of interactions that lead to an increase of energy by more than a factor of 3 have been excluded from the figure for the sake of clarity.

TABLE 1

Statistics of Certain Stellar Populations Created in the Cluster Simulation and from the Field Binaries

Population	Cataclysmic Variables		DWDs ($P < 1$ day)		Binaries
	Total	20 (15) Gyr	Total	20 (15) Gyr	20 (15) Gyr
Cluster	15	2 (3)	67	14 (17)	465 (1310)
Field	17	11 (9)	53	24 (24)	4676 (4700)

Notes.—Columns labeled "Total" refer to the total number of that population (CVs or short-period DWDs) created during the simulation or expected from the same binaries evolved in the field evolved for 20 Gyr. The remaining columns show the number of CVs, short-period DWDs, and binaries present in the model and field populations at 20 Gyr (numbers in parentheses are for an age of 15 Gyr).

hard binary, then the number increases to about 90%. Thus, our self-consistent simulation with unequal-mass binaries agrees with previous studies (of the equal-mass case; e.g., Hut 1984) that "hard binaries get harder"—Heggie's Law. Before we can claim to have verified Heggie's Law for the general case, we must be careful to consider the non-point-mass effects of the *N*-body model. For example, we have not included exchanges that increased the binding energy so much that it led to a collision or merger (this is much more likely to occur for initially hard binaries). As such, the finding that hard binaries get harder in 90% of the interactions is likely to be a lower limit. A full investigation into the behavior of exchange interactions is beyond the scope of this current study.

4. STELLAR POPULATIONS

In Table 1 we show the total number of CVs and short-period DWD binaries produced during the cluster simulation. These are compared to the numbers we expect from the identical 5000 primordial binaries evolved outside of the cluster environment, i.e., a *field* population. The field binaries are evolved using the binary star evolution (BSE) algorithm (Hurley et al. 2002). This algorithm is identical to that used in NBODY4 and thus any differences between the characteristics of the field and cluster populations are directly attributable to the latter being exposed to interactions with other cluster stars.

In Table 1 we also show the number of binaries, CVs, and short-period DWDs in the model cluster at the end of the simulation. Numbers at 15 Gyr, just before core collapse, are compared to what is expected from the field population.

The depletion of binaries in the cluster is the result of a number of processes. Many binaries simply escape from the cluster, while others are significantly affected by perturbations from nearby stars. These perturbations may cause the binary to become unbound, in the case of initially wide binaries. Alternately, perturbations may bring the two stars closer together, making mass transfer and/or a merger more likely. Binaries also can become involved in three- and four-body interactions that destroy the original binary (or binaries) but generally produce a new binary via an exchange so that the overall binary number is not affected.

The number of binaries in the corresponding field population decreases with time both because of binary breakup due to supernovae and because of Roche lobe overflow–initiated mergers. These processes also affect the cluster binaries. In the cluster, mass segregation acts to shield binaries from evaporation across the tidal boundary; low-mass single stars preferentially reside in the outer regions of the cluster and suffer a much higher rate of evaporation. Also, encounters in which a binary is destroyed often lead to the ejection of a single star from the cluster (Aarseth 1996). These competing processes explain why we find that the binary fraction of the cluster does not decrease with time. Hurley et al. (2005) found a similar result in the case of binary-rich open cluster models. It has been suggested by Ivanova et al. (2005) that the combination of stellar evolution and dynamical interactions (binary-single and binary-binary) leads to a rapid depletion of the binary population in the core of a dense star cluster. We see no sign of this effect in our simulation.

4.1. *Cataclysmic Variables*

A field population of 5000 binaries with initial conditions identical to those we start with makes 17 CVs during a 20 Gyr span. Only 11 of these are still "alive" (i.e., still transferring mass from a RD to a WD) at an age of 20 Gyr. The remaining six objects are what we would call "short-lived" CVs. These binaries only existed as CVs for lifetimes of the order of 10–100 Myr. The reason for these relatively short lives is the relatively high masses of the main-sequence secondary (donor) stars: typically greater than 0.7 $M_\odot$ or so.

In standard binary evolution the CV phase comes to an end for one of three possible reasons. The first (and simplest) possibility is that the secondary evolves off the main sequence. A second possibility is that the mass-transfer rate may become high enough that the WD primary cannot steadily burn the accreted material. In this case, it swells up to become a giant, producing a common envelope (CE) binary and a merger of the stars. The third possibility allows the MS secondary star to steadily transfer material to the WD until the envelope of the MS star becomes deeply convective (this occurs for low-mass MS stars below about 0.66 $M_\odot$ for $Z = 0.001$). In this case, for a mass ratio (secondary/primary) greater than 0.695 (Hurley et al. 2002), mass transfer switches to a dynamical timescale, and it is assumed that the two stars coalesce.

To produce a close MS-WD binary in which the MS star will subsequently evolve to fill its Roche lobe, it is generally assumed that a CE event is required (Paczyński 1976). In fact, the theory of CE evolution as we know it today was driven by the need to explain the existence of CVs, such as U Gem (Smak 1976). CE evolution occurs when mass transfer develops on a dynamical timescale. In the BSE algorithm (and NBODY4), this equates to the donor star, generally a giant, having an appreciable convective envelope and a mass ratio, q, exceeding some critical value, $q_{\rm crit} \simeq 0.7$. If the conditions for dynamical mass transfer are met, then the envelope of the giant overfills the Roche lobes of both stars, leaving the giant core and the secondary star contained within a common envelope. Owing to orbital friction these will spiral together and transfer energy to the envelope with an efficiency $\alpha_{\rm CE}$. If this process releases sufficient energy to drive off the entire envelope, the outcome will be a close binary consisting of the giant core (i.e., the proto-WD) and the secondary; otherwise, it leads to coalescence of the two objects.

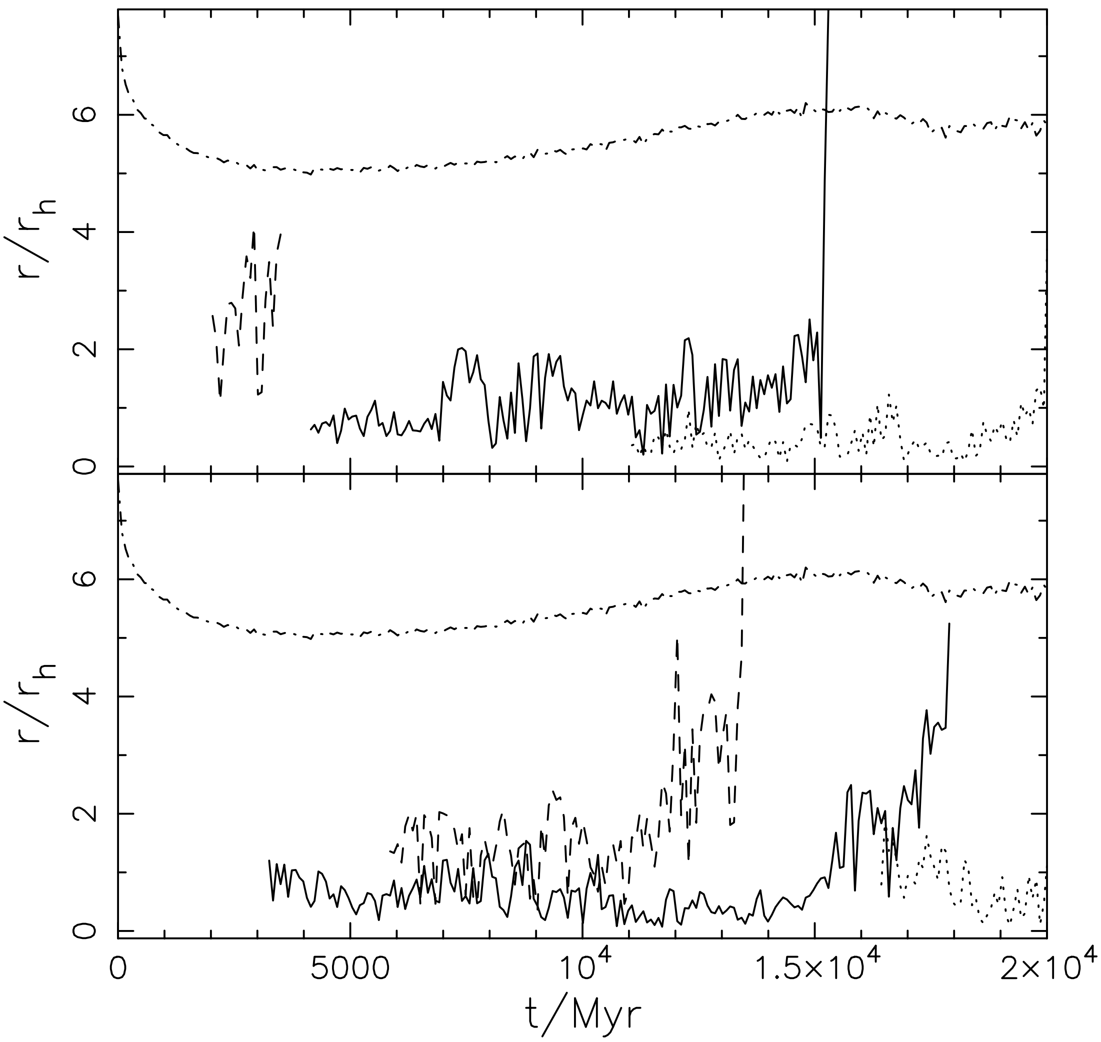

FIG. 6.—Radial evolution of the four CVs that escaped from the cluster simulation—CV3 (*dashed line, top panel*), CV6 (*solid line, bottom panel*), CV7 (*solid line, top panel*), and CV11 (*dashed line, bottom panel*)—and the two long-lived CVs that remain in the cluster at 20 Gyr—CV12 (*dotted line, top panel*) and CV14 (*dotted line, bottom panel*). See Table 2 for details of these CVs, including their start times. Also shown is the tidal radius of the cluster (*dot-dashed line in both panels*).

Compared to the 11 field CVs expected at 20 Gyr, we see from Table 1 that the cluster has only two. Only one of these was predicted by the BSE algorithm for the primordial binaries. What has happened to the 10 long-lived CVs that have gone missing in the cluster?

Four of these were produced as CVs but subsequently escaped from the cluster, at times of 3.6, 13.5, 15.3, and 17.9 Gyr. In Figure 6 we show the evolution of the radial position of these four CVs within the cluster and also include the two long-lived CVs that remain in the cluster at 20 Gyr (see Table 2). We note that the position data were only recorded every 80 Myr during the simulation, so there is scope for significant change in the radial position between successive data points. This is especially true in the central regions, where the crossing time is of the order of a few Myr. However, we can clearly see that the CVs spend the majority of their time near, or inside, the half-mass radius, and those that escape do so gradually, as opposed to being ejected violently in a single event. In particular, each of the escaping CVs does so at a velocity of 2 km s^{-1} or less. Another two of the *missing* CVs escaped from the cluster as detached MS-WD binaries prior to the onset of mass transfer. One escaped very early in the simulation while it was still an MS-MS binary.

The remaining three *missing* CVs all merged in CE events. In each of these three cases, it was *N*-body dynamics that drove the binary stars together; perturbations prior to the onset of CE evolution reduced the orbital separation so that, compared to evolution as a field binary, there was not enough orbital energy available to drive off the envelope before the stars merged. Why did this happen? It is true that the final stages are the most energetically crucial for determining the pre-CE separation. However, if the stars are brought closer together by perturbations, then the giant will fill its Roche lobe earlier in its evolution. That means that it has a comparatively smaller core radius (less time to grow) and a larger mass (less mass loss); i.e., the envelope mass is greater. Also, the separation at Roche lobe overflow will be smaller because the stars were pushed closer together and also because there was less mass loss from the giant. Both of these factors make it harder for the binary to survive CE. As a result, a close MS-WD pair—a proto-CV—was not created.

Table 2 lists the properties of the 15 CVs created during the cluster simulation. Start and end times for the CV phase are given along with an explanation as to why the CV phase ended, if indeed it did. In the field, CVs were active for a total of 147.7 Gyr, while in the cluster only 48.5 Gyr of CV activity was

TABLE 2
Properties of the Cluster CV Population

No. (1)	T_i (2)	$M_{\rm MS}$ (3)	$M_{\rm WD}$ (4)	WD (5)	P (6)	T_f (7)	$M_{\rm MS}$ (8)	P (9)	Notes (10)	Field (11)
1	1789	1.66	0.24	He	0.48	1805	1.59	0.27	Contact	No
2	1863	1.35	0.32	He	0.34	1930	1.26	0.20	Contact	Yes
3	2032	0.11	0.34	He	0.06	3557	0.05	0.10	Escape	Yes
4	2764	1.28	0.27	He	0.76	2778	1.23	0.55	Contact	Yes
5	3155	1.21	0.47	CO	0.61	3197	1.03	0.35	Contact	Yes
6	3212	0.15	0.71	CO	0.06	17930	0.01	0.50	Escape	Yes
7	4127	0.13	0.26	He	0.06	15325	0.01	0.41	Escape	Yes
8	4290	0.94	0.30	He	0.31	4394	0.80	0.35	Contact	Yes
9	4601	1.09	0.58	CO	0.83	4710	0.66	0.48	Merge	Yes
10	4776	0.98	0.24	He	0.35	4880	0.90	0.26	Contact	Yes
11	5781	0.12	0.27	He	0.06	13514	0.02	0.27	Escape	Yes
12	10928	0.36	0.73	CO	0.12	20000	0.02	0.25	...	Yes
13	14806	0.67	0.77	CO	0.24	14834	0.66	0.22	Merge	No
14	16366	0.15	0.33	He	0.07	20000	0.04	0.15	...	No
15	16840	0.74	0.91	CO	0.42	16940	0.66	0.32	Merge	No

Notes.—Col. (1) gives an ID number for each CV, and this is followed by the time at which the CV evolution began (T_i; col. [2]). The corresponding masses of the MS star and the WD are given in cols. (3) and (4), respectively. In col. (5) the WD type is shown (He = helium, CO = carbon/oxygen). The orbital period of the CV at birth is given in col. (6). Col. (7) shows the time at the end of the CV phase (T_f) up to a maximum of 20,000 Myr. This is followed by the mass of the MS star and the period of the CV at T_f (cols. [8] and [9]). Col. (10) provides an indication of how the CV phase ended. Note that in the case of the CV escaping the cluster, T_f is the escape time and not the time at which the CV phase ends; evolution continues after escape but is not followed by the simulation. In col. (11) we state whether or not the CV has a field-star analog.

observed. The cluster environment is clearly very harsh for CVs, and while they may be created more often, they expire more quickly. At any given instant, CVs will be about 3 times less likely to be discovered in our simulated cluster than in an equivalent field population.

Remarkably, four of the cluster CVs do not have a field-star analog; i.e., their existence was not predicted after evolving the primordial binaries with the BSE algorithm. Two of these four did evolve from primordial binaries, but only after their evolution pathways were significantly altered by perturbations from other stars.

The first CV with no field-star analog is CV1. The BSE algorithm predicted that this binary would initiate a mass-transfer phase after 1490 Myr of evolution when the more massive star was on the subgiant branch. This phase proceeded steadily until the donor star evolved to become a fully fledged giant. Because it developed a deep convective envelope, mass transfer from the giant became dynamical, and a CE occurred. In the pure BSE case, the stars then merged during the CE. However, when evolved within the cluster this binary had its orbital separation reduced by weak perturbations from nearby stars. Mass transfer was initiated earlier than expected, but with the donor star still on the subgiant branch. An extended period of mass transfer prior to the donor star becoming a giant meant that this star had a reduced envelope mass at the onset of the CE, and this allowed a short-period MS-WD binary to emerge from the event. Subsequently, this binary evolved to experience a short-lived phase of CV evolution (as described in Table 2).

The other CV that owes it existence to orbital perturbations experienced by the pre-CV binary is CV14. In this case, BSE predicted that the primordial binary would evolve through a CE event to produce a close MS-WD binary. However, the separation was not close enough for mass transfer to commence within 20 Gyr. In the cluster, the binary had a reduced separation (relative to its field counterpart) at the onset of CE, and this resulted in a post-CE binary that was close enough for a CV phase to begin at an age of 16.4 Gyr. This long-lived CV is one of the two still active in the cluster at 20 Gyr.

CV13 and CV15 are extraordinary. They evolved from binaries produced in exchange interactions. Such CVs are of great interest, of course, because *they have no analogs in the field*. The binary that evolved to become CV15 was created in a four-body interaction when the cluster was 15.2 Gyr old. Involved in this interaction was a primordial binary comprising MS stars of mass 0.37 and 0.74 $M_\odot$, with an eccentricity of 0.98 and an orbital period of 2×10^4 days. The other binary involved had been created previously in an exchange interaction and contained a 0.91 $M_\odot$ CO WD and a 0.71 $M_\odot$ MS star in a 5×10^4 day orbit. The four-body interaction was brief and resulted in the WD and the 0.74 $M_\odot$ MS star forming a bound pair with $P = 4302$ days and an eccentricity of 0.97. This new binary was resident in the core and suffered weak perturbations from nearby stars. These drove the eccentricity as high as 0.99 and caused the orbit to become chaotic (Mardling & Aarseth 2001). The orbit subsequently circularized—with further interactions hastening the process—with a period of 0.52 days. The MS star then evolved to fill its Roche lobe at a cluster age of 16.8 Gyr. The CV phase was short-lived: only about 100 Myr of mass transfer occurred. After 100 Myr the MS star mass was reduced to 0.66 $M_\odot$ with an appreciable convective envelope. The mass ratio at this time was 0.72, in excess of the lower bound for dynamical timescale mass transfer ($q = 0.695$), and therefore the two stars were allowed to coalesce. If the mass of the WD had been just slightly higher then, according to the evolution prescription, dynamical mass transfer would have been avoided, and a long-lived CV phase would have resulted.

The evolution pathway of CV13 was similar except that this binary formed in a three-body exchange interaction in which a single 0.77 $M_\odot$ CO WD ejected a 0.3 $M_\odot$ star from a primordial binary to form a new binary with a 0.67 $M_\odot$ MS star. This occurred at a cluster age of 8.9 Gyr. The resulting binary was subject to orbital perturbations from other stars, which caused the orbit to become chaotic before tidal forces could circularize it. The close binary appeared as a short-lived CV at 14.8 Gyr.

We emphasize the remarkable aspect about the pathways for producing these two dynamical CVs: *neither involved CE evolution. Thus, no such objects exist in the field*. It is to be expected

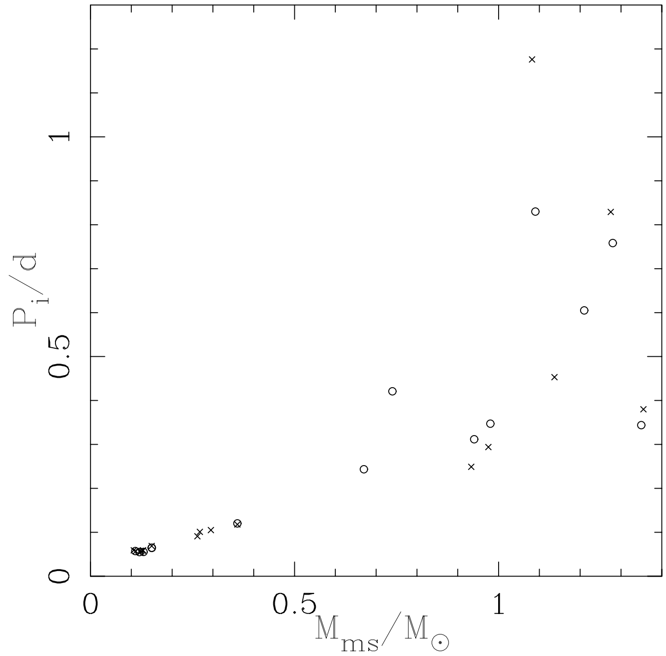

FIG. 7.— Orbital period and MS star mass at the onset of the CV phase for field CVs (*crosses*) and cluster CVs (*circles*).

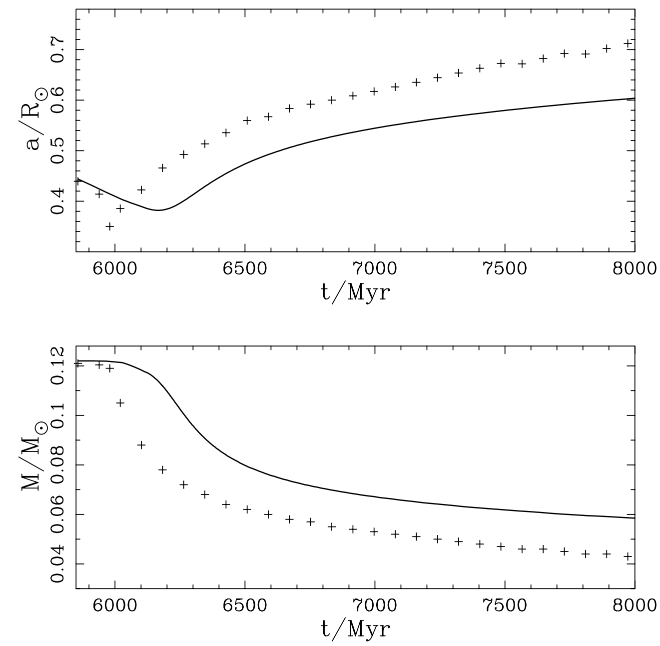

FIG. 8.—Evolution of semimajor axis (*top*) and MS star mass (*bottom*) for CV11. The evolution for the cluster CV (*plus signs*) is compared to the evolution for the same binary evolved in the field (*solid line*). Note that only the early evolution is shown.

that exchange interactions are more likely to make CVs in which the MS star mass is greater than 0.7 $M_{\odot}$, as opposed to CVs with low-mass donors. This is because exchange interactions tend to eject the least massive star from a multiple star system. Thus, CVs produced via dynamical interactions are more likely to be short-lived.

Figure 7 compares the distribution of orbital periods and MS donor masses for field and cluster CVs at birth. There is no apparent difference between the two CV populations. However, when we look at the evolution of the CVs on a case-by-case basis, we do see evidence for the cluster environment accelerating CV evolution. In Figure 8 we show the early evolution of cluster CV11 and compare this to the evolution of its field-star analog. In both instances, the CV was born with a period of 0.06 days after the binary had evolved for 5.8 Gyr. The WD was composed of helium and had a mass of 0.27 $M_{\odot}$, while the MS star had a mass of 0.12 $M_{\odot}$. The progenitor of the WD was a 1.2 $M_{\odot}$ star. Prior to becoming a WD-MS binary, the system was a relatively massive object in the cluster, and mass segregation had caused it to reside near the core of the cluster. At the time that the cluster binary commenced CV evolution, it was still within a few core radii of the cluster center, and shortly afterward it experienced a perturbation that brought the two stars closer together. This is the cause of the sharp dip in the semimajor axis evolution of the cluster CV at about 6 Gyr. We can see from Figure 8 that this significantly increased the mass-transfer rate and accelerated the evolution of the CV compared to the field-star case.

Another interesting example involves CV6: a long-lived cluster CV that was born at an age of 3.2 Gyr. The progenitor of this CV was a primordial binary comprising stars of mass 2.87 and 0.15 $M_{\odot}$ in an eccentric orbit with a 1995 day period. According to the BSE algorithm, the more massive star (in the field) should fill its Roche lobe after 370 Myr when it is on the asymptotic giant branch (AGB). This leads to CE evolution, and the result is a 0.73 $M_{\odot}$ CO WD in a circular binary with the low-mass MS star. The orbital separation is 1.6 $R_{\odot}$. A CV phase begins at an age of 10.5 Gyr.

In the cluster the MS-MS binary received a perturbation to its orbit that increased the orbital eccentricity and slightly reduced the separation. As a result, the CE phase occurred earlier than expected but with the more massive star still on the AGB. The separation at the onset of the CE phase was less than in the BSE case: 128 $R_{\odot}$ compared to 147 $R_{\odot}$. The outcome is a 0.71 $M_{\odot}$ WD separated by 1.2 $R_{\odot}$ from its MS companion. This explains why the cluster CV was born much earlier than if it had been residing in the field.

Figure 9 shows the period distribution of the CVs that are "observed" in the simulation. Also shown is the period distribution for the progenitor binaries of these same CVs. The cluster is observed at intervals of about 80 Myr, and any CV present during an observation contributes a count in the distribution.

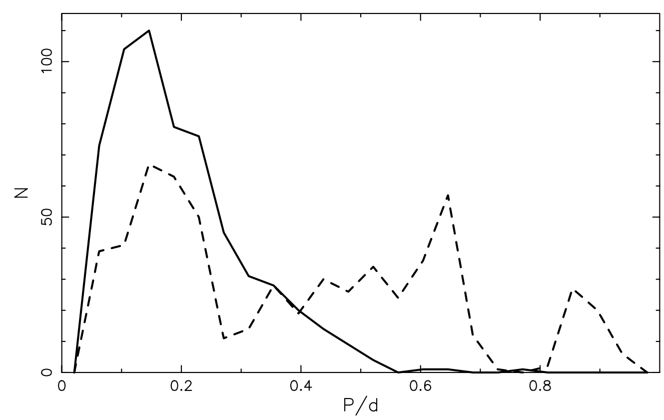

FIG. 9.—Period distribution of all CVs observed in the simulation (*solid line*). The cluster is observed at intervals of about 80 Myr, and any CV present during an observation contributes a count in the distribution. As such, a particular CV can contribute more than one count, and long-lived CVs will contribute many counts. Also shown is the period distribution of MS-WD binaries that will evolve to become CVs (*dashed line*).

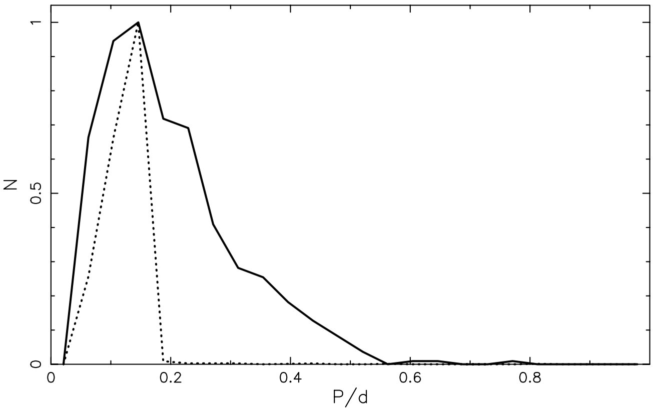

FIG. 10.—Similar to Fig. 9, but now the period distribution of all CVs in the simulation has been normalized (*solid line*), and we also show the corresponding distribution for CVs evolved from the primordial binaries in the field (*dotted line*). The latter has also been sampled at intervals of 80 Myr.

As such, a particular CV or progenitor can contribute more than one count, and long-lived CVs will contribute many counts. It is possible that we see a deficit of progenitor systems at around the 5–6 hour period mark, although statistically this is difficult to confirm. The progenitor binaries of CVs will be perturbed in the same way as the CVs themselves, so we should expect the two distributions to be similar. What is clear from Figure 9 is that the cluster CVs exhibit no analog of the pronounced deficit of systems with periods in the 2–3 hour range observed for field CVs (Downes et al. 2001)—one of the key results of this paper. There are today just a handful of confirmed CVs (those with the photometric, spectrographic, and outburst properties of CVs) with known orbital periods. Thus, our prediction is not yet testable, but it will be as statistics improve. In Figure 10 we compare the normalized period distribution of the model cluster CVs with that expected from our model field population (the 5000 primordial binaries evolved outside of the cluster environment). We see that the latter has a fairly sharp cutoff at about 0.2 days. Field CVs with periods greater than this are expected (and "observed" if we evolve many binaries in a full population synthesis), but these would have main-sequence donor stars with masses greater than about 0.3 $M_\odot$. In this case the orbital evolution of such systems would be dominated by magnetic braking of the main-sequence star.

In the binary evolution algorithm of Hurley et al. (2002) magnetic braking does not operate on stars with masses less than 0.3 $M_\odot$, as these are deemed to be fully convective. Thus, the orbital evolution of CVs with low-mass donors is governed by gravitational radiation. That we do not see CVs with periods greater than 0.2 days in our field distribution shown in Figure 10 reflects the fact that CVs dominated by magnetic braking evolve more quickly than those dominated by gravitational radiation and that we only have a small number of CVs to sample. We note that conducting a full binary population synthesis using the Hurley et al. (2002) code reveals a period gap at about 0.2 days, although the precise location of the gap is model dependent and can be altered by changing the mass below which magnetic braking is assumed to cease operating. It is important to realize that in the cluster simulation the binaries are evolved using the same algorithm, and that in comparison the cluster CVs show no sign of a period gap. The long-lived CVs in the cluster population have MS donor masses of less than 0.3 $M_\odot$. Perturbations to the orbits have disturbed the period distribution significantly, resulting in CVs with periods greater than 0.2 days.

4.2. *Other Simulations*

N-body simulations of open clusters have been performed in the past using the same techniques that we employed here. These have illustrated the role of the cluster environment in enhancing and shaping the nature of stellar populations such as DWD binaries and blue stragglers (Shara & Hurley 2002; Hurley et al. 2005). These simulations involved 20,000–30,000 stars for the initial models and evolved for about 6–7 Gyr before dissolving. While these simulations did experience core collapse, their core densities never exceeded a few hundred stars pc^{-3}. In these open cluster models there was no enhancement of the CV population of the cluster compared to the analog field-star population. Furthermore, there were no cases of CV formation via an exchange interaction, and no cases of significantly accelerated CV evolution.

We have also looked at a simulation of a low-density cluster that started with $N = 200,000$ to see how the CV population is affected. This simulation was initiated with 5000 binaries drawn from the same distributions as the primordial binaries in the 100,000 star simulation. It was evolved subject to the same standard Galactic tidal field defined in § 2. The total mass of the initial model was 100,020 $M_\odot$, and the tidal radius was 64 pc. The half-mass radius was 8.2 pc, and the initial velocity dispersion of the stars was 4.5 km s^{-1}.

This model was evolved to an age of 13.5 Gyr when about 100,000 stars remained. The half-mass radius at this point was 9.2 pc, and the core radius was 2.7 pc. In Figure 11 we show the evolution of the core and half-mass radii number densities for this model. We see that when the simulation was stopped, the number density of stars in the core of the cluster was only about 60 stars pc^{-3}. Considering that the half-mass relaxation timescale of the starting model was 2.5 Gyr and remained at or above this value for the majority of the simulation, the model at 13.5 Gyr is not dynamically old and is certainly not approaching core collapse. This model was not evolved past 13.5 Gyr because it was not deemed to be of sufficient interest to justify the computational expense.

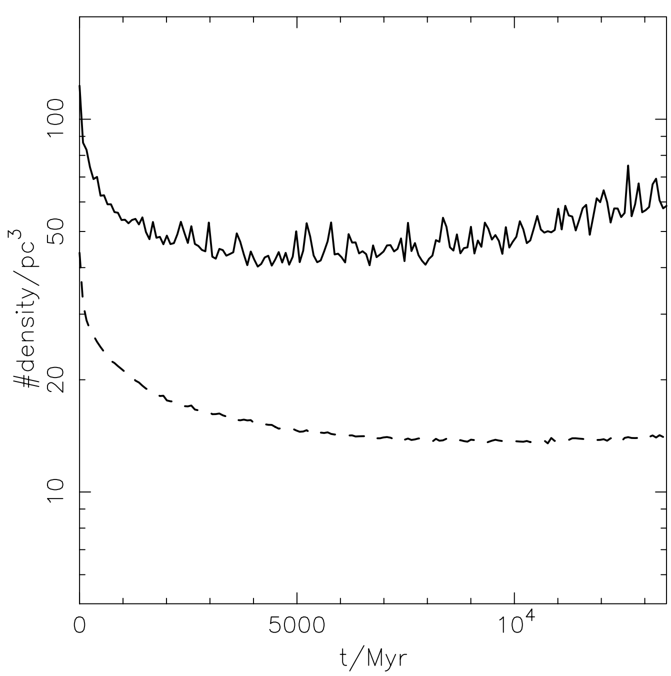

FIG. 11.—Evolution of the number density of stars in the core (*solid line*) and within the cluster half-mass radius (*dashed line*) for the 200,000 star model.

From the 5000 primordial binaries in the 200,000 star model, field evolution predicts nine long-lived CVs present at 13.5 Gyr. The corresponding *N*-body model has three. Of the missing CVs, five escaped the cluster: two as CVs and three before becoming MS-WD binaries. The remaining *missing* CV merged during CE evolution because of a slight orbital perturbation experienced prior to the onset of the CE. None of the three CVs present at 13.5 Gyr experienced significantly accelerated evolution subsequent to beginning its CV phase. One of these CVs did begin its CV phase about 300 Myr earlier than expected as a result of an interaction with a third star. Thus, orbital perturbations did affect two of the CVs in this simulation. The low density of the cluster ensured that there were no dynamical (or exchange) CVs formed in this simulation.

The comparison of these three simulations—100,000 stars at relatively high density versus 20,000, 30,000, and 200,000 stars at very low density—emphasizes that the total number of stars in a cluster simulation is *not* the key parameter in determining the evolution of the cluster or of its populations. Rather, it is the cluster density that controls the number and evolution of binary stars in general, and CVs in particular.

5. DISCUSSION

Our 100,000 star simulation stands out from previous simulations of open clusters and from the 200,000 star simulation we have mentioned, because, importantly, it reaches a high central density at an age similar to that of the globular clusters in our Galaxy. As such, the model provides a unique insight into the behavior of stellar populations in globular clusters. For the first time, we have witnessed the formation of CVs in exchange interactions. We have also seen substantial modification of CV-forming binaries owing to perturbations from other cluster stars. There is a clear link between core density and the incidence of accelerated CV evolution in the various models we have looked at.

Although we found that the CV population in the 100,000 star model was altered in relation to a comparable field population, the number of CVs made by the model was about the same as expected from the field. The effect of the cluster environment on the nature of stellar populations is more noticeable if we examine the DWD binaries in the model. Table 1 shows us that more short-period DWDs (those with orbital periods of 1 day or less) are made by the cluster than expected from the field binaries. Inspecting the ratio of short-period DWDs to binaries at an age of 20 Gyr, we see that the cluster is 5–6 times more abundant in short-period DWDs than the field. This is compared to a factor of 2 enhancement for the relative CV numbers at 20 Gyr. However, the short lives of the cluster CVs mean that, at any given time, we will discover 3 times fewer CVs in the cluster than in the field.

For DWDs that evolve from primordial binaries, the progenitor binary must have contained two stars more massive than the cluster turnoff mass; for CVs this is only true for one of the stars. Also, DWDs themselves are on average more massive than CVs. Thus, DWDs (including pre-DWD evolution) are more likely to spend time in or near the cluster center, and hence they are more susceptible to orbital perturbations.

A DWD is also more likely to emerge from an exchange interaction than a CV simply because DWDs contain two relatively heavy components compared to only one for CVs (at least in the case of the long-lived variety of CV). These reasons explain why DWDs show a greater degree of dynamical modification than CVs in our 100,000 star simulation and why the numbers of DWDs is enhanced in open cluster simulations while CVs are not. It is only in the 100,000 star simulation that reached high density at a late age, when the cluster turnoff mass is relatively low, that we have started to see dynamically modified, and manufactured, CVs.

The most important effect that living inside a star cluster has on the evolution of binary stars is the cumulative hardening of hard binaries and the softening of soft binaries. Binaries that become close enough to transfer mass begin to do so at epochs, and at rates that are completely different from those of their field-star analogs. This leads to qualitatively different evolutionary pathways for a significant fraction of the binary stars in clusters. These close binaries, in turn, control the dynamical evolution of their host clusters. It is no exaggeration to say that the evolution of every close binary star in a cluster is intimately linked to the dynamics of all the stars in the cluster. This feedback between dynamics and evolution must be accounted for in any simulation of dense star clusters.

6. SUMMARY

We have described the self-consistent dynamical evolution of a relatively dense cluster composed of 100,000 stars, of which 5000 were primordial binaries. Over the 20 Gyr of the simulation, the cluster core density increased by 2 orders of magnitude, and very hard binaries became the dominant binary species. Binaries dominated the dynamics of the cluster core for the entire 20 Gyr of the simulation.

CVs spent the vast majority of their time within a half-mass radius of the cluster center. Those CVs that escaped the cluster did so gradually, suffering numerous distant encounters that eventually led to their liberation. Utterly novel CVs, formed by exchange reactions and never undergoing a common envelope phase, were detected for the first time. Exchange reactions produced CVs with more massive secondaries than those seen in the field: such CVs tended to be short-lived. Cluster CVs were hardened by encounters with field stars. This tended to increase mass-transfer rates, hasten evolution, and sometimes led to CV destruction by forcing a merging event. At any given time, we will see 3 times fewer active CVs in the cluster than in the field.

On the basis of this simulation we make a very testable prediction: the period distribution of cluster CVs will not show the prominent 2–3 hour minimum seen in field CVs. Dynamical effects dominate the evolution of cluster CVs, and these tend to smear out the so-called period gap in dense star clusters.

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