The Astrophysical Journal, 616:346–356, 2004 November 20

INFLUENCE OF AN INTERNAL MAGNETAR ON SUPERNOVA REMNANT EXPANSION

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Received 2002 October 10; accepted 2004 July 29

ABSTRACT

Most of the proposed associations between magnetars and supernova remnants suffer from age problems. Usually, supernova remnant ages are determined using some approximation for the Sedov-Taylor supernova phase, which yields a relation between radius and age for a fixed energy of the explosion (generally assumed to be $\sim 10^{51}$ ergs). Those ages do not generally agree with the characteristic ages of the (proposed) associated magnetars. We show in this work that a faster expansion results when the energy injected into the supernova remnant by magnetar spin-down is taken into account, thus helping to improve the matches between characteristic ages and supernova remnant ages. However, the magnetar velocities inferred from observations would make some associations inviable if correct. Since characteristic ages may not be good age estimators after all, their influence on the likelihood of the association may not be as important. In this work, we perform simple numerical simulations of supernova remnant expansion with internal magnetars and apply them to the observed sample of objects. A short initial spin period, thought to be important for the very generation of the magnetic field, is shown to be quite relevant to the modified expansion of the remnant. We finally analyze all proposed associations on a case-by-case basis, addressing the likelihood of each one, according to this perspective. We consider a larger explosion energy and reassess the characteristic age issue, and conclude that ~50% of the associations can be real, provided that soft gamma repeaters and anomalous X-ray pulsars are magnetars.

Subject headings: stars: magnetic fields — stars: neutron — supernova remnants

1. INTRODUCTION

Soft gamma repeaters (SGRs) and anomalous x-ray pulsars (AXPs) are two classes of objects tentatively identified as magnetars, neutron stars with magnetic fields above the quantum critical threshold $B_c = 4.41 \times 10^{13}$ G. In the magnetar model proposed by Duncan & Thompson (1992), which we adopt throughout this work, dynamo action in a fast-rotating proto–neutron star amplifies the "seed" magnetic field (10^{11}–10^{13} G) by up to a factor of 1000. The simplest estimate of the magnetic field intensity of actual objects is obtained using the measurements of their rotational periods (P) and period derivatives ($\dot{P}$), through the well-known expression

$$B \sin \chi = \left(\frac{3c^3 I P \dot{P}}{8\pi R_{\rm psr}^6} \right)^{1/2}, \qquad (1)$$

where B is the magnetic field intensity, $R_{\rm psr}$ is the stellar radius ($\sim 10^6$ cm), χ is the angle between the rotational and the magnetic field axes, I is the moment of inertia ($\sim 10^{45}$ g cm^2), and c is the speed of light. Their ages can also be estimated by calculating the characteristic age,

$$\tau = \frac{P}{2\dot{P}}, \qquad (2)$$

but both estimations assume continuous spin-down driven by magnetic dipole braking, an assumption that has been questioned by several authors (Marsden et al. 2001; Harding et al. 1999; van Paradijs et al. 1995; Chaterjee et al. 2000; Kouveliotou et al. 1999), using different arguments, regarding AXPs and SGRs. The numbers derived for a selected sample of SGRs and AXPs are shown in Table 1. Since most of the objects are younger than 10^4 yr, it seems reasonable to look for the supernova remnant (SNR) that was originated in the same supernova explosion that gave birth to the compact object. In fact, almost all known SGRs and AXPs have been tentatively associated with some SNR. However, the likelihood of those associations is often called into question because the age estimated by most authors for those SNRs is generally larger than the characteristic age of the associated magnetar. Generally speaking, the age of an SNR is estimated from a theoretical relationship between SNR radius, interstellar density, and initial kinetic energy. The last quantity is the most difficult one to determine and is usually fixed to 10^{51} ergs, a number that has plenty of observational and theoretical support.

The attempts to match SNRs and compact objects often lead to the "age" and "velocity" problems of SGRs and AXPs. Both problems are related to the relatively low characteristic ages found for these objects. The SNRs associated with them seem to be systematically older, leading to the so-called age problem. In addition, some SGRs and AXPs have been found to lie beyond, at, or near the border of their proposed SNRs, and if they are young, the implied velocities must be much higher than those found for ordinary pulsars (see Hurley 1999 for a discussion about this specific issue). As seen in Tables 1 and 2, there is a large uncertainty in these estimates, which is mainly due to uncertainties in the distances. This implies uncertainties in the radii on which age estimates are based through models of the expansion.

Several authors have addressed magnetar-SNR associations. Some of them have discarded either the magnetar hypothesis or the proposed association on the basis of the considerations of the age and/or velocity. This became known as the "nature versus nurture" debate, where it is proposed either that the features of SGRs and AXPs are a result of their special nature (see Thompson & Duncan 1996) or that the environment is actually responsible, nurturing the otherwise ordinary neutron stars with a fossil accretion disk (Marsden et al. 2001; Chaterjee

TABLE 1
DATA FOR SGRs AND AXPs

Magnetar	P (s)	$\dot{P}$ (10^{-12})	τ (kyr)	B $(10^{14}$ G$)$	d (kpc)
SGRs					
SGR 1806−20	7.48[1]	83[1]	1.4[1]	8.0[1]	17[2], 14[1]
SGR 1900+14	5.16[3]	50 – 140[4]	0.58 – 1.6[4]	5.1 – 8.6[4]	7[3], 5[5]
SGR 0526−66	8.00[6]	...	...	...	55[5]
SGR 1627−41	6.4?[7]	...	...	...	11[8]
SGR 1801−23	...	...	...	...	...
AXPs					
AXP 1048−5937	6.45[9]	15–40[4]	2.6–6.8[4]	5.1–8.6[4]	10.6[10], >2.8[4]
AXP J1709−4009	11[4,11]	19[11], 22.5[12]	9.2[11], 7.7[12]	4.6[11], 5.0[12]	10[9], >8[4]
AXP 1841−045	11.8[13]	47[14], 41[13]	3.9[14], 4.5[13]	7.6[14], 7.0[13]	7[14]
AXP 2259+586	6.98[10,11]	0.74[10], 0.49[11]	150[10], 230[11]	0.73[10], 0.59[11]	6.2[10]
AXP J1845−0258	6.97[15]	...	...	...	8.5[15]
Other					
PSR J1846−0258	0.324[16]	7.1[16,17]	0.72[16,17]	0.5[16,17]	...

REFERENCES.—(1) Kouveliotou et al. 1998; (2) Kouveliotou et al. 1994; (3) Kouveliotou et al. 1999; (4) Mereghetti 1999; (5) Hurley et al. 1999; (6) Duncan & Thompson 1992; (7) Woods et al. 1999; (8) Corbel et al. 1999; (9) Chakrabarty et al. 2001; (10) van Paradijs et al. 1995; (11) Kaspi et al. 1999; (12) Israel et al. 1999; (13) Gotthelf & Vasisht 1997; (14) Vasisht & Gotthelf 1997; (15) Torii et al. 1998; (16) Gotthelf et al. 2000; (17) Mereghetti et al. 2002.

et al. 2000; van Paradijs et al. 1995). To contribute to the debate, we must take a closer look at the associations themselves.

Associations between pulsars and SNRs are generally examined according to the following criteria: (1) positional coincidence in sky, (2) distance estimates, (3) age estimates, and (4) evidence for interaction between the neutron star and the SNR. Additional criteria may be derived from these, such as an estimate of the projected velocity on the sky of the compact object. Gaensler et al. (2001) performed a comprehensive study of positional coincidences for magnetar-SNR associations. Distances can be determined by neutral hydrogen column density measurements, evidence of association with another astronomical object (e.g., H II region or molecular cloud) whose distance is already known, or through surface brightness–diameter (Σ-D) relations. This last method is not considered reliable enough because it ignores the effect of the density and structure of the local interstellar medium, resulting in a large spread of the values around the best fit, as shown in Case &

TABLE 2
DATA FOR ASSOCIATED SNRs

SNR	t_a (kyr)	d (kpc)	R (pc)	β
Associated to SGRs				
G10.0−0.3	0.2–30[1]	13–16[2]	14–19[2]	0.0[3], 0.6[4]
G42.8+0.6	0.2–30[1]	3–9[1]	11–31[1]	1.2[3], 1.4[1,5]
N49	5–16[1,5]	55[6], 50[7,8]	7.1–8[1], 8.5[6]	0.6[3], 1.0[5]
G337.0−0.1	0.2–30[1]	11[9,5,10]	2.4[9,1]	1.7[5], 2.3[1,10]
G6.4−0.1	>2.4[1]	1.2–3[1], 3.5–4[9]	7–17.5[1], 21–24[9]	0.1[1]
Associated with AXPs				
G287.8−0.5	0.2–30[1]	2.5–2.8[1]	9.1–10.2[1]	2.2[1]
G346.6−0.2	0.2–30[1]	3–5[1], 11[5]	4.4–7.3[1]	1.7[1]
G27.4+0.0	2[5], <3[11]	6– 7.5[9,12]	3.5–4.4[9,1], 4.7[13]	<0.25[5], 0.1[1],
G109.1−1.0	3–20[11]	4–5.6[11,14]	16–24[1], 18–25[14]	0.3[3], 0.2[1], <0.2[5]
G29.6+0.1	0.2–30[1]	8.5[3], <20[11,5,15]	6.5–9.8[1]	0.1[1], <0.25[5]
Associated with Other				
G29.7−0.3	1.8–7[16]	9–21[9]	3.9–10.7[9,17]	0.0[16]

REFERENCES—(1) Marsden et al. 2001; (2) Corbel et al. 1997; (3) Chakrabarty et al. 2001; (4) Kulkarni et al. 1994; (5) Gaensler et al. 2001; (6) Matthewson et al. 1983; (7) Shull 1983; (8) Vancura et al. 1992; (9) Green 1998; (10) Corbel et al. 1999; (11) Mereghetti 1999; (12) Sanbonmatsu & Helfand 1992; (13) Vasisht & Gotthelf 1997; (14) Rho & Petre 1997; (15) Gaensler et al. 1999; (16) Mereghetti et al. 2002; (17) Gotthelf et al. 2000.

Battacharya (1998). Some of the SNRs of the proposed associations host plerions, usually taken to be indicative of the presence of a neutron star, but most do not host detectable plerions at all. Finally, measurements of H I column density are known to be subject to systematic errors and usually are not better than 50%, implying similar uncertainties in the distance determinations. Radio pulsars may have their distance estimated using the dispersion measure; however, SGRs and AXPs have not yet been detected in radio waves. There are reasons to believe they cannot emit radio pulses, either because they are magnetars, in which the huge magnetic field induces photon splitting rather than pair creation (Baring & Harding 1998, but see Camilo et al. 2000), or because they are accretors, since accretion quenches radio emission (Chaterjee et al. 2000; van Paradijs et al. 1995).

SNR age estimates are usually derived from knowledge of the radius by means of an approximate evolution of expanding blast waves. We see in next section that those approximations (which generally ignore the previous regime[s] of the expanding remnant) can induce a substantial error when the blast waves change regime. In addition, in practice the approximations force a reduction of the number of actual variables by linking the explosion energy, the local interstellar medium density, and the ejected mass. The present local density of the interstellar medium can be estimated within a 50% uncertainty, although this may not be representative of the structure of the interstellar medium prior to the supernova explosion. For example, explosions that occurred inside an H II region (low-density medium) and have run into a nearby molecular cloud (high-density medium) would be misleading, since the observer would actually estimate the molecular cloud density only. Even inside an H II region it would be likely that density is not constant. Situations such as these can be modeled (see a good review on this in Truelove & McKee 1999), but the point is that the unknown structure of the presupernova environment introduces one more possible systematic error in the age estimates that has to be considered. The ejected mass can be estimated from the observation of how much Fe was produced, coupled to models of supernova yields, when the SNR is young. Explosion energy is usually considered to be within a factor of 2 of the canonical value of 10^{51} ergs because of previous observational and theoretical knowledge. The review of Hamuy (2003) considers the energy range $(0.5-5.5)\times10^{51}$ ergs for classical Type II supernovae from a sample of a dozen of well-observed events. However, these estimates also contribute their own uncertainties because the type of explosion event giving rise to an SGR/AXP is completely unknown. Since the radius suffers from the same relative uncertainty as the distance, the combined uncertainty on the age estimate will not be smaller than a factor of 2, and probably much larger than that. We must recall that it is not uncommon to be quite unsure about the phase a given SNR is in and that near the transition time between phases approximations do not strictly hold. More accurate relations can be used in these cases, reducing systematic errors, although generally most authors feel that the rather small gain in precision is not worth the trouble.

Blast-wave *velocity* estimates could also be used to determine ages and radii. Knowledge of both radius and velocity would determine the phase of the expansion, removing an important source of uncertainties. Unfortunately, such estimates must come from X-ray measurements that are difficult to make, and up to now they could not be obtained for any of the proposed associations.

Finally, there is evidence of interaction between the neutron star and the SNR in some of the proposed associations that show plerions (or filled-center morphology). In addition, one of the associations was reported as presenting a jetlike feature (Kouveliotou et al. 1998).

Given the several sources of uncertainties, the overall reliability of the described estimates is not very good. However, several papers have addressed the association issue and drawn some conclusions about the nature of AXPs and SGRs. In this work, we show that at least one important factor for those estimates is still missing, namely, the injection of energy by the internal magnetar, and we analyze the proposed associations bearing that important factor in mind. The next section is dedicated to SNR expansion considerations, § 3 introduces the effects of the injected energy on the SNR, while § 4 deals with an application of those considerations to the observed sample. In § 5 we discuss our results.

2. SUPERNOVA REMNANT EXPANSION

Supernova explosions eject several solar masses to the interstellar medium, which constitute the SNR. Along the evolution, mass from the swept interstellar medium will be added at a rate $4\pi R^2\dot{R}\rho$, where R is the SNR radius (blast-wave front), $\dot{R}$ is the SNR expansion velocity, and ρ represents the local interstellar medium density, which we take as a constant for simplicity (calculations including power-law density functions have been performed by Truelove & McKee 1999, which is also a good review reference about the nonradiative expansion phases of SNRs). The initial expansion velocity is set by the total kinetic energy of the explosion, E, and the ejected mass, $M_{\rm ej}$. The interstellar medium will not greatly affect the expansion until the swept mass becomes $\sim M_{\rm ej}$.

Another approximation that holds initially is that very little energy (compared to the huge kinetic energy of the SNR) is lost, so that energy can be considered a constant. In this way, with constant mass and energy and negligible external pressure, the velocity is also (approximately) constant. These conditions define the "free expansion" phase. An approximate expression for the evolution of the radius can be found in Truelove & McKee (1999):

$$R \simeq 0.46 \text{ pc } E_{51}^{1/2} M_{10}^{-1/2} t_2 \times \left(1 + 0.011 E_{51}^{3/4} M_{10}^{-5/4} t_2^{3/2} n_1^{1/2}\right)^{-2/3}, \tag{3}$$

where $n = \rho m_{\rm H}^{-1} \mu^{-1}$ is the number density of the interstellar medium, $m_{\rm H}$ is the hydrogen mass, and $\mu = 1.4$ is the mean molecular weight of the interstellar gas. Energy is scaled in units of 10^{51} ergs ($E_{51} = E/10^{51}$ ergs), mass in units of 10 $M_\odot$ ($M_{10} = M/10\ M_\odot$), number density in units of 1 cm^{-3} ($n_1 = n/1$ cm^{-3}), and age in units of 100 yr ($t_2 = t/10^2$ yr). The velocity at which the blast-wave front expands is approximately

$$\dot{R} \simeq 4500 \text{ km s}^{-1}\ E_{51}^{1/2} M_{10}^{-1/2} \times \left(1 + 0.011 E_{51}^{3/4} M_{10}^{-5/4} t_2^{3/2} n_1^{1/2}\right)^{-5/3}. \tag{4}$$

As M increases, so does the ram pressure of the interstellar medium, slowing down the SNR expansion. The increasing mass and pressure must then be taken into account. The SNR enters the Sedov-Taylor phase, named after the works of Sedov

(1959) and Taylor (1950) on pressure-driven explosions. To describe the SNR expansion, one should solve the equation

$$\frac{d}{dt}\left(\frac{3}{4}M\dot{R}\right) = 4\pi R^2 p, \tag{5}$$

the internal pressure being

$$p = (\gamma - 1)U\left(\frac{3}{4\pi R^3}\right), \tag{6}$$

where γ is the adiabatic index (5/3 for an ideal gas, 4/3 for a relativistic one) and U is the internal energy of the gas inside the internal cavity formed by the remnant, whose mass concentrates in a thin shell (its thickness will be neglected). As the total energy is roughly constant, one may use

$$U = E - \frac{9}{32}M\dot{R}^2. \tag{7}$$

In the Sedov-Taylor phase $M_{\rm ej} \ll M$, so one can use this to obtain the well-known analytic solution

$$R \simeq \left(\xi\frac{E}{n}\right)^{1/5} t^{2/5}, \tag{8}$$

where $\xi = 2.02$ in the exact solution; we found 1.77 (both for $\gamma = 5/3$). By construction, this solution disregarded the previous phase. Truelove & McKee (1999) have calculated a corrected (though still approximate) expression, which is

$$R \simeq 12.5 \text{ pc } M_{10}^{1/3} n_1^{-1/3} \times \left(E_{51}^{1/2} M_{10}^{-5/6} n_1^{1/3} t_4 - 0.051\right)^{2/5}, \tag{9}$$

with age given in units of 10^4 yr ($t_4 = t/10^4$ yr). The correspondent expression for velocity is

$$\dot{R} \simeq 490 \text{ km s}^{-1}\ E_{51}^{1/2} M_{10}^{-1/2} \times \left(E_{51}^{1/2} M_{10}^{-5/6} n_1^{1/3} t_4 - 0.051\right)^{-3/5}. \tag{10}$$

Although we have considered the constancy of the total energy, the SNR has in fact been slowly radiating away. As the SNR expands, its temperature decreases because it depends on the blast-wave velocity, following the well-known relation for strong shocks,

$$T = \frac{3}{16}\frac{\rho}{k_{\rm B} n}\dot{R}^2, \tag{11}$$

where $k_{\rm B}$ is Boltzmann's constant.

Eventually, T will reach 10^7 K, where the dominating cooling process changes from thermal bremsstrahlung to line emission, which is much more efficient in radiating away the energy. Therefore, the adiabatic approximation ceases to be accurate and the SNR enters the "snowplow" phase. As a rough approximation to estimate when the Sedov-Taylor phase ends, it is usual to estimate how much time it would take to reduce the thermal energy of the SNR to zero, considering just radiative losses. Starting from the energy loss per particle,

$$\frac{d}{dt}\left(\frac{3}{2}k_{\rm B}T\right) = -n\Lambda, \tag{12}$$

where $\Lambda = 1.6 \times 10^{-19}\zeta T^{-1/2}$ ergs cm^3 s^{-1} is a simple cooling function appropriate for ionized gas at temperatures $10^7 \text{ K} \lesssim T \lesssim 10^5$ K and $\zeta = 1$ is a metallicity factor parameterized for solar abundances, one can integrate that equation and equate the result to the dynamical time $R/\dot{R} \simeq 5t/2$, obtaining

$$t_{\rm ST-SP} \simeq 19 \times 10^3 \text{ yr } E_{51}^{3/14} n_1^{-4/7} + 510 \text{ yr } E_{51}^{-1/2} M_{10}^{5/6} n_1^{-1/3}. \tag{13}$$

It has been suggested that SNRs are very difficult to detect after 20 kyr (Braun et al. 1989). The similarity of the values of "fading time" and the transition from the Sedov-Taylor to the snowplow phase may be taken as indicative that they are related by a factor of order unity. Thus, we adopt equation (13) as the rough limit of visibility of an SNR. Since our intention is to address associations between SNRs and magnetars, we do not explore further the expansion of SNRs in the snowplow phase because SNRs would be detectable only with difficulty in that phase or afterward.

3. ENERGY INJECTION BY A MAGNETAR

The energy loss of a pulsar is usually taken as arising from a rotating magnetic dipole approximation,

$$L_{\rm psr} = \frac{B^2 R_{\rm psr}^6 \sin^2\chi}{6c^3}\left(\frac{2\pi}{P}\right)^4 = 4\pi^2 I\frac{\dot{P}}{P^3}. \tag{14}$$

That energy is held by a young SNR, which will contain in its internal cavity most of the relativistic particles and/or electromagnetic waves emitted by the central object. Either way, this cavity would thus be filled by a relativistic gas, pushing the SNR from the inside. Considering that the magnetic field, moment of inertia, and χ are constants, they can be absorbed together with other factors into a new constant $K = B^2 R_{\rm psr}^6 \sin^2\chi/6Ic^3$. After integrating equation (14) the period evolution can be expressed as

$$P = \sqrt{8\pi^2 K t + P_0^2}, \tag{15}$$

with P_0 being the initial period of the pulsar. Two additional constants are defined:

$$\tau_0 = \frac{P_0^2}{8\pi^2 K} = 0.6 \text{ days}\left(\frac{B}{10^{14}\text{ G}}\right)^{-2}\left(\frac{P_0}{1 \text{ ms}}\right)^2 \tag{16}$$

is the initial timescale for deceleration, and

$$L_0 = KI\left(\frac{2\pi}{P_0}\right)^4 = 3.85 \times 10^{47} \text{ ergs s}^{-1} \times \left(\frac{B}{10^{14}\text{ G}}\right)^2\left(\frac{P_0}{1 \text{ ms}}\right)^{-4} \tag{17}$$

is the initial rate of energy loss. We note that $L_0\tau_0^2 \propto B^{-2}$ and $L_0\tau_0 \propto P_0^{-2}$. With these relations and equation (15), we can rewrite equation (14) as

$$L_{\rm psr} = L_0\left(1 + \frac{t}{\tau_0}\right)^{-2}. \tag{18}$$

Total injected energy is just the integral of equation (18) from the initial instant $t_0 < \tau_0$ to the present time,

$$E_{\rm inj}(t) = \frac{L_0}{t^{-1} + \tau_0^{-1}} = 2.0 \times 10^{52} \text{ ergs } \left(\frac{P_0}{1 \text{ ms}}\right)^{-2} \times \left[1.6 \times 10^{-3} \left(\frac{B}{10^{14} \text{ G}}\right)^{-2} \times \left(\frac{P_0}{1 \text{ ms}}\right)^{2} \left(\frac{t}{1 \text{ yr}}\right)^{-1} + 1\right]. \quad (19)$$

The birth of a magnetar in the supernova explosion would inject a large amount of energy in much the same way, provided that some minimal conditions are fulfilled. For this application, equations (16) and (17) have already been scaled to the typical magnetar parameters. We remark that unless magnetars are born with very short periods ($\sim$1 ms), their magnetic fields would *not* be expected to grow enough to cross the critical quantum boundary B_c, according to the model of magnetar formation of Duncan & Thompson (1992). While pulsars can be born with such short periods, they are not required to do so. The difference of 2 orders of magnitude in the magnetic field strength between a typical pulsar ($\sim 10^{12}$ G) and a "typical" magnetar ($\sim 10^{14}$ G) translates to 4 orders of magnitude in both τ_0 and L_0 values. This in turn implies a dramatically different influence on SNR expansion: a magnetar will inject most of its rotational energy into the internal cavity of the SNR within a day, while a pulsar would take tens or hundreds of years, depending on its initial period and magnetic field. Also, a magnetar will inject typically 10^4 times more energy than a pulsar, and, more remarkably, that energy is a factor of 10–20 greater than the kinetic energy of an ordinary SNR. This situation is quite reminiscent of the suggestion of Ostriker & Gunn (1971) about that transfer of energy being the very cause of a supernova event.

Gravitational radiation losses are usually larger than rotating magnetic dipole ones for pulsars with very short periods; hence, any estimate of the initial period based on the electromagnetic dipole torque will actually yield a number corresponding roughly to the period of transition between gravitational and magnetic dipole dominance. In fact, a simple estimate can be obtained equating equation (14) to the expression for gravitational wave–carried energy loss from Shapiro & Teukolsky (1983),

$$L_{\rm grav} = \frac{32}{5} \frac{G I^2 \epsilon^2}{c^5} \left(\frac{2\pi}{P}\right)^6, \quad (20)$$

where G is the familiar gravitational constant and ϵ is the oblateness of the neutron star. The result is

$$P_{\rm tran} = 16 \sqrt{\frac{3G}{5}} \frac{I \pi \epsilon}{R_{\rm psr} c B \sin\chi} = 0.34 \text{ ms} \left(\frac{B}{10^{14} \text{ G}}\right)^{-1} \left(\frac{\epsilon}{10^{-4}}\right), \quad (21)$$

which shows that for a magnetar the large magnetic dipole losses would not be much affected by gravitational losses. In other words, the initial period will effectively be the same period that has allowed the magnetic field intensity to increase above B_c, and almost all rotational energy will be injected into the SNR. However, it must be acknowledged that there is room to consider competition between dipole radiation and gravitational radiation by r-modes. Doing a very simple analysis from the work of Owen et al. (1998), we would expect r-mode gravitational waves to be more efficient than dipole radiation in extracting rotational energy from the rapidly rotating magnetar for $P \leq 4$ ms. A recent work by Watts & Andersson (2002) suggests that, when initial fallback on the neutron star is taken into account, the r-mode instability never fully develops in magnetars. Other studies concerning the damping of the r-modes by intense magnetic fields point in the same direction (Rezzolla et al. 2001). We simply assume that the gravitational wave output is negligible compared to the electromagnetic one.

Earlier works have addressed pulsar energy injection on a SNR (Luz & Berry 1999; van der Swaluw et al. 2001), but they have not explored the case of a magnetar source. It can be easily seen that the scenario built by van der Swaluw et al. (2001) cannot hold when magnetars are considered, since they have assumed the energy injected by the pulsar to be smaller than the kinetic energy of the SNR from the very beginning.

The introduction of magnetar-injected energy changes the equations describing the expansion of the SNR in the following ways: equation (6) remains the same, but $\gamma = 4/3$ could be used to represent the dominance of the injected energy over the initial kinetic energy of the SNR; equation (7) picks up a new term, becoming

$$U = E - \frac{9}{32} M \dot{R}^2 + \frac{L_0}{t^{-1} + \tau_0^{-1}}. \quad (22)$$

However, in this simple formulation of the problem only the blast-wave front is described, and not the full problem of two expanding gases. The energy injection is so quick that we consider the entire SNR to be instantaneously reacting to it. That is not strictly true, but since the internal shock (the inner cavity boundary) would be near the external shock (the SNR boundary) in less than 100 yr (that is, much before the transition to the Sedov-Taylor phase starts), this simplification will not greatly affect the Sedov-Taylor and later phases. We only consider $\gamma = 5/3$ throughout this work. While the works of Luz & Berry (1999) and van der Swaluw et al. (2001) studied the injection of energy by a pulsar into an SNR, we adopted simplifications that are not well suited to be combined with their methods and results. However, the general results obtained in our work are not expected to be affected by these approximations.

Since the reversal and internal shocks have not been modelled in this work (nor their expected interaction), it is not easy to dismiss completely the possibility of an early radiative phase developing. However, according to Truelove & McKee (1999) the reverse shock can radiate just a few percent of the explosion energy, which does not affect the dynamics of the blast wave significantly. When energy injection is included, extrapolating from the work of Luz & Berry (1999) the internal shock will catch up the external shock in $\sim$100 yr, while the SNR is still in the free expansion phase. Because of the high temperature of the postshock gas, the emission will be dominated by free-free processes, for which $\Lambda \propto T^{1/2}$. Using this relation in equation (2) would result in a cooling time that is nearly constant in that phase and always smaller than the dynamical time, thus preventing an effective cooling of the SNR.

We have performed numerical simulations to find solutions for the set of equations describing the position and velocity of the blast-wave front. The energy injected by an internal magnetar was first ignored to test the simulation engine and then varied according to the discussion and expressions given above. Qualitatively, the results for the case with energy

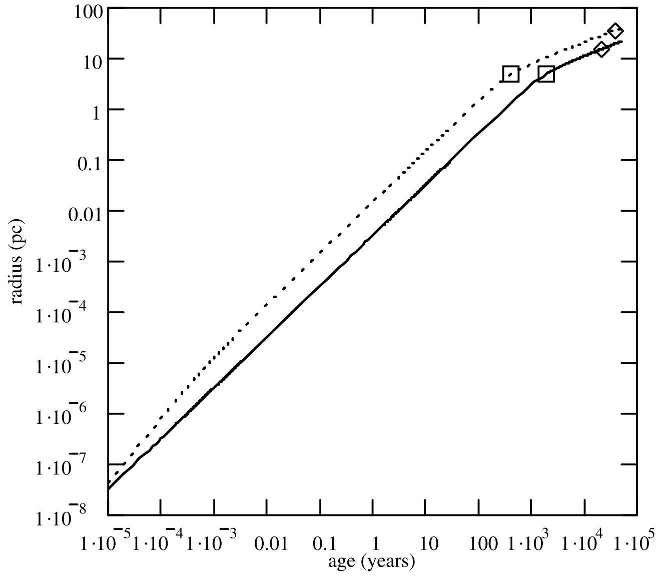

FIG. 1.—Evolution of the radius of a supernova with $M_{\rm ej} = 10\ M_\odot$, $n = 1\ {\rm cm}^{-3}$, and $E = 10^{51}$ ergs. The solid line represents the case without energy injection and the dotted line represents the case with energy injection by a magnetar with $B = 5 \times 10^{14}$ G and $P_0 = 1$ ms. Squares mark the transition to the Sedov-Taylor phase and diamonds mark the transition to the snowplow phase, beyond which the curves are no longer valid.

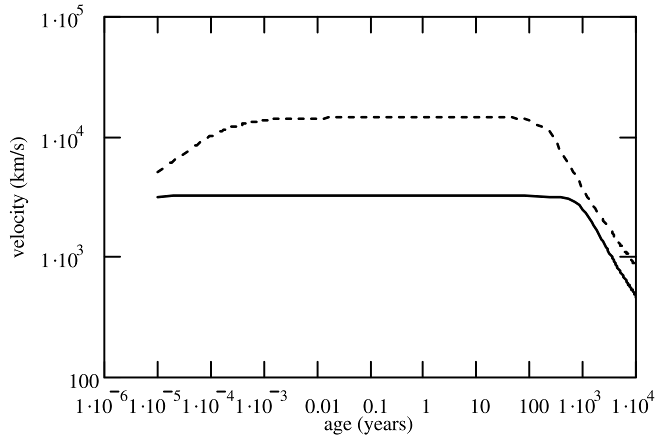

FIG. 2.—Expansion velocity evolution for a supernova with $M_{\rm ej} = 10\ M_\odot$, $n = 1\ {\rm cm}^{-3}$, and $E = 10^{51}$ ergs. The solid line represents the case without energy injection and the dotted line represents the case with energy injection by a magnetar with $B = 5 \times 10^{14}$ G and $P_0 = 1$ ms.

injection are similar to the ones in the case without energy injection, albeit with explosion energy set to the initial kinetic energy plus injected energy (from the internal magnetar). The final results are nearly identical because of the extremely short time needed for the injected energy to overcome the initial kinetic energy. Our numerical results differ from the expressions shown in equations (9)–(13) in only one point, namely, our values for R and $\dot{R}$ in the free expansion phase differ from Truelove & McKee (1999) as if the energy were reduced by a factor of 1.14. This factor comes from the approximation adopted in § 2, specifically the difference in ξ values in equation (8). This factor must multiply the energy if our values are to be compared to Truelove & McKee (1999). Throughout this work, however, we adopt our own results without corrections, because the discrepancy is not large, and certainly it does not affect our considerations.

The numerical results can be appreciated in Figures 1 and 2. In Figure 1 we show the evolution of $R(t)$ for both cases (with and without energy injection). In Figure 2 we show the evolution of $\dot{R}(t)$ for both cases. We can now proceed to analyze the proposed associations within the magnetar-driven supernova hypothesis, as expected from dynamo considerations.

4. ANALYSIS OF PROPOSED ASSOCIATIONS

The proposed associations between would-be magnetars and SNRs are shown in Tables 1 and 2. We analyze them without considering the injection of energy by a magnetar first. This situation is shown in Figure 3, where we show the range of radii and ages associated with each SNR, the characteristic ages of the magnetars, and the radius evolution for two different cases (*thick solid lines*): a "low density/low mass" evolution scenario, in which $M_{\rm ej} = 8\ M_\odot$ and $n = 0.01\ {\rm cm}^{-3}$, and a "high density/high mass" evolution scenario, with $M_{\rm ej} = 30\ M_\odot$ and $n = 10\ {\rm cm}^{-3}$. For both extreme cases, the explosion energy is held fixed to 10^{51} ergs. The position of a given SNR should be between these two extremes, unless quite different explosion energies are considered. Some SNRs have no reliable age estimate (Marsden et al. 2001); for those cases we adopted an arbitrary range of 0.2–30 kyr, which is the possible range for Galactic SNRs.

An inspection of Figure 3 shows that for several associations, SNR and magnetar ages are not compatible and some magnetar ages are not compatible with the SNR expansion in environments with standard values for n. This has been used to justify the model of AXPs and SGRs being born in regions of higher density than radio pulsars (Marsden et al. 2001), or to dismiss the association altogether.

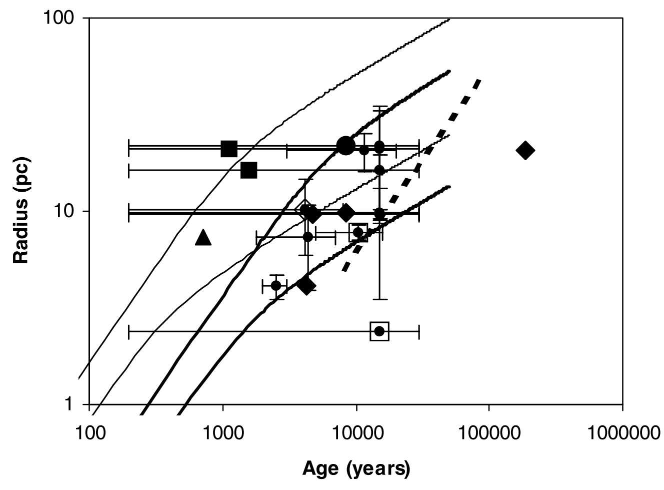

FIG. 3.—Comparison of age and radius ranges of the proposed associations with the models constructed with and without energy injection. The thick solid lines represent the evolution of the radius for the two standard extreme cases (low density/low mass and high density/high mass scenarios, see text). The thin solid lines represent the evolution with energy injection for the two extreme cases. The thick dashed line is the approximate end of the Sedov-Taylor phase for the model without energy injection. SGRs (*filled squares*) and AXPs (*filled diamonds*) with estimated $\dot{P}$ values are placed according to the median of their characteristic age ranges and the median of the radius ranges of the associated SNRs. SGRs (*open squares*) and AXPs (*open diamonds*) without estimated ages are placed according to the median values of their associated SNRs. SNRs are small dots placed at the median values of the ranges (shown as error bars). For SNRs with unreliable ages, we assumed an arbitrary range 0.2–30 kyr. The new association proposed for AXP 1709−4009 is marked as a filled circle. PSR J1846−0258 is marked as a triangle.

Varying the ejected mass value will displace those curves diagonally (in free expansion phase only) but will not meaningfully affect the position of the curves in the Sedov-Taylor phase. In any case, quite unreasonable values of both M_{ej} and n should be invoked to maintain the validity of some associations. The other simple way to displace the curves (in both phases) is by changing explosion energy.

In Figure 3 we also show the curves including the energy injection by a magnetar with $B = 5\times10^{14}$ G and $P_0 = 1$ ms. The displacement of curves helps to attribute lower values to the density than before to all associations, and it makes it possible to ''save'' some of the otherwise untenable associations. Thus, the injection of energy could be behind the age discrepancy, as SNRs truly associated with magnetars would have expanded faster than expected. In other words, an SNR that has been born with an internal magnetar will seem older than it actually is. The actual relation between the true age (t_t) and the apparent one (t_a) can be obtained from equation (9), for the Sedov-Taylor phase, considered for both the standard energy value (E_a) and the one including energy injected (E_t), as

$$t_t = \sqrt{\frac{E_a}{E_t}}\,t_a. \tag{23}$$

Typical figures would be in the range $t_t \sim 0.2{-}0.3t_a$.

Likewise, other timescales will be shifted. The transition from the free expansion to the Sedov-Taylor phase will occur sooner, because of higher initial velocities. The transition to the snowplow phase will occur at 36 kyr after the supernova explosion (see eq. [13]), using $E_t = 20E_a$. Although the precise numbers are not important, this is just meant to show that an SNR with energy injection could be visible for more time than an ordinary SNR, while at the same time appearing to be younger. The time a neutron star will take to catch up its SNR (t_{cross}) before the transition to the snowplow phase (while it is still visible) can be roughly estimated from equation (9) and the neutron star (assumed to be constant) velocity $v = 10^2 v_2$ km s^{-1}, resulting in

$$t_{cross} = 6.5\times10^5\ \mathrm{yr}\ v_2^{-5/3}E_{51}^{1/3}n_1^{-1/3}, \tag{24}$$

where we have ignored the small negative term in equation (9) for the sake of simplicity. From this last equation it can be seen that only a very fast magnetar can catch up its SNR shell. For example, if $v_2 = 10$, as suggested for some associations, and $E_{51} = 20$, then $t_{cross} \simeq 38$ kyr. Of course, the proximity of the neutron star to the SNR shell can induce a ''re-energization'' of the shell, extending the visibility of the SNR to later ages (see Shull et al. 1989), but we do not address this possibility here.

Writing the distance of the neutron star to the center of the SNR as $r(t) = \beta(t)R(t)$, the projected quantity β becomes an observable parameter. Inverting equation (24) and inserting β, we can find an expression for the minimal neutron star velocity needed to reach a relative displacement β at age t_4,

$$v = 1.2\times10^3\ \mathrm{km\ s^{-1}}\beta t_4^{-3/5}E_{51}^{1/5}n_1^{-1/3}. \tag{25}$$

If the neutron star velocity happens to be transverse to the line of sight, then equation (25) gives the actual velocity of the neutron star.

4.1. *Analysis of Magnetar Candidates with Known* $\dot{P}$

We analyze the proposed associations taking into account the energy injected by the internal magnetar, addressing the plausibility of the association, and the nature of the compact object. Similar studies have been published by several authors (Mereghetti 1999; Marsden et al. 2001; Gaensler et al. 2001), based mainly on position and velocity considerations. Those works have disregarded magnetar characteristic ages in favor of SNR estimated ages. However, their conclusions are not always in agreement. According to our above discussion, we shall in turn disregard the *ages* of the SNRs, because if they were born with an internal magnetar their ages are overestimated according to equation (23).

The novel feature of our approach is just the fact that if the AXPs and SGRs are indeed magnetars, following the model of Duncan & Thompson (1992) for magnetar formation (which is based on a dynamo action to make possible the growth of the magnetic field beyond ordinary pulsar range) they will inject sufficient energy into their SNRs to affect their expansions. The remnants reach a larger size in less time than considered by previous authors. Although our simulation engine is probably too crude to provide a reliable estimate of the time after which the injection of energy becomes less efficient, more sophisticated simulations have not been tested to pin down this number. To be safe, it is enough to assume injection times below 1 month because they certainly provide an optimal coupling between injected energy and the SNR kinetic energy. Since the main effect of the injected energy is to add to the initial kinetic energy of the SNR, any pulsar that can inject most of its rotational energy in less than 1 month will provide an SNR evolution as if the kinetic energy were the initial plus the injected energy. The injection time for the smallest B of the AXP/SGR sample (AXP 2259+586) is lower than 5 days, considering $P_0 \sim 1$ ms. Even initial periods as large as 3 ms would provide injection times close to or within the 1 month figure, which reassures one as to the effectiveness of the coupling. On the other hand, ordinary pulsars with lower magnetic fields would produce injection times greater than or equal to decades, which are not as dramatic.

For simplicity, and the lack of a good estimate of the actual initial periods of AXPs and SGRs, we assume all of them to be born with $P_0 = 1$ ms, which in turn determines that the injected energy ($\sim2\times10^{52}$ ergs at $t \gg \tau_0$) will be essentially the same, regardless of the actual value of B (from eq. [19]), as long as $\tau_0 \lesssim$ 1 yr. Because of this fact, we only consider two extreme situations to analyze the associations: the low density/low mass evolution scenario, where $M_{ej} = 8\,M_\odot$ and $n = 0.01$ cm^{-3}, and the high density/high mass evolution scenario, with $M_{ej} = 30\,M_\odot$ and $n = 10$ cm^{-3}. The actual situation for each case should be bracketed between these values. The results can be appreciated in Figure 4, in which the range of radius and characteristic ages values for each association is explicitly shown. Our assessment of each association is as follows:

SGR 1806−20/G10.0−0.3.—The probability of alignment by chance is ~0.5% (Marsden et al. 2001). G10.0−0.3 was considered to not be an SNR at all (Gaensler et al. 2001; Chakrabarty et al. 2001), although Green (1998) lists it in his catalog. A cluster of stars is close to the line of sight to this association (Fuchs et al. 1999), so either the SGR or the SNR may be physically related to it. In our model, considering the characteristic age of SGR 1806−20 as the true age of the association, it can be seen that the entire range of radius values lie between the two extreme scenarios (Fig. 4), with an indication of a mid- to low-density interstellar medium. The magnetar transverse velocity implied is high, 4000–6500 km s^{-1}, if $\beta = 0.5$ (Kulkarni et al. 1994). However, if $\beta \sim 0$ (Chakrabarty et al. 2001) the velocity cannot be inferred. G10.0−0.3 would be entering the Sedov-Taylor phase. Dropping altogether the

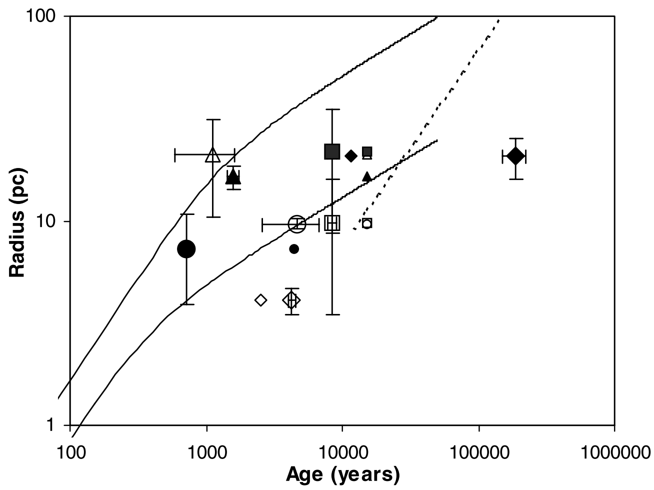

FIG. 4.—Comparison of the radii and ages of the proposed associations with the expansion model with energy injection for the objects with an estimated $\dot{P}$. The solid lines represent the evolution according to our model. Large symbols are placed according to the median of characteristic ages and radii ranges. Small symbols are placed according to the median of estimated SNR ages and radii. Error bars indicate ranges of characteristic ages and SNR radii. The dotted line is the approximate end of the Sedov-Taylor phase (including energy injection). Filled symbols represent likely associations and open symbols represent the unlikely ones (see text). The symbols are: open triangle (SGR 1900+14), filled triangles (SGR 1806−20), open circles (AXP 1048−5937), open squares (AXP 1709−4009), open diamonds (AXP 1841−045), filled diamonds (AXP 2259+586), filled squares (AXP 1709−4009, new association), and filled circles (PSR J1846−0258).

characteristic age as a good age estimator, the association could be as old as 15 kyr, with $v \simeq 500$ km s^{-1} ($\beta = 0.5$). This association can be considered as true if SGR 1806−20 is a magnetar, $\beta \sim 0$, and G10.0−0.3 is confirmed as an SNR.

SGR 1900+14/G42.8+0.6.—The probability of random alignment is ~4% (Gaensler et al. 2001). A pulsar was recently discovered near this position (Lorimer & Xilouris 2000) that could be related to the SNR, although its characteristic age is 38 kyr. Our model would allow for the association if the true distance were on the lower $\frac{1}{3}$ of the range and the true age were on the upper $\frac{1}{2}$ of the quoted range, but the extremely high velocity implied ($\geq$8000 km s^{-1}) precludes that. Disregarding characteristic age, the range would be extended up to 6–40 kyr, depending on true distance and interstellar medium density, although the magnetar velocity would still be high (800–2000 km s^{-1}). Wang et al. (2002) suggested that SGR 1900+14 was born in the 4 B.C. supernova and that the discrepancy between characteristic age and the proposed age of 2 kyr is due to dynamical evolution with braking index $\simeq$2, but they did not offer a good reason to explain how the SNR disappeared from view in just 2 kyr. We conclude that this association is not convincing (if SGR 1900+14 is a magnetar) unless a mechanism to provide a very high velocity of the magnetar exists.

AXP 1048−5937/G287.8−0.5.—The probability of chance alignment is ~16% (Marsden et al. 2001; Gaensler et al. 2001). Data about G287.8−0.5 were considered unreliable by Gaensler et al. (2001). Nevertheless, if the SNR is confirmed as such our model indicates that it is entering the Sedov-Taylor phase, in a high-density interstellar medium. Low ages are preferred. Again, the very high velocities implied (4500–8500 km s^{-1}) argue against the association. In this case, even disregarding the characteristic age does not improve the plausibility of the association. We conclude that this association is unlikely on any grounds.

AXP J1709−4009/G346.6−0.2.—The random alignment probability is ~10% (Marsden et al. 2001) to ~30% (Gaensler et al. 2001). Our model could allow the association if the interstellar medium has high density and the true distance is in the upper 25% of the values in the range. Once more, the high velocity required for $\beta = 1.7$ (3000–2000 km s^{-1}) is in excess of the known pulsar population. Disregarding the characteristic age would only help very slightly to avoid the "velocity problem." We therefore consider this association unlikely.

AXP J1709−4009/G346.5−0.1.—Gaensler et al. (2001) suggest this association instead of the previous one. While the newly identified SNR G346.5−0.1 awaits to be confirmed, we can analyze the association in the same fashion as we have been doing. The probability of random alignment is ~10% (Gaensler et al. 2001). Our model allows the association if the true distance is in the upper 80% of the values in the range. Once more, the high velocity required for $\beta = 1.2$ (1700–4800 km s^{-1}) exceeds the typical pulsar velocity. Disregarding the characteristic age would allow the age range to go up to ~50 kyr (~800 km s^{-1}) at the transition to the snowplow phase. We consider this association more likely, requiring a mid- to high-density medium. We note that our solutions with injection are not better than the old situation (without energy injection), so the association can be true even if AXP J1709−4009 is not a magnetar.

AXP 1841−045/G27.4+0.0.—The chance alignment probability is $\simeq$0.01% (Gaensler et al. 2001). In our model, this association would require either an exceedingly low density interstellar medium or a negligible energy injection. Equivalently, the association would require $P_0 > 6$ ms, or age $\leq$600 yr (but with $v \geq 1600$ km s^{-1}). Even then, the characteristic age would have to be disregarded. Contrary to most AXPs and SGRs, the characteristic age of AXP 1841−045 is *higher* than the SNR estimated age. The association is possible only if AXP 1841−045 is not a magnetar with short P_0.

AXP 2259+586/CTB 109.—The probability of random alignment is ~0.05% (Gaensler et al. 2001). The characteristic age of AXP 2259+586 indicates that its associated SNR would be in the snowplow phase, where it is unlikely to be detected. Given the low probability of chance alignment, this age discrepancy (this is the other AXP that has a characteristic age higher than SNR age) again argues against the characteristic age as a good estimate of the true age. Removing this parameter allows for our model a wide range of possible age values, from 1 to 30 kyr. The requirement of reasonable magnetar velocity would limit ages to $\geq$6 kyr. Moreover, this range is already allowed by the expansion without energy injection. The association is probably true for both models.

4.2. *Analysis of Magnetar Candidates with Unknown* $\dot{P}$

For this subsample of objects we do not have information about their characteristic ages or magnetic field strength. Therefore, the task here is to verify whether the allowed range of ages of our model is compatible with reasonable magnetar velocities. The results are shown in Figure 5.

SGR 0526−66/N49.—The random alignment probability is ~0.7% (Gaensler et al. 2001). The allowed range of ages is 0.5–3.5 kyr, which implies very high velocities, $v > 1800$ km s^{-1}. If this association is true, then SGR 0526−66 would have to be another magnetar born with a long P_0, as suggested for AXP 1841−045.

AXP J1845−0258/G29.6+0.1.—The chance alignment probability is ~0.2% (Gaensler et al. 2001). The allowed range of ages is 1–13 kyr, which implies 3500 km s$^{-1} > v > 300$ km s^{-1}

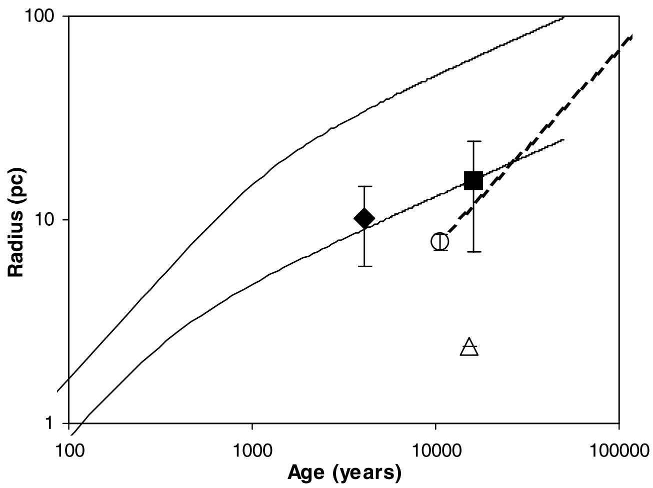

FIG. 5.—Comparison of the radii and ages of the proposed associations with the expansion model with energy injection for the objects without estimated $\dot{P}$. The solid lines represent the evolution according to our model. Symbols are placed according to the median of the estimated SNR ages and radii. Error bars indicate ranges of SNR ages and radii. The dotted line is the approximate end of the Sedov-Taylor phase (including energy injection). Filled symbols represent likely associations and open symbols represent the unlikely ones (see text). The symbols are: open triangle (SGR 1627−41), open circle (SGR 0526−66), filled diamond (AXP 1845−0258), and filled square (SGR 1627−41).

for the largest distance of the range and 0.4–1.5 kyr (3600 km $s^{-1} > v >$ 1000 km s^{-1}) for the opposite extreme of the distance range. This association can be true with or without energy injection. If AXP J1845−0258 is a magnetar, then the higher end of values for the distance and age of G29.9+0.1 are preferred.

SGR 1627−41/G337.0−0.1.—The probability of random alignment is ∼5% (Gaensler et al. 2001). The allowed age range of our model for this association (150–300 yr) implies that $v >$ 15, 000 km s^{-1}, ruling out the magnetar hypothesis unless $P_0 \geq$ 6 ms, as discussed for AXP 1841−045. Even so, the velocity problem still holds. Thus, we find the association unlikely.

SGR 1801−23/G6.4−0.1.—SGR 1801−23 is just a candidate SGR, and its position is not well determined. The distance to G6.4−0.1 is also uncertain. Given these data, it is not surprising that our model allows for wide ranges of ages. Within the high end of the distance range, ages can be 2–35 kyr (1000 km $s^{-1} > v >$ 70 km s^{-1}). In the low end of the distance range, ages allowed are 0.45–2 kyr (1500 km $s^{-1} > v >$ 350 km s^{-1}). Thus, the association is likely, and this holds even if SGR 1801−23 is not a magnetar at all. However, we remark that this result is quite dependent on the position determination.

5. DISCUSSION

We have analyzed in this work the proposed SGR-AXP/SNR associations. Previous analyses of the associations arrived at different results. According to Marsden et al. (2001), all associations can be considered likely. Gaensler et al. (2001) contend that only AXP J1845−0258/G29.6+0.1, AXP 1841−045/G27.4+0.0, and AXP 2259+586/CTB 109 could be valid. Ankay et al. (2001) considered SGR 1806−20/G10.0−0.3 and SGR 0526−66/N 49 as plausible, in addition to the three already mentioned by Gaensler et al. (2001).

Within dynamo-generated magnetic field scenarios, it is expected that if magnetars exist and are born in supernova explosions, they can inject enough energy to enhance the expansion of their associated SNRs. If AXPs and SGRs are indeed magnetars, their associated SNRs should appear older than the ages derived from standard expansion models. We analyzed all the proposed associations and have come to the following results:

1. If the characteristic age of the neutron star is regarded as the true age of the association, then SGRs and AXPs may not be magnetars at all (or the model of Duncan & Thompson 1992 is not correct regarding the magnetar origin), since only one case (SGR 1806−20/G10.0−0.3) has shown good agreement within the model and even the true nature of G10.0−0.3 was put in doubt.
2. If characteristic ages are in turn ignored (see Harding et al. 1999; Gaensler et al. 2001; Marsden et al. 2001; Mereghetti et al. 2002, among others), then two SGRs (of five) and three AXPs (of five with proposed associations) seem to be associated within our model, so there is a ~50% general agreement. It should be noted that two associations that agreed within our model (AXP J1845−0258/G29.6+0.1 and AXP 2259+586/CTB 109) were already believed to be true by previous works (Gaensler et al. 2001; Marsden et al. 2001; Ankay et al. 2001), meaning that uncertainties in distance are large enough to allow for both possible scenarios (standard and with energy injection).
3. AXP 1841−045/G27.4+0.0 and SGR 0526−66/N49 can only be considered as true associations if the magnetars were born with $P_0 \geq 6$ ms because in that way the injected energy would be insufficient to directly affect SNR expansion.

These results from our model are tied to the dynamical evolution of magnetars. While we have assumed for simplicity the standard magnetic dipole braking with braking index equal to 3, several proposals have been put forward that argue for different braking models: fossil or fallback accretion disks (Marsden et al. 2001; Chaterjee et al. 2000; van Paradijs et al. 1995), episodes of relativistic wind emission (Harding et al. 1999), a different constant braking index (Wang et al. 2002), and magnetic field decrease (Duncan & Thompson 1992; Colpi et al. 2000). Since the measurements of braking indices for five young pulsars revealed that all but one have significant departures from the canonical value, it should not come as a surprise that magnetars do not follow the standard model of spin-down. The main effects of alternative models are to change the estimates of magnetic field strength and spin-down age. Nonetheless, the overall influence on our model would be very small, since τ_0 can be increased up to 1 month without appreciable modifications in the results and rotational energy is not dependent on spin-down models. That is why we feel justified in leaving item 1 above out of the discussion. Spin-down models that allow for ages significantly smaller or larger than the conventional characteristic age are *required* to explain certain associations, if they are true ones, either within our model or considering the standard scenario.

Supposing that our results from item 2 above represent the actual situation, we could ask, where are the SNRs that are associated with the other SGRs and AXPs? The answer could be that they have not yet been detected, since they generally lie within regions of coexistence of SNRs, H II regions, and other types of objects (variable stars, young stars, molecular clouds, etc.). Including AXP 0142+614 and AXP J0720−312 in the sample, we would have slightly less than 50% of the true associations identified. On the other hand, few of the latter can be considered firm, because of several other factors. For example, the angular size of SNRs are often quoted without uncertainties, which is certainly an understatement given the difficulty of recognizing an SNR and assessing its shape and size.

It is worth remarking that a similar analysis, ignoring the effects of the injection of energy and not considering conventional characteristic ages, can be performed to analyze the same associations and will result in different age and velocity ranges than those obtained here. Indeed, even associations considered unlikely by us may turn out to be likely. This is because the standard energy SNRs would take more time to reach the observed sizes, allowing higher ages for the associations than in our model and thus producing lower velocities than those we found. Nevertheless, that scenario requires nonstandard spin-down and negligible injection of energy by the neutron star, no matter whether it is a magnetar or not.

It is interesting to check that the most important factor in deciding on the likelihood of associations is in fact β. As β increases, associations are considered increasingly unlikely. However, Gvaramadze (2002) points to the possibility that an SNR expanding in a region with anisotropic interstellar medium densities will be distorted and/or expands faster in one or more directions. This means that the geometrical center of the SNR can be displaced from the actual explosion site. Although it is difficult to take this effect into account quantitatively, one should be cautious not to dismiss it entirely. It is possible that one or more of the proposed associations are affected by this effect, which can increase or decrease β randomly. This effect will be more accentuated for SNRs that have been expanding in low-density regions (“bubbles”) amidst high-density “walls,” and increases with age. The first possibility is the case for the most massive stars, which do not live long enough to move far away from the sites where they were born, while their stellar winds create a low-density cavity. The second possibility means we must expect β estimates to be a little more scattered as age (and thus radius) increases.

Besides studying associations including AXPs and SGRs, we also examined the association PSR J1846−0258/G29.7−0.3 (Gotthelf et al. 2000; Mereghetti et al. 2002), which also suffers from the age problem, since it has the smallest characteristic age known among pulsars (723 yr), while the SNR age was estimated as at least 1800 yr. The magnetic field of this pulsar is $\sim 5 \times 10^{13}$ G, considering magnetic braking spin-down, slightly above the quantum critical field and thus marginally qualifying as a magnetar (at least for the purposes of the present work). It was not detected at radio wavelengths, but only in X-rays. The association is considered very likely, since the pulsar is located at the geometrical center of the SNR, coincident with a radio/X-ray nebula, which is probably powered by the pulsar. Proceeding as in the cases analyzed in the previous sections, we see from Figure 4 that the range of radii quoted in the literature is nearly coincident with the range allowed by our two extreme cases, considering the characteristic age. This way, there are no preferences for high or low densities or ejected mass. If, as before, we ignore altogether the characteristic age, it is found that the allowed age range is 250–700 yr in the case of high $M_{\rm ej}$ or n, and 650–6000 yr in the case of low $M_{\rm ej}$ or n. Intermediate values of the parameters $M_{\rm ej}$ and n would provide intermediate ranges of ages. The placement of the neutron star at the geometrical center of the SNR implies low velocities or alignment between the velocity vector and the line of sight. It is important to note that Mereghetti et al. (2002) find a braking index ~ 1.9 and an age ~ 1700 yr for this pulsar, and in this case the magnetic field can be below the quantum critical one. Nonetheless, we consider this association to represent further evidence against the consideration of the characteristic age (without braking index information) as a good age estimate.

We have left for a future work (M. P. Allen & J. E. Horvath 2004, in preparation) the study of an alternative origin scenario for magnetars, the collapse of a white dwarf star induced by accretion or the merging of a binary system. Simulations for this scenario (Fryer et al. 1999) reveal that $\sim 0.1\ M_\odot$ should be ejected, with an explosion energy of 10^{50} ergs, implying higher initial velocities from the very beginning. It remains to be seen to what extent the associations can be attributed to this rare type of events.

Finally, we would like to point out that magnetars lose very little rotational energy through gravitational waves when compared to typical pulsars, unless r-modes can play an important role, which does not seem to be the case according to Watts & Andersson (2002) and Rezzolla et al. (2001). Statistical studies on pulsar gravitational wave detectability, such as that performed by Regimbau & de Freitas Pacheco (2000), shall not be affected by this consideration, at least for the current wide-band detectors (LIGO/VIRGO), because of the scarcity of magnetars.

We are grateful to G. Medina-Tanco for discussions about SNR blast-wave physics and to A. Watts for directing us to recent developments on r-mode instability. This work was supported by FAPESP (São Paulo State, Brazil) and CNPq (Brazil).

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