THE ASTROPHYSICAL JOURNAL, 611:1080–1083, 2004 August 20

ON THE RATE OF DETECTABILITY OF INTERMEDIATE-MASS BLACK HOLE BINARIES USING *LISA*

CLIFFORD M. WILL
Department of Physics, McDonnell Center for the Space Sciences, Washington University, St. Louis, MO 63130;[1]
and Groupe Gravitation Relativiste et Cosmologie, Institut d'Astrophysique de Paris, 75014 France
Received 2004 March 28; accepted 2004 May 4

ABSTRACT

We estimate the rate at which the proposed space gravitational-wave interferometer *LISA* could detect intermediate-mass black hole (IMBH) binaries, that is, binaries containing a black hole in the mass range 10–100 $M_\odot$ orbiting a black hole in the mass range 100–1000 $M_\odot$. For 1 yr integrations leading up to the innermost stable circular orbit and a signal-to-noise ratio of 10, we estimate a detection rate of only 1 per million years for 10 $M_\odot$/100 $M_\odot$ binaries. The estimate uses the method of parameter estimation via matched filtering, incorporates a noise curve for *LISA* established by the *LISA* Science Team that is available online, and employs an IMBH formation rate model used by Miller in 2002. We find that the detectable distance is relatively insensitive to *LISA* arm lengths or acceleration noise but is roughly inversely proportional to *LISA* position errors and varies roughly as $T^{1/2}$, where T is the integration time in years. We also show that while all IMBH systems in this mass range may be detected in the Virgo cluster up to 40 yr before merger, none can be detected there earlier than 400 yr before merger. An extended *LISA* mission that enabled 10 yr integrations could detect IMBH systems at the Virgo cluster 1000 yr before merger, and systems in Galactic globular clusters 1 million yr before merger. We compare and contrast these estimates with earlier estimates by Miller.

Subject headings: black hole physics — gravitation — gravitational waves

1. INTRODUCTION

In recent years, intermediate-mass black holes (IMBHs), holes with masses between hundreds and thousands of solar masses, have attracted considerable interest, both as relics of the evolution and structure of globular clusters and as possible sources of gravitational radiation for both ground- and space-based laser interferometers (Miller & Colbert 2004). Miller (2002) has estimated the rate of detectable IMBH binaries by the proposed space antenna *LISA* to be given by

$$R \approx 7\times10^{-3}\left(\frac{h}{0.7}\right)^3\left(\frac{f_{\rm tot}}{0.1}\right)\left(\frac{\mu}{10\ M_\odot}\right)^{1/2} \times\left(\frac{M_{\rm max}}{100\ M_\odot}\right)^{3/2}\left(\ln\frac{M_{\rm max}}{M_{\rm min}}\right)^{-1}\ {\rm yr}^{-1}, \quad (1)$$

where h is the Hubble parameter in units of 100 km s^{-1} Mpc^{-1}, $f_{\rm tot}$ is the total fraction of globular clusters that have IMBHs, μ and M are the reduced mass and total mass of the binary, respectively, and $M_{\rm min}$ and $M_{\rm max}$ denote the range over which such IMBHs may exist in globular clusters.

A key ingredient in calculating this rate is the distance to which *LISA* could detect binary IMBH in-spirals for a given signal-to-noise ratio (S/N). Miller (2002) used estimates of S/Ns for a binary in-spiral derived by Flanagan & Hughes (1998) from semianalytic *LISA* noise curves. We have recalculated this distance using standard methods based on matched filtering for a binary in-spiral and using an up-to-date *LISA* noise curve available online.[2] We find significantly smaller distances and rates reached for a given S/N and with significantly different dependencies on mass from those obtained by Miller (2002). Figure 1 shows distances reached for $S/N = 10$ and 1 yr of integration leading up to merger, as a function of total mass M and reduced mass parameter $\eta = m_1 m_2/M^2$ ($0 < \eta \leq 1/4$). For $M = 100\ M_\odot$ and reduced mass $\mu = \eta M = 10\ M_\odot$, we find a distance $D_L \approx 18$ Mpc, 11 times smaller than Miller's equation (19) (we assume that all masses are suitably redshifted). The rate of detection we find is given by

$$R \approx 1.0\times10^{-6}\left(\frac{h}{0.7}\right)^3\left(\frac{f_{\rm tot}}{0.1}\right)\left(\frac{\mu}{10\ M_\odot}\right)^{19/8} \times\left(\frac{M_{\rm max}}{100\ M_\odot}\right)^{13/4}\left(\ln\frac{M_{\rm max}}{M_{\rm min}}\right)^{-1}\ {\rm yr}^{-1}. \quad (2)$$

We also analyze *LISA*'s reach for IMBH systems at epochs earlier than the year leading up to merger. Figure 2 shows distances as a function of years before merger for various IMBH systems, for $S/N = 10$ and 1 yr of integration. While all systems in this mass range are detectable in the Virgo cluster (at 19 Mpc) within 40 yr of merger, *none* is detectable there earlier than 400 yr before merger, and at 1000 yr before merger, only systems closer than 7 Mpc are detectable. On the other hand, we find that for systems in this mass range, the distance reached varies roughly as the square root of the integration time, so that an extended *LISA* mission that enabled 10 yr integrations could reach roughly 3 times farther,

The primary reason for this downward revision in the reach of *LISA* for IMBHs is our use of a more current sensitivity curve for *LISA*, whose noise level is substantially higher than the older Flanagan-Hughes curve used by Miller. The rest of this paper gives the details to support this conclusion. In § 2 we determine the distance reached by *LISA* using standard S/N

[1] Permanent address.
[2] The SCG was originally written by Shane Larson and can be found online at http://www.srl.caltech.edu/~shane/sensitivity/MakeCurve.html.

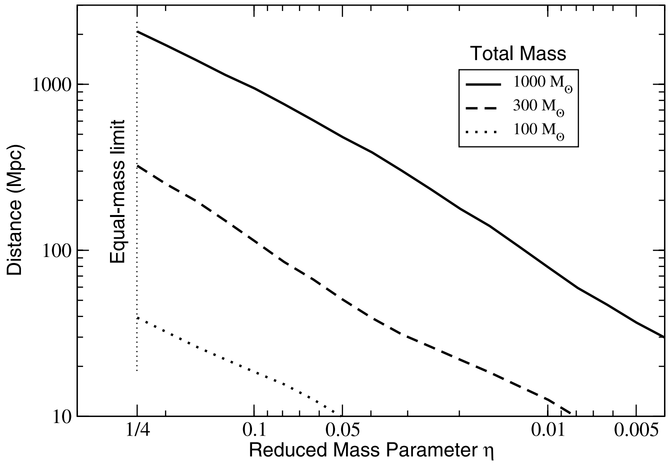

FIG. 1.—Distances in megaparsecs reached by *LISA* for an IMBH binary in-spiral, for $S/N = 10$ and 1 yr of integration prior to merger.

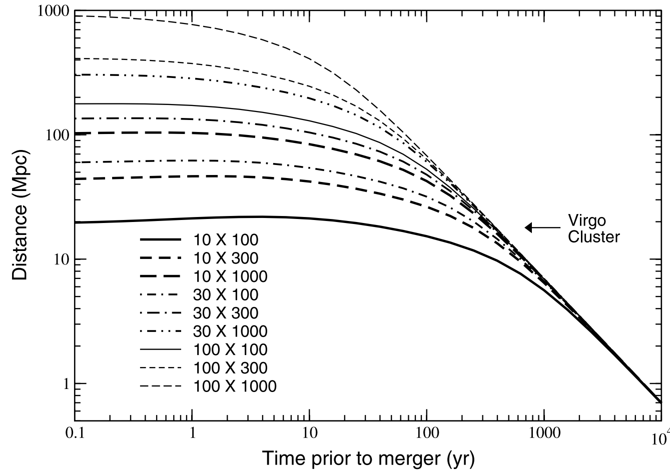

FIG. 2.—Distances in megaparsecs reached by *LISA* for an IMBH binary in-spiral, for $S/N = 10$ and 1 yr of integration vs. years before merger, for various black hole pairs (in $M_\odot$).

calculations, and in § 3 we use the method of Miller (2002) to estimate the detection rate of systems in the final year of in-spiral. In § 4 we discuss the results.

2. CALCULATION OF DISTANCE REACHED BY *LISA*

The S/N ρ for a gravitational-wave signal whose Fourier transform is $\tilde{h}(f)$ in a detector of noise spectral density $S_n(f)$ is given by

$$\rho(h)^2 \equiv 4 \int_{f_i}^{f_f} \frac{\left|\tilde{h}(f)\right|^2}{S_n(f)}\, df, \tag{3}$$

where f_i and f_f are the initial and final frequencies between which the signal is integrated (see Cutler et al. 1993; Cutler & Flanagan 1994; Finn 1992; Finn & Chernoff 1993; Poisson & Will 1995 for details of the method of matched filtering). In the "restricted post-Newtonian" approximation, in which the wave amplitude is given by the quadrupole approximation and the wave phase is given by a high-order post-Newtonian expression, $\tilde{h}(f)$ is given by

$$\tilde{h}(f) = \begin{cases} Af^{-7/6}e^{i\Psi(f)}, & 0 < f < f_{\max}, \\ 0, & f > f_{\max}, \end{cases} \tag{4}$$

where $f_{\max}$ is the largest frequency for which the wave can be described by the restricted post-Newtonian approximation, often taken to correspond to waves emitted at the innermost stable circular orbit (ISCO) before the bodies plunge toward each other and merge. A useful approximation for this frequency is (we use units in which $G = c = 1$)

$$f_{\max} = \left(6^{3/2}\pi M\right)^{-1}, \tag{5}$$

where M is the total mass of the system. After averaging over all angles, the amplitude A is given by

$$A = \frac{1}{\sqrt{30}\pi^{2/3}} \frac{\mathcal{M}^{5/6}}{D_L}, \tag{6}$$

where D_L is the luminosity distance of the source and $\mathcal{M} = \eta^{3/5}M$ is the "chirp" mass. In general relativity, the phasing function $\Psi(f)$ can be given to high post-Newtonian order but is not relevant for our purposes. Combining equations (3), (4), and (6), we obtain the expression for D_L:

$$D_L = \sqrt{\frac{2}{15}} \frac{\mathcal{M}^{5/6}}{\rho} \frac{1}{\pi^{2/3}} B(7)^{1/2}, \tag{7}$$

where

$$B(7) = \int_{f_i}^{f_f} \frac{f^{-7/3}}{S_n(f)}\, df. \tag{8}$$

For a quasi-circular binary in-spiral, the initial and final frequencies can be related to the integration time T using the expression obtained from gravitational radiation damping of the orbit at quadrupolar order (which is sufficiently accurate for our purpose),

$$u_i = u_f \left(1 + \frac{256}{5} \frac{T}{\mathcal{M}} u_f^{8/3}\right)^{-3/8}, \tag{9}$$

where $u = \pi \mathcal{M} f$.

In this paper, we adopt sensitivity curves for *LISA* developed independently by Larson et al. (2000) and Armstrong et al. (1999) (see Fig. 3). The two methods are in substantial agreement, and the former has been summarized in the Sensitivity Curve Generator (SCG), available online.[3] The sensitivity curves incorporate sources of instrumental noise in *LISA*, such as laser shot noise, acceleration noise, and thermal noise, coupled with a *LISA* transfer function that takes into account the effect of the finite time of propagation of the laser beams during the passage of the gravitational waves. We assume the case in which all three arms of *LISA* are of equal length, and we assume that averages over angles and polarizations have been done. The SCG permits various choices to be made for *LISA* instrumental parameters and has an option to include an estimate for confusion noise resulting from a background of Galactic white dwarf binaries (Bender & Hils 1997); this background is included in the analysis but in fact plays no significant role for the late stage of IMBH in-spiral because the

[3] See footnote 2.

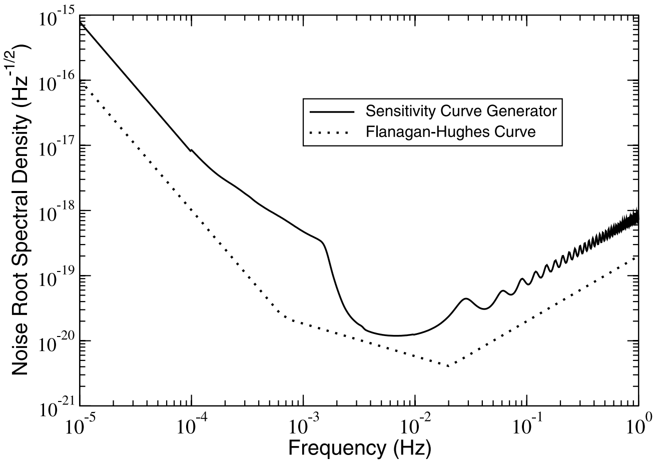

FIG. 3.—Root noise spectral density for *LISA*.

signals are predominantly at high frequency, well above the white dwarf band.

We consider IMBH systems containing one black hole in the mass range 10–100 $M_\odot$ and a companion black hole in the mass range 100–1000 $M_\odot$. Figure 1 shows the results for the distance as a function of reduced mass parameter for various total masses, assuming an S/N of 10 with a 1 yr integration time ending at the ISCO (or at the termination of the *LISA* sensitivity curve, whichever comes first). The distances, in megaparsecs, can be fitted by the approximate formula

$$\begin{aligned} D_L &= 190\left(\frac{10}{\rho}\right)\left(\frac{\mathcal{M}}{100\,M_\odot}\right)^{15/8}\left(\frac{T}{1\text{ yr}}\right)^{1/2}\text{ Mpc} \\ &= 40\left(\frac{10}{\rho}\right)(4\eta)^{9/8}\left(\frac{M}{100\,M_\odot}\right)^{15/8}\left(\frac{T}{1\text{ yr}}\right)^{1/2}\text{ Mpc}. \end{aligned} \tag{10}$$

This scaling can be understood as follows: the waves from IMBHs in the final year of in-spiral are mainly at the high-frequency end of the *LISA* sensitivity band, in which $S_n(f) \propto f^2$. In addition, for these masses and year-long integrations, the second term in equation (9) dominates, so that $f_i = (1/8\pi)(5/T)^{3/8}\mathcal{M}^{-5/8} \ll f_f$. Consequently, the integral $B(7) \sim f_i^{-10/3} \sim \mathcal{M}^{25/12}T^{5/4}$. Substituting this into equation (7), we obtain the scaling in $\mathcal{M}$ shown in equation (10). A $T^{1/2}$ integration-time scaling actually fits the curves somewhat better than a $T^{5/8}$ scaling inferred from the analytic estimate. Equation (10) underestimates the distances at the low-mass end, e.g., for $\eta \sim 1/10$ and $M \sim 100\ M_\odot$, by about 20%.

It is straightforward to show that the distance reached is independent of the *LISA* acceleration noise but is inversely proportional to the *LISA* position noise error budget. Halving the position noise doubles the distance reached. The distance is weakly dependent on *LISA* arm length, varying by between 10% and 40% for factor of 2 changes in arm length; varying arm length does not alter the overall level of the high-frequency end of the noise curve but instead moves the locations of the peaks in the oscillations of the transfer function and raises the floor of the noise curve near 10^{-3} Hz.

We obtain similar results using a sensitivity curve derived by Finn & Thorne (2000) based on work of the *LISA* Mission Definition Team (see reference 44 of Finn & Thorne 2000). This curve closely matches the curve from the SCG except for the oscillations in the noise root spectral density at high frequency resulting from the *LISA* transfer function, an effect of the finite arm length. The oscillations have the effect of reducing somewhat the distances inferred from the SCG for a given S/N compared to those from the Finn-Thorne curve, for sources at high frequency.

We also calculate the distances that can be reached in a 1 yr integration for IMBH systems much earlier than the ISCO stage (see Fig. 2). All systems detected within about 40 yr of merger can be detected at the Virgo cluster (19 Mpc) with S/N = 10 and 1 yr of integration. However, at earlier epochs the reach decreases dramatically, so that by 400 yr *no* systems can be seen at Virgo, and by 1000 yr before merger, the distance reached for S/N = 10 is in the range of 5.6–7 Mpc for all IMBH mass ranges considered. This is well short of the Virgo cluster. For systems detected 10^6 yr before merger, the distance reached is 7 kpc, independent of mass. This behavior can be understood analytically. For observation epochs long before the ISCO, one can show that the initial and final frequencies observed are related by $f_i \approx f_f(1 + T/T')^{-3/8}$, where T is the integration time and T' is the epoch before merger, with $T \ll T'$. Then, in a region where the noise spectral density is dominated by the behavior $S_n(f) \approx \alpha f^n$, one can show using equations (7) and (8) that

$$D_L \approx \frac{2}{5}\frac{(8\pi)^{n/2}}{\rho}\left(\frac{T}{\alpha}\right)^{1/2}\left(\frac{T'}{5}\right)^{(3n-4)/16}\mathcal{M}^{[5(4+n)]/16}. \tag{11}$$

For epochs earlier than 1000 yr before merger, the systems are in the low-frequency regime in which $S_n(f) \propto f^{-4}$ (apart from white dwarf confusion noise); hence, $D_L \approx (1/32\pi^2)(T/\alpha)^{1/2}(T'\rho)^{-1}$, independent of chirp mass. With $\alpha = 6.09 \times 10^{-51}$ Hz^3 giving a good fit to the low-frequency *LISA* instrumental noise, we obtain

$$D_L \approx 7T^{1/2}(1000/T')(10/\rho)\text{ Mpc}, \tag{12}$$

which perfectly matches the large-epoch behavior in Figure 2. The $T^{1/2}$ behavior is expected for what are observations of an essentially continuous wave source. These results also are far more pessimistic than the estimates made by Miller & Colbert (2004), which suggested detecting IMBH binaries in the Virgo cluster 1000 yr before merger and in the local system of globular clusters 10^6 yr before merger. On the other hand, because of the $T^{1/2}$ behavior, such binaries *could* be reached in an extended *LISA* mission that enabled 10 yr integration times.

3. RATE OF DETECTABLE MERGERS

To calculate the rate of in-spirals detectable by *LISA* in the last year before merger, we follow the method outlined by Miller (2002). The rate is given by

$$R = \frac{4\pi}{3}\int D(M)^3\nu(M)n_{\rm gc}f(M)\,dM, \tag{13}$$

where $D(M)$ is the distance reached for a given S/N and integration time as a function of total mass (holding the reduced mass μ fixed); $\nu(M) = 10^{-10}\mu^{-1}M$ yr^{-1} is the rate at which smaller black holes merge with black holes of mass M in a given cluster; $n_{\rm gc} = 8\ h^3$ Mpc^{-3} is the number density of globular clusters in the local universe; and $f(M) = [f_{\rm tot}/\ln(M_{\rm max}/M_{\rm min})]M^{-1}$ is the fraction of globular clusters harboring black holes with mass M per mass interval dM, in the range $M_{\rm min} < M < M_{\rm max}$, with $\int f(M)\, dM = f_{\rm tot}$. We assume $M_{\rm max} \gg M_{\rm min}$. Substituting these formulae along with equation (10) into equation (13), we obtain equation (2).

4. DISCUSSION

Both the distance reached by *LISA* and the estimated rate of in-spirals detected differ markedly from the estimates given by Miller (2002), namely, $D_L \approx 200(\mu/10\ M_\odot)^{1/2}(M/100\ M_\odot)^{1/2}$ Mpc and equation (1). The distance derived by Miller is larger and the mass dependence is different than in equation (10) because he appears to have adopted equation (B7) from Flanagan & Hughes (1998), which actually applies only to equal-mass systems. For the low masses relevant to IMBH in the year leading up to the merger, the high-frequency end of the *LISA* noise spectrum is dominant, where the dependence on frequency (and hence on M) is steeply increasing. But the underlying dependence is on *chirp* mass, so for a given total mass, there is still a strong dependence on the reduced mass parameter, which is not reflected in equation (B7) of Flanagan & Hughes (1998). In addition, the noise root spectral density curve modeled by Flanagan & Hughes (1998) is lower by a factor of about 3–5 than that given by the SCG (Fig. 3). Their noise curve was based on an out-of-date description of the *LISA* mission. Distance estimates using the Flanagan-Hughes curve will therefore automatically be larger and rates will be higher than those using the SCG.

We have assumed throughout that the in-spiral orbits are quasi-circular, that is, circular apart from the adiabatic in-spiral due to radiation reaction. In reality, IMBH binary orbits are likely to be highly eccentric (Gültekin et al. 2004). This will increase the average gravitational-wave flux for a given orbital period and could add harmonics in frequency bands in which *LISA*'s noise may be lower; on the other hand, it will increase the number of parameters to be estimated in the matched filter, which raises the threshold needed to achieve detection. It is not clear whether the net effect of these complicated and possibly offsetting effects will improve *LISA*'s reach for IMBH binaries or not. In any event, they are beyond the scope of this paper.

We are grateful for useful discussions with Cole Miller and Scott Hughes, who have confirmed the conclusions reached here. We thank the Institut d'Astrophysique de Paris for its hospitality during the academic year 2003–2004. This work was supported in part by the National Aeronautics and Space Administration under grant NAG5-10186 and the National Science Foundation under grants PHY 00-96522 and PHY 03-53180.

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