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DETERMINATION OF THE CORONAL DENSITY STRATIFICATION FROM THE OBSERVATION OF HARMONIC CORONAL LOOP OSCILLATIONS

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ABSTRACT

The recent detection of multiple harmonic standing transverse oscillations in coronal loops by Verwichte et al. is of special importance, as it allows one to obtain information on the longitudinal density variation in loops. Verwichte et al. detected the simultaneous presence of both the fundamental and the first-overtone mode in two coronal loops. Here we point out that the ratio of the period of the fundamental mode to the period of the overtone mode differs from 2 in loops with longitudinal density stratification. Conversely, the difference between this ratio and 2 can be used as a seismological tool to obtain information about the density scale height in loops.

Subject headings: Sun: corona — Sun: magnetic fields — Sun: oscillations

1. INTRODUCTION

Theoretical investigations of magnetohydrodynamic (MHD) waves in photospheric and coronal magnetic flux tubes date back to at least the 1970s (e.g., Defouw 1976; Spruit 1982; Edwin & Roberts 1983; Roberts et al. 1984). These studies all used simple uniform-equilibrium models for the flux tubes. The avalanche of detections of MHD waves by space-borne instruments (the *Solar and Heliospheric Observatory*, the *Transition Region and Coronal Explorer*) over the last decade has given a tremendous boost to the subject. Several wave types have been identified, and they are all believed to carry valuable information about the plasma through which they propagate, opening up the possibility of coronal seismology (Uchida 1970; for reviews, see Roberts 2000; Aschwanden 2004). The interpretation of transverse coronal loop oscillations in terms of standing kink oscillations of a magnetic flux tube, for example, has lead to an estimate of the magnetic field strength of around 13 ± 9 G (Nakariakov & Ofman 2001). For the standing transverse oscillations, the interpretation relies to a large extent on theoretical results obtained for uniform-equilibrium models. Surprisingly, these theoretical results seem to offer a good explanation as far as periods are concerned. MHD waves of fully one- and two-dimensional models have been computed recently with the principal aim of understanding the rapid damping of the observed transverse oscillations. Until very recently, only single-mode oscillating loops were known. Multimode oscillations were detected for the first time by Verwichte et al. (2004). They found that two loops were oscillating in both the fundamental and the first-overtone mode. According to the theory of MHD waves, for uniform loops the ratio of the period of the fundamental to the period of the first overtone is exactly 2. Although the ratios found by Verwichte et al. are 1.81 and 1.64 and thus clearly differ from 2, they conclude from an analysis of the errors that the observed values are not in conflict with the expected value of 2. In our view, however, there should not be any concern about the values' differing from 2. Rather, they are a source of important new information.

Higher harmonics need not oscillate at a multiple of the fundamental frequency. Guitar players know very well that there are good and bad strings. Strings can be bad because, for example, the mass distribution along the string is not perfectly homogeneous. Bad strings make it impossible to tune one's guitar well. When, for example, pressing the fret at half the string length, the frequency should be exactly double that of the fundamental. If it is not, all that one can do is tune the guitar so that the fundamentals sound right, but then the high tones on those strings will sound false.

The loops under study by Verwichte et al. (2004) stretch out up to a distance of about 1 density scale height. It should thus not come as a surprise that they seem to be tuned badly. Longitudinal density stratification might not be the only cause of detuning; noncircular curvature of the loop, variable cross section, etc., could influence the ratio of the frequencies of the harmonics as well.

We have recently computed eigenmodes of longitudinally stratified loops (Andries et al. 2005) and shown that to a good approximation, the frequency is unaffected by longitudinal density stratification itself, depending only on the weighted mean density. The weight function is, not surprisingly, given by the square of the fundamental sine, as this represents the longitudinal wave energy distribution along the loop. Likewise, it could be anticipated that the second harmonic is only influenced by the mean density when weighted by the square of the second sine. Clearly, different harmonics are influenced differently by the density distribution along the loop, and, by inversion, observation of the frequencies of different harmonics can provide information about the possible density distribution along the loop.

In this Letter, we use the method from Andries et al. (2005) to compute the frequencies of the fundamental and the first overtone in longitudinally stratified loops. We show that the stratification of the density in the longitudinal direction has a different effect on the fundamental than it does on the first overtone. The main result is that in a stratified atmosphere, the oscillation period of the fundamental should be less than double that of the second harmonic. We use the observed values of the period ratio to deduce very reasonable values for the density scale height.

2. ANALYSIS

We use the equilibrium model and the method to compute MHD oscillations as outlined in Andries et al. (2005). The equilibrium configuration is a straight pressureless cylindrical

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flux tube with a constant magnetic field. The density varies in both the radial and the axial directions. The aim of this Letter is to elucidate the effect of longitudinal density stratification on the periods. To single out this effect, we have adopted a piecewise-constant equilibrium density profile in the radial direction:

$$\rho(r, z) = \begin{cases} \rho_i(z), & \text{if } r < R, \\ \rho_e(z), & \text{if } r > R. \end{cases}$$

This choice eliminates damping due to resonant absorption and allows for analytic solutions in the internal and external regions. In an ongoing large-scale numerical investigation, MHD waves are computed for equilibrium models with continuous variations of the density in both the radial and longitudinal directions. For the numerical calculations shown in this Letter we have used $\rho_i/\rho_e = 2$, but the results are found to be independent of this parameter.

Since the equilibrium quantities are independent of φ and t, the perturbed quantities can be set proportional to $\exp[i(m\varphi - \omega t)]$ because no coupling between different Fourier modes can occur. Here m (an integer) is the azimuthal wavenumber and ω is the oscillation frequency. For the kink modes under consideration $m = 1$, but as it has no influence on the derivations, a general m is used in the expressions. The linearized MHD equations then lead to the following set of partial differential equations for the radial component of the Lagrangian displacement, ξ_r, and the Eulerian perturbation of the total pressure p_T:

$$L_{\rm A}\frac{1}{r}\frac{\partial(r\xi_r)}{\partial r} = \left(\frac{m^2}{r^2} - \frac{\mu}{B^2}L_{\rm A}\right)p_T, \quad \frac{\partial p_T}{\partial r} = L_{\rm A}\xi_r,$$

where $L_{\rm A}$ is the Alfvén operator,

$$L_{\rm A} = \rho\omega^2 + \frac{B^2}{\mu}\frac{\partial^2}{\partial z^2} = \rho\left(\omega^2 + v_{\rm A}^2\frac{\partial^2}{\partial z^2}\right)$$

with $v_{\rm A}^2 = B^2/\rho\mu$ the square of the Alfvén speed.

These equations differ from those for the longitudinally invariant model in that here $L_{\rm A}$ operates on functions of the longitudinal coordinate, so that the r- and z-dependences cannot be readily separated. By expressing the solutions in the two homogeneous regions as a sum of eigenmodes of the local Alfvén operator, the internal and external solutions can be readily obtained and represented as a sum of separable terms. The z-dependence of the perturbed quantities can be expressed as a sine series, and in practice all computations are done on the sine-series coefficient vectors. The longitudinal equilibrium density profile is expressed as a sine series plus a constant term taking into account the density at the footpoints:

$$\rho(r, z) = \rho_0(r)\left[1 + \sum_{n=1}^{+\infty}\alpha_n(r)\sin\left(\frac{n\pi}{L}z\right)\right],$$

where L is the length of the coronal loop, taken to be $100R$ in the calculations, which, in accordance with observations, corresponds to the thin-tube regime, where the results are rather independent of the parameter L/R. Acting on the sine-series coefficient vectors, the Alfvén operator is expressed as a matrix operator:

$$L_{\rm A} = \rho_0\omega^2\left(I + \sum_{n=1}^{+\infty}\alpha_n S_n\right) - \frac{B^2}{\mu}\partial_z^2$$
$$= \rho_0\left[\omega^2\left(I + \sum_{n=1}^{+\infty}\alpha_n S_n\right) - v_{\rm A,0}^2\partial_z^2\right]$$

with $v_{\rm A,0}$ the Alfvén speed at the footpoints of the field lines, I the unit matrix, ∂_z^2 the diagonal matrix $(\partial_z^2)_{kl} = (k\pi/L)^2\delta_{kl}$, and the matrix S_n representing the multiplication with $\sin(n\pi z/L)$, that is, S_{nkl} is the lth coefficient in the series of $\sin(n\pi z/L) \times \sin(k\pi z/L)$. The dispersion relation is obtained as the determinant of an infinite system of algebraic equations representing the matching conditions for each of the sine components of $p_T(R, z)$ and $\xi_r(R, z)$ at the loop surface $r = R$.

In the absence of longitudinal stratification, the matching conditions reduce to a block diagonal system and the dispersion relation decomposes into a dispersion relation for each sine component. In the thin-tube limit $L \gg R$, these can be solved analytically and yield the well-known solutions

$$\omega = \frac{k\pi}{L}\left(\frac{2}{\rho_i + \rho_e}\right)^{1/2}B.$$

As $k = 1$ corresponds to the fundamental and $k = 2$ to the first overtone, it is clear that the ratio of the periods of the fundamental and the first overtone is equal to 2 in an unstratified model. Although this is only exactly 2 in the limit $R/L \to 0$, the dispersive modifications due to finite L/R are much smaller than the effects of longitudinal stratification in the case of realistic L/R-values for coronal loops.

From a linear expansion of the dispersion relation, in Andries et al. (2005) we derived a linear expression for the frequency shift due to the presence of density stratification:

$$\delta\omega = -\frac{1}{2}\omega\sum_n \alpha_n S_{nkk}.$$

This result is somewhat dependent on the normalization of the frequency, which could be chosen to vary with α_n. From the definition of S_{nkk} it can be found that the frequency is independent of α_n when the normalization is done in terms of the weighted mean density, where the weight function is the square of the kth sine (for more details, see Andries et al. 2005). In any event, as $S_{n11} \neq S_{n22}$ the stratification can be seen to act differently on the frequency of the fundamental and that of the first overtone. Moreover, the ratio of the two periods is independent of the aforementioned normalization. In particular, for the fundamental and the first overtone we obtain

$$\frac{P_1}{P_2} = \frac{\omega_2}{\omega_1} = \frac{\omega_{2,0}}{\omega_{1,0}}\frac{1 - \frac{1}{2}\sum_n \alpha_n S_{n22}}{1 - \frac{1}{2}\sum_n \alpha_n S_{n11}} = 2\,\frac{1 - \frac{1}{2}\sum_n \alpha_n S_{n22}}{1 - \frac{1}{2}\sum_n \alpha_n S_{n11}}.$$

The ratio can thus be seen to be 2 in an unstratified loop but to differ from this value in a stratified loop.

For computations in which the amount of stratification is outside the linear domain, we have used an exponentially stratified atmosphere, that is, $\rho(h) = \rho_0\exp(-h/H)$, where h is the height in the solar atmosphere and H is the density scale height.

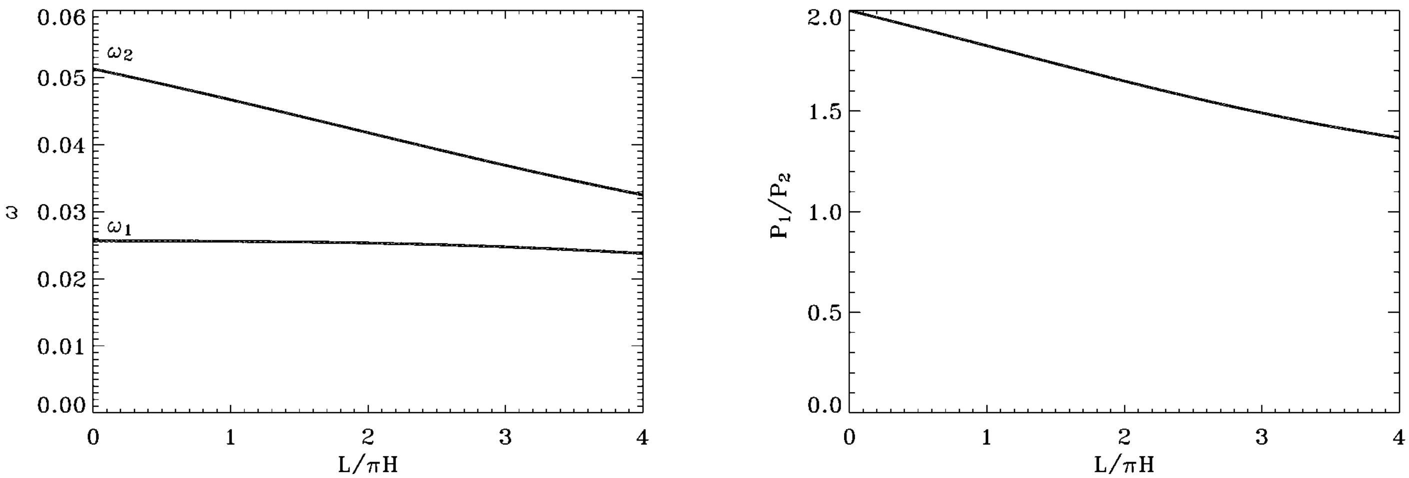

FIG. 1*a* FIG. 1*b*

FIG. 1.—(*a*) Frequency of the fundamental kink mode and its first harmonic for a tube with length 100*R* and density contrast 2, as a function of the density stratification $L/\pi H$. The frequencies are normalized with respect to the mean density weighted by $\sin^2(\pi z/L)$. (*b*) Ratio of the periods P_1/P_2 or frequencies ω_2/ω_1 of the fundamental kink mode and its first harmonic for the same tube.

This stratification function is projected on a semicircular loop of length L with a consequent height of L/π, that is, $\rho(z) = \rho_0 \exp[-L \sin(\pi z/L)/\pi H]$. Thus, the relative height of the loop as compared with the density scale height $L/\pi H$ is used as a measure of the stratification and as the independent variable in the figures. By the projection, $L/\pi H$ determines the values of all the α_n. Whereas Andries et al. (2005) included only the α_1 parameter in the computations (and consequently used α_1 as the independent variable on the plots), we have here included all α_n up to α_9 and used $L/\pi H$ as the independent variable. In fact, from α_9 on, inclusion of higher α_n only changes the frequencies by less than 1 part in 1000.

We emphasize that the use of a simple exponentially stratified density is not an essential point in the method that we are presenting. In principle, much more complicated models could be used for the stratification. Moreover, the final aim would be to also include other equilibrium parameters, such as variable cross sections of the flux tube and noncircular curvature, which might influence the period ratio. In general, the detection of n modes would allow for an equilibrium model including $n - 1$ parameters. As at this moment only two modes are observed, a one-parameter model is just the best we can do.

Figure 1*a* shows how the oscillation frequencies of both the fundamental and the first harmonic depend on the amount of stratification. While the stratification is increased we have normalized with the weighted mean density as weighted by the fundamental $\sin^2(\pi z/L)$. The fundamental frequency (*lower curve*) is therefore approximately invariant, but the frequency of the harmonic depends on the stratification, as the density weighted by $\sin^2(2\pi z/L)$ changes with varying $L/\pi H$. For $L/\pi H = 0$, the frequency of the harmonic is indeed exactly double that of the fundamental, but this does not remain true for higher values of $L/\pi H$.

Figure 1*b* shows the ratio of the two former curves and is independent of the aforementioned normalization. In fact, Figure 1*b* is totally independent of all other loop parameters such as the magnetic field and the values of the density (our computations have also shown that the density contrast has no influence on the results). Consequently, Figure 1*b* presents a one-to-one relationship between the ratio of the periods and the density stratification parameter.

Figure 2 shows essentially the same one-to-one relation as Figure 1*b*, but now in terms of the seismologically relevant variables, viz., the difference from 2 of the observed period ratio as the independent variable (hence $2 - P_1/P_2$) and the density scale height (normalized with respect to the loop height, hence $H\pi/L$) as the dependent variable. The period ratios observed in two cases by Verwichte et al. (2004) are 1.81 ± 0.25 and 1.64 ± 0.23.[2] When they are subtracted from 2, this provides us with the seismologically relevant observables of 0.19 ± 0.25 and 0.36 ± 0.23, as indicated in Figures 2*a* and 2*b*, respectively. The first measurement im particular suffers from extremely large error bars, even allowing for negative values, which indicates the possibility of larger density at the loop tops. Because of the large error bars and the strongly nonlinear character of the curve in Figure 2, the error bars on the estimated values for the density scale height are not symmetric. For the first case, with a loop height of 70 Mm (according to Verwichte et al. 2004), we obtain an estimate for the density scale height of around 65 Mm and, because of the upper bound of the error bar, most probably not below 27 Mm. As indicated above, the first measurement does not exclude the possibility of heavier plasma at the loop tops, but that possibility is certainly limited, as the correspondingly negative density scale height is most probably lower than −190 Mm. This can be translated to the statement that the density at the top is unlikely to exceed that at the footpoints by more than 44%. The error bars on the second measurement are much smaller. With a loop height of 73 Mm, this leads to a much more reasonably confined estimate of around 36 Mm, and most likely within 20–99 Mm.

Although the current data set, containing only two items, is certainly too limited to obtain strong quantitative results, especially given the considerable error bars on the observations and the poor condition of the problem, we believe that we have demonstrated an important new tool for coronal seismology. Future observations with possibly higher accuracy could improve the practical value of this method. At this stage we can only be encouraged by the fact that the two observations

[2] The error bars on the ratios are estimated as $\sigma_{P_1/P_2} = P_1/P_2[(\sigma_{P_1}/P_1)^2 + (\sigma_{P_2}/P_2)^2]^{1/2}$.

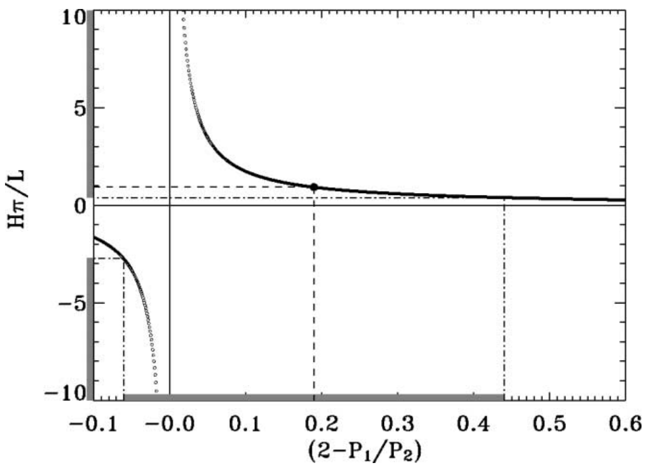

FIG. 2*a*

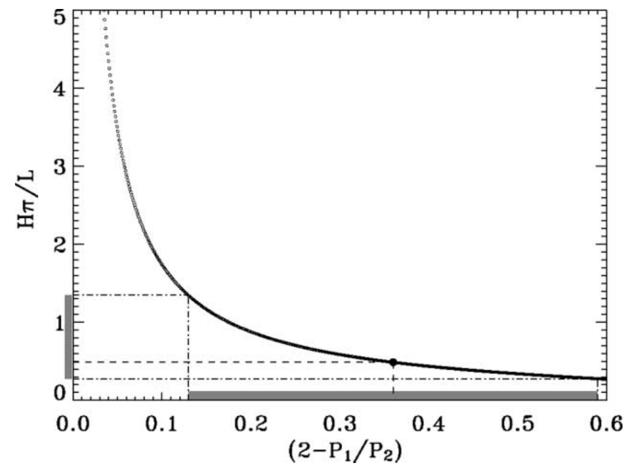

FIG. 2*b*

FIG. 2.—Inversion of Fig. 1*b*: the density scale height (normalized with respect to the loop height, hence $H\pi/L$) as a function of the seismological observable $2 - P_1/P_2$. (*a*) Error bars on the observed values and the estimated scale height for the first observation. As the error bar on the measurement includes the origin, the estimate includes the possibility of a $\pm\infty$ scale height, i.e., no stratification. (*b*) Same as (*a*), but for the second observation.

are in line with our expectations of a density scale height of around 50 Mm.

3. CONCLUSIONS

We have shown that the ratio of the periods of the fundamental kink mode of a coronal loop and of its first harmonic is lower than 2 if there is longitudinal density stratification along the loop. The observed ratio can be used to provide estimates of the density scale height in the solar atmosphere.

We have applied our findings to the two known simultaneous observations of the fundamental kink mode and its harmonic in coronal loops. Both observations yield ratios that are lower then 2, but only in the second case is the deviation from 2 significant. Hence, the first case cannot exclude the possibility of heavier plasma at the loop tops, but it does limit that possibility considerably. Both observations are consistent with an expected scale height of about 50 Mm. For the second case we obtain a reasonably confined estimate for the density scale height of between 20 and 99 Mm.

More, and more accurate, simultaneous observations of the fundamental kink mode and its harmonic in coronal loops are necessary before this method can be used to provide accurate quantitative results. We thus wish to emphasize that the observation of different harmonics in the same coronal loop provides an important new tool for the determination of the longitudinal density profiles specifically, and for coronal seismology in general.

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