The Astrophysical Journal, 604:108–115, 2004 March 20

THE EVOLUTION OF DIFFUSE RADIO SOURCES IN GALAXY CLUSTERS

Ping-Hung Kuo, Chorng-Yuan Hwang, and Wing-Huen Ip
Institute of Astronomy, National Central University, Chung-Li 32054, Taiwan;
d882001@astro.ncu.edu.tw, hwangcy@astro.ncu.edu.tw, wingip@astro.ncu.edu.tw
Received 2003 February 19; accepted 2003 December 2

ABSTRACT

We investigate the evolution and number distribution of radio halos in galaxy clusters. Without reacceleration or regeneration, the relativistic electrons responsible for the diffuse radio emission will lose their energy via inverse Compton and synchrotron losses in a rather short time, and radio halos will have lifetimes ~0.1 Gyr. Radio halos could last for ~Gyr if a significant level of reacceleration is involved. The lifetimes of radio halos would be comparable with the cosmological time if the radio-emitting electrons are mainly the secondary electrons generated by pion decay following proton-proton collisions between cosmic-ray protons and the thermal intracluster medium within the galaxy clusters. Adopting both observational and theoretical constraints for the formation of radio halos, we calculate the formation rates and the comoving number density of radio halos in the hierarchical clustering scheme. Comparing with observations, we find that the lifetimes of radio halos are ~Gyr. Our results indicate that a significant level of reacceleration is necessary for the observed radio halos and the secondary electrons may not be a dominant origin for radio halos.

Subject headings: cosmology: theory — galaxies: clusters: general — radio continuum: general

1. INTRODUCTION

Diffuse radio emission from galaxy clusters is a rare phenomenon. These radio sources, which usually possess large sizes and steep spectra, are called radio halos if they permeate the cluster centers and radio relics if they are located in cluster peripheral regions. Observations found that radio halos exist only in the clusters that show X-ray substructures (Feretti 2000). Since a galaxy cluster possessing X-ray substructures indicates that it is under ongoing merging, it is expected that the formation of radio halos is closely related to the merging process of galaxy clusters.

The diffuse radio emission of galaxy clusters is believed to be produced by the synchrotron radiation of relativistic electrons. Nonetheless, the sources of these relativistic electrons are still unclear. Cluster merging is a very violent event and releases a large amount of energy ($\sim 10^{64}$ ergs); this leads cluster mergers to be a very favorable mechanism for the production of the relativistic particles. However, relativistic electrons lose energy on the timescale of order $\sim 10^8$ yr because of inverse Compton and synchrotron losses; this suggests that without reacceleration radio halos in galaxy clusters might be transient features associated with a major merger and would have lifetimes ~0.1 Gyr (De Young 1992; Tribble 1993).

The numerical simulations of cluster mergers (e.g., Miniati et al. 2000; Ricker & Sarazin 2001) showed that the intracluster medium (ICM) is seriously disturbed by merging. Violent turbulence generated by mergers must play an important role in the reacceleration of relativistic electrons (Sarazin 2001). Considering reacceleration for the relativistic electrons, a two-phase model proposed by Brunetti et al. (2001) successfully reproduces the radial steepening of the spectral index, the radio spectrum steepening at high frequencies, and the hard X-ray excess in the Coma Cluster (see also Schlickeiser, Sievers, & Thiemann 1987; Liang, Dogiel, & Birkinshaw 2002). In a similar study, Kuo, Hwang, & Ip (2003) showed that the "age" of Coma C might be ~1 Gyr; this indicates that the lifetimes of radio halos could be ~Gyr if a significant level of reacceleration is involved.

The secondary electron model first proposed by Dennison (1980) assumes that relativistic electrons are produced from the pion decay following collisions between the cosmic-ray protons and the thermal ions of the ICM. It has been recognized that the diffusion time of cosmic-ray protons is comparable with the cosmological time, so cosmic-ray protons are confined within galaxy clusters for the lifetimes of the clusters (Völk, Aharonian, & Breitschwerdt 1996; Berezinsky, Blasi, & Ptuskin 1997; Schlickeiser et al. 1987). If radio halos are formed from the secondary electrons (e.g., Blasi & Colafrancesco 1999; Miniati et al. 2001a), their lifetimes would be comparable with the cosmological time. The significantly different timescales of radio halos could have discernible effects on their number distribution and thus could discriminate on the origins of radio halos.

The formation rates of radio halos in galaxy clusters can be estimated from the hierarchical model. Press & Schechter (1974, hereafter PS) derived a mass function to evaluate the comoving number density of bound virialized objects, but this function does not specify the formation epochs of the objects. To solve this problem, Lacey & Cole (1993, hereafter LC) derived a distribution function of formation epochs by using the merger probabilities in the framework of PS formalism (see also Bond et al. 1991; Bower 1991). Based on the formalism of LC, Kitayama & Suto (1996a, hereafter KS) proposed another distribution function of formation epochs in a similar but slightly different manner. We follow the formalism of KS but modify it to suit the situation for forming radio halos.

In this paper we investigate the evolution and number distribution of radio halos with different lifetimescales. Three typical lifetimes that are representatives of different origins for radio halos are considered (i.e., 0.1 Gyr, 1 Gyr, and the cosmological time). We estimate the different number distributions and compare the results with observations to determine the valid models that are responsible for the origin of radio halos.

This paper is planned as follows. In § 2 we discuss the conditions for radio halos forming in galaxy clusters. In § 3 we describe the methods based on the formalism proposed by KS for calculating the formation rates and comoving number density of radio halos. In § 4 we compare the modeling results of radio halos with different lifetimes with observations, and in § 5 we give our discussion and conclusions.

2. FORMATION CRITERIA

Radio halos are not found in low X-ray luminosity clusters and are present only in massive clusters with high X-ray luminosity and high temperature (Giovannini, Tordi, & Feretti 1999). This fact indicates that there might be a threshold mass for galaxy clusters to form radio halos. It has been recognized that radio halos are strongly correlated with cluster mergers (Feretti 2000). Nonetheless, many cluster mergers do not possess radio halos. Buote (2001) studied the dynamical states of clusters possessing radio halos and found that radio halos form preferentially in massive clusters experiencing violent mergers that have seriously disrupted the cluster core. Disrupting the cluster core in the merging process should be an important factor for forming radio halos. According to above discussion, we assume two criteria for a cluster merger to form a radio halo: (1) the cluster mass must be greater than or equal to a threshold mass and (2) the merging process must be violent enough to disrupt the cluster core. Under these two conditions, mergers might generate sufficient primary electrons or secondary electrons from cosmic-ray protons to form radio halos.

We use observational data (Giovannini et al. 1999; Feretti 2000; Kempner & Sarazin 2001) to determine the threshold mass. We found that the A548b cluster has the lowest temperature $\sim$2.4 keV (Giovannini et al. 1999), which is presumed to have the smallest mass from the well-known mass-temperature relation for galaxy clusters (e.g., Mulchaey 2000; Rosati, Borgani, & Norman 2002). Using the observed mass-temperature relationship (Evrard, Metzler, & Navarro 1996; Horner, Mushotzky, & Scharf 1999)

$$M = 5 \times 10^{13}\ h^{-1} \left(\frac{T_{\rm X}}{1\ \rm keV} \right)^{1.5} M_\odot, \qquad (1)$$

we found that the A548b cluster has a mass of $\approx 2.7 \times 10^{14}\ M_\odot$ for $h = 0.7$ or $3.7 \times 10^{14}\ M_\odot$ for $h = 0.5$. Since very few clusters with lower temperatures have been found to possess radio halos, the threshold mass may be on the order of $\sim 10^{14}\ M_\odot$. We choose $10^{14}\ M_\odot$ to be the threshold mass $M_{\rm th}$ for cluster mergers to form radio halos. For comparison, we also consider different $M_{\rm th}$ (5×10^{13} and $5 \times 10^{14}\ M_\odot$) in our calculation.

The condition for the disruption of cluster cores have been determined from dynamical simulation. Salvador-Solé, Solanes, & Manrique (1998) found that a cluster with mass M experiencing a merging process would disrupt its core structure when the relative mass increase $\triangle M/M$ exceeds a certain threshold $\triangle_m \equiv (\triangle M/M)_{\rm threshold} = 0.6$. We use these two conditions, $M_{\rm th} = 10^{14}\ M_\odot$ and $\triangle_m = 0.6$, as the criteria for cluster mergers to form radio halos.

3. FORMULATION

3.1. *Formation Rates*

The Press-Schechter mass function (PS) is used to model the cluster number density and its evolution. The comoving number density of clusters in the mass range $M \sim M + dM$ at time t is given by

$$n_{\rm ps}(M, t)dM = \sqrt{\frac{2}{\pi}} \frac{\rho_0}{M} \frac{\delta_c(t)}{\sigma^2(M)} \left| \frac{d\sigma(M)}{dM} \right| \exp\left[-\frac{\delta_c^2(t)}{2\sigma^2(M)} \right] dM, \qquad (2)$$

where ρ_0 is the present mean density of the universe, $\delta_c(t)$ is the critical density threshold for a spherical perturbation to collapse by the time t, and $\sigma(M)$ is the present rms density fluctuation smoothed over a region of mass M. We adopt the expressions of $\delta_c(t)$ summarized in Randall & Sarazin (2001) for different cosmological models. For $\sigma(M)$, we use an approximate formula proposed by Kitayama & Suto (1996b) for the cold dark model fluctuation spectrum and choose the value of σ_8 from the Ω_0-σ_8 constraint derived from the present cluster abundance: $\sigma_8 \Omega_0^{0.45} = 0.53$ (for $\Omega_\Lambda = 0$) and $\sigma_8 \Omega_0^{0.53} = 0.53$ (for $\Omega_0 + \Omega_\Lambda = 1$) (Pen 1998). The parameter $\Omega_0 \equiv \rho_0/\rho_c$ is the ratio of the present mean density to the critical density $\rho_c = 3H_0^2/(8\pi G)$ and $\Omega_\Lambda \equiv \Lambda/(3H_0^2)$, where Λ is the cosmological constant. The Hubble constant is defined as $H_0 = 100\ h$ km s^{-1} Mpc^{-1}. Three different cold dark matter (CDM) models are considered in our analysis: a standard model (SCDM) ($\Omega_0 = 1$, $\Omega_\Lambda = 0$, $h = 0.5$, and $\Gamma = 0.5$), an open model (OCDM) ($\Omega_0 = 0.3$, $\Omega_\Lambda = 0$, $h = 0.7$, and $\Gamma = 0.2$), and a low-density flat model (ΛCDM) ($\Omega_0 = 0.3$, $\Omega_\Lambda = 0.7$, $h = 0.7$, and $\Gamma = 0.2$), where Γ is the shape parameter defined by Sugiyama (1995).

The formation rate for an object with mass M_f ($M_f > M_i$) formed from initial mass between M_i and $M_i + dM_i$ in unit time at t is given by (LC; KS)

$$r(M_i \to M_f; t)dM_i \equiv \frac{1}{\sqrt{2\pi}} \frac{1}{[\sigma^2(M_i) - \sigma^2(M_f)]^{3/2}} \times \left[-\frac{d\delta_c(t)}{dt} \right] \left| \frac{d\sigma^2(M_i)}{dM_i} \right| dM_i. \qquad (3)$$

As mentioned in § 2, for a cluster merger with mass greater than the threshold mass $M_{\rm th}$ to form a radio halo, the cores of the progenitor subclusters must have been disrupted in the merging process. Assume that a cluster of mass M ($M \geq M_{\rm th}$) is formed from the merging process of two subclusters M_1 and M_2, where $M_1 \geq M_2$. For M_1 to have significant core disruption during the merging process, values of M_2 have to be greater than some threshold mass, $M_2 \geq \triangle_m M_1$, according to Salvador-Solé et al. (1998). Since $M_1 \geq M_2$ and $M = M_1 + M_2 \geq (1 + \triangle_m) M_1$, we obtain the mass range of M_1: $M/2 \leq M_1 \leq M/(1 + \triangle_m)$. The quantity M_2 can be the accumulated mass of the merged subclusters in multiple merging. The formation rate of radio halos in cluster mergers with mass $M \geq M_{\rm th}$ at time t is given by

$$\begin{aligned} R_f(M, t) &= \int_{M_a}^{M_b} r(M' \to M; t)dM', \\ &= \sqrt{\frac{2}{\pi}} \left[-\frac{d\delta_c(t)}{dt} \right] \\ &\times \left[\frac{1}{[\sigma^2(M_b) - \sigma^2(M)]^{1/2}} - \frac{1}{[\sigma^2(M_a) - \sigma^2(M)]^{1/2}} \right], \end{aligned} \qquad (4)$$

where $M_a = M/2$, and $M_b = M/(1 + \triangle_m)$.

3.2. *Cumulative Comoving Number Density*

The comoving number density of radio halos that form with cluster mass $M \sim M + dM$ at time $t_f \sim t_f + dt_f$ and survive until a latter time t is

$$n_{\rm rh}(M, t_f, t)\, dM\, dt_f = \begin{cases} n_{\rm ps}(M, t_f) R_f(M, t_f) P_s(M, t_f, t)\, dM\, dt_f & \text{if } t_f \le t \le t_f + t_{\rm rh}, \\ 0 & \text{if } t > t_f + t_{\rm rh}, \end{cases} \quad (5)$$

where P_s is the survival probability defined by KS,

$$P_s(M, t_1, t_2) = P(M' < (1 + \triangle_m)M, t_2 | M, t_1), \quad (6)$$

which is the probability that a cluster merger of mass M at t_1 will survive to have mass M' less than $(1 + \triangle_m)M$ at t_2, and $t_{\rm rh}$ is the lifetime of radio halos. The lifetimes of radio halos may be slightly different from one another, but we ignore the deviation of the lifetime in calculation for simplicity. A survival merger increases its mass only by accretion without disrupting its core structure during the lifetime of its radio halo. We note that a radio halo might survive a new core-disruption merger even its host cluster is destroyed; the survival radio halo is treated as a new radio halo possessed by the new merger in our scheme.

The cumulative comoving number density of radio halos with cluster mass $\ge M (\ge M_{\rm th})$ at time t can be evaluated as

$$\begin{aligned} n_{\rm rh}(\ge M, t) = & \int_M^\infty \int_{t - t_{\rm rh}}^t n_{\rm rh}(M', t_f, t)\, dM'\, dt_f \\ & + \int_{M_{\rm L}}^M \int_{t - t_{\rm rh}}^t n_{\rm ps}(M', t_f) R_f(M', t_f) \\ & \times P(M \le M'' < (1 + \triangle_m)M', t | M', t_f)\, dM' dt_f, \end{aligned} \quad (7)$$

where

$$P(M \le M'' < (1 + \triangle_m)M', t | M', t_f) = P(M'' \ge M, t | M', t_f) - P(M'' \ge (1 + \triangle_m)M', t | M', t_f),$$

and

$$M_L = \begin{cases} M/(1 + \triangle_m) & M > (1 + \triangle_m)M_{\rm th}, \\ M_{\rm th} & M_{\rm th} \le M \le (1 + \triangle_m)M_{\rm th}. \end{cases}$$

The symbol M' is the cluster mass at t_f and M'' the cluster mass at t. If $t_{\rm rh}$ equals the cosmological time, the term $t - t_{\rm rh}$ in the time integral is replaced by 0. The second term in the right-hand side of equation (7) represents the number density of radio halos that form with cluster mass less than M but increase their mass to greater than M by accretion at the time t.

4. RESULTS AND COMPARISONS WITH OBSERVATIONS

The evolution of the total comoving number density of radio halos is shown in Figure 1. We note that there is a maximum at $z \sim 0.4\text{–}0.5$ in the evolution of the total number density of radio halos for clusters with mass $M \ge 10^{14} M_\odot$ with $t_{\rm rh} = 0.1$ and 1 Gyr in the ΛCDM and OCDM models; there is no such distribution maximum if we consider only the clusters with mass $M \ge 10^{15}\ M_\odot$. This may represent that before the period, the formation of many clusters with mass $\ge 10^{14}\ M_\odot$ are significantly due to violent merger of subclusters with mass less than $10^{14}\ M_\odot$; after the period, the violent-merger rates are very small and the formation of the clusters with mass $\ge 10^{14}\ M_\odot$ are mainly due to accretion of subclusters with mass less than $10^{14}\ M_\odot$. But many massive clusters ($M \ge 10^{15}\ M_\odot$) are still mainly due to violent merger of subclusters.

Giovannini et al. (1999) searched radio halo candidates in the NRAO VLA Sky Survey from a sample of X-ray–bright

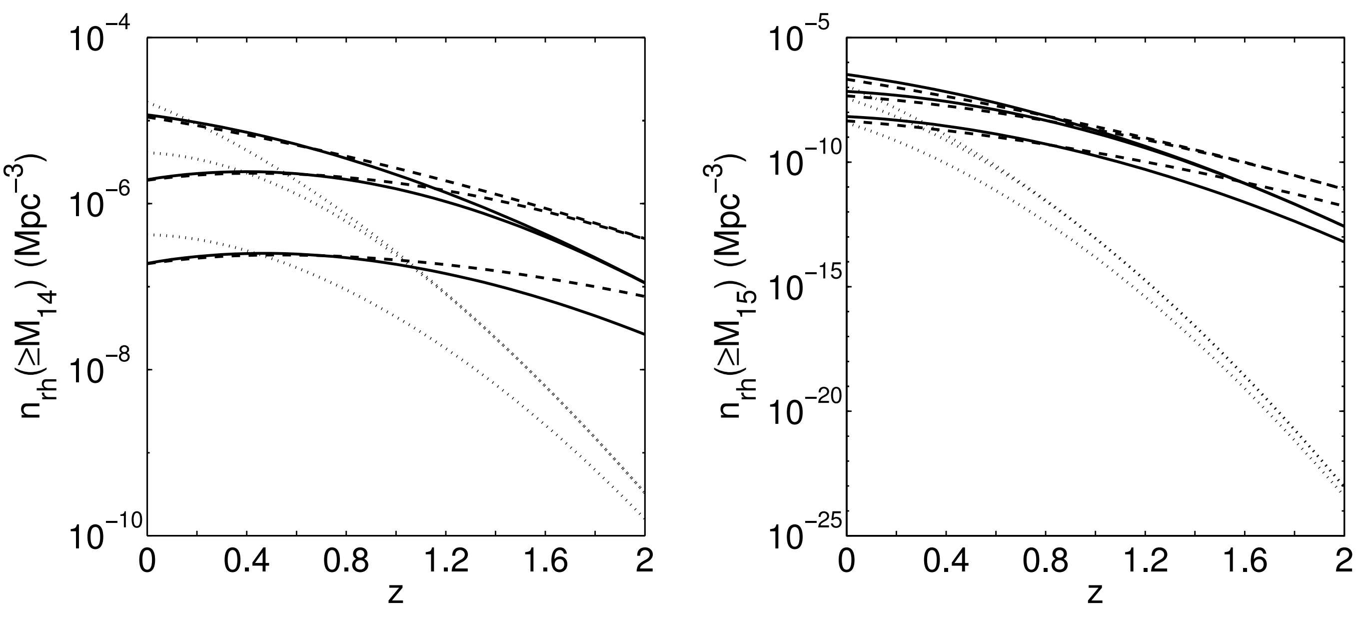

Fig. 1.—(*a*) Evolution of the total number density of radio halos with cluster mass $M \ge M_{14}$, where $M_{14} = 10^{14}\ M_\odot$. Three different cosmological models, SCDM (*dotted curves*), OCDM (*dashed curves*), and ΛCDM (*solid curves*) with three representative lifetimes of radio halos: 0.1 and 1 Gyr, and the cosmological time (*bottom to top*) are shown. (*b*) Same as (*a*), but for $M \ge 10^{15}\ M_\odot$.

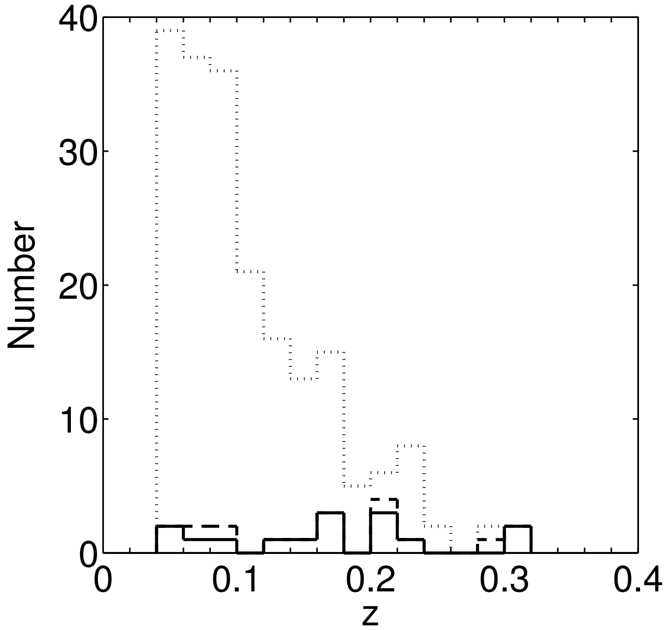

FIG. 2.—Distributions of the X-ray clusters inspected (*dotted line*) and radio halos detected (*solid and dashed lines*) by Giovannini et al. (1999). The dashed line includes halo sources that have uncertainty in their detection.

clusters presented by Ebeling et al. (1996). They found that the percentage of galaxy clusters possessing diffuse radio sources is 6%–9% for $L_X \leq 10^{45}$ ergs s^{-1} and 27%–44% for $L_X > 10^{45}$ ergs s^{-1}. Here L_X is the luminosity in the 0.1–2.4 keV energy band. The redshift distributions of radio halos and the parent X-ray cluster population inspected by Giovannini et al. (1999) are shown in Figure 2. In this figure we have corrected two uncertain halos (A754 and A2219) as confirmed ones and eliminated the uncertain halo in A2390, which is found to be a minihalo (Bacchi et al. 2003). For $L_X > 10^{45}$ ergs s^{-1}, the corrected percentage of galaxy clusters possessing diffuse radio sources is 28%–41%.

In Tables 1 and 2 we show the number density ratios of radio halos to clusters in different cosmological models with different lifetimes of radio halos. To compare with the observations, we divide the mass of clusters possessing radio halos into two ranges, $M_{\rm th} \leq M < 10^{15}$ and $M \geq 10^{15}$ $M_\odot$. The mass $M \sim 10^{15}$ $M_\odot$ roughly corresponds to the luminosity $L_X \sim 10^{45}$ ergs s^{-1} (e.g., Rosati et al. 2002). We note that the ratios are a function of redshift and the percentages in Tables 1 and 2 are estimated at low $z \leq 0.4$ so that they can be compared with the observations. Obviously, the theoretical percentages of radio halos with $t_{\rm rh} = 0.1$ Gyr are much lower than the observational results for $L_X > 10^{45}$ ergs s^{-1}. In other words, we would expect to observe many fewer radio halos in high X-ray luminosity clusters if radio halos were transient phenomena with lifetimes $\sim$0.1 Gyr. On the other hand, the expected percentages are much higher than observational results for those radio halos with $t_{\rm rh}$ = cosmological time; we would have observed much more radio halos if radio halos had the cosmological lifetime. We find that only radio halos with $t_{\rm rh} = 1$ Gyr can produce results roughly matching the observations in two mass ranges. Because of the limits of present instruments, the detection of radio halos in high-luminosity clusters is easier than that in low-luminosity ones. Thus the percentage of radio halos for $L_X > 10^{45}$ ergs s^{-1}, 28%–38%, may be more robust than that for $L_X \leq 10^{45}$ ergs s^{-1}. We note that different cosmological models have some effects on the calculated ratios, but the effects are too small to cause confusion.

In Figures 3, 4, and 5 we show the redshift distributions of the ratios of radio halos to the galaxy clusters. It is obvious that in all three different cosmological models only the results with the lifetime $t_{\rm rh} = 1$ Gyr roughly fit the observations at $z \leq 0.18$. We note that the number of the cluster sample for $z > 0.18$ is too small, and this may result in a large deviation. In particular, people tend to observe only high luminous clusters at high redshifts, and the ratio of radio halos is higher in high luminous clusters; the high ratios observed at $z > 0.18$ thus do not represent the real ratio of radio halos to the total clusters.

In Figure 6 we show the evolution of the ratios up to $z = 2$. For simplicity, we only show the results of the ΛCDM models; other cosmological models show similar results. We find that the ratios of the model with cosmological lifetime always increase, the model of 1 Gyr lifetime show a distribution peak at $z = 1.6$, and the ratios of the 0.1 Gyr models decrease as the universe evolves. These results can also be used as an indicator of the origin of the radio sources if we have precise measurements and statistics for clusters at high redshifts.

To investigate the effects of the threshold mass $M_{\rm th}$, we have adopted different $M_{\rm th}$ on our models. In Table 3 we show the ratios of radio halos to the galaxy clusters in the mass range $M_{\rm th} \leq M < 10^{15}$ $M_\odot$ for $M_{\rm th} = 5 \times 10^{13}$ $M_\odot$ and 5×10^{14} $M_\odot$ at $z \leq 0.4$. The ratios for clusters with mass $\geq 10^{15}$ $M_\odot$ are not affected by the low-mass threshold. We find that the ratio distributions of $M_{\rm th} = 5 \times 10^{13}$ and 5×10^{14} $M_\odot$ are not very different from that of $M_{\rm th} = 10^{14}$ $M_\odot$.

5. DISCUSSION AND CONCLUSIONS

The distribution of galaxy clusters possessing radio halos provides a strong constraint on the origins of radio halos. About 28%–38% of clusters with $L_X > 10^{45}$ ergs s^{-1} possess radio halos. Since these X-ray–luminous clusters are generally very massive and have undergone multiple mergers during their formation, it is expected that these massive clusters would have accumulated a large amount of cosmic-ray protons from their formation history (e.g., Miniati et al. 2001b). The cosmic-ray protons would be confined in the clusters longer than the cosmological time, and the lifetimes of radio halos would be comparable with the cosmological time in the secondary electron model. If the secondary electron model were applicable, most of these massive clusters should possess radio halos; our results show that the percentage of clusters possessing radio halos should be greater than 70%. This is inconsistent with the observations, in which only $\sim$35% of these massive clusters possess radio halos. According to these results, the secondary electrons do not seem to be the dominant origin of the radio halos.

On the other hand, if radio halos were transient phenomena associated with a single acceleration event, such as a major merger shock, they would have lifetimes $\sim$0.1 Gyr. Because

TABLE 1
THE RATIOS OF GALAXY CLUSTERS CONTAINING DIFFUSE RADIO SOURCES: OBSERVATIONS

Observations	$L_X \leq 10^{45}$ ergs s^{-1} (%)	$L_X > 10^{45}$ ergs s^{-1} (%)
Halos	$\sim$4	$\sim$28–38
Halos+relics	$\sim$6–9	$\sim$28–41

TABLE 2

THE RATIOS OF GALAXY CLUSTERS CONTAINING DIFFUSE RADIO SOURCES: MODELS

Models	$M_{th} \leq M < 10^{15}\ M_\odot$ SCDM (%)	OCDM (%)	ΛCDM (%)	$M \geq 10^{15}\ M_\odot$ SCDM (%)	OCDM (%)	ΛCDM (%)
0.1 Gyr	~2	~1	~1	~4	~2	~2
1 Gyr	~14	~10	~9	~38	~24	~21
Cosmological time	~38	~40	~37	~80	~77	~70

of the short lifetimes of the sources, radio halos would be hardly observable even in the massive clusters. The observed percentage ~35% is thus too high to explain in the hierarchical clustering formation model.

According to the results presented in § 4, the lifetimes of radio halos may be ~Gyr. As mentioned in § 1, relativistic electrons in the ICM lose energy on the timescale of order ~10^8 yr because of the inverse Compton and synchrotron losses. This indicates that a significant level of reacceleration is necessary to support the relativistic electrons against radiative losses and to maintain radio halos to last for ~Gyr (Brunetti et al. 2001; Kuo et al. 2003).

As discussed in § 4, the percentage of radio halos for $L_X > 10^{45}$ ergs s^{-1}, ~28%–38%, is more robust. We here investigate the effects of the lifetime of the radio halos, the dividing mass, and the threshold mass ratio $\triangle_m$ on our results. Only the ΛCDM model is considered. First, the lifetimes of radio halos can affect the ratios of the radio halos. The percentage of radio halos with $t_{rh} = 1$ Gyr is ~21% and seems to be lower than the observational results, ~28%–38%, as shown in Tables 1 and 2. However, we note that the lifetime of the radio halo is of ~Gyr and could be slightly longer or shorter than 1 Gyr, and the difference of the assumed lifetime could affect the predicted percentage of the radio halo. For example, if a longer lifetime for the radio halo, $t_{rh} = 1.5$ Gyr, is assumed, then the percentage of the radio halo would become ~32% and would be in agreement with the observational results. Second, the dividing mass at $M = 10^{15}\ M_\odot$ may be lower than the realistic mass corresponding to $L_X = 10^{45}$ ergs s^{-1}. For example, the A773 cluster with $L_X \sim 1.01 \times 10^{45}$ ergs s^{-1} has mass in the range $1.25-2.08 \times 10^{15}\ M_\odot$ (Govoni et al. 2001). In Table 4 we show the percentages corresponding to different dividing masses. Obviously, the results with $t_{rh} = 1$ Gyr are improved, but those with t_{rh} = cosmological time are worse if a higher and more realistic dividing mass is taken. Third, different threshold $\triangle_m$ also affect our results. The results with different $\triangle_m$ are shown in Table 5. The values of $\triangle_m$ strongly affect the results for $t_{rh} = 1$ Gyr and t_{rh} = cosmological time. For $\triangle_m = 0.7$, the results with t_{rh} = cosmological time seems to be close to the observational results; this implies that the radio halos would only be generated in the mergers with two nearly equal-mass progenitors if the secondary electrons were the dominant origin for forming radio halos. For $\triangle_m = 0.5$, the ratio of the radio halos with $t_{rh} = 1$ Gyr is consistent with the observational results. Obviously, the $\triangle_m$ is a dominant factor in the determining the ratio of the radio halos. Thus it is important to investigate in detail the exact value of $\triangle_m$ to clarify the question.

Note that we have assumed that a merger with mass greater than a threshold mass will form a radio halo if its relative mass increase exceeds a threshold, $\triangle_m$. It might be possible that some mergers that satisfy our criteria did not form radio halos. However, according to the study of Buote (2001) for ~30

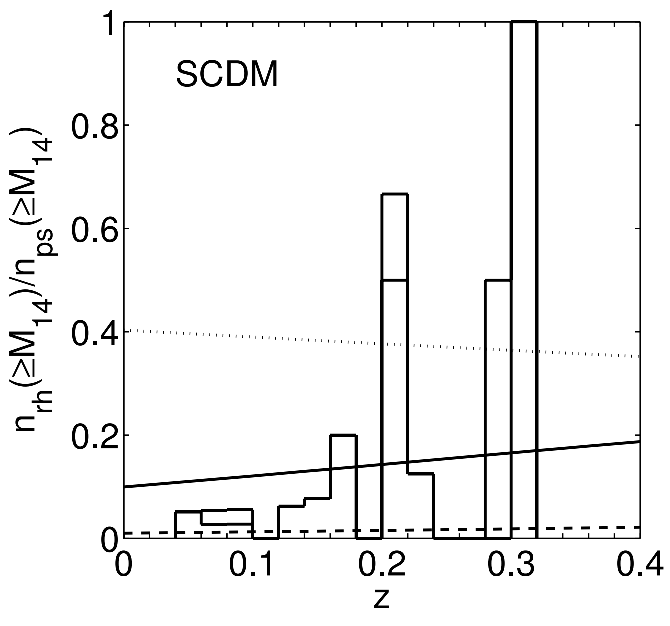

FIG. 3.—Distributions of the ratios of the total number density of radio halos to that of galaxy clusters in the SCDM model. The results for three representative lifetimes of radio halos: 0.1 Gyr (*dashed curves*), 1 Gyr (*solid curves*), and the cosmological time (*dotted curves*) are shown. The histograms show the observational results of Fig. 2; the higher histogram includes the uncertain halos.

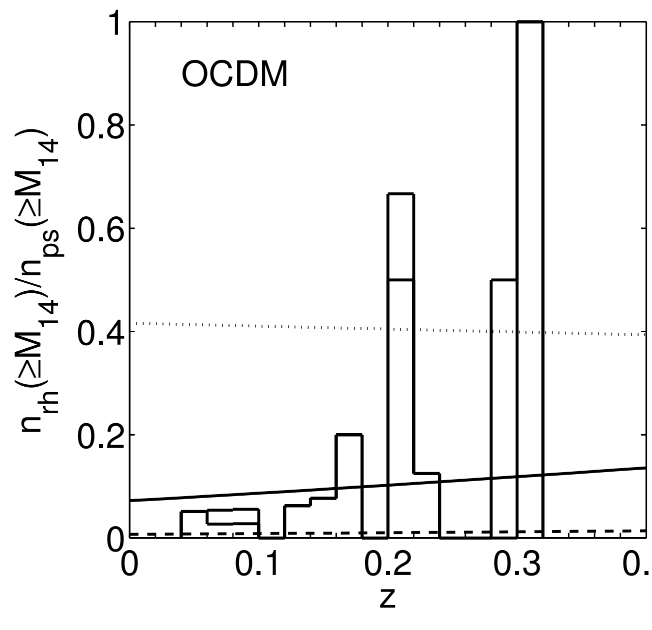

FIG. 4.—Same as Fig. 3, but for the OCDM model

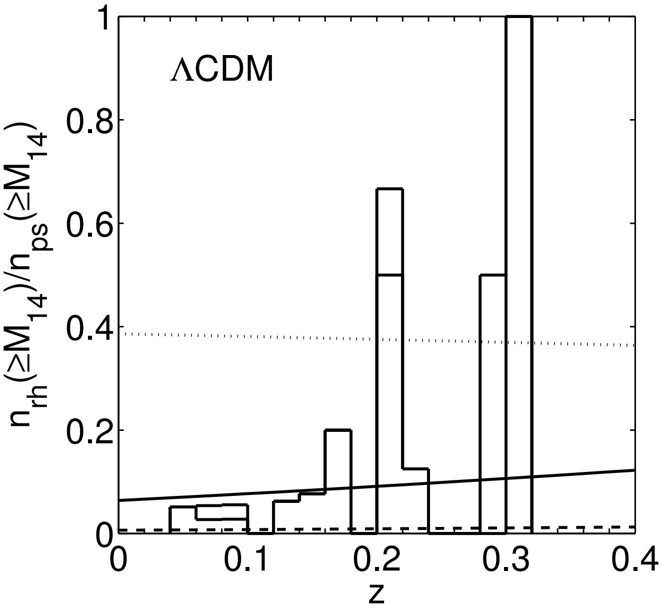

FIG. 5.—Same as Fig. 3, but for the ΛCDM model

bright X-ray clusters, the number of the unaccounted mergers should be very small and thus have little effect on our results.

Magnetic fields could be also an important parameter in determining the radio powers of the cluster radio halos. It is very difficult and complicated to determine this parameter. Magnetic fields $\leq 0.4\ \mu$G were found for several clusters using the inverse Compton models for the hard X-ray excess (Fusco-Femiano et al. 2003). Clarke, Kronberg, & Böhringer (2001) found that the cluster fields are typically around 5–10 μG using rotation measurements. Furthermore, during a cluster merging process, the magnetic fields might be amplified by a factor of $\sim$20 on small scales (Roettiger, Stone, & Burns 1999) and might have a nonnegligible effect on the radio emission. However, we note that both the magnetic field energy and the relativistic particle energy are provided by the merging energy, which is a function of the merger mass. In other words, the cluster mass should be a more fundamental parameter in determining the radio powers of the radio halos (Buote 2001; Govoni et al. 2001). Since we have considered the merger mass as a main parameter in our calculation, we thus do not treat the magnetic field as an independent parameter.

We could try to match the results obtained from the secondary electron model ($\sim$70%) with the observations ($\sim$35%) by assuming that about half of the radio halos produced by the secondary electrons are unobservable. However, Miniati et al. (2001a) showed that for a cluster of given mass the radio power from the secondary electrons could vary by almost an order of magnitude in their simulation. We note that this level of variation, as also detected in observation (Bacchi et al. 2003), is not large enough to make a detectable radio halo source to become nondetectable, especially for high X-ray luminosity clusters at low redshifts. It is thus unlikely to apply some observational effects to lower the radio halo fraction of the secondary electron model to match the observational results.

The radio halo fraction of galaxy clusters have been estimated in some recent studies. Fujita & Sarazin (2001) estimated the fraction of cluster radio halos based on the radiative energy loss timescale of the relativistic electrons. This timescale is similar to our case for primary electrons without reacceleration and can account for only $\sim$10% of observations. To match the observations, they assumed that even a rather weak merger should also trigger a radio halo. This assumption seems to be in contradiction with the observations, which showed that only massive clusters experiencing violent mergers can have halos (Buote 2001), as we have discussed in § 2. Ensslin & Röttgering (2002) estimated the cluster radio halo luminosity function by assuming a radio halo fraction $f_{\rm rh} = \frac{1}{4}$ for all clusters. However, observations showed that the halo fraction is very different for low- and high-luminosity X-ray clusters, and it is inconsistent with current observations to assume a constant radio halo fraction for all clusters. Giovannini et al. (1999) have noted that the lack of diffuse radio sources in low X-ray luminosity clusters is real because of their low redshifts; Bacchi et al. (2003) have also stressed that the correlation between the halo radio power and the cluster X-ray luminosity is applied only to clusters showing major mergers and therefore cannot be generalized to all clusters. The other more realistic fraction adopted by Ensslin & Röttgering (2002) is estimated by assuming that a cluster can possess a radio halo if its mass increases by more than 40% of its present mass within half a dynamical timescale. In their work, the fraction for clusters with mass of $10^{15}\ M_\odot$ is 0.32. This seems to agree with the observational results for $L_X > 10^{45}$ ergs s^{-1}. However, the fraction for clusters with mass of $10^{14}\ M_\odot$ and that for clusters with mass of $10^{13}\ M_\odot$ are 0.26 and 0.22, respectively. These results indicate that the radio halo ratio for $L_X \leq 10^{45}$ ergs s^{-1} should be greater than 20% and therefore are in conflict with the observations. We also note that the effects of the secondary electron model on the radio halo fraction were not considered in previous studies.

Our results are based on the assumption that the diffuse radio emission is associated with cluster merger events. We note that radio galaxies, starbursts, and active galactic nuclei (AGNs) might also contribute significant relativistic electrons to the ICM of the clusters. However, the high-energy ($\geq$1 GeV) relativistic electrons injected by these sources would become

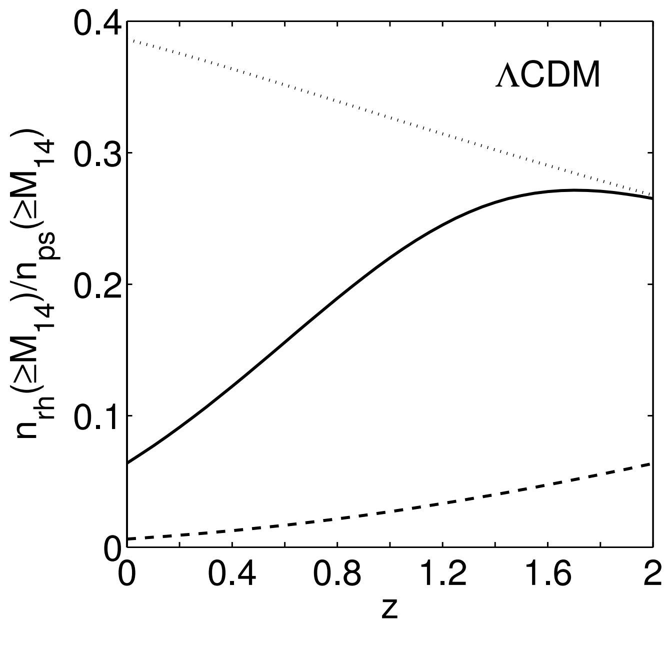

FIG. 6.—Evolution of the ratios of the total number density of radio halos to that of galaxy clusters in the ΛCDM model. The results for three representative lifetimes of radio halos: 0.1 Gyr (*dashed curves*), 1 Gyr (*solid curves*), and the cosmological time (*dotted curves*) are shown.

TABLE 3

THE RATIOS OF GALAXY CLUSTERS CONTAINING DIFFUSE RADIO SOURCES IN THE MASS RANGE $M_{th} \leq M < 10^{15}\ M_{\odot}$ UNDER DIFFERENT M_{th}

	$5 \times 10^{14} M_{\odot}$			$5 \times 10^{13}\ M_{\odot}$		
M_{th}	SCDM (%)	OCDM (%)	ΛCDM (%)	SCDM (%)	OCDM (%)	ΛCDM (%)
0.1 Gyr	∼3	∼2	∼2	∼1	∼1	∼1
1 Gyr	∼22	∼16	∼14	∼12	∼8	∼8
Cosmological time	∼39	∼41	∼38	∼35	∼37	∼34

TABLE 4

THE RATIOS OF GALAXY CLUSTERS CONTAINING DIFFUSE RADIO SOURCES IN THE MASS RANGE $M \geq M'$ IN ΛCDM

M'	$1 \times 10^{15}\ M_{\odot}$ (%)	$1.2 \times 10^{15}\ M_{\odot}$ (%)	$1.5 \times 10^{15}\ M_{\odot}$ (%)	$2 \times 10^{15}\ M_{\odot}$ (%)
0.1 Gyr	∼2	∼3	∼3	∼3
1 Gyr	∼21	∼23	∼25	∼29
Cosmological time	∼70	∼73	∼77	∼82

TABLE 5

THE RATIOS OF GALAXY CLUSTERS CONTAINING DIFFUSE RADIO SOURCES FOR DIFFERENT $\triangle_m$ IN THE MASS RANGE $M \geq 10^{15}\ M_{\odot}$ IN ΛCDM

$\triangle_m$	0.5 (%)	0.6 (%)	0.7 (%)
0.1 Gyr	∼3	∼2	∼2
1 Gyr	∼32	∼21	∼14
Cosmological time	∼95	∼70	∼49

invisible in a very short time and cannot directly account for the extended radio emission. On the other hand, the low-energy ($\leq$1 GeV) relativistic electrons can survive a longer time and become more extended via diffusion. A cluster merger event, as suggested in this paper for the diffuse radio emission phenomena, can produce merger shocks and MHD turbulence that can reaccelerate the survived low-energy relativistic electrons and reignite the radio emission in a more extended region. In this respect, our results are also applicable even when the relativistic electrons are originally injected from radio galaxies, starburst galaxies, or AGNs.

We note that the effects of the inverse Compton loss rising with increasing redshift may reduce the ratio of radio halos at high redshifts, but the results at low redshifts are only slightly affected, and the conclusions are still tenable.

We thank an anonymous referee for helpful comments. This work was partially supported by the National Science Council of Taiwan under NSC 92-2112-M-008-023 and by the Ministry of Education of Taiwan through the CosPA Project 92-N-FA01-1-4-5.

REFERENCES

Bacchi, M., Feretti, L., Giovannini, G., & Govoni, F. 2003, A&A, 400, 465
Berezinsky, V. S., Blasi, P., & Ptuskin, V. S. 1997, ApJ, 487, 529
Blasi, P., & Colafrancesco, S. 1999, Astropart. Phys., 12, 169
Bond, J. R., Cole, S., Efstathiou, G., & Kaiser, N. 1991, ApJ, 379, 440
Bower, R. J. 1991, MNRAS, 248, 332
Brunetti, G., Setti, G., Feretti, L., & Giovannini, G. 2001, MNRAS, 320, 365
Buote, D. A. 2001, ApJ, 553, L15
Clarke, T. E., Kronberg, P. P., & Böhringer, H. 2001, ApJ, 547, L111
Dennison, B. 1980, ApJ, 239, L93
De Young, D. S. 1992, ApJ, 386, 464
Ebeling, H., Voges, W., Böhringer, H., Edge, A. C., Huchra, J. P., & Briel, U. G. 1996, MNRAS, 281, 799
Ensslin, T. A., & Röttgering, H. 2002, A&A, 396, 83
Evrard, A. E., Metzler, C. A., & Navarro, J. F. 1996, ApJ, 469, 494
Feretti, L. 2000, in IAU Symp. 199, The Universe at Low Radio Frequencies, ed. A. Pramesh Rao (San Francisco: ASP), 123
Fujita, Y., & Sarazin, C. L. 2001, ApJ, 563, 660
Fusco-Femiano, R., Dal Fiume, D., Orlandini, M., De Grandi, S, Molendi, S., Feretti, L., Grandi, P., & Giovannini, G. 2003, in Matter and Energy in Clusters of Galaxies, ed. S. Bowyer, & C.-Y. Hwang (San Francisco: ASP), 109
Giovannini, G., Tordi, M., & Feretti, L. 1999, NewA, 4, 141
Govoni, F., Feretti, L., Giovannini, G., Böhringer, H., Reiprich, T. H., & Murgia, M. 2001, A&A, 376, 803
Horner, D. J., Mushotzky, R. F., & Scharf, C. A. 1999, ApJ, 520, 78
Kempner, J. C., & Sarazin, C. L. 2001, ApJ, 548, 639
Kitayama, T., & Suto, Y. 1996a, MNRAS, 280, 638 (KS)
———. 1996b, ApJ, 469, 480
Kuo, P.-H., Hwang, C.-Y., & Ip, W.-H. 2003, ApJ, 594, 732
Lacey, C., & Cole, S. 1993, MNRAS, 262, 627 (LC)
Liang, H., Dogiel, V. A., & Birkinshaw, M. 2002, MNRAS, 337, 567
Miniati, F., Ryu, D., Kang, H., Jones, T. W., Cen, R., & Ostriker, J. P. 2000, ApJ, 542, 608
———. 2001a, ApJ, 559, 59
Miniati, F., Jones, T. W., Kang, H., & Ryu, D. 2001b, ApJ, 562, 233
Mulchaey, J. S. 2000, ARA&A, 38, 289
Pen, U.-L. 1998, ApJ, 498, 60
Press, W. H., & Schechter, P. 1974, ApJ, 187, 425 (PS)
Randall, S. W., & Sarazin, C. L. 2001, ApJ, 548, 60
Ricker, P. M., & Sarazin, C. L. 2001, ApJ, 561, 621
Roettiger, K., Stone, J. M., & Burns, J. O. 1999, ApJ, 518, 594
Rosati, P., Borgani, S., & Norman, C. 2002, ARA&A, 40, 539
Salvador-Solé, E., Solanes, J. M., & Manrique, A. 1998, ApJ, 499, 542
Sarazin, C. L. 2001, in Merging Processes in Clusters of Galaxies, ed. L. Feretti, I. M. Gioia, & G. Giovannini (Dordrecht: Kluwer), 1
Schlickeiser, R., Sievers, A., & Thiemann, H. 1987, A&A, 182, 21
Sugiyama, N. 1995, ApJS, 100, 281
Tribble, P. C. 1993, MNRAS, 263, 31
Völk, H. J., Aharonian, F. A., & Breitschwerdt, D. 1996, Space Sci. Rev., 75, 279