The Astrophysical Journal, 599:L57–L60, 2003 December 20

COSMOLOGICAL MAGNETIC FIELD GENERATION BY THE WEIBEL INSTABILITY

R. Schlickeiser and P. K. Shukla[1]
Institut für Theoretische Physik, Lehrstuhl IV: Weltraum und Astrophysik, Ruhr-Universität Bochum,
D-44780 Bochum, Germany; rsch@tp4.rub.de, ps@tp4.rub.de
Received 2003 July 29; accepted 2003 November 10; published 2003 December 3

ABSTRACT

Hydrodynamical simulations of a cold dark matter universe imply relative motion of fully ionized gaseous baryonic matter. Using the dispersion relation for the Weibel instability in unmagnetized plasmas with colliding ion-electron streams, we investigate the instability threshold conditions for the Weibel instability. The intense relative streaming is subject first to the longitudinal two-stream instability resulting in an anisotropic bi-Maxwellian electron-ion distribution function. The Weibel instability operates as a secular instability of this bi-Maxwellian in high Mach number ($M > 43$) flows, and it generates magnetic fields as purely growing modes. The maximum cosmological field strength generated by this mechanism is 3.4×10^{-7} G.

Subject headings: cosmology: theory — large-scale structure of universe — magnetic fields — plasmas

1. INTRODUCTION

The processes leading to the magnetization of the intergalactic medium are not yet known (Kronberg 2002). Hydrodynamical simulations of a cold dark matter universe with a cosmological constant are currently considered to be the most successful theory for cosmological structure formation (e.g., Springel & Hernquist 2003; Cen & Ostriker 1999). Large-scale structures in the universe, such as filaments and sheets of galaxies, evolve by the gravitational collapse of initially overdense regions giving rise to an intense relative motion of fully ionized gaseous matters. Because the sound speed $c_s = v_{\rm th}(m_e/m_p)^{1/2} = v_{\rm th}/43$ in a hot (with the temperature $T_0 = 10^7 T_7$ K) intergalactic medium is much smaller than the electron thermal speed, gaseous shock structures result. We investigate the physical conditions under which the shock waves efficiently generate long-lived quasi-static cosmological magnetic fields due to the Weibel instability, an idea first aired, albeit briefly, by Gruzinov (2001).

2. WEIBEL BEAM-PLASMA INSTABILITY

2.1. *Growth Rate and Threshold Condition for the Weibel Instability*

The Weibel instability operates in initially unmagnetized plasmas. Whereas the original work of Weibel (1959) considered a bi-Maxwellian electron distribution function, later work has dealt with a wide variety of anisotropic plasma distributions including in particular a waterbag distribution function (Yoon & Davidson 1987; Silva et al. 2002). In order to represent as close a situation as in the cosmological structure formation, we start here from an initial configuration of two interpenetrating collisionless electron-ion streams with different densities and speeds at the center of the plasma mass system. Such a beam-plasma system is unstable against purely growing electromagnetic perturbations, as has been shown earlier by Akhiezer et al. (1975) and Medvedev & Loeb (1999).

For the case of a cold beam of the density n_b and the bulk velocity u propagating through a hot electron-ion Maxwellian distribution of the density n_e, the dispersion relation reads (Akhiezer et al. 1975, eq. [6.1.5.15])

$$\omega^2 = -\omega_{be}^2\left(\frac{k^2u^2}{k^2c^2+\omega_{be}^2} - \frac{k^2 v_{{\rm th},e}^2 v_{{\rm th},p}^2}{\omega_{pe}^2 v_{{\rm th},p}^2 + \omega_{pi}^2 v_{{\rm th},e}^2}\right), \tag{1}$$

with the plasma frequencies $\omega_{be} = (4\pi e^2 n_b/m_e)^{1/2}$, $\omega_{pe} = (4\pi e^2 n_e/m_e)^{1/2} = 5.64 \times 10^4 \sqrt{n_e}$ s^{-1}, $\omega_{pp} = (4\pi e^2 n_e/m_p)^{1/2} = (m_e/m_p)^{1/2}\,\omega_{pe} = \omega_{pe}/43$, and the thermal speeds $v_{{\rm th},e} = (k_{\rm B}T_e/m_e)^{1/2}$ and $v_{{\rm th},p} = (k_{\rm B}T_p/m_p)^{1/2}$. Equation (1) reveals that purely growing ($\omega^2 = -\Gamma^2$) oscillations result if

$$\frac{u^2}{k^2c^2+\omega_{be}^2} > \frac{v_{{\rm th},e}^2 v_{{\rm th},p}^2}{\omega_{pe}^2 v_{{\rm th},p}^2 + \omega_{pi}^2 v_{{\rm th},e}^2}. \tag{2}$$

For an equal temperature plasma $T_e = T_p = T_0$, equation (2) reduces to

$$\frac{u^2}{k^2c^2+\omega_{be}^2} > \frac{v_{\rm th}^2}{2\omega_{pe}^2}, \tag{3}$$

where $v_{\rm th} = (k_{\rm B}T_0/m_e)^{1/2} = 1.23 \times 10^4\, T_7^{1/2}$ km s^{-1}. Equation (3) is equivalent to the wavenumber restriction

$$k^2 < k_0^2 = \frac{\omega_{pe}^2}{c^2}\left(\frac{2u^2}{v_{\rm th}^2} - \alpha\right), \tag{4}$$

where $\alpha = n_b/n_e$ denotes the density contrast. In order to have real wavenumbers, the condition

$$u \geq u_c = \left(\frac{n_b}{2n_e}\right)^{1/2} v_{\rm th} = \left(\frac{\alpha}{2}\right)^{1/2} v_{\rm th} = 0.71\alpha^{1/2} v_{\rm th}$$
$$= 8.73 \times 10^3 \alpha^{1/2} T_7^{1/2} \text{ km s}^{-1} \tag{5}$$

has to be fulfilled. Apparently, for *any given* density contrast $\alpha = n_b/n_e$, this type of Weibel instability sets in, provided that the streaming velocity (eq. [5]) is larger than u_c and that the beam is cold, implying that $u \gg w_e$, where w_e denotes the thermal speed of the beam distribution, not to be identified with $v_{\rm th}$.

[1] Also at the Department of Physics, Umeå University, SE-90187 Umeå, Sweden.

In the wavenumber regime $|k| < |k_0|$, equation (1) yields the growth rate

$$\Gamma = \omega_{be} k \sqrt{\frac{u^2}{k^2c^2 + \omega_{be}^2} - \frac{v_{\rm th}^2}{2\omega_{pe}^2}}. \quad (6)$$

The growth rate attains its maximum value

$$\Gamma_{\max} = \frac{\alpha v_{\rm th}\omega_{pe}}{2^{1/2}c}\left(\frac{2^{1/2}u}{\alpha^{1/2}v_{\rm th}} - 1\right)^{3/4} = \frac{\alpha v_{\rm th}\omega_{pe}}{2^{1/2}c}\left(\frac{u}{u_c} - 1\right)^{3/4} \quad (7)$$

at the wavenumber

$$k_{\max} = \frac{\omega_{pe}}{c}\alpha^{1/4}\sqrt{\frac{2^{1/2}u}{v_{\rm th}} - \alpha^{1/2}} = \frac{\omega_{pe}}{c}\alpha^{1/2}\sqrt{\frac{u}{u_c} - 1}. \quad (8)$$

Obviously, $k_{\max}$ is always smaller than k_0. Numerically, for typical intergalactic plasma densities $n_e = 10^{-4}n_{-4}$ cm^{-3} and a streaming velocity of $u = 2u_c$ the maximum growth rate is

$$\Gamma_{\max} = 16.4T_7^{1/2}n_{-4}^{1/2}\alpha \text{ s}^{-1}, \quad (9)$$

implying an exponential growth time of electromagnetic fields, given by

$$\tau = \Gamma_{\max}^{-1} \simeq 0.061T_7^{-1/2}n_{-4}^{-1/2}\alpha^{-1} \text{ s}, \quad (10)$$

which is much smaller than any relevant cosmological timescale.

2.2. *Conditions for a Primary Weibel Instability: Comparison with Langmuir Instability*

The Weibel instability will only be the primary relaxation mechanism for the initial beam-plasma configuration if its maximum growth rate is much larger than the growth rate of all other plasma instabilities. In an unmagnetized equal temperature beam-plasma configuration, the only competing plasma instability is the longitudinal (electrostatic, Langmuir) two-stream instability. For the beam-plasma system, the maximum growth rate of the Langmuir instability in our notation is (Medvedev & Loeb 1999, eq. [B7])

$$\Gamma_{\max,\,\rm Langmuir} \simeq \frac{3^{1/2}}{2^{4/3}}\alpha^{1/3}\omega_{pe}. \quad (11)$$

A comparison of equations (7) and (11) shows that the maximum growth rate of the Weibel instability is larger than the maximum growth rate of the Langmuir instability if

$$\alpha\left(\frac{u}{u_c} - 1\right)^{9/8} > \left(\frac{27}{32}\right)^{1/4}\left(\frac{c}{v_{\rm th}}\right)^{3/2} = 115T_7^{-3/2}. \quad (12)$$

In the opposite case,

$$\alpha\left(\frac{u}{u_c} - 1\right)^{9/8} < \left(\frac{27}{32}\right)^{1/4}\left(\frac{c}{v_{\rm th}}\right)^{3/2} = 115T_7^{-3/2}, \quad (13)$$

the Langmuir instability would be the primary relaxation mechanism, and the Weibel instability would then set in as a secular instability. As we show next, this second case seems to be relevant in the intergalactic medium.

2.3. *Primary Beam-Plasma or Secular Weibel Instabilty*

If we identify the regions of large streaming velocities with strong shocks, we can express the streaming velocity $u = Mv_{\rm th}/43$ in terms of the shock's Mach number. Equation (12) then becomes

$$\alpha\left(\frac{M}{30\alpha^{1/2}} - 1\right)^{9/8} > \left(\frac{27}{32}\right)^{1/4}\left(\frac{c}{v_{\rm th}}\right)^{3/2} = 115T_7^{-3/2}. \quad (14)$$

When the shocks are strong and nonradiative, we can set the density contrast equal to its maximum value $\alpha = 4$ and obtain

$$M \geq 60\left[\frac{(27/32)^{1/4}}{4}\left(\frac{c}{v_{\rm th}}\right)^{3/2}\right]^{8/9} = 1192T_7^{-4/3}. \quad (15)$$

Shocks with such high Mach numbers do not occur in intergalactic space as a result of the cosmological structure formation, as Figure 1 of Miniati (2002) indicates.

Also for radiative shocks, where the density contrast α can be larger than 4, the equation (14) is difficult to fulfil. Taking, e.g., a value of $\alpha = 16$, equation (14) requires

$$\left(\frac{M}{120} - 1\right)^{9/8} > 7.2T_7^{-3/2}, \quad (16)$$

implying Mach numbers larger than $695T_7^{-4/3}$, which do not occur (Miniati 2002). According to these considerations, the Weibel instability occurs as a secular instability after the Langmuir instability has been fully developed.

3. SECULAR WEIBEL INSTABILITY

The outcome of the electrostatic Langmuir instability is well known (e.g., Donaldson 1972; Grognard 1975). By resonant wave-particle interactions the beam distribution quickly relaxes toward the plateau distribution (with $\partial f_e/\partial p_\parallel = 0$) in the direction of the beam motion. Nonresonant interactions then provide some energy exchange between the background plasma and the plateaued distribution in the $p_\parallel$-phase space direction. We, therefore, may well characterize the nonrelativistic plasma distribution after the Langmuir relaxation as the anisotropic bi-Maxwellian

$$f_0 = \frac{n_e}{(2\pi)^{3/2}u_0^2u_3}\exp\left(-\frac{v_0^2}{2u_0^2} - \frac{v_3^2}{2u_3^2}\right), \quad (17)$$

where $u_3 = v_{\rm th}$ has to be identified with the thermal electron speed of the background plasma, and $u_0 = u$ represents the plateaued distribution.

If written in this form, equation (17) corresponds exactly to equation (5) of Weibel (1959), who studied the stability of this configuration against transverse waves. In the limit $\omega/(u_3k) =$

$\omega/(v_{\rm th}k) > 1$, corresponding to $u > v_{\rm th}$, the dispersion relation in our notation reads

$$\omega^4 - (\omega_{p,e}^2 + k^2c^2)\omega^2 - \omega_{p,e}^2 u^2 k^2 = 0, \tag{18}$$

which implies purely growing modes with the growth rate

$$\Gamma_4 \simeq \frac{u\omega_{p,e}k}{(\omega_{p,e}^2 + k^2c^2)^{1/2}}. \tag{19}$$

The condition $\omega/(v_{\rm th}k) > 1$ restricts the wavenumber to values smaller

$$|k| < k_{s,\,\rm max} = u\omega_{p,e}/(v_{\rm th}c), \tag{20}$$

so that the maximum growth rate of the secular Weibel instability is

$$\Gamma_{4,\,\rm max} = \frac{u\omega_{p,e}}{c} = 0.54 n_{-4}^{1/2} M T_7^{1/2}\ {\rm s}^{-1}. \tag{21}$$

The miminum growth time of the secular Weibel instability, $\tau_s = \Gamma_{4,\,\rm max}^{-1} = 2M^{-1}n_{-4}^{-1/2}T_7^{-1/2}$ s, although long compared to the Langmuir growth time, still is much smaller than any cosmological timescale, indicating that after the Langmuir relaxation there is still plenty of time left for the development of the secular Weibel instability.

The instability condition $u > v_{\rm th}$ is equivalent to

$$M > 43 \tag{22}$$

for the shock's Mach number. Figure 1 of Miniati (2002) shows that there are some shocks with $M > 43$, although they are quite rare; only a few percent of the thermal energy in the simulations is mediated through such strong shocks.

4. SATURATED MAGNETIC FIELD VALUE

The nonlinear saturated amplitude of the magnetic field due to the Weibel streaming instability can be estimated following the arguments of Medvedev & Loeb (1999). The free streaming of particles across the magnetic field lines is suppressed once the magnetic field amplitude has grown to a level that the particle's gyroradii in the excited magnetic fields, viz., $\rho = u/\Omega_e$, are comparable to the characteristic scale length of unstable modes, $k_{s,\,\rm max}^{-1}$, yielding with equation (20)

$$B \simeq \frac{m_e uck_{s,\,\rm max}}{e} = \sqrt{4\pi n_e m_e}\,\frac{u^2}{v_{\rm th}}. \tag{23}$$

The above condition can be rewritten as

$$\frac{B^2/8\pi}{n_e m_e u^2} = \frac{u^2}{2v_{\rm th}^2}. \tag{24}$$

Direct computer simulations of the instability in both nonrelativistic and relativistic electron plasmas (Califano et al. 1998; Kazimura et al. 1998; Yang, Arons, & Langdon 1994; Wallace & Epperlein 1991) confirm that the saturation occurs at slightly subequipartition values of B, giving

$$\frac{B_s^2/8\pi}{n_e m_e u^2} = \eta\,\frac{u^2}{2v_{\rm th}^2}, \tag{25}$$

with $\eta \simeq 0.01$–0.1. Using $\eta = 0.01$ we then obtain for the saturated magnetic field strength

$$B_s \simeq \eta^{1/2}B = 0.1\sqrt{4\pi n_e m_e}\,\frac{u^2}{v_{\rm th}} = 0.1\sqrt{4\pi n_e m_e}\,\frac{m_e}{m_p}\,v_{\rm th}M^2$$
$$= 1.3\times 10^{-7}T_7^{1/2}\left(\frac{M}{43}\right)^2 n_{-4}^{1/2}\ {\rm G}, \tag{26}$$

which is mainly determined by the shock's Mach number. Consequently, the secular Weibel instability provides saturated magnetic field values over a rather wide range being determined by the distribution of Mach numbers of shock waves from cosmological structure formation with values larger than the instability equation (22), i.e., $M > 43$. It is known (e.g., Miniati 2002) that Mach numbers larger than 70 are extremely rare. Taking $M = 70$ as an upper limit, the maximum field strength according to the equation (26) is $B_{s,\,\rm max} \simeq 3.4\times 10^{-7}$ G. This magnetic field value is consistent with the upper limit to the magnetic fields $\leq 10^{-6}$ G in large-scale filaments and sheets, derived from rotation measure observations (Rye, Kang, & Biermann 1998).

5. SUMMARY AND CONCLUSIONS

We have demonstrated that high Mach number flows generate magnetic fields in the intergalactic medium because of the Weibel instability involving interpenetrating electron-ion streams. This intense relative streaming of fully ionized baryonic matters occurs as a result of evolving large-scale structures in the universe, such as filaments and sheets of galaxies, as indicated by current hydrodynamical simulations of a cold dark matter universe. The intense relative streaming is subject first to the longitudinal two-stream instability resulting in an anisotropic bi-Maxwellian electron-ion distribution function. The Weibel instability operates as secular instability of this bi-Maxwellian and generates magnetic fields as purely growing modes. Since the *e*-folding time of the secular instability is of the order of seconds, it is likely that free energy stored in the bi-Maxwellian plasma particle distribution can generate electron currents that are required for exciting purely growing magnetic fields rapidly.

When the spatial scale of the excited fields is of the order of the electron gyroradius, the magnetic fields saturate because of the magnetic trapping of electrons in the wave potential. The saturated magnetic field turns out to be a fraction of microgauss at subequipartition level in cosmological environments. Our mechanism supports the earlier conjecture (Rye et al. 1998) that cosmic magnetic fields are strongly correlated with the large-scale structure of the universe.

Because the magnetic field strengths reach subequipartition values, it implies that the current hydrodynamic simulations of gas flows have to be extended to fully magnetohydrodynamic simulations. Moreover, the forming gaseous shock structures will also be modified by the presence of the magnetic fields. This will change the microphysical details (e.g., Vainio &

Schlickeiser 1999) of the diffusive acceleration process of energetic charged particles at intergalactic shocks (Kang, Rye, & Jones 1996; Kang, Rachen, & Biermann 1997; Ensslin et al. 1998; Loeb & Waxman 2002; Keshet et al. 2003). Nonthermal radiation (synchrotron and inverse Compton radiation) from the accelerated electrons will be signatures of gaseous shock structures in the universe.

We are grateful to the referee for her/his constructive and helpful comments, in particular on the importance of the longitudinal two-stream instability. This research was partially supported by the Deutsche Forschungsgemeinschaft through the Sonderforschungsbereich 591. This Letter was completed when R. S. was a visiting professor at the University of California at Riverside Institute of Geophysics and Planetary Physics. He thanks G. P. Zank, Director, for his sponsorship and kind hospitality.

REFERENCES

Akhiezer, A. I., Akhiezer, I. A., Polovin, R. V., Sitenko, A. G., & Stepanov, K. N. 1975, Plasma Electrodynamics (Oxford: Pergamon)
Califano, F., Pegoraro, F., Bulanov, S. V., & Mangeney, A. 1998, Phys. Rev. E, 57, 7048
Cen, R., & Ostriker, J. P. 1999, ApJ, 514, 1
Donaldson, R. C. 1972, Methods in Nonlinear Plasma Theory (New York: Academic)
Ensslin, T. A., Biermann, P. L., Klein, U., & Kohle, S. 1998, A&A, 332, 395
Grognard, R. J.-M. 1975, Australian J. Phys., 28, 731
Gruzinov, A. 2001, ApJ, 563, L15
Kang, H., Rachen, J. P., & Biermann, P. L. 1997, MNRAS, 286, 257
Kang, H., Rye, D., & Jones, T. W. 1996, ApJ, 456, 422
Kazimura, Y., Sakai, J. I., Neubert, T., & Bulanov, S. V. 1998, ApJ, 498, L183
Keshet, U., Waxman, E., Loeb, A., Springel, V., & Hernquist, L. 2003, ApJ, 585, 128
Kronberg, P. P. 2002, Phys. Today, 55, 40
Loeb, A., & Waxman, E. 2000, Nature, 405, 156
Medvedev, M., & Loeb, A. 1999, ApJ, 526, 697
Miniati, F. 2002, MNRAS, 337, 199
Rye, D., Kang, H., & Biermann, P. L. 1998, A&A, 335, 19
Silva, L. O., Fonseca, R. A., Tange, J. W., Mori, W. B., & Dawson, J. M. 2002, Phys. Plasmas, 9, 2458
Springel, V., & Hernquist, L. 2003, MNRAS, 339, 289
Vainio, R., & Schlickeiser, R. 1999, A&A, 343, 303
Wallace, J. M., & Epperlein, E. M. 1991, Phys. Fluids, 3, 1579
Weibel, E. 1959, Phys. Rev. Lett., 2, 83
Yang, T.-H., Arons, J., & Langdon, A. B. 1994, Phys. Plasmas, 1, 3059
Yoon, P. H., & Davidson, R. C. 1987, Phys. Rev. A, 35, 2718