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# BARNARD'S STAR AND THE M DWARF TEMPERATURE SCALE

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## ABSTRACT

The recently measured angular diameter of Barnard's star, together with its large and precise parallax, and a spectral energy distribution that extends from the near-ultraviolet to almost 12 $\mu$m establish some of the star's fundamental properties—we find a bolometric luminosity $L = (3.46 \pm 0.17) \times 10^{-3}\ L_{\odot}$, radius $R = 0.200 \pm 0.008\ R_{\odot}$, and effective temperature $T_{\rm eff} = 3134 \pm 102$ K. Accurate knowledge of those parameters helps in turn to constrain the star's metallicity and mass. Although it is evidently possible to estimate bolometric fluxes with good accuracy from photometry alone, angular diameters present more of a challenge, and we examine alternative methods for determining them, namely, through the use of the Barnes-Evans relation and the infrared flux method. We find further evidence that even "state-of-the-art" M dwarf models, which appear to yield good results for the effective temperatures, nevertheless underestimate the radii of the actual stars.

*Key words:* stars: fundamental parameters — stars: individual (Barnard's star) — stars: late-type — stars: low-mass, brown dwarfs

## 1. INTRODUCTION

The M dwarf temperature scale has been a subject of some interest for several decades now, especially since the development of detectors with suitable sensitivity in the infrared made it possible to acquire the relevant observational data. Early work by Wing (1967) resulted in estimated effective temperatures of 3200 and 2300 K for Barnard's star and Wolf 359, respectively; these were derived on the basis of scanner measurements of the near-infrared spectrum. Shortly thereafter, Greenstein, Neugebauer, & Becklin (1970) obtained *HKL* photometry of Wolf 359 and BD +4°4048B, and after correcting for line and band blocking for $\lambda < 1\ \mu$m, they fitted blackbody energy distributions to arrive at estimates of the effective temperatures. Their approach formed the basis for a study by Veeder (1974) of 145 nearby late-type stars. Subsequently, two detached M dwarf eclipsing binary systems, YY Gem (Leung & Schneider 1978) and CM Dra (Lacy 1977b), yielded valuable information not only on M dwarf masses, but also absolute dimensions, with the result that, given the bolometric luminosities, effective temperatures could be found directly. (More recently, Delfosse et al. 1999 discovered a third such system, GJ 2069A.) Reid & Gilmore (1984) attempted to recalibrate the effective temperature scale for $T_{\rm eff} < 3500$ K by addressing certain deficiencies inherent in the technique introduced by Greenstein et al. (1970): they showed that failure to account for $H_2O$ absorption between the infrared filter passbands could result in an overestimate of $T_{\rm eff}$ by about 90 K at $T_{\rm eff} = 3000$ K. Berriman & Reid (1987) conducted systematic spectroscopic observations of 11 M dwarfs redward of 2.2 $\mu$m and concluded that spectra covering the wavelength range 1 $\mu{\rm m} \leq \lambda \leq 4\ \mu$m are essential in deriving accurate luminosities and possibly effective temperatures for M dwarfs. Berriman, Reid, & Leggett (1992, hereafter BRL) updated some of the photometry on which the Berriman & Reid (1987) temperature scale was based and recalculated the effective temperatures: for the most part, only relatively small changes resulted, and they concluded that their new scale, which gave results close to those found by Veeder (1974), remains valid. It agrees rather well with an independent determination by Bessell (1995).

Within the past few years, a number of interferometers capable of resolving nearby M dwarfs have come into service. As one of the Sun's nearest neighbors, Barnard's (1916) "small star with large proper motion" is an obvious target for such instruments; indeed, it is attractive for a number of reasons. As one of the brightest examples of its class, and favorably located near the celestial equator, Barnard's star has probably received more observational attention than any other M dwarf. Its parallax is very well determined: we adopt the value $\pi_{\rm trig} = 545.4 \pm 0.3$ mas determined by Benedict et al. (1999) using Fine Guidance Sensor 3 on the *Hubble Space Telescope*. Its radial velocity is almost as precisely determined: $V_R = -110.85 \pm 0.23$ km s$^{-1}$ (Marcy & Benitz 1989). Its proper motion remains the largest known, Barnard's (1916) value of 10″.36 yr$^{-1}$ in position angle 359°.7 having recently been refined by *Hipparcos* (Perryman et al. 1997) to $\mu = 10{,}368.6 \pm 2.1$ mas yr$^{-1}$ in position angle 355°.58 ± 0°.07. The space velocity relative to the Sun therefore has components $(U,\ V,\ W) = (141.4 \pm 0.2,\ 4.0 \pm 0.1,\ 19.9 \pm 0.1)$ km s$^{-1}$, with $U$ positive in the anticenter direction. The resultant (142.8 km s$^{-1}$) is unusually high for an old disk star and has sometimes been taken as an indication that Barnard's star should properly be assigned to the halo population.

## 2. THE SPECTRAL ENERGY DISTRIBUTION

The spectral energy distribution (SED) for Barnard's star (LHS 57 = Gl 699) was constructed in a piecewise fashion

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TABLE 1
CONSTRUCTION OF THE SPECTRAL ENERGY DISTRIBUTION

| Observatory/Instrument | Reference No. | Exposure (s) | Date | Wavelength Interval ($\mu$m) | Spectral Resolution | Source |
|---|---|---|---|---|---|---|
| *IUE*/SWP | 26459 | 24660 | 1985 Jul 23 | 0.12–0.20 | 2.67 | Archive |
| *IUE*/LWR | 08283 | 7200 | 1980 Jul 18 | 0.18–0.33 | 2.67 | Archive |
| WHT/ISIS | 173530402 | 20 | 1997 Jul 12 | 0.37–0.52 | 4 | Archive |
| DDO | ... | 300 | 2001 Sep 16 | 0.52–0.62 | 8 | Authors |
| USNOFS | ... | ... | 1989 Sep | 0.62–0.83 | 6 | SL |
| UH2.2/KSPEC | ... | ... | 1994 Jul | 0.84–1.18 | 20 | SL |
| | | | | 1.18–1.45 | 25 | SL |
| | | | | 1.45–1.82 | 30 | SL |
| | | | | 1.92–2.42 | 40 | SL |
| *ISO*/PHT-S | 031101503 | 620 | 1996 Sep 23 | 2.47–4.87 | ... | Archive |
| | | | | 5.84–11.62 | ... | Archive |

NOTE.—(SL) Leggett et al. 1996

according to the information provided in Table 1. The *IUE* archival data were processed using the most recent pipeline calibrations. A three-point boxcar smoothing was applied to the resulting spectra to yield an effective wavelength resolution of 8 Å. Only the data blueward of 3070 Å were consistent with zero flux; they are not included in the SED.

The 0.37–0.52 $\mu$m data were taken from the William Herschel Telescope (WHT) archives and were calibrated using a single spectrophotometric standard star. The 0.52–0.62 $\mu$m data were obtained using the Cassegrain spectrograph on the 1.88 m telescope at the David Dunlap Observatory (DDO) by the authors. A 100 line mm$^{-1}$ grating provided an effective spectral resolution of 8 Å. Data reduction was performed in a standard way using IRAF.[3] The data were wavelength-calibrated using FeNe and FeAr comparison lamps and placed on an energy scale using a single photometric standard star taken immediately afterward.

While the relative shape of the spectral response curve or sensitivity function derived from a flux standard is normally fairly well determined—apart from the edges—the absolute flux level can be uncertain by a significant amount. The WHT and DDO data were scaled so that the *B* and *V* apparent magnitudes generated synthetically using appropriate filter transmission curves and compared with the spectrum of Vega gave magnitudes generally consistent within a few hundredths of the photometric magnitudes given by Leggett et al. (1996) and provided in Table 2. At the same time, a function of order unity was used to scale the data in the vicinity where these two spectra joined in order to make the transition appear smoother. This scaling had no effect on the magnitudes provided in Table 2. The optical filter transmission profiles were taken from the Mosaic filters at the NOAO Web site,[4] while the infrared filter transmission profiles were taken from the Canada-France-Hawaii Telescope Web site.[5]

The piecewise construction of the SED between 0.62 and 2.42 $\mu$m is described by Leggett et al. (2000); we followed the same scaling algorithm. (Between 0.62 and 0.72 $\mu$m, however, the University of Hawaii data had to be scaled slightly, in order to ensure that the synthetic *R* magnitude agreed with the photometry listed in Table 2.)

The *ISO* data, 2.47–4.87 $\mu$m and 5.84–11.62 $\mu$m, were requested and processed using version 9.1 of the *ISO* Pipeline software (2000 December). Only a single bad channel was observed in the data and it was removed. The predominantly zodiacal background (redward of 7$\mu$m) was removed from the "star plus background" using the prescription provided in Ábrahám et al. (1997). The uncertainty in the integrated flux of the SED is estimated to be less than 5%, based on the uncertainties quoted by Leggett et al. (1996) and from the *ISO* data.

## 3. PROPERTIES OF BARNARD'S STAR

### 3.1. *Luminosity*

The bolometric flux $\mathcal{F}$ at Earth is readily obtained by integration of the SED. Assuming (1) negligible flux to the blue of 3074 Å, (2) linear behavior of the SED in the few narrow windows left uncovered by observations, and (3) a $\lambda^{-4}$ dependence of $f_\lambda$ for $\lambda > 11.62$ $\mu$m, it is found that

$$\mathcal{F} = (3.30 \pm 0.16) \times 10^{-11}\ \mathrm{W\ m^{-2}}.$$

[The "Rayleigh-Jeans tail" is accounted for by adding $\lambda f_\lambda(11.62\ \mu\mathrm{m})/3$ to the result of the numerical integration from 0.3074 to 11.62 $\mu$m, but it provides a correction of less than 0.22% to the bolometric luminosity.] With this, and the adopted value of the trigonometric parallax (given above), the luminosity of Barnard's star is $(1.33 \pm 0.066) \times 10^{24}$ W, or in the customary units, $L = (3.46 \pm 0.17) \times 10^{-3}\ L_\odot$, where the solar luminosity $L_\odot = 3.845 \times 10^{26}$ W. With $M_{\mathrm{bol},\odot} = +4.74$,

TABLE 2
BROADBAND PHOTOMETRY OF BARNARD'S STAR

| Filter | Apparent Magnitude | *m* from SED |
|---|---|---|
| *B* | 11.28 | 11.28 |
| *V* | 9.55 | 9.52 |
| *R* | 8.34 | 8.42 |
| *I* | 6.77 | 6.76 |
| *J* | 5.27 | 5.37 |
| *H* | 4.77 | 4.81 |
| *K* | 4.51 | 4.52 |
| *L* | 4.20 | 4.27 |
| *L′* | 4.18 | 4.18 |

[3] IRAF is distributed by the National Optical Astronomy Observatory, which is operated by the Association of Universities for Research in Astronomy, Inc., under cooperative agreement with the National Science Foundation.
[4] See http://www.noao.edu/kpno/mosaic/filters/filters.html.
[5] See http://www.cfht.hawaii.edu/Instruments/Filters/cfhtir.html.

TABLE 3

Bolometric Corrections

| Filter | BC (mag) |
|---|---|
| $B$ | −4.07 |
| $V$ | −2.34 |
| $R$ | −1.13 |
| $I$ | +0.44 |
| $J$ | +1.94 |
| $H$ | +2.44 |
| $K$ | +2.70 |
| $L$ | +3.01 |
| $L'$ | +3.03 |

this amounts to $M_{\rm bol,*} = +10.89$, in fair agreement with Veeder's (1974) empirical $M_{\rm bol}$-$M_{\rm K}$ relation, which yields $M_{\rm bol} = 10.98$. Since $M_{V,*} = 13.23$, the bolometric correction at $V$ is then $M_{\rm bol,*} - M_{V,*} = -2.34$ mag. Bolometric corrections in other passbands are listed in Table 3.

### 3.2. *Effective Temperature*

The effective temperature of Barnard's star is established by combining the bolometric flux with the result of a recent measurement by Lane, Boden, & Kulkarni (2001) of the star's angular diameter, using the Palomar Testbed Interferometer. They obtained a uniform-disk angular diameter $\theta_{\rm UD} = 0.987 \pm 0.04$ mas; this is of course a lower limit on the limb-darkened stellar angular diameter, so that for constant bolometric flux $\mathcal{F}$, it corresponds to the maximum $T_{\rm eff}$. Since

$$T_{\rm eff} = 2\left(\frac{\pi\mathcal{F}}{1\ L_\odot}\right)^{1/4}\left(\frac{1\ R_\odot}{\theta}\right)^{1/2} T_\odot \approx \frac{3155\ {\rm K}}{\theta^{1/2}({\rm mas})}, \qquad (1)$$

where $T_\odot = 5777$ K is the solar effective temperature and $R_\odot$ is the solar radius, then for $\theta = \theta_{\rm UD}$ it follows that $T_{\rm eff} \leq 3175 \pm 102$ K. Lane et al. (2001) arrived at a limb-darkened angular diameter $\theta_{\rm LD} = 1.026 \pm 0.04$ mas by use of a linear limb-darkening law together with limb-darkening coefficients from Claret, Díaz-Cordovés, & Giménez (1995). For various reasons, we have followed here a different approach. J. P. Aufdenberg (2002, private communication) has computed limb-darkening corrections in the $H$ and $K$ bands directly from radiation field data, using a model atmosphere whose parameters approximate well those of the atmosphere of Barnard's star: his methods are described in Aufdenberg, Hauschildt, & Baron (2002). The mean value of the ratio $\theta_{\rm LD}/\theta_{\rm UD}$ is found to be 1.026, so that $\theta_{\rm LD} = 1.013$ mas, and from equation (1), $T_{\rm eff} = 3134 \pm 102$ K. The corresponding stellar radius is $0.200 \pm 0.008\ R_\odot$.

### 3.3. *Mass and Metallicity*

Several empirical mass-luminosity relations can be found in the literature, including those by Henry & McCarthy (1993), Henry et al. (1999), and Delfosse et al. (2000). All of them indicate that the mass of Barnard's star is in the range $0.15\ M_\odot \leq M \leq 0.17\ M_\odot$. Delfosse et al. (2000) find a very tight correlation between mass and $K$-band luminosity, independent of metallicity. For Barnard's star, with $M_K = 8.19$, their mass-$M_K$ relation gives $M = 0.159\ M_\odot$: that value is adopted here. With $R = 0.200\ R_\odot$, it follows that $\log g = 5.04$. The metallicity is unknown, although there is nothing in the gross appearance of the spectrum to suggest that the star is notably metal-poor—Leggett et al. (1996) estimated $[{\rm M/H}] = -0.5$ from photometry alone. Tout et al. (1996) presented a set of fitting formulae based on a grid of zero-age main-sequence models computed using a code described by Pols et al. (1995). For a mass of $0.159\ M_\odot$, and an assumed $[{\rm M/H}] = 0.0$, these yield $R = 0.195\ R_\odot$, $L = 3.33 \times 10^{-3}\ L_\odot$, $T_{\rm eff} = 3138$ K, $M_{\rm bol} = 10.94$, and $\log g = 5.06$, in rather good agreement with the results obtained above. Because there is no way to reproduce these parameters with a model of lower metallicity, it would seem that Barnard's star has essentially solar metal abundance, its space motion notwithstanding. All the same, it is important to check whether these models, which employ a simplified equation of state, represent adequately the mean component of YY Gem, whose properties have been derived by Torres & Ribas (2002). Here, unfortunately, the agreement is much less satisfactory: a model of the appropriate mass and metallicity underestimates the radius by about 10% and overestimates the effective temperature by about 130 K. Even the models of Baraffe et al. (1998), based on more sophisticated input physics, do not fully reproduce the properties of Barnard's star: interpolation in their grid suggests that a star of $0.159\ M_\odot$ and solar metallicity has $T_{\rm eff} = 3180$ K, but a radius of $0.18\ R_\odot$ for any plausible age exceeding 1 Gyr. More metal-deficient models are, of course, too hot. In fact, Torres & Ribas (2002) showed that this general behavior is typical of the nine sets of models with which they compared the observations of YY Gem, commenting that "all models underestimate the radius by up to 20% and that most overestimate the effective temperature by 150 K or more." In the circumstances, the best approach to finding [M/H] is to compare directly the observed SED with synthetic spectra derived from NextGen model atmospheres (Hauschildt, Allard, & Baron 1999) computed for $T_{\rm eff} = 3100$ K and $\log g = 5.0$. Such comparisons are not entirely straightforward, because known deficiencies in those models, especially in the $B$ and $V$ bands, introduce complications. We find that there is little to choose between $[{\rm M/H}] = -0.3$ and $[{\rm M/H}] = 0.0$, with the latter weakly favored. Lower metallicities are clearly ruled out, but the possibility of nonsolar abundance ratios should be investigated before a final answer can be given.

## 4. DISCUSSION

The temperature derived above is a true effective temperature, and because it is based on direct observations of the relevant parameters, it is definitive. The uncertainty of just over 100 K is such that the effective temperature of Barnard's star is more precisely known than that of either component of YY Gem (Leung & Schneider 1978), and it is at least as well established as the effective temperatures of the CM Dra pair (Lacy 1977b; Chabrier & Baraffe 1995). It is consistent with the results of BRL, who obtained $T_{\rm eff} = 3150$ K for Barnard's star itself, and it is in excellent agreement with Bessell's (1995) temperature scale, according to which a dM4 star has $T_{\rm eff} = 3130$ K.

Reid & Gilmore (1984) arrived at $T_{\rm eff} = 3250$ K by calculating the total flux blueward of $K$ (neglecting the $U$ band) from the broadband photometry. Then, assuming (in line with the conclusions of Bopp, Gehrz, & Hackwell 1974), that for $\lambda > 2.2\ \mu$m the SED is essentially that of a blackbody,

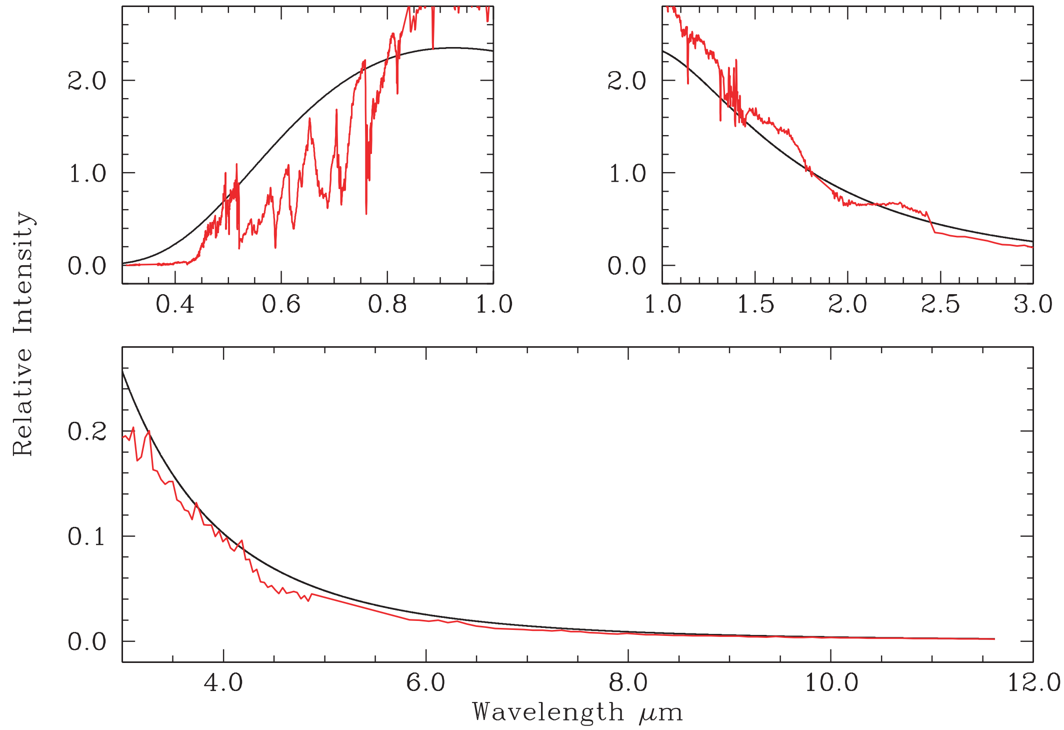


FIG. 1.—SED of Barnard's star in units of $10^{-11}$ W m$^{-2}$ $\mu$m$^{-1}$ (*red*) and a 3135 K blackbody (*black*) plotted in three frames: 0.3–1.0, 1.0–3.0, and 3.0–11.6 $\mu$m

they arrived at an estimate of the bolometric flux. Next, they normalized a Planck function to the observed monochromatic flux at $K$, while requiring that the area under the Planck curve equal the bolometric flux. In effect, their procedure yields two equations in the unknowns $\phi$ (the angular diameter) and $T_{\rm eff}$, which can then be solved for the desired quantities. The quality of the result depends on how well the bolometric flux can be estimated from photometry and upon the validity of the assumption that the blackbody flux density at 2.2 $\mu$m is equal, or nearly so, to the $K$-band flux density. In the case of Barnard's star at least, the latter assumption has proved to be very sound: the blackbody flux density at $K$ is within 3% of that observed, as can be seen from Figure 1, in which the flux density from a 3135 K blackbody is plotted over the stellar SED (the normalization is such that the areas under both curves are equal).

Later work by Berriman & Reid (1987) showed that the assumption of blackbody behavior of the SED beyond 2.2 $\mu$m is an oversimplification. Their spectroscopic observations in that wavelength region revealed the presence of strong $H_2O$ absorption in the 3 $\mu$m window in the spectra of the coolest dwarfs, and in all cases, the spectral energy distributions for $\lambda > 4$ $\mu$m were found to decline less steeply than $\lambda^{-4}$. Neglect of the steam bands near 3 $\mu$m results in an overestimate of the bolometric flux, and hence, in an overestimate of $T_{\rm eff}$ for the later types. Using their spectra to provide a better accounting of the energy distribution out to 4 $\mu$m, they arrived at a new estimate of 3100 K for the effective temperature of Barnard's star. Finally, with new photometry that, in the particular case of interest here, resulted in fairly significant reductions of the $V$, $J$, and $H$ magnitudes, BRL arrived at $T_{\rm eff} = 3150$ K.

It may seem that a more direct approach would be to simply fit a Planck function to the SED, with the constraint that the bolometric fluxes be equal. The fitting function is just

$$f_\lambda = \frac{a}{\lambda^5[\exp(b/\lambda) - 1]},$$

where

$$a = 2\pi hc^2 \frac{\mathcal{F}}{\sigma T_{\rm eff}^4}$$

and $b = hc/kT_{\rm eff}$; the symbols have their usual meanings. The fit should be carried out for $\lambda_{\rm min} > 4$ $\mu$m, where the stellar SED might reasonably be approximated by a Planck function. The data comprise $N$ data triplets ($\lambda_i$, $f_{\lambda,i}$, $\sigma_i$), where $\sigma_i$ is the uncertainty in $f_{\lambda,i}$, and the value of $N$ depends on the choice of $\lambda_{\rm min}$, the short-wavelength cutoff. We compute the value of $f_{\lambda,i}$ (fit) for each $\lambda_i$ and for values of $T_{\rm eff}$ ranging from, for example, 2500 to 3500 K. For each choice of $T_{\rm eff}$, we evaluate the merit function

$$\chi_\nu^2 = \frac{\chi^2}{\nu} = \frac{1}{\nu}\sum_{i=1}^{N} w_i[f_{\lambda,i} - f_{\lambda,i}(\text{fit})]^2$$

with weights $w_i$ given by $w_i = 1/\sigma_i^2$: the number of degrees of freedom $\nu = N - 1$.

The result of this exercise is unsatisfactory. For example, the choice $\lambda_{\rm min} = 4$ $\mu$m results in a minimum of $\chi_\nu^2$ for $T_{\rm eff} = 3360$ K, and no value of $\lambda_{\rm min} > 4$ $\mu$m yields an effective temperature sufficiently low to satisfy the constraint $T_{\rm eff} < 3176$ K, discussed above. This outcome is not surprising, given the observation by BRL that $f_\lambda$ falls off less steeply than $\lambda^{-4}$ for $\lambda > 4$ $\mu$m: forcing a fit to the shallower slope effectively moves the peak of the Planck function blueward, yielding a higher value of $T_{\rm eff}$.

### 4.1. *Application of the Barnes-Evans Relation*

Barnes & Evans (1976) drew attention to the tight correlation between the unreddened color $(V-R)_0$ (on the *UBVRI* system) of a star and a surface brightness parameter given by

$$F_V = 4.2207 - 0.1V_0 - 0.5 \log \phi' = \log T_{\rm eff} + 0.1C,$$

where $V_0$ is the unreddened apparent visual magnitude, $\phi'$ is the limb-darkened stellar angular diameter in milliarcseconds, and $C$ is the bolometric correction at $V$. Their result was founded in part on earlier work by Wesselink (1969), who found a similar relation between the surface brightness and the color index $(B-V)_0$ and applied it to the study of several stars covering a broad range of spectral types and luminosity classes. Noting that the utility of the index $(B-V)_0$ is highly dubious for $(B-V)_0 > 1.5$, Barnes & Evans claimed that the $F_V$, $(V-R)$ relationship is "well defined for the entire range of stellar temperatures, without dependence on luminosity class," although their calibrating sample, which comprised mainly late-type giants whose angular diameters had been measured at lunar occultations, included only a single main-sequence point cooler than the Sun—the mean component of the detached, eclipsing binary YY Gem, for which an angular diameter was derived from the results of a photometric study of that system by Kron (1952). Subsequently, Barnes, Evans, & Parsons (1976) studied the applicability of the $F_V$, $(V-R)_0$ relation to a sample of stars with spectral types from O5 to G2 and luminosity classes from V to Ia and confirmed that, for these early to intermediate types at least, the relation is valid independent of surface gravity. Further work by Barnes, Evans, & Moffett (1978) led to an improved calibration through the addition of 40 new stars to the original 52 star calibrating sample and strengthened the assertion "that the $F_V$, $(V-R)_0$ relation is independent of luminosity class and applicable to all spectral types O4–M8, S, and C."

According to Lacy (1977a), Barnard's star has $(V-R)_J = 1.83$, which transforms (Bessell 1983) to $(V-R)_C = 1.22$, in excellent agreement with the value given by Bessell (1990). At this color, the surface brightness parameter is given by Barnes et al. (1978) as

$$F_V = 3.841 - 0.321, \quad (V-R)_J = 3.254.$$

Since

$$F_V = \log T_{\rm eff} + 0.1C,$$

where $C = -2.34$ mag from above, then $T_{\rm eff} = 3073$ K.

Recently, Beuermann et al. (1999) demonstrated that the visual surface brightness of K and M giants and dwarfs with approximately solar metal abundances is not altogether insensitive to surface gravity. Specifically, they found that the visual surface brightness of early M dwarfs is higher than that of their giant counterparts, with a maximum difference reaching $0.30 \pm 0.09$ mag at type M0. However, the difference diminishes among later types and disappears altogether at M5 or so (although it may reverse sign at still later spectral types). This suggests that our use of the single relation given by Barnes et al. (1978) is not unreasonable, given the MK type of Barnard's star. In any case, Beuermann et al. (1999) present relations between the $K$-band surface brightness and the color indexes $V-K$ and $V-I_C$ and apply them to Barnard's star, obtaining $\log(R/R_\odot) = -0.72$ from the former and $-0.73$ from the latter. These correspond to $\phi' = 0.970$ mas and $\phi' = 0.948$ mas, respectively, and a mean value of $\phi' = 0.96$ mas. The corresponding effective temperature is 3220 K.

### 4.2. *The Infrared Flux Method*

The infrared flux method had its origins in work by Gray (1967, 1968), who used the energy conservation relation

$$R = r\sqrt{F_{\nu,\oplus}/F_{\nu,*}},$$

where $r$ is the distance to the star of interest, $R$ is its radius, and $F_{\nu,\oplus}$ and $F_{\nu,*}$ are, respectively, the monochromatic fluxes at Earth and at the star, to find the radii of several nearby stars with well-known parallaxes. The emergent flux is found through the use of a model atmosphere. Subsequently, Blackwell & Shallis (1977) argued that, in the infrared, the stellar surface flux is rather insensitive to the effective temperature, so that even a poor choice for the effective temperature of the model atmosphere will yield a good result for the angular diameter $\theta = 2R/r$. This can then be combined with the bolometric flux $\mathcal{F}$ to obtain an improved estimate for $T_{\rm eff}$ and hence an improved value of $\theta$ from the infrared flux. In principle, the process can be continued until convergence is achieved. We have applied the method to the infrared ($\lambda > 3$ $\mu$m) SED of Barnard's star, using smoothed synthetic spectra from NextGen model atmospheres to obtain the surface flux densities at the roughly 100 wavelengths at which the stellar SED is sampled in this range. Each wavelength point yields a value for the effective temperature: the mean then guides the choice of model atmosphere whose SED is used as input to the next iteration. Surprisingly, perhaps, our implementation of this procedure does not yield satisfactory results. Because the synthetic spectra generally well represent the stellar SED in the wavelength range of interest (see Fig. 2), we conclude that the difficulty is due to the temperature resolution of the NextGen model grid (100 K). A much simpler and more effective approach exploits the fact, noted above, that at 2.179 $\mu$m the stellar monochromatic surface flux density is well approximated by that of the blackbody whose temperature is equal to the star's effective temperature. The blackbody monochromatic flux density is easily computed, so that, with the apparent $K$ magnitude of Barnard's star and its bolometric flux $\mathcal{F}$, and starting from a first estimate $T_{\rm eff} = 3300$ K, the method converges to a final value of $T_{\rm eff} = 3091$ K in 14 iterations. This is identical, within the errors, to the result obtained earlier. Applying the technique to another well-observed M dwarf, Kapteyn's star, for which we estimate (using published photometry) $\mathcal{F} = 2.52 \times 10^{-11}$ W m$^{-2}$, we quickly arrive at $T_{\rm eff} = 3535$ K. The corresponding angular diameter is 0.696 mas, in good agreement with the value $0.692 \pm 0.06$ mas measured by Ségransan et al. (2003), using the Very Large Telescope Interferometer.

## 5. CONCLUSIONS

The effective temperature of Barnard's star is found to be $3134 \pm 102$ K; the precision of that result equals or surpasses that of the best temperature determinations for late-type dwarfs in detached, eclipsing binary systems. Because such systems are very rare, it appears that their utility will diminish as instruments capable of resolving relatively bright,

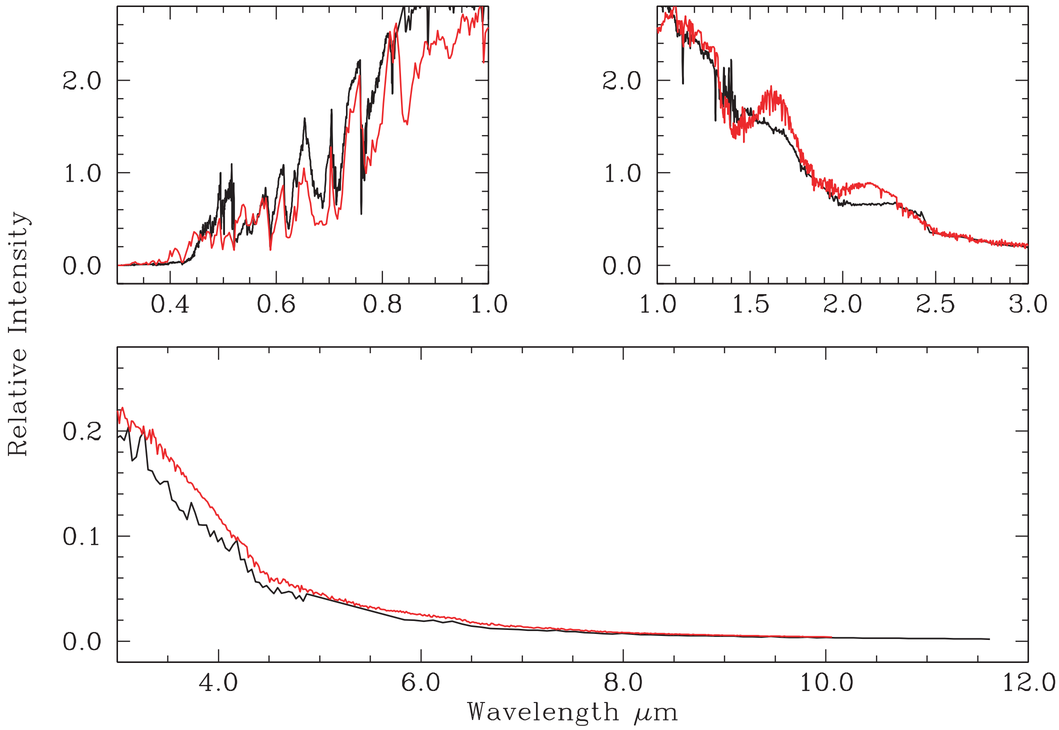


Fig. 2.—SED of Barnard's star in units of $10^{-11}$ W m$^{-2}$ $\mu$m$^{-1}$ (*black*) and a 3100 K, log $g = 5.0$, and solar metallicity model taken from Hauschildt et al. (1999; *red*) in the same wavelength regions as Fig. 1. The synthetic model was smoothed using a 15-point moving average to more closely approximate the resolution of the stellar data at near-infrared wavelengths.

nearby M dwarfs are brought to bear on the problem. The effective temperature found here agrees well with the results of BRL and with Bessell's (1995) calibration. Barnard's star appears, perhaps somewhat surprisingly, given its space motion, to have near-solar metal abundance: this follows not only from a comparison of its SED with synthetic spectra but also from the fact that more metal-deficient dM models fail to adequately predict the gross properties of the star. Both the Barnes-Evans relation and the infrared flux method seem capable of yielding good estimates of dM angular diameters and hence of the effective temperatures if the bolometric fluxes are known. There is, however, evidence that even the most sophisticated models available underestimate the radius of Barnard's star—a quiescent, isolated object; this is yet another example of a chronic problem that merits theoretical attention.

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