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PROBING THE SPACETIME AROUND SAGITTARIUS A* WITH RADIO PULSARS

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ABSTRACT

The supermassive black hole at the Galactic center harbors a bound cluster of massive stars that should leave neutron star remnants. Extrapolating from the available data, we estimate that $\sim$1000 radio pulsars may currently orbit Sgr A* with periods of $\lesssim$100 yr. Optimistically, 1–10 of the most luminous of these pulsars may be detectable with current telescopes in periodicity searches at frequencies near 10 GHz, where the effects of interstellar scattering are alleviated. Long-term timing observations of such a pulsar would clearly reveal its Keplerian motion and possibly show the effects of relativistic gravity. We briefly discuss how pulsar timing can be used to study the dynamical and interstellar environment of the central black hole and speculate on the prospects for astrometric observations of an orbiting pulsar.

Subject headings: black hole physics — Galaxy: center — pulsars: general

1. INTRODUCTION

Ten years of near-infrared observations of the Galactic center have revealed the proper motions of nearly two dozen stars within 0$''$.5 of the compact radio source Sgr A*. Significant astrometric accelerations measured for eight members of the Sgr A* stellar cluster point to a common center of gravity coincident with the position of Sgr A* and imply a central mass of $(3-4)\times10^6\ M_\odot$ (Ghez et al. 2003b; Schödel et al. 2003). Stars S0-2 and S0-16 have the most compact orbits yet identified, with respective periods of $\simeq$15 and $\simeq$30 yr, eccentricities of $\simeq$0.88 and $\simeq$0.95, and comparable pericenter distances of $\simeq$100 AU (Schödel et al. 2002; Ghez et al. 2003b; Eisenhauer et al. 2003). If the central mass is confined within 100 AU, the implied density is $\gtrsim10^{16}\ M_\odot\ \mathrm{pc}^{-3}$, which essentially rules out existing models alternative to the hypothesis that the central object is a supermassive black hole (BH; Maoz 1998; Ghez et al. 2003b; Schödel et al. 2003).

Evidence from the near-infrared spectrum of S0-2 (Ghez et al. 2003a) and the integrated spectrum within $\simeq$0$''$.5 of Sgr A* (Genzel et al. 1997; Eckart et al. 1999; Figer et al. 2000; Gezari et al. 2002) suggest that the observed Sgr A* stellar cluster is largely composed of luminous ($\sim10^4\ L_\odot$), early-type (O9 to B0) stars. If these stars are near the main sequence, their masses are 10–20 $M_\odot$. How these stars came to reside so near the supermassive BH remains a puzzle; for discussions and references, see Genzel et al. (2003) and Ghez et al. (2003b). Nevertheless, the existence of a cluster of massive stars tightly bound to Sgr A* has important implications.

Stars of mass 10–20 $M_\odot$ have nuclear lifetimes of $\sim10^7$ yr and leave neutron star (NS) remnants. Therefore, we expect a significant number of NSs to be bound to Sgr A* in orbits similar to those of the observed cluster stars, as well as in more compact orbits. Source confusion has so far inhibited the discovery of stars with orbital periods of $\lesssim$10 yr about Sgr A* (Genzel et al. 2003; Ghez et al. 2003b), although we anticipate that massive stars and NSs populate this region. The most exciting possibility is that some of the NSs orbiting Sgr A* may be detectable radio pulsars, an idea first considered in the prescient article by Paczyński & Trimble (1979). In § 2, we estimate the total number of *normal* radio pulsars (i.e., those with surface magnetic field strengths of $\sim10^{11}-10^{13}$ G) that may currently orbit the central BH with periods of $\lesssim$100 yr.

Radio-wave scattering in the interstellar plasma poses the largest obstacle to discovering pulsars near Sgr A*, where the column density of free electrons is very high. At observing frequencies of $\simeq$1 GHz, pulsed emission from this vicinity suffers severe temporal broadening, prohibiting the detection of pulsars as periodic sources (Cordes & Lazio 1997). Relatively high frequencies of $\gtrsim$10 GHz are required to alleviate the effects of scattering. These issues are addressed in § 3, in which we estimate the number of pulsars orbiting Sgr A* that may be detectable with current telescopes.

The Keplerian motion of a pulsar orbiting Sgr A* would be clearly apparent in its long-term timing properties. Relativistic gravity may introduce measurable deviations from the best-fit Keplerian timing solution, depending on the orbital parameters and timing precision. Various arrival-time delays and secular effects are quantified in § 4. In § 5 we consider what pulsar timing can teach us about the accretion flow onto the Galactic BH and the stellar dynamical environment and investigate the possibility of astrometrically monitoring an orbiting pulsar.

2. RADIO PULSARS ORBITING SGR A*

The observed Sgr A* stellar cluster occupies the central $\simeq$4000 AU about the BH. Although the observational census is incomplete, this volume likely contains at least several dozen massive stars. However, it is not the present population of massive stars but rather their predecessors that are the progenitors of radio pulsars orbiting Sgr A*. Since the origin of the observed cluster stars is unknown, we can only speculate on the history of the population of massive stars in this region. It is plausible that cluster stars are steadily or episodically replenished as they evolve and leave NS remnants. This might be the case if the mechanism that feeds stars into the central $\simeq$4000 AU is linked to significant star formation activity on larger scales. Observational evidence suggests that the central $\sim$200 pc of the Galaxy is a region of past and current star formation, with an average massive-star formation rate as large as $\sim10^{-3}\ \mathrm{yr}^{-1}$, or $\sim$10% of the rate in the entire Galaxy (Mezger et al. 1999; Launhardt et al. 2002).

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A plausible initial guess is that there are roughly as many pulsars within 0″.5 of Sgr A* as there are massive stars. Thus, perhaps several tens of radio pulsars with ages of $\lesssim 10^7$ yr orbit near the black hole. A simple extension of this argument provides an estimate of the total number of active radio pulsars orbiting Sgr A*, including those much too faint to be detected by current telescopes. Assume that over a time of $\gtrsim 10^8$ yr an average number of $\sim$10–100 NS progenitors orbit Sgr A* with semimajor axes of $\lesssim$4000 AU and periods of $P_{\rm orb} \lesssim 100$ yr. A stellar lifetime of $\sim 10^7$ yr then implies a NS birthrate of $\sim 10^{-6}$ to 10^{-5} yr^{-1}. We further suppose that a large fraction of NSs turn on as radio pulsars shortly after birth. Radio emission terminates when a pulsar crosses the "death line" in the $\log P_p$–$\log \dot{P}_p$ plane, where P_p is the pulse period (e.g., Rudak & Ritter 1994 and references therein). A coarse inspection of pulsar statistics in the $\log P_p$–$\log \dot{P}_p$ plane shows that a median terminal age for normal radio pulsars is probably $\sim 10^8$ yr, depending on the model for the death line and neglecting magnetic-field decay (e.g., Bhattacharya et al. 1992; Rudak & Ritter 1994; Tauris & Konar 2001). Therefore, we predict that $\sim$100–1000 radio pulsars presently orbit Sgr A* with $P_{\rm orb} \lesssim$ 100 yr and ages of $\lesssim 10^8$ yr.

Typical NS "kick" speeds of $v_k \simeq$ 100–300 km s^{-1} are inferred from the proper motions of $\sim$100 isolated pulsars in the Galactic disk (e.g., Hansen & Phinney 1997; Arzoumanian et al. 2002). Since the stars we are considering have orbital speeds of $v_{\rm orb} \gtrsim 1000$ km s^{-1}, an impulsive NS kick will usually cause only a small fractional change ($\sim v_k/v_{\rm orb}$) in the orbital parameters. Therefore, the distributions of pulsar orbital parameters should be similar to those of their progenitors. For a stellar number density $n(r) \propto r^{-3(1+q)/2}$ about Sgr A*, the differential period distribution is $p(P_{\rm orb}) \propto P_{\rm orb}^{-q}$ if the velocity distribution is isotropic (e.g., Schödel et al. 2003). Genzel et al. (2003) find that $q \simeq 0$ at $\lesssim 10''$ from Sgr A*, so that $p(P_{\rm orb})$ is approximately flat. If there are $\sim$1000 radio pulsars with $P_{\rm orb}$ uniformly distributed over, e.g., 1–100 yr, then $\sim$100 pulsars may have $P_{\rm orb} \lesssim 10$ yr.

3. PULSAR DETECTION

Scattering of radio waves by fluctuations in the free-electron density causes angular broadening of radio images and temporal smearing of pulsed emission. Toward the Galactic center, observed scattering diameters of $\simeq 1''(\nu/1\ {\rm GHz})^{-2}$ for Sgr A* and nearby OH masers imply a scattering timescale of $\sim(300\ {\rm s})(\nu/1\ {\rm GHz})^{-4}$ (Lazio & Cordes 1998a, 1998b), where ν is the observing frequency. Pulsars with $P_p \lesssim 1$ s would thus be undetectable as periodic sources for $\nu \simeq 1$ GHz. At $\nu \gtrsim$ 5 GHz, the scattering time is $\lesssim P_p$, but the flux density is reduced according to the declining power-law spectrum, $S_\nu \propto \nu^{-\alpha}$, followed by most pulsars. Cordes & Lazio (1997; see also Kramer et al. 2000) find that near Sgr A* the pulsed flux is maximized at $\nu \simeq 10$ GHz for $P_p \simeq 1$ s, with a weak dependence on P_p and α. At higher frequencies, the pulsed fraction of the flux is $\simeq$1, and the pulse duty cycle, ϵ, approaches its intrinsic value; $\epsilon \simeq 0.05$ is typical for normal pulsars.

The minimum detectable flux density is

$$S_{\rm min} \simeq C S_n [\epsilon/(N_p \Delta\nu t_{\rm int})]^{1/2}$$

(e.g., Dewey et al. 1985), where $C \simeq 10$ is the signal-to-noise ratio threshold, S_n is the noise flux from the telescope and sky, $\Delta\nu$ is the bandwidth, $N_p = 2$ is the number of polarizations, and $t_{\rm int}$ is the integration time. For the 100 m Robert C. Byrd Green Bank Telescope[3] operating at $\nu \gtrsim 10$ GHz, the noise flux is $S_n \simeq$ 25–50 Jy toward the Galactic center, and $\Delta\nu \simeq$ 1 GHz. We then find that

$$S_{\rm min} \simeq (20\text{–}40\ \mu{\rm Jy}) \left(\frac{\epsilon}{0.05}\right)^{1/2} \left(\frac{t_{\rm int}}{1\ {\rm hr}}\right)^{-1/2}. \qquad (1)$$

From Green Bank, the Galactic center can be observed for up to $\simeq$8 hr day^{-1} (S. Ransom 2004, private communication), so that sensitivities of $S_{\rm min} \simeq 10\ \mu$Jy may be possible.

Observed Galactic pulsars have intrinsic 400 MHz luminosities of 1–10^4 mJy kpc^2, with a cumulative distribution $f(>L_{400}) \simeq L_{400}^{-1}$ (e.g., Lyne et al. 1985, 1998; Taylor et al. 1993), where L_{400} is in mJy kpc^2 and is assumed to be $\ll 10^4$. Pulsar luminosities are typically defined by $L_\nu = D^2 S_\nu$ (e.g., Taylor & Manchester 1977), where D is the distance. For a pulsar with $S_\nu \propto \nu^{-\alpha}$ at the 8 kpc distance of Sgr A* (Eisenhauer et al. 2003), we have $L_{400} \simeq 64 S_\nu x^\alpha$ mJy kpc^2, where $x = \nu/0.4$ GHz and S_ν is in mJy. The distribution, $p(\alpha)$, of measured spectral slopes is roughly Gaussian over $\alpha =$ 0–4.0, with a peak at $\alpha = 1.5$–2 (Lorimer et al. 1995; Maron et al. 2000). Utilizing the above pulsar statistics, along with the steady state assumption of the last section, we now proceed to estimate the detectable fraction of pulsars near Sgr A*; our approach is quite similar to that of Cordes & Lazio (1997).

At high frequencies, the detection of shallow-spectrum pulsars is favored (e.g., Johnston et al. 1992; Wex et al. 1996). It is encouraging that $\simeq$10% of pulsars with a measured spectrum have $\alpha =$ 0–1, although $p(\alpha < 1)$ is poorly constrained. We can crudely estimate the fraction $f(>S_{\rm min})$ with flux densities greater than $S_{\rm min}$ by restricting to the range $\alpha =$ 0–1 and assuming a flat distribution, $p(\alpha < 1) = 0.1$. After evaluating a simple integral, we find that

$$f(>S_{\rm min}) \simeq 5\% \left(\frac{S_{\rm min}}{10\ \mu{\rm Jy}}\right)^{-1} \left(\frac{\ln x}{\ln 25}\right)^{-1} \qquad (2)$$

when $x \gg 1$. If $p(\alpha < 1)$ is taken to be a linear or quadratic function that vanishes at $\alpha = 0$, each of which is consistent with the data, then $f(>S_{\rm min}) \simeq$ 2%–3% at 10 GHz and scales with frequency approximately as $(\ln x)^{-(n+1)}$, where $n = 1$ (linear) or 2 (quadratic). If $\simeq$20% of pulsars are beamed toward the Earth (e.g., Lyne & Manchester 1988), we conclude that $\sim$1% of the active pulsars near Sgr A*, or as many as 1–10 (see § 2), may be detectable with current technology. These pulsars would most likely have been born in the past several million years. The planned Square Kilometer Array[4] (SKA) will easily achieve sensitivities of $\lesssim 1\ \mu$Jy and could perhaps ultimately find $\gtrsim$100 orbiting pulsars.

4. PULSAR TIMING AND RELATIVITY

The dynamics of a pulsar orbiting Sgr A* are revealed through analysis of the pulse arrival times. The Newtonian dynamical signature of the supermassive BH should be clearly evident in a small segment of the orbit via the acceleration of the pulsar. After a full orbit, a best-fit Keplerian timing model is obtained. Residuals of the Keplerian solution contain further dynamical information, including contributions from relativistic gravity. In this section, we quantify various arrival-time delays and secular effects and crudely assess their measurability.

[3] See http://www.gb.nrao.edu/GBT/.
[4] See http://www.skatelescope.org/.

TABLE 1
Pulse Arrival-Time Delays

Delay[a]	Amplitude	Width	References
Roemer[b]	$\sim(1\ \mathrm{day})M_{6.5}^{1/3}P_1^{2/3}\sin i$	$\sim(1\ \mathrm{yr})P_1$	1
Einstein[c]	$\sim(1\ \mathrm{hr})M_{6.5}^{2/3}P_1^{1/3}e$	$\sim(1\ \mathrm{yr})P_1$	1
First-order Shapiro[d]	$\sim(30\ \mathrm{s})M_{6.5}\lvert\ln[(1-e)(1-\sin i)]\rvert$	$\sim(1\ \mathrm{yr})P_1(1-e)^{3/2}(\cos i)^{1/2}$	1
Second-order Shapiro[e]	$\sim(0.1\ \mathrm{s})M_{6.5}^{5/3}P_1^{-2/3}(1-e)^{-1}/\cos i$	$\sim(1\ \mathrm{yr})P_1(1-e)^{3/2}\cos i$	2, 3
Frame dragging[f]	$\sim(0.1\ \mathrm{s})M_{6.5}^{5/3}P_1^{-2/3}(1-e)^{-1}\chi/\cos i$	$\sim(1\ \mathrm{yr})P_1(1-e)^{3/2}\chi\cos i$	2, 3, 4, 5

[a] The dimensionless variables used are $M_{6.5} = M_{\rm BH}/10^{6.5}\ M_\odot$ and $P_1 = P_{\rm orb}/1$ yr. For simplicity, we have adopted $\omega = 90°$ in estimating the amplitudes and widths.
[b] Light travel time across the orbit. The Keplerian orbit is evident in the Roemer delay.
[c] Combined effect of time dilation and the gravitational redshift.
[d] Lowest order relativistic propagation delay in the gravitational field of a point mass.
[e] Next highest order contribution to the propagation delay that is independent of the BH spin.
[f] Contribution to the net propagation delay due to the BH spin, in the special case in which the spin direction is parallel to the orbital angular momentum of the pulsar. Here $0 < \chi < 1$ is the dimensionless spin parameter, where $\chi = 1$ corresponds to an extreme Kerr BH.

References.—(1) Damour & Taylor 1992; (2) Dymnikova 1986; (3) Goicoechea et al. 1992; (4) Laguna & Wolszczan 1997; (5) Wex & Kopeikin 1999.

4.1. *Time Delays and Secular Effects*

Each pulse arrival time, t, at the solar system barycenter is related to the pulsar proper time, t', by $t - t_0 = t' + \sum_i \Delta_i$, where t_0 is a reference time and the Δ_i are variable delays due to the Keplerian motion and relativity. The delays are functions of t' and the following Keplerian parameters: (1) BH mass $M_{\rm BH}$, (2) orbital period $P_{\rm orb}$, (3) eccentricity e, (4) inclination i, and (5) longitude of pericenter ω. When frame-dragging is considered, we must also specify the magnitude and direction of the BH spin. Each delay may be characterized by an amplitude, the difference between the maximum and minimum values of Δ_i, and a width, the timescale over which the delay shows the largest variation.

In Table 1 we quantify the five delays with the largest expected amplitudes in the special case in which $\omega = 90°$, so that superior conjunction—when the pulsar is farthest behind the BH—coincides with pericenter passage. This simplifies the analysis and illustrates roughly the dependence on eccentricity. Dimensionless variables used in Table 1 are $M_{6.5} = M_{\rm BH}/10^{6.5}\ M_\odot$ and $P_1 = P_{\rm orb}/1$ yr. Not included in Table 1 are the delays due to aberration of the beamed pulsar emission (e.g., Smarr & Blandford 1976) or the bending of light rays in the gravitational field of the BH (Doroshenko & Kopeikin 1995; Wex & Kopeikin 1999). Each of these delays has an amplitude of $\lesssim 1$ ms for a wide range of $P_{\rm orb}$, e, and i.

Several secular processes cause changes in the orbital elements over long timescales. Emission of gravitational radiation by an orbiting pulsar causes the semimajor axis to shrink on a timescale of $\sim(10^{13}\ \mathrm{yr})M_{6.5}^{-2/3}P_1^{8/3}(1-e^2)^{7/2}$ (e.g., Taylor & Weisberg 1989), which is typically too long to be of interest. The geodetic precession rate of the pulsar spin axis is $\simeq 0\overset{\circ}{.}1 M_{6.5}^{2/3}P_1^{-2/3}(1-e^2)^{-1}$ orbit^{-1} (e.g., Weisberg et al. 1989). This is evident as a long-term change in the pulse profile, and its measurement depends on, among other things, the precise geometry of the pulsar beam. For $e \gtrsim 0.9$ and $P_1 \sim 1$ it is conceivable that geodetic precession could be detected after several orbits. We now estimate the Newtonian and relativistic contributions to the secular apsidal precession of the pulsar orbit.

Suppose that surrounding the BH is a spherically symmetric distribution of matter, in the form of mostly low-mass stars and compact objects (e.g., § 5.2). For a density profile $\rho \propto r^{-\gamma}$, the extended mass enclosed within a radius r is $M_e(r) \propto r^{3-\gamma}$ ($\gamma < 3$). If $M_e(a)/M_{\rm BH} \ll 1$ for an orbit with semimajor axis a, the Newtonian contribution to the change in ω per orbit is[5]

$$\Delta\omega_{\rm N} = \frac{M_e(a)}{M_{\rm BH}}\frac{(1-e^2)^{3-\gamma}}{e}\int_0^{2\pi} d\phi\,\frac{\cos\phi}{(1+e\cos\phi)^{3-\gamma}}, \quad (3)$$

where ϕ is the true anomaly for the unperturbed elliptical trajectory. If $\gamma = 2$ (a plausible choice) the above integral is analytic, and we have

$$\Delta\omega_{\rm N} = -2\pi\frac{M_e(a)}{M_{\rm BH}}\frac{1-e^2}{e^2}\left[\frac{1}{(1-e^2)^{1/2}} - 1\right], \quad (4)$$

where the minus sign indicates retrograde precession, which is generally the case for $\gamma < 3$. For example, if $\gamma = 2$, $M_e(a)/M_{\rm BH} = 0.01$, and $e = 0.9$, we find that $\Delta\omega_{\rm N} \simeq 1°$.

The net apsidal precession rate also includes two relativistic contributions. In the Schwarzschild spacetime, the prograde advance per orbit is (e.g., Weinberg 1972)

$$\Delta\omega_{\rm S} \simeq +0\overset{\circ}{.}23 M_{6.5}^{2/3}P_1^{-2/3}(1-e^2)^{-1}. \quad (5)$$

For a spinning BH, frame dragging introduces an additional contribution (e.g., Jaroszynski 1998b; Wex & Kopeikin 1999):

$$\Delta\omega_{\rm FD} \simeq -27'' M_{6.5}P_1^{-1}(1-e^2)^{-3/2}\chi\cos\psi, \quad (6)$$

where ψ is the angle between the angular momentum vectors of the BH and the orbit and $0 < \chi < 1$ is the dimensionless BH spin parameter.

4.2. *Remarks on Measurability*

The degree to which different contributions to the timing residuals can be resolved depends on the precision and number of measured arrival times. Typical precisions are $\delta t \sim (10^{-3}$–$10^{-2})P_p$, or $\sim$1–10 ms for $P_p \simeq 1$ s. If one average arrival time

[5] We determined the precession rate by computing the orbit-averaged rate of change of the Laplace-Runge-Lenz vector (e.g., Goldstein 1980), $\boldsymbol{e} = (GM_{\rm BH})^{-1}\boldsymbol{v}\times\boldsymbol{h} - \boldsymbol{r}/r$, where $\boldsymbol{h} = \boldsymbol{r}\times\boldsymbol{v}$. Weinberg (1972) uses the same technique to calculate the general relativistic precession rate.

is measured each day the pulsar is observed and $N \sim 100$ arrival times are measured per orbit ($P_{\rm orb} \gtrsim 1$ yr), then we expect an rms timing precision of $\epsilon \sim \delta t/\sqrt{N} \lesssim 1$ ms over 1 orbital period. However, various potential sources of error may limit the net timing precision in a single orbit to $\sim P_p$. In particular, stochastic "timing noise," which is largest for the youngest, most luminous pulsars, may introduce net residuals of $\gtrsim 0.1 P_p$ after several years (e.g., Arzoumanian et al. 1994). In general, a precise assessment of the measurability of arrival-time delays and secular effects requires detailed simulations that cover a large parameter space. Here we present some simple statements regarding the detection of the delays listed in Table 1, as well as apsidal precession.

The amplitudes of the Einstein and first-order Shapiro delays given in Table 1 suggest that these effects should be easily measurable over a wide range in orbital parameters. This may indeed be the case for the Shapiro delay, allowing for an independent determination of $\sin i$, as long as perturbations to the orbit by stellar encounters (see § 5.2) do not have a significant impact. However, the Einstein delay can only be measured if the orbit undergoes appreciable apsidal precession (e.g., Blandford & Teukolsky 1976; Damour & Taylor 1992). In effect, extraction of the Einstein delay requires that the change in $\cos\omega$ be resolved sufficiently, which is generally more difficult than measuring $\dot{\omega}$ alone (see below). For a characteristic rms timing precision of $\epsilon \lesssim P_p$, the second-order Shapiro and frame-dragging delays may be just at the threshold of detectability for $P_1 \simeq 1$, unless the orbit is highly inclined and eccentric.

The rough scaling of the fractional error in the measured value of $\dot{\omega}$ is (e.g., Blandford & Teukolsky 1976)

$$\left|\frac{\delta\dot{\omega}}{\dot{\omega}}\right| \sim 10^{-3}\,\frac{k\epsilon}{N_{\rm orb}e\Delta\omega_d\sin i}M_{6.5}^{-1/3}P_1^{-2/3}, \qquad (7)$$

where k is a dimensionless factor that depends on the initial values of the Keplerian parameters, $N_{\rm orb}$ is the number of orbits over which the pulsar is monitored, $\Delta\omega_d$ is the net apsidal advance per orbit in degrees, and ϵ is in seconds. Even under rather unfavorable circumstances, where, e.g., $k \sim 100$, it might be possible to obtain a $\lesssim 10\%$ measurement of $\dot{\omega}$ in only two or three orbits if $\Delta\omega_d \sim 1$, thus facilitating the detection of the Einstein delay. However, the various contributions to $\dot{\omega}$ cannot be determined independently. The degeneracy could be broken if two or more pulsars were monitored over several orbits; such an ambitious project probably must wait for the SKA.

5. DISCUSSION

Here we address three additional topics pertaining to observations of a radio pulsar orbiting Sgr A*. We first discuss how pulsar timing can be used to probe the physics of the accretion flow onto the BH. This is followed by a short investigation of the effects of gravitational interactions between an orbiting pulsar and the surrounding cluster of stars and remnants. Finally, we consider the prospects of astrometrically monitoring an orbiting pulsar.

5.1. *The Interstellar Plasma around Sgr A**

Timing observations of a pulsar orbiting Sgr A* can be used to derive the properties of the local interstellar plasma. As the pulsar moves, photons trace many different lines of sight through the plasma to the observer. A gradient in the free-electron density on the scale of the pulsar orbit introduces dispersive and refractive pulse arrival-time delays that vary over the orbital period.[6] Each of these effects has a characteristic frequency dependence, pointing to a need for multifrequency observations. Because the detection of shallow-spectrum pulsars is favored at high frequencies (see § 3), there is a good chance that a pulsar detected at 10 GHz will also be detected at 15–20 GHz for the same sensitivity.

Chandra observations of the Galactic center (Baganoff et al. 2003) indicate that the density and temperature of the electrons at $\simeq 1''$ ($\simeq 8000$ AU) from Sgr A* are $n_e \sim 100$ cm^{-3} and $kT_e \simeq 1$–2 keV. The gravitational potential of the BH exceeds the thermal energy of the plasma inside the Bondi (1952) radius, $R_{\rm B} \sim GM_{\rm BH}/c_s^2$, where $c_s \sim (kT_e/m_p)^{1/2}$ is the thermal speed, assuming equipartition between electrons and protons, and m_p is the proton mass. For the plasma around Sgr A*, we find that $R_{\rm B} \sim 10^4$ AU. Within $R_{\rm B}$, $n_e(r)$ depends on the physics of the accretion flow. We adopt $n_e(r) \sim (10^2\ {\rm cm}^{-3})(10^4\ {\rm AU}/r)^{\beta}$, where models predict $\beta \simeq 1$–1.5 (e.g., Melia & Falcke 2001; Yuan et al. 2003).

The plasma frequency is $\nu_p = (n_e e^2/\pi m_e)^{1/2} \simeq (90\ {\rm kHz})(10^4\ {\rm AU}/r)^{\beta/2}$ for the density profile given above. The index of refraction of the plasma is $\xi(\nu) \simeq 1 - (\nu_p/\nu)^2/2$ for $\nu \gg \nu_p$. Radiation at different frequencies propagates through the medium with different group velocities of $c\xi(\nu)$, so that the arrival time of a pulse is frequency dependent. The difference in arrival times at frequencies ν_1 and ν_2 is proportional to $(\nu_1^{-2} - \nu_2^{-2})$DM, where DM is the dispersion measure, the column density of free electrons along the path of the pulse. Modulation of the DM over the orbital period of a pulsar bound to Sgr A* can thus be used to constrain the density profile of the plasma within the orbit.

Refraction due to the large-scale gradient of the electron density causes a net angular deflection of individual pulses that reach the observer. Consequently, there will be a variable *geometrical* time delay as the pulsar orbits. We assume that a light ray is refracted impulsively as it passes its closest approach, b, to Sgr A*, the scattering-screen or thin-lens approximation. The deflection angle is $\theta_p \sim \beta[\nu_p(r=b)/\nu]^2/2$, pointing *away* from Sgr A*. From the thin-lens geometry, the resulting excess propagation time, compared to that of a straight-line path, is $\sim r\theta_p^2/2c \propto \nu^{-4}$, where r is the orbital radius. For an orbital period of 10 yr, inclination of $i = 80°$, and eccentricity of $e = 0.9$, the minimum possible impact parameter is $a(1-e)\cos i \simeq 10$ AU, where $\nu_p(10\ {\rm AU}) \simeq 16$ MHz for $\beta = 3/2$. At $\nu = 10$ GHz, the amplitude and width of the refractive delay are, respectively, ~ 0.1 s and ~ 10 days. The strong frequency dependence of the refractive delay distinguishes it from dynamical effects.

5.2. *Dynamics of the Sgr A* Cluster*

A given orbiting pulsar will interact gravitationally with an unknown number of normal stars, other NSs, white dwarfs, and stellar-mass BHs. An important prediction of our work is that thousands of NSs born over the past ~ 1 Gyr may orbit Sgr A* with periods of $\lesssim 100$ yr. In addition, Miralda-Escudé & Gould (2000) suggest that $\sim 10^4$ stellar-mass ($\simeq 10\ M_\odot$) BHs may have migrated because of dynamical friction to the central $\simeq 1$ pc about Sgr A*; extending their results, we find

[6] Here we are considering the gradient in the spatially averaged electron density near Sgr A*. This is distinct from the small-scale turbulent density fluctuations that are responsible for angular and pulse broadening (see § 3).

that perhaps $\sim 10^2$ such BHs may reside in the central $1''$. Chanamé & Gould (2002) have discussed the possibility of using positional information for $\sim$50 millisecond radio pulsars within a few parsecs of Sgr A* to indirectly infer the presence of a large population of stellar-mass BHs.

For a star orbiting Sgr A*, many weak gravitational perturbations accumulate over a relaxation time to yield a significant net change in the orbital parameters.[7] In the absence of resonant effects, it is straightforward to show that near Sgr A* the relaxation time is $\tau_{\rm rel} \sim (10N_s)^{-1}(M_{\rm BH}/M_s)^2 P_{\rm orb}$, where M_s and N_s are the typical mass and total number, respectively, of the perturbing stars (e.g., Rauch & Tremaine 1996). If $M_{\rm BH}/M_s \simeq 10^6$ and $N_s = 10^3$–10^4, we find that $\tau_{\rm rel} \sim (10^7$–$10^8)P_{\rm orb}$. Resonant angular-momentum relaxation (Rauch & Tremaine 1996), which causes variation in only the orbital eccentricity and orientation angles, can act on a much shorter timescale of $\tau_{\rm rel,\,res} \sim (M_{\rm BH}/M_s)P_{\rm orb} \sim 10^6 P_{\rm orb}$.

Random-walk fluctuations in the orbital energy and angular momentum lead to an rms fractional change in 1 orbit of approximately $(P_{\rm orb}/\tau)^{1/2} \sim 10^{-3}$ to 10^{-4}, where τ is either the conventional or resonant relaxation time. The resulting change in a/c may be $\sim$100–1000 lt-s for $P_{\rm orb} \sim 10$ yr. Gravitational perturbations by stars and remnants may have measurable consequences for the timing analysis of a pulsar orbiting Sgr A*, possibly even masking the important relativistic time delays discussed in § 4. With the SKA it may be possible to obtain a large sample of regularly timed, orbiting radio pulsars. If evidence of random gravitational encounters is found in the timing properties of a significant fraction of these pulsars, important clues could be extracted regarding the number, masses, and velocities of the Sgr A* cluster members.

5.3. *Radio Astrometry*

It would be an extreme challenge to image a pulsar orbiting Sgr A* with an interestingly short orbital period. The projected apocenter separation would be less than $a(1+e)/D = (20\ {\rm mas})M_{6.5}^{1/3}P_1^{2/3}(1+e)$. Very long baseline interferometry (VLBI) would be required to resolve a faint pulsar next to the very bright Sgr A* ($\sim$1 Jy from 1 to 10 GHz; Melia & Falcke 2001). Such observations would have to be conducted at $\nu \gtrsim 10$ GHz to sufficiently reduce the scattering diameter of Sgr A* (see § 3). With a current VLBI sensitivity of $\simeq$1 mJy, probably no orbiting pulsars would be detectable (see § 3). The SKA will have the requisite sensitivity, and the current design specifications call for an angular resolution of $\simeq$10 mas at 10 GHz and, of course, higher astrometric resolution. Astrometric precision smaller than $\simeq$0.1 mas at $\sim$10 GHz may not be attainable from the surface of the Earth (e.g., Chatterjee et al. 2004). The advent of highly sensitive space- or Moon-based VLBI instruments is then a necessary step toward $\lesssim 10\ \mu$as astrometry for Sgr A* pulsars.

If $\Delta\omega_d$ is the net apsidal advance per orbit in degrees, then the angular shift of the apocenter position in 1 orbit is

$$\Delta\theta_a < (0.34\ {\rm mas})\Delta\omega_d M_{6.5} P_1^{2/3}(1+e). \qquad (8)$$

For a large eccentricity and $P_1 \sim 10$, $\Delta\theta_a$ could be $\sim$1 mas. It is at least conceivable that apsidal precession could be detected from the ground after several orbits.

If the orbit is inclined with respect to the BH spin, frame dragging causes the line of nodes, Ω, on the plane of the sky to advance in one orbit by an amount (e.g., Jaroszynski 1998b)

$$\Delta\Omega_{\rm FD} \simeq 9'' M_{6.5} P_1^{-1}\left(1-e^2\right)^{-3/2}\chi \sin\psi, \qquad (9)$$

where $\chi \sin\psi$ is the projection of the spin onto the orbital plane. The corresponding apocenter shift is expected to be no larger than a few tens of microarcseconds. If the contributions from frame dragging to apsidal (see § 4) and nodal precession were measured, it would be possible to determine the magnitude and direction of the BH spin, but this is a highly unlikely prospect.

The effects of gravitational lensing by the BH may be important when the pulsar is near superior conjunction. For lensing by a point mass, two images are produced, one inside and one outside the Einstein radius, $\theta_{\rm E} \simeq (4GM_{\rm BH}r/c^2D^2)^{1/2}$, where $r \ll D$ is the orbital radius near superior conjunction. For $r = 100$–1000 AU, we find $\theta_{\rm E} \simeq 0.5$–1.5 mas. Since $\theta_{\rm E}$ is small, we consider here only the image outside $\theta_{\rm E}$. At superior conjunction, the angular separation between the pulsar and its image is $\delta\theta \simeq \theta_{\rm E}^2 D/b$, where $b = r\cos i$ is the impact parameter (e.g., Jaroszynski 1998a). We then find that $\delta\theta \sim 20\ \mu{\rm as}/\cos i$, which is $\sim$1 mas for $i = 89°$. Therefore, gravitational lensing has a small astrometric signature in typical situations.

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[7] This is due to the granularity of the stellar distribution. Long-term orbital precession, as discussed in § 4.1, is due to the smoothed potential of the extended stellar mass component.

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