SELF-SIMILAR SOLUTIONS FOR THE INTERACTION OF RELATIVISTIC EJECTA WITH AN AMBIENT MEDIUM

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ABSTRACT

We find self-similar solutions to describe the interaction of spherically symmetric ejecta expanding at relativistic speeds with an ambient medium having a power-law density distribution. Using these solutions, the time evolution of the Lorentz factor of the outer shock is derived as a function of the explosion energy, the mass of the ejecta, and parameters for the ambient medium. These solutions are an ultrarelativistic version of the solutions for the circumstellar interaction of supernova ejecta obtained by Chevalier and extensions of the relativistic blast wave solutions of Blandford & McKee.

Subject headings: gamma rays: bursts — hydrodynamics — relativity — shock waves — supernovae: general

1. INTRODUCTION

Self-similar solutions for fluid flows involving shock waves propagating at relativistic speeds have been presented in the literature (e.g., Blandford & McKee 1976; Hidalgo & Mendoza 2005; Nakayama & Shigeyama 2005). Most of them are ultrarelativistic versions of the already known nonrelativistic self-similar solutions. Blandford & McKee (1976) derived solutions for an ultrarelativistic spherical blast wave enclosed by a strong shock, which is initiated by the release of a large amount of energy within a small volume. If the released energy is not very large, the resulting nonrelativistic flow is described by the well-known solution discovered independently by Sedov (1959) and Taylor (1950). The implosion of a spherical strong shock wave had been solved independently by Landau and Stanyukovich (see, e.g., Zel'dovich & Raizer 2002) and by Guderly (1942) in nonrelativistic cases. Hidalgo & Mendoza (2005) derived self-similar solutions for the same problem in the relativistic limit. Nakayama & Shigeyama (2005) discovered solutions for the emergence of an ultrarelativistic shock wave at the surface of a star. For the case of a nonrelativistic shock wave the same problem had been posed by Gandel'man & Frank-Kamenetskii (1956) and solved by Sakurai (1960).

Here we present a self-similar solution for the interaction of spherical ejecta expanding at ultrarelativistic speeds with the ambient medium. When the ejecta expand at nonrelativistic speeds, self-similar solutions are known to exist (Chevalier 1982). Chevalier discovered solutions for the flows resulting from the collision of freely expanding spherical matter with the ambient medium. In the relativistic limit, self-similar solutions describing the shocked ambient media with power-law density distributions had already been obtained by Blandford & McKee (1976) with the shock Lorentz factor evolving with time t as $t^{-m/2}$. The constant m can be determined when the nature of the explosion is specified. This paper discusses self-similar solutions for describing shocked ejecta as a result of a collision with the ambient medium. This solution will be relevant to extremely high energy supernovae originating from compact progenitors with small masses.

The density distribution of the ejecta used here is suggested by analytical solutions (Nakayama & Shigeyama 2005) for the breakout of an ultrarelativistic shock wave from a hydrostatic atmosphere and its nonrelativistic versions (Sakurai 1960; Matzner & McKee 1999). The outer layers of the resulting flow in a nonrelativistic solution expand at velocities greater than the speed of light. They have a density distribution that agrees with numerical calculations for the same problem using a special relativistic hydrodynamic code (Nakamura & Shigeyama 2004). Thus, we use the results as the inner boundary conditions of the present self-similar solutions. We describe the relation of our solution to results obtained by Blandford & McKee (1976) in some detail.

In the next section, the density distribution of freely expanding ejecta as a result of shock breakout is discussed. Self-similar flows in the shocked ejecta and ambient medium are obtained in § 3. The behavior of these two flows at the contact surface is discussed in § 4. Using the obtained solution, the shock propagation in the ambient medium is discussed in § 5. The results are summarized in § 6.

2. RELATIVISTIC EJECTA FROM SHOCK BREAKOUT

The velocity v_e of freely expanding ejecta at a distance r from the explosion site is written as $v_e = r/t$ at a time t after the explosion. The corresponding Lorentz factor is $\gamma_e = \left(1 - v_e^2\right)^{-1/2}$. We set the speed of light equal to unity throughout this paper. In the relativistic limit, self-similar solutions for the emergence of a shock wave at the stellar surface (Nakayama & Shigeyama 2005) suggest that the density in the outer layer of ejecta has a power-law distribution in terms of the Lorentz factor as

$$\rho_e = \frac{\rho_e'}{\gamma_e} = \frac{a}{t^3 \gamma_e^n}, \tag{1}$$

where ρ_e is the density of ejecta measured in the comoving frame and ρ_e' is that in the fixed frame; that is, $\rho_e' = \rho_e \gamma$ (hereafter prime denotes densities that are measured in the fixed frame). Here the nondimensional parameter n can be estimated to be ~ 1.1 if the stellar envelope is in radiative equilibrium and the adiabatic index is 4/3. The constant a has dimensions of mass. On the other hand, a self-similar solution for the nonrelativistic shock breakout in a plane-parallel geometry (Sakurai 1960; Fields et al. 2002) results in a density distribution in the same form with $n \sim 2.6$. Although this self-similar solution assumes nonrelativistic flow,

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the resulting density distribution agrees with that obtained by numerical calculations solving special relativistic hydrodynamics equations (Nakamura & Shigeyama 2004) even for large Lorentz factors $\gamma_e > 10$. The transition between these solutions occurs when the ratio of the pressure to the mass density P/ρ at the shock front becomes of the order of unity. Thus, the outermost layers should be governed by the solution in the relativistic limit.

3. FLOW IN THE SHOCKED REGION

Flows located between two shock fronts are considered by connecting two self-similar solutions. Suppose that the shock front in the ambient medium propagates at a speed V_1 and the shock front in the ejecta at V_2. If we define $\alpha = \hat{\gamma}/(\hat{\gamma} - 1)$, where $\hat{\gamma}$ is the adiabatic index, the flow in each of the shocked regions is described by the following equations:

$$(\alpha P + \rho)\frac{d \ln \gamma}{dt} + \frac{dP}{dt} = \gamma^{-2}\frac{\partial P}{\partial t}, \qquad (2)$$

$$\frac{d \ln(P^{\alpha-1}\gamma^{\alpha})}{dt} = -\frac{\alpha}{r^2}\frac{\partial r^2 v}{\partial r}, \qquad (3)$$

$$\frac{\partial \rho'}{\partial t} + \frac{1}{r^2}\frac{\partial r^2 \rho' v}{\partial r} = 0, \qquad (4)$$

where the pressure is denoted by P, the density ρ, the velocity v, the corresponding Lorentz factor γ, and the density in the fixed frame $\rho' = \rho\gamma$. In the shocked ambient medium, the ultrarelativistic equation of state is assumed. Thus, $\hat{\gamma} = 4/3$ and the energy density ϵ per unit mass is written as $\epsilon = 3P/\rho$ by neglecting the rest-mass energy density. These approximations lead to the same equations as presented in Blandford & McKee (1976). On the other hand, it is found that the rest-mass energy should not be ignored in the shocked ejecta to have a compression shock. Otherwise, the jump conditions would result in an expansion shock for $m < 1$. Then the energy density becomes $\epsilon = P/[(\hat{\gamma} - 1)\rho] + 1$ in this region and the term including the density in the left-hand side of equation (2) is not neglected.

3.1. Jump Conditions at the Shock Front in the Ejecta

The density, pressure, and velocity have discontinuities at the shock front. Relations to connect these quantities at both sides of the shock front are given here. The mass flux ρu^{μ} and the energy momentum tensor $T^{\mu\nu}$ change across the shock propagating at the velocity of V_2 according to the following formulae:

$$\left(\rho_e u_e^{\mu} - \rho_2 u_2^{\mu}\right)n_{\mu} = 0, \qquad (5)$$

$$\left(T_e^{\mu\nu} - T_2^{\mu\nu}\right)n_{\nu} = 0. \qquad (6)$$

Subscript "e" refers to values in the freely expanding ejecta at radius R_2 defined as

$$R_2(t) = \int_0^t d\tau\, V_2(\tau), \qquad (7)$$

while subscript "2" refers to values in the shocked matter at the same radius R_2. The variables n_{μ}, u^{μ}, and $T^{\mu\nu}$ have been defined as

$$n_{\mu} \equiv \Gamma_2(-V_2,\ 1,\ 0,\ 0), \quad u^{\mu} \equiv \gamma(1,\ v,\ 0,\ 0), \qquad (8)$$

$$T^{\mu\nu} \equiv \rho h u^{\mu} u^{\nu} + P\eta^{\mu\nu}, \qquad (9)$$

with the metric $\eta^{\mu\nu} \equiv \mathrm{diag}(-1,\ 1,\ 1,\ 1)$. Here Γ_2 denotes the Lorentz factor of the shock and is assumed to evolve as a function of time t as

$$\Gamma_2^2 = At^{-m} \qquad (10)$$

with constants A and m. The enthalpy h is denoted by other thermodynamic variables as $h \equiv \epsilon + P/\rho$. Equation (5) yields

$$\rho_e\gamma_e(v_e - V_2) = \rho_2\gamma_2(v_2 - V_2). \qquad (11)$$

Equation (6) can be rewritten as

$$\rho_e\gamma_e^2(v_e - V_2) = \rho_2 h_2\gamma_2^2(v_2 - V_2) + P_2 V_2, \qquad (12)$$

$$\rho_e\gamma_e^2 v_e(v_e - V_2) = \rho_2 h_2\gamma_2^2 v_2(v_2 - V_2) + P_2. \qquad (13)$$

Equations (11)–(13), together with the relation $\gamma_e = (m + 1)^{1/2}\Gamma_2$ at $r = R_2$, lead to the following equation:

$$\alpha x^3 + (\alpha - 2)\sqrt{m + 1}x^2 - (\alpha - 2)x - \alpha\sqrt{m + 1} = 0, \quad (14)$$

where $x = \gamma_2/\Gamma_2$. This equation has only one positive solution between 1 and $(m + 1)^{1/2}$ for $m > -1$. If this solution is denoted as $x = \sqrt{q}$, the Lorentz factor, density, and pressure at the front in the shocked matter will be derived as

$$\gamma_2^2 = q\Gamma_2^2, \qquad (15)$$

$$\rho_2' = \frac{mq}{(m + 1)(q - 1)}\rho_e\gamma_e, \qquad (16)$$

$$P_2 = \frac{m\rho_e}{\alpha(q - 1) + 2}\left(1 - \sqrt{\frac{q}{m + 1}}\right). \qquad (17)$$

These relations are used to obtain the boundary conditions for flows in the shocked ejecta. If the constant m is smaller than unity, as is usually the case, the compression is so weak that the thermal energy cannot dominate the rest-mass energy in the shocked ejecta. Therefore, we cannot use the ultrarelativistic equation of state there, while the ultrarelativistic equation of state is always a good approximation in the shocked ambient medium as long as the shock Lorentz factor is much larger than unity.

3.2. Self-similar Flow in the Shocked Ejecta

It is convenient to define self-similar variables for the pressure $F(\xi)$, the Lorentz factor $G(\xi)$, and the density $H(\xi)$ in the shocked ejecta as

$$P = \frac{m\rho_e}{\alpha(q - 1) + 2}\left(1 - \sqrt{\frac{q}{m + 1}}\right)F(\xi), \qquad (18)$$

$$\gamma^2 = q\Gamma_2^2 G(\xi), \qquad (19)$$

$$\rho' = \frac{mq}{(m + 1)(q - 1)}\rho_e\gamma_e H(\xi). \qquad (20)$$

The similarity variable ξ is defined as

$$\xi = \left[1 + 2(m + 1)\Gamma_2^2\right]\left(1 - \frac{r}{t}\right), \qquad (21)$$

following Blandford & McKee (1976).

To obtain equations governing the self-similar evolution, the transformation of coordinates from (r, t) to (Γ_2, ξ) is performed using the following relations of derivatives:

$$\frac{\partial}{\partial \ln t} = -m \frac{\partial}{\partial \ln \Gamma_2^2} + \left[(m+1)(2\Gamma_2^2 - \xi) + 1\right]\frac{\partial}{\partial \xi}, \quad (22)$$

$$t\frac{\partial}{\partial r} = -\left[1 + 2(m+1)\Gamma_2^2\right]\frac{\partial}{\partial \xi}, \quad (23)$$

$$\frac{d}{d \ln t} = -m \frac{\partial}{\partial \ln \Gamma_2^2} + (m+1)\left(\frac{1}{qG} - \xi\right)\frac{\partial}{\partial \xi}. \quad (24)$$

Then we obtain, from equations (2)–(4),

$$2(m+1)(1 + qG\xi)\frac{d \ln F}{d\xi} - (m+1)(\alpha + I)(1 - qG\xi)\frac{d \ln G}{d\xi}$$
$$= [mn - (\alpha + I)m - 6]qG, \quad (25)$$

$$2(\alpha - 1)(m+1)(1 - qG\xi)\frac{d \ln F}{d\xi} - \alpha(m+1)(1 + qG\xi)\frac{d \ln G}{d\xi}$$
$$= [(1 - \alpha)mn + \alpha m + 2\alpha - 6]qG, \quad (26)$$

$$2(m+1)(1 - qG\xi)\frac{d \ln H}{d\xi} - 2(m+1)\frac{d \ln G}{d\xi}$$
$$= -(mn - m - 2)qG, \quad (27)$$

where $I = I(\xi) = \rho/P$. The boundary conditions are given at the shock front as

$$G(1) = F(1) = H(1) = 1. \quad (28)$$

The solution is obtained by numerically integrating equations (25)–(27) from the shock front $\xi = 1$ to the contact discontinuity $\xi = \xi_c = 1/qG(\xi_c)$.

3.3. Jump Conditions at the Shock Front in the Ambient Medium

The density, Lorentz factor, and pressure change across the shock wave propagating at the speed $V_1 = \left(1 - 1/\Gamma^2\right)^{1/2}$ according to the following relations:

$$\rho_1' = \frac{\rho_i \Gamma^2}{2}, \quad \gamma_1^2 = \frac{\Gamma^2}{2}, \quad P_1 = \frac{2\rho_i \Gamma^2}{3}, \quad (29)$$

where subscript "1" refers to values in the shocked fluid at the shock front while subscript "i" refers to values in the unshocked ambient medium at the shock front. The shock Lorentz factor is denoted by Γ. This Γ evolves with time in the same way as Γ_2, i.e., $\Gamma^2 = Bt^{-m}$. The density in the ambient medium is assumed to have a power-law distribution as $\rho = br^{-k}$, where b is a constant. Thus, self-similar variables for the pressure $f(\chi)$, Lorentz factor $g(\chi)$, and density $h(\chi)$ are defined as

$$P = \frac{2\rho_i \Gamma^2}{3}f(\chi), \quad (30)$$

$$\gamma^2 = \frac{\Gamma^2}{2}g(\chi), \quad (31)$$

$$\rho' = 2\rho_i \Gamma^2 h(\chi). \quad (32)$$

Here the similarity variable χ has been introduced as

$$\chi = \left[1 + 2(m+1)\Gamma^2\right]\left(1 - \frac{r}{t}\right). \quad (33)$$

3.4. Self-similar Flow in the Shocked Ambient Medium

The same procedure as in § 3.2 leads to equations governing a self-similar flow in the shocked ambient medium as

$$\frac{d \ln f}{gd\chi} = \frac{4[2(m-1) + k] - (m+k-4)g\chi}{(m+1)(4 - 8g\chi + g^2\chi^2)}, \quad (34)$$

$$\frac{d \ln g}{gd\chi} = \frac{(7m + 3k - 4) - (m+2)g\chi}{(m+1)(4 - 8g\chi + g^2\chi^2)}, \quad (35)$$

$$\frac{d \ln h}{gd\chi} = \frac{2(9m + 5k - 8) - 2(5m + 4k - 6)g\chi + (m+k-2)g^2\chi^2}{(m+1)(4 - 8g\chi + g^2\chi^2)(2 - g\chi)}, \quad (36)$$

in the relativistic limit. The boundary conditions are

$$f(1) = g(1) = h(1) = 1. \quad (37)$$

These are exactly the same equations as presented in Blandford & McKee (1976).

4. FLOW AT THE CONTACT SURFACE

The density distribution has a discontinuity at the contact surface defined as $\xi = \xi_c$ and $\chi = \chi_c$, where $G(\xi_c)\xi_c = 1/q$ and $g(\chi_c)\chi_c = 2$, while the velocity and pressure are continuous. The fact that the velocities of the two flows at the contact surface are the same indicates that the ratio of the Lorentz factors of the two shock fronts should become

$$\frac{\Gamma^2}{\Gamma_2^2} = \frac{B}{A} = 2q\frac{G(\xi_c)}{g(\chi_c)}, \quad (38)$$

according to equations (19) and (31). The continuous pressure at the contact surface requires that the pressures at both sides of the contact surface evolve with time in the same manner. Thus, we obtain from equations (18) and (30)

$$m = \frac{6 - 2k}{n + 2}. \quad (39)$$

This indicates that $k < 4 + n/2$ because $m > -1$. In addition, the pressure equilibrium yields a relation of constants that have been introduced in the preceding sections as

$$\frac{a}{bA^{1+n/2}} = \frac{4q[\alpha(q-1) + 2]G(\xi_c)f(\chi_c)}{3mg(\chi_c)F(\xi_c)}$$
$$\times \left(\frac{m+1}{\xi_c}\right)^{n/2}\left(1 - \sqrt{\frac{q}{m+1}}\right)^{-1}. \quad (40)$$

This relation is used to discuss the propagation of the outer shock in the ambient medium.

A blast wave in a uniform ambient medium with power supply was treated by Blandford & McKee (1976). They assumed that the power is supplied by a stationary source located at $r = 0$ and varies with time as a power law, $L = L_0 t^s$. If $s = (2n - 8)/(n + 2)$, then the flow in the shocked ambient medium evolves in exactly the same way as described in Blandford &
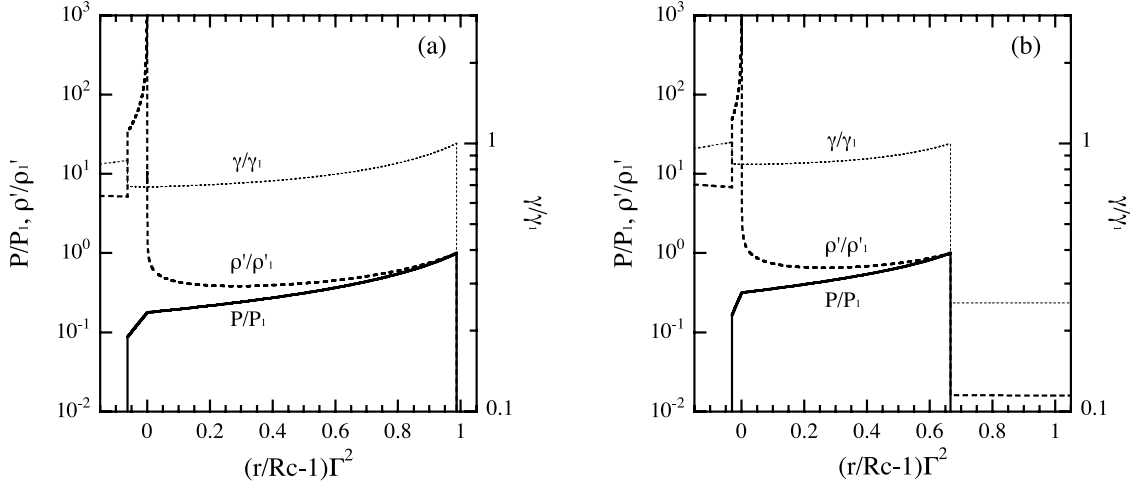
FIG. 1.— (a) Profiles of the pressure P (solid line), the density ρ (thick dashed line), and the gas Lorentz factor γ (thin dashed line) in the interaction region between freely expanding ejecta with $n = 1.10$ and the ambient medium with a power-law distribution (ρr^{-2}). The physical variables are normalized to their values at $r = R_1$, where subscript "1" refers to the value at the outer shock front. The radius at the contact surface is denoted by R_c. (b) Profiles of the same quantities as in (a), but for freely expanding ejecta ($E_{ex} = 10^{53}$ ergs and $M_{ej} = 13\ M_\odot$) at time $t = 100$ s.

McKee (1976). They also argued that the adiabatic impulsive solution is appropriate for the flow with m in the range of $m > 3$ or < -1. Since $m > -1$ in our problem, the condition $m > 3$ restricts the value of n to be less than $-2k/3$ through equation (39). Thus, the values of $n = 1.1$ and 2.6 indicate a continuous supply of energy.

When $(n, k) = (1.1, 2)$, equation (39) yields $m = 2/(n+2) \sim 0.65$ and then, from equation (14), $q \sim 1.06$. Performing numerical integration of equations (25)–(27) and (34)–(36) with these parameters, boundary conditions (eqs. [28] and [37]), and $\hat{\gamma} = 5/3$ in the shocked ejecta, we find $\xi_c \sim 0.95$ and $(f_c, g_c, F_c, G_c) \sim (0.18, 0.47, 2.1, 0.98)$. Here the subscript "$c$" of each variable indicates that the value is taken at the contact surface. Results are shown in Figure 1a. When $(n, k) = (2.6, 2)$, the same procedure leads to $m = 2/(n+2) \sim 0.43$, $q \sim 1.05$, $\xi_c \sim 0.97$, and $(f_c, g_c, F_c, G_c) \sim (0.32, 0.70, 2.0, 0.98)$. Results are shown in Figure 1b.

5. PROPAGATION OF SHOCK IN STELLAR WIND

Nakayama & Shigeyama (2005) investigated the evolution of an ultrarelativistic shock wave in a plane-parallel atmosphere and derived self-similar solutions. The resulting energy spectrum of the ejected matter with an explosion energy E_{ex} and an ejected mass M_{ej} for a radiative stellar envelope can be expressed as

$$\frac{M(>\varepsilon)}{M_{ej}} \propto \left(\frac{E_{ex}}{M_{ej}}\right)^{5.75} \varepsilon^{-2.10}, \qquad (41)$$

where $M(>\varepsilon)$ denotes the ejected mass that has a particle energy per nucleon greater than $\varepsilon = m_p(\gamma_e - 1)$. Here the proportionality constant is not specified by Nakayama & Shigeyama (2005) because their solutions cannot trace the flow to the free expansion phase in which the equation of state is no longer ultrarelativistic.

Using the relation $dM = 4\pi r^2 \rho_e \gamma_e dr$ we can convert the energy distribution given by equation (41) into the density distribution of the ejecta as a function of γ_e and t:

$$\rho_e = a\gamma_e^{-n} t^{-3} \propto \left(\frac{E_{ex}}{M_{ej}}\right)^{5.75} M_{ej} \gamma_e^{-1.10} t^{-3}. \qquad (42)$$

This formula indicates $a \propto E_{ex}^{5.75} M_{ej}^{-4.75}$ in equation (1), $n = 1.10$, and $m = 0.645$ from equation (39). Eliminating A in equations (38) and (40) leads to

$$\Gamma^2 = \left\{ \frac{3m(2q)^{n/2}}{2[\alpha(q-1)+2]} \left(\frac{\xi_c}{m+1}\right)^{n/2} \right.$$
$$\left. \times \left(1 - \sqrt{\frac{q}{m+1}}\right) \frac{F_c G_c^{n/2}}{f_c g_c^{n/2}} \frac{a}{b} \right\}^{2/(n+2)} t^{-m}. \qquad (43)$$

Substituting the expression for a into equation (43) with the exponents m and n obtained above, we then have

$$\Gamma \propto \left(\frac{a}{b}\right)^{1/(n+2)} t^{-m/2} \propto E_{ex}^{1.85} M_{ej}^{-1.53} t^{-0.322}. \qquad (44)$$

We can discuss the time evolution of the Lorentz factor of a shock in a steady stellar wind more definitely if the proportionality constant in equation (41) is specified. This is the case for Type Ic supernovae (SNe Ic) discussed by Fields et al. (2002) and Nakamura & Shigeyama (2004). According to the results of special relativistic hydrodynamic calculations by Nakamura & Shigeyama (2004; see also Fields et al. 2002), the energy distribution of SN Ic ejecta is given by

$$\frac{M(>\varepsilon)}{M_{ej}} = C\left(\frac{E_{ex}}{10^{51}\ \text{ergs}}\right)^{3.4} \left(\frac{M_{ej}}{1\ M_\odot}\right)^{-3.4} \left(\frac{\varepsilon}{10\ \text{MeV}}\right)^{-3.6}, \qquad (45)$$

where C is a nondimensional constant ($\sim 2 \times 10^{-4}$) slightly depending on stellar models. Thus, the density distribution of the ejecta becomes

$$\rho_e = 2.26 \times 10^{-8} CM_{ej} \left(\frac{E_{ex}}{10^{51}\ \text{ergs}}\right)^{3.4} \left(\frac{M_{ej}}{1\ M_\odot}\right)^{-3.4} \gamma_e^{-2.6} t^{-3}. \qquad (46)$$

Accordingly, the constant a is expressed in terms of the parameters of the explosion as

$$a = 2.26 \times 10^{-8} CM_{ej} \left(\frac{E_{ex}}{10^{51}\ \text{ergs}}\right)^{3.4} \left(\frac{M_{ej}}{1\ M_\odot}\right)^{-3.4}. \qquad (47)$$

On the other hand, the constant b is expressed in terms of the mass-loss rate $\dot{M}$ and the wind velocity v_{wind} as $b = \dot{M}/4\pi v_{\mathrm{wind}}$. Substituting the expression for a obtained above and values for nondimensional functions at the contact surface into equation (43), Γ is expressed in terms of physical quantities characterizing the explosion and ambient medium as

$$\Gamma = 6.3 \times 10^1 \left(\frac{b}{10^{10} \text{ g cm}^{-1}} \right)^{-0.22} \left(\frac{E_{\mathrm{ex}}}{10^{53} \text{ ergs}} \right)^{0.74}$$
$$\times \left(\frac{M_{\mathrm{ej}}}{1 \ M_\odot} \right)^{-0.52} \left(\frac{t}{1 \text{ s}} \right)^{-0.22}. \tag{48}$$

6. SUMMARY

We have found self-similar solutions for the collision of spherical ejecta freely expanding at relativistic speeds with the ambient medium when the ejecta have a density distribution in the form of equation (1) and the ambient medium has a power-law density distribution in terms of the distance from the explosion site with the exponent $-k$. Solutions can be obtained as long as $k < 4 + n/2$. It is found that the rest-mass energy is always comparable to the thermal energy in the shocked ejecta, while the rest-mass energy becomes negligible in the shocked ambient medium.

Profiles of the flows in the shocked regions are presented in two cases in which the ejecta are results of a shock breakout from a stellar surface embedded in a steady wind. The time evolution of the Lorentz factor of the outer shock front is derived as a function of physical parameters characterizing the explosion and the environment.

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