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MODEL-INDEPENDENT RECONSTRUCTION OF THE PRIMORDIAL POWER SPECTRUM FROM *WILKINSON MICROWAVE ANISTROPY PROBE* DATA

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ABSTRACT

Reconstructing the shape of the primordial power spectrum in a model-independent way from cosmological data is a useful consistency check on what is usually assumed regarding early universe physics. It is also our primary window to unknown physics during the inflationary era. Using a power-law form for the primordial power spectrum $P_{\rm in}(k)$ and constraining the scalar spectral index and its running, in 2003 Peiris and coworkers found that the first-year *Wilkinson Microwave Anistropy Probe* (*WMAP*) data seem to indicate a preferred scale in $P_{\rm in}(k)$. We use two complementary methods, the wavelet band power method of Mukherjee & Wang and the top-hat binning method of Wang, Spergel, & Strauss, to reconstruct $P_{\rm in}(k)$ as a free function from cosmic microwave background (CMB) data alone (*WMAP*, CBI, and ACBAR), or from CMB data together with large-scale structure data (2dFGRS and PCSz). The shape of the reconstructed $P_{\rm in}(k)$ is consistent with scale invariance, although it allows some indication of a preferred scale at $k \sim 0.01$ Mpc^{-1}. While consistent with the possible evidence for a running of the scalar spectral index found by the *WMAP* team, our results highlight the need of more stringent and independent constraints on cosmological parameters (the Hubble constant in particular) in order to more definitively constrain deviations of $P_{\rm in}(k)$ from scale invariance without making assumptions about the inflationary model.

Subject headings: cosmic microwave background — cosmological parameters — cosmology: theory — large-scale structure of universe

1. INTRODUCTION

The long-anticipated first-year *Wilkinson Microwave Anistropy Probe* (*WMAP*) data (Bennett et al. 2003; Spergel et al. 2003) have very interesting implications for the state of cosmology today. On one hand, *WMAP* results on cosmological parameters are consistent with and refine previous constraints from various independent and complementary observations. On the other hand, the data seem to indicate a preferred scale in the primordial scalar power spectrum $P_{\rm in}(k)$ (Peiris et al. 2003), with or without complementary large-scale structure data, although not at high significance. If true, this would contradict the simple assumption of scale invariance of $P_{\rm in}(k)$ made by most researchers in cosmology and the prediction of the simplest inflationary models, but would be consistent with earlier lower significance findings of Wang & Mathews (2002), Mukherjee & Wang (2003a, hereafter MW03a), and Mukherjee & Wang (2003b).

With *WMAP*, cosmic microwave background (CMB) data continue to be fully consistent with inflation (Guth 1981; Kolb & Turner 1990; Hu & Dodelson 2002; Peebles & Ratra 2003). *WMAP* reveals new evidence for inflation from the anticorrelation between CMB temperature and polarization fluctuations near l of 150. Focus is now shifting toward distinguishing between the different inflationary models. The simplest models of inflation predict a power-law primordial matter power spectrum (e.g., Linde 1983; Freese, Frieman, & Olinto 1990; La & Steinhardt 1991). Thus, some efforts have been focused on constraining slow-roll parameters (Liddle & Lyth 1992; Leach et al. 2002; Barger, Lee, & Marfatia 2003), evaluated at a certain epoch during inflation, or at the Hubble crossing time of a certain scale usually chosen to be at the center of the scales probed by observations. The primordial power spectrum that results from slow-roll inflation can be computed to high accuracy in terms of these parameters. The *WMAP* team fits to observables that can be written as derivatives of the slow-roll parameters. However, there are also viable models of inflation that predict primordial power spectra that cannot be parametrized by a simple power law (e.g., Holman et al. 1991a, 1991b; Linde 1994; Wang 1994; Randall, Soljacic, & Guth 1996; Adams, Ross, & Sarkar 1997; Lesgourgues, Polarski, & Starobinsky 1997). In such models, features in $P_{\rm in}(k)$ can result from unusual physics during inflation (Chung et al. 2000; Enqvist & Kurki-Suonio 2000; Lyth, Ungarelli, & Wands 2002). The assumption of a power law $P_{\rm in}(k)$ could then lead to our missing the discovery of the possible features in the primordial matter power spectrum and erroneous estimates of cosmological parameters (Kinney 2001).

WMAP results underscore the importance of model-independent measurements of the shape of the primordial power spectrum (Wang, Spergel, & Strauss 1999). In this paper we reconstruct $P_{\rm in}(k)$ as a free function using two different methods, the wavelet band power method of MW03a and the top-hat binning method of Wang et al. (1999). We briefly describe our methods in § 2. Section 3 contains our results. Section 4 contains a summary and discussions.

2. METHODS

Both the methods that we have used to reconstruct $P_{\rm in}(k)$ are essentially binning methods in which we can write $P_{\rm in}(k)$ as

$$P_{\rm in}(k) = \sum_i \alpha_i f_i(k) , \tag{1}$$

where the $f_i(k)$'s are functions of wavenumber k and the α_i's are constant. We use equation (1), instead of $P_{\rm in}(k) = Ak^{n_S-1}$, to parametrize $P_{\rm in}(k)$ as an arbitrary function. Our $P_{\rm in}(k)$ parameters are the coefficients α_i, instead of the normalization parameter A, the power-law spectral index, n_S, and its running, $dn_S/d\ln k$ (Kosowsky & Turner 1995).

This allows us to expand the CMB temperature angular power spectrum as follows:

$$\begin{aligned} C_l(\{\alpha_i\},\ \boldsymbol{s}) &= (4\pi)^2 \int \frac{dk}{k} P_{\rm in}(k) |\Delta_{Tl}(k,\ \tau=\tau_0)|^2 \\ &= \sum_i \alpha_i \int \frac{dk}{k} f_i(k) |\Delta_{Tl}(k,\ \tau=\tau_0)|^2 \\ &\equiv \sum_i \alpha_i C_l^i(\boldsymbol{s})\ , \end{aligned} \tag{2}$$

where the cosmological model–dependent transfer function $\Delta_{Tl}(k,\ \tau=\tau_0)$ is an integral over conformal time τ of the sources that generate CMB temperature fluctuations, τ_0 being the conformal time today, and $\boldsymbol{s}$ represents cosmological parameters other than the α_i's. We use CAMB[1] to compute the CMB angular power spectra, in a form such that for given cosmological parameters other than the α_i's, the $C_l^i(\boldsymbol{s})$ are computed, so that there is no need to call CAMB when we vary only the α_i's.

The choice of the basis functions $f_i(k)$ in equation (1) differs in the two methods, as described below.

2.1. *The Wavelet Band Power Method*

In this method, we are using wavelets essentially as band-pass filters. The wavelet band powers of the primordial power spectrum are given by

$$P_j = \frac{1}{2^j} \sum_{n=-\infty}^{\infty} \left|\hat{\psi}\left(\frac{n}{2^j}\right)\right|^2 P_{\rm in}(k_n) \tag{3}$$

(MW03a). The wavelet band power window functions in k-space, $|\hat{\psi}(k/2^j)|^2$, are the modulus squared of the Fourier transforms of the wavelet basis functions of different j (dilation index). The translation index has been integrated out as we Fourier transformed the basis functions. The resulting band powers P_j's are thus the ban- averaged Fourier power spectrum.

The wavelet band power window functions, $\left|\hat{\psi}(k/2^j)\right|^2$, are plotted in Figure 1 of MW03a. Figure 2 of MW03a shows the window functions in CMB multipole l-space that these functions map on to.

If the primordial density field is a Gaussian random field, the P_j's, which represent the variance of wavelet coefficients of scale j, are uncorrelated:

$$\frac{\langle P_j P_{j'}\rangle}{P_j P_{j'}} = 1 \tag{4}$$

(e.g., Mukherjee, Hobson, & Lasenby 2000).

Furthermore, the primordial power spectrum $P_{\rm in}(k)$ can be reconstructed as a smooth function from the wavelet band power P_j's as follows:

$$\hat{P}_{\rm in}(k) = \sum_j P_j \left|\hat{\psi}\left(\frac{k}{2^j}\right)\right|^2 \tag{5}$$

(Fang & Feng 2000; MW03a), i.e., $\alpha_i = P_i$, and $f_i(k) = |\hat{\psi}(k/2^i)|^2$ in equation (1). MW03a have shown that equation (5) gives excellent estimates of $P_{\rm in}(k)$ at the centers of the wavelet window functions. Smooth wavelets work best in this method. We have chosen the wavelet Daubechies 20 (Daubechies 1992). Our results are insensitive to the choice of the particular wavelet among smooth wavelets.

The wavelet band power method is an optimal binning method, in which the locations of the bands are not arbitrary. Here the position and momentum spaces are decomposed into elements that satisfy $\Delta x \Delta k \sim 1$, with $\Delta x \propto 1/k$ and $\Delta k/k = \log_{10} 2$. Thus, on small length scales (large k), Δx is small, and on large length scales, Δx is large, and since the wavelet bases are complete, one cannot have more independent bands than used here (Fang & Feng 2000). We choose to estimate 11 P_j's that cover the k range probed by current data. The P_j's outside of this k range are set equal to their adjacent P_j's.

Note that although the P_j's are mutually uncorrelated by construction, the P_j's estimated from CMB data (the measured CMB temperature anisotropy angular power spectrum C_l bands) will be somewhat correlated because of the cosmological model–dependent nonlinear mapping between the wavenumber k and the CMB multipole number l and because of correlations with cosmological parameters.

We have chosen to use the wavelet band power method in this paper, since the first-year *WMAP* data seem to be relatively well fitted by a smooth $P_{\rm in}(k)$ and are not yet sensitive to very sharp features (see Fig. 7 of Peiris et al. 2003). The direct wavelet expansion method of Mukherjee & Wang (2003b) is more suited to reconstructing an unknown function with possible sharp features on scales smaller than $\log_{10} 2$.

2.2. *The Top-Hat Binning Method*

We also use the top-hat binning method of Wang et al. (1999) to parametrize $P_{\rm in}(k)$. We write

$$P_{\rm in}(k) = \begin{cases} \alpha_1\ , & k < k_0\ , \\ \alpha_i\ , & k_{i-1} < k < k_i\ , \\ \alpha_n\ , & k > k_n\ . \end{cases} \tag{6}$$

The k_i's are chosen to be uniformly spaced in $\log k$ as in Wang et al. (1999). In this method, the basis functions $f_i(k)$ in equation (1) are top-hat window functions: $f_i(k) = 1$ for $k_{i-1} < k < k_i$, and $f_i(k) = 0$ elsewhere. The boundary conditions at the minimum and maximum k-values are similar to what we imposed in the wavelet band power method.

We choose the centers of the top-hat window functions to coincide with the central k-values of the wavelet band power window functions, so that the results from the two methods can be compared for consistency.

The advantage of this binning is its simplicity. The disadvantage is that the reconstructed $P_{\rm in}(k)$ is a *discontinuous* step function, which might introduce additional degeneracies with cosmological parameters.

We have included this binning method primarily to cross check the results of the wavelet band power method.

[1] Similar to CMBFAST, CAMB can be used to compute the CMB and matter power spectra from a given set of cosmological parameters and primordial power spectrum. For details, see http://camb.info.

3. RESULTS

We work with CMB temperature anisotropy data from *WMAP* (Bennett et al. 2003), complemented at $l > 800$, and up to an $l_{\max}$ of 2000, by data from the Cosmic Background Imager (CBI; Pearson et al. 2003) and the Arcminute Cosmology Bolometer Array Experiment (ACBAR; Kuo et al. 2002), and large-scale structure (LSS) power spectrum data from the Two Degree Field Galaxy Redshift Survey (2dFGRS; Percival et al. 2002) and the Point Source Catalog Redshift (PSCz) Survey (Hamilton & Tegmark 2002). We use the data covariance matrices and window functions provided by the disfferent experimental teams. For CMB data, we marginalize analytically over known beamwidth and calibration uncertainties (Bridle et al. 2002). For LSS data, we assume that the galaxy power spectrum is a multiple of the underlying matter power spectrum and marginalize analytically over a linear bias (Lewis & Bridle 2002). Both CMB and LSS data depend on $P_{\rm in}(k)$ and on the cosmological parameters. Note, however, that as pointed out by Elgaroy, Gramann, & Lahav (2002), it is hard to detect features in the primordial power spectrum at $k < 0.03\,h\,{\rm Mpc}^{-1}$ using LSS data.

We estimate the $P_{\rm in}(k)$ parameters α_i's (see eq. [1]), together with the Hubble constant H_0, baryon density $\Omega_b h^2$, cold dark matter density $\Omega_c h^2$, and reionization optical depth $\tau_{\rm ri}$. We assume Gaussian adiabatic scalar perturbations in a flat universe with a cosmological constant. We do not use tensor modes in this paper, since current data are not sensitive to tensor contributions. We make use of the *WMAP* constraint on $\tau_{\rm ri}$, derived for a ΛCDM cosmology from *WMAP*'s TE polarization data (Kogut et al. 2003), by imposing a Gaussian prior on $\tau_{\rm ri}$, $p(\tau_{\rm ri}) \propto \exp[-(\tau_{\rm ri} - 0.17)^2/(2\sigma^2_{\tau_{\rm ri}})]$, with $\sigma_{\tau_{\rm ri}} = 0.04$ (this error estimate includes systematic and foreground uncertainties).

For a power-law primordial fluctuation spectrum, $P_{\mathscr{R}} = 2.95 \times 10^{-9} A (k/k_0)^{n_S - 1}$ (Spergel et al. 2003). For an arbitrary primordial power spectrum, we define

$$P_{\rm in}(k) = \frac{P_{\mathscr{R}}}{2.95 \times 10^{-9}} \; . \tag{7}$$

We use $k_0 = 0.05$ Mpc^{-1} when quoting parameter constraints for a power-law model.

We use the Markov Chain Monte Carlo (MCMC) technique to estimate the likelihood functions of the parameters (Knox, Christensen, & Skordis 2001; Kosowsky, Milosavljevic, & Jimenez 2002; Lewis & Bridle 2002; Verde et al. 2003[2]). The use of MCMC is necessitated by the large number of parameters, and it is free of the interpolation errors expected in the conventional and much slower grid method. At its best, the MCMC method scales approximately linearly with the number of parameters. The method samples from the full posterior distribution of the parameters, and from these samples the marginalized posterior distributions of the parameters can be estimated.

Table 1 lists the mean values and marginalized 1 σ confidence limits of the cosmological parameters estimated using four different models for $P_{\rm in}(k)$, as well as $\chi^2_{\rm eff} = -2 \ln \mathscr{L}$, where $\mathscr{L}$ is the likelihood of each model. Both the wavelet band power model and the top-hat binning model allow $P_{\rm in}(k)$ to be an arbitrary nonnegative function. The scale-invariant model assumes $P_{\rm in}(k) = A$ (A is a constant), while the power-law model assumes $P_{\rm in}(k)$ to be a power law, $P_{\rm in}(k) = A k^{n_S - 1}$ (A and n_S are constants). The only priors used are a Gaussian prior on $\tau_{\rm ri}$, $p(\tau_{\rm ri}) \propto \exp[-(\tau_{\rm ri} - 0.17)^2/(2\sigma^2_{\tau_{\rm ri}})]$, with $\sigma_{\tau_{\rm ri}} = 0.04$ as discussed earlier, and a weak age prior of the age of the universe $t_0 > 10$ Gyr. Also, we do not use tensor modes in this paper, since current data are not sensitive to tensor contributions.

Figure 1 shows the reconstructed $P_{\rm in}(k)$ from the two different methods discussed in § 2, using CMB temperature anisotropy data. The dotted line in each panel indicates the scale-invariant model that fits these data.[3]

From Table 1 we see that the power-law model differs by $\Delta\chi^2_{\rm eff} = 8$ from the wavelet band power model, and by

[2] See also Neil (1993) at ftp://ftp.cs.utoronto.ca/pub/~radford/review.ps.gz.

[3] The uncertainty in $P_{\rm in}(k)$ in the wavelet band power method is calculated using $\sigma^2_{P_{\rm in}(k)} = (1/N) \sum_N [\sum_j (P_j - \bar{P}_j) |\hat{\psi}(k/2^j)|^2]^2$, where the average is over the MCMC samples and $\bar{P}_j$ denotes the mean or expectation value of the wavelet band power.

TABLE 1

Parameters Estimated from CMB and LSS Data

$P_{\rm in}(k)$ Model	$P_{\rm in}(k)$ Parameters	$\Omega_b h^2$	$\Omega_m h^2$	h	$\tau_{\rm ri}$	$\chi^2_{\rm eff}$
			CMB Data Only			
Wavelet band power	See Fig. 1	0.0180 ± 0.0038	0.143 ± 0.029	0.575 ± 0.082	0.185 ± 0.045	980.04
Top-hat binning	See Fig. 1	0.0185 ± 0.0031	0.129 ± 0.029	0.617 ± 0.080	0.175 ± 0.044	977.81
Scale-invariant	$A = 0.893 \pm 0.050$	0.0237 ± 0.0006	0.123 ± 0.015	0.710 ± 0.044	0.173 ± 0.036	988.29
Power-law	$A = 0.799 \pm 0.117$	0.0228 ± 0.0012	0.116 ± 0.016	0.713 ± 0.044	0.136 ± 0.054	987.92
	$n_S = 0.974 \pm 0.028$					
			CMB and LSS Data			
Wavelet band power	See Fig. 2	0.0187 ± 0.0031	0.136 ± 0.021	0.601 ± 0.069	0.191 ± 0.047	1037.86
Top-hat binning	See Fig. 2	0.0189 ± 0.0019	0.134 ± 0.016	0.597 ± 0.049	0.164 ± 0.047	1034.80
Scale-invariant	$A = 0.883 \pm 0.050$	0.0238 ± 0.0006	0.121 ± 0.007	0.714 ± 0.022	0.170 ± 0.032	1044.33
Power-law	$A = 0.836 \pm 0.107$	0.0233 ± 0.0010	0.120 ± 0.007	0.707 ± 0.026	0.147 ± 0.055	1043.68
	$n_S = 0.985 \pm 0.028$					

Notes.—CMB temperature anisotropy data are from *WMAP*, CBI, and ACBAR. LSS power spectrum data are from the 2dFGRS and PSCz galaxy redshift surveys. The numbers of data points in the different data sets used are 899 (*WMAP*), 4 (CBI), 7 (ACBAR), 32 (2dFGRS), and 22 (PSCz).

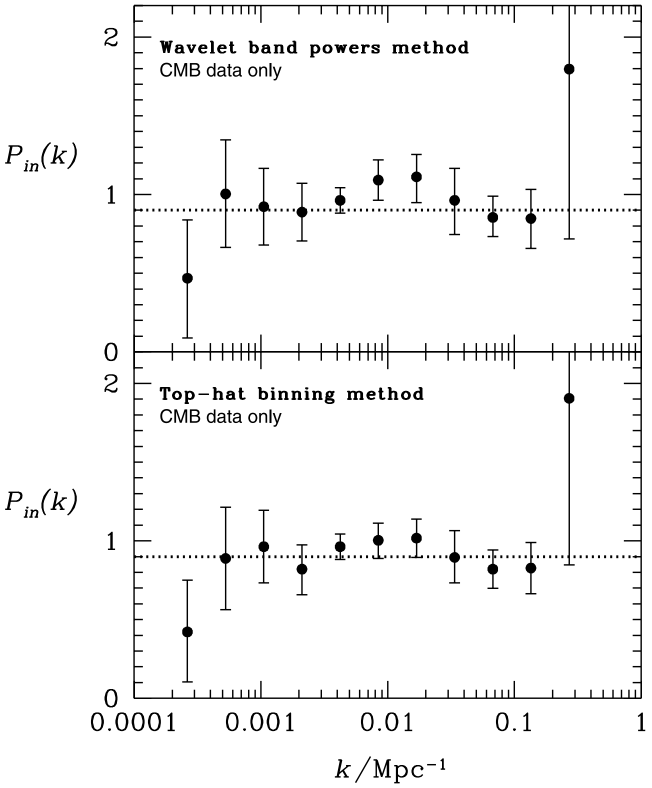

FIG. 1.—Reconstructed $P_{\rm in}(k)$ with 1 σ error bars from the two different methods discussed in § 2, using only CMB data. The dotted line indicates the scale-invariant model that best fits the data.

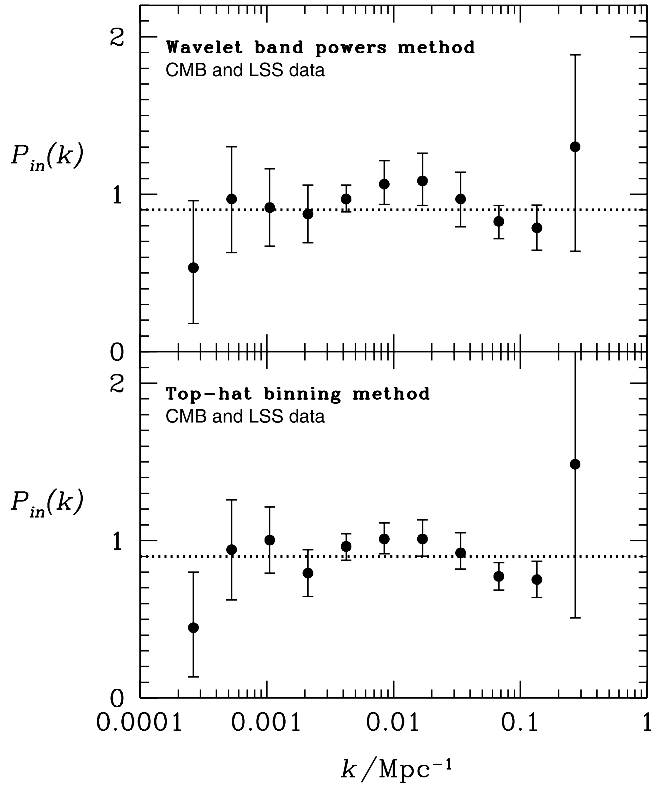

FIG. 2.—Same as Fig. 1, but using CMB data and LSS data

$\Delta\chi^2_{\rm eff} = 10$ from the top-hat binning model. Since the difference in the number of degrees of freedom is approximately 9, the power-law model is disfavored at approximately $\sim$0.7 and $\sim$1 σ compared to the wavelet band power model and the top-hat binning model, respectively. Also, note that the power-law model is favored over the scale-invariant model at less than 1 σ.

Given the estimated wavelet band power P_j's with their full covariance matrix C_{P_j}, we can treat the P_j's as data and compute $\chi^2 \equiv \Delta^T C_{P_j}^{-1} \Delta$, Δ being the difference between the estimated P_j's and the P_j's corresponding to the fitted power-law and scale-invariant spectra. The χ^2's turn out to be 8 and 7 for the power-law and scale-invariant parametrization, respectively. In the top-hat binning case, we can treat the estimated top-hat bin amplitudes as data and compute a similarly defined χ^2. We find that the corresponding χ^2 is 9 for both power-law and scale-invariant parametrizations. Thus, similar significances for deviation of the reconstructed $P_{\rm in}(k)$ from the simpler parametrizations are indicated in this way also. In general, note that the low levels of significance are also due to the large number of degrees of freedom that we are allowing for in the analysis here, and it is clear from the figure which points do not contribute much to the χ^2.

Figure 2 shows the reconstructed $P_{\rm in}(k)$ from the two different methods discussed in § 2, using CMB temperature anisotropy data as above together with LSS data from the 2dFGRS and PSCz galaxy redshift surveys. The constraints on cosmological parameters are listed in Table 1.

The results of fitting the same CMB and LSS data to a scale-invariant model and a power-law model are shown in Table 1. The power-law model differs by $\Delta\chi^2_{\rm eff} = 6$ from the wavelet band power model,[4] and by $\Delta\chi^2_{\rm eff} = 9$ from the top-hat binning model. Since the difference in the number of degrees of freedom is approximately 9, the power-law model is disfavored at approximately $\sim$0.4 and $\sim$0.9 σ compared to the wavelet band power model and the top-hat binning model, respectively. The power law model is again favored over the scale-invariant model at less than 1 σ.

As discussed previously, the estimated parameters that describe the power spectrum as an arbitrary function (wavelet band powers or the top-hat bin amplitudes) can be treated as data and compared with the fitted power-law and scale-invariant spectra by computing a χ^2 using the full covariance matrix of the estimated parameters (wavelet band powers or the top-hat bin amplitudes). In the wavelet band power method, the χ^2 for both power-law and scale-invariant parametrizations is 6. Similarly, in the top-hat case the χ^2's are 10 and 12 for the power-law and scale-invariant parametrization, respectively. These indicate similar significances for deviation of the reconstructed spectrum from the simpler parametrizations.

Figure 3 shows the reconstructed $P_{\rm in}(k)$ using the wavelet band power method, compared with the $P_{\rm in}(k)$ constraints derived using the *WMAP* team's constraints on A, n_S and $dn_S/d\ln k$ for $P_{\rm in}(k) = A(k/k_0)^{n_S-1}$ (Peiris et al. 2003; Spergel et al. 2003) (*shaded region*).[5] The shaded region in

[4] Since the two methods give very similar estimates of $P_{\rm in}(k)$, the smaller $\chi^2_{\rm eff}$ of the top-hat binning method seems to indicate that the wavelet band power method has not yet sampled the parameter values with the smallest possible $\chi^2_{\rm eff}$.

[5] The *WMAP* constraints in Fig. 3 are similar to those in Fig. 2 of Peiris et al. (2003; which considers tensor contributions), but the $P_{\rm in}(k)$ parameter constraints are taken from Table 8 of Spergel et al. (2003) since we do not consider tensor contributions in this paper.

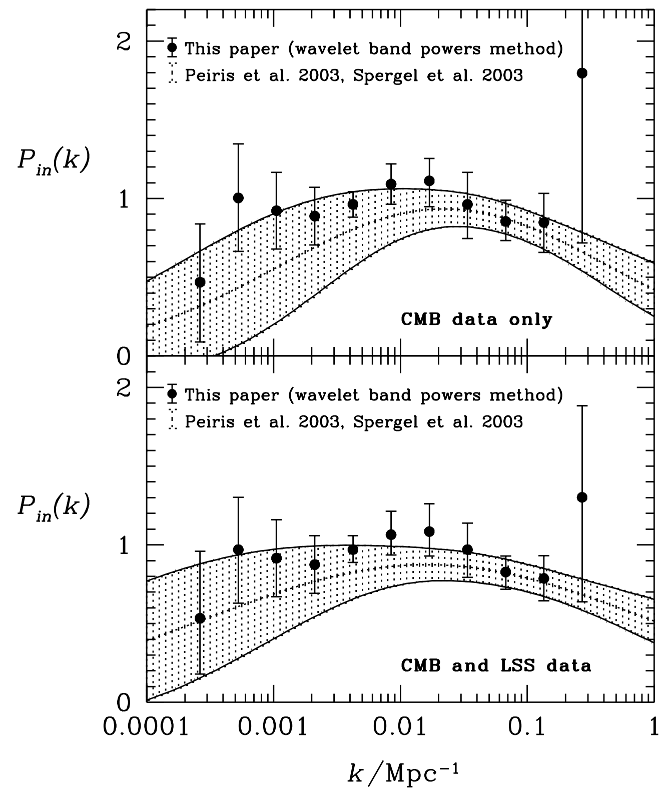

FIG. 3.—Reconstructed $P_{\rm in}(k)$ using the wavelet band power method (with 1 σ error bars), compared with the $P_{\rm in}(k)$ constraints derived using the *WMAP* team's constraints on A, n_S, and $dn_S/d\ln k$ for $P_{\rm in}(k) = A(k/k_0)^{n_S-1}$ (*shaded region*; Peiris et al. 2003; Spergel et al. 2003). We have not included the covariances among A, n_S, and $dn_S/d\ln k$.

Figure 3 is only meant to illustrate roughly the *WMAP* team's constraints, since we have not included the covariances among A, n_S, and $dn_S/d\ln k$ estimated by them (these are not publicly available). Clearly, our results are consistent with the *WMAP* results within 1 σ.

We note that the cosmological parameters are relatively well constrained even when the primordial power spectrum is reconstructed as a free function. In the MCMC method, the parameters are allowed to vary within wide limits. We have monitored convergence and mixing as advocated in Verde et al. (2003). The amplitude of the band on the smallest scale (centered at $k \sim 0.2$ Mpc^{-1}) is essentially unconstrained in both the wavelet band power and top-hat binning methods when using just CMB data. Using CMB and LSS data, the amplitude of this band gets constrained. The amplitude in the band centered at $k \sim 0.0003$ $^{-1}$ is single-tailed toward larger values, but well constrained within the prior. Besides these bands, the power in all the other bands and the cosmological parameters are all well constrained and have close to Gaussian one-dimensional marginalized distributions. Parameter constraints from the full n-dimensional distribution are somewhat weaker, as expected, but consistent with the one-dimensional marginalized distributions.

We find only slight evidence for a preferred scale at $k \sim 0.01$ Mpc^{-1} in the primordial power spectrum from current data (Figs. 1 and 2). This apparent deviation from scale invariance of $P_{\rm in}(k)$ accompanies a slightly low Hubble constant and nonvanishing reionization optical depth $\tau_{\rm ri}$ (see Table 1). Inclusion of tensor contributions in the analysis would also increase the effect (as the data would then be consistent with reduced power on large scales, which can be filled in by tensor contributions, as also noted in Seljak, McDonald, & Makarov 2003). The data do not require this deviation, however, and within parameter degeneracies appear consistent with scale invariance, as well as with a slight red tilt (see Table 1). Note that the current data are consistent with the tensor-to-scalar ratio $T/S = 0$, and the fit is not improved by including T/S as a parameter (Spergel et al. 2003). Therefore, current data do not require a tensor contribution. However, this does not imply that a nonzero tensor contribution is ruled out. Similarly, deviation of the primordial power spectrum from scale invariance is not ruled out at present, although limits can be placed on such deviations (see Figs. 1 and 2). We would be able to better distinguish between these models if cosmological parameters could be constrained to better accuracy. We have not included Lyα, weak-lensing, and supernovae data, since these have larger uncertainties at present.

4. SUMMARY AND DISCUSSION

Reconstructing the shape of the primordial power spectrum $P_{\rm in}(k)$ in a model-independent way from cosmological data is a useful consistency check on what is usually assumed regarding early universe physics. It is also our primary window to unknown physics during inflation. We have used two methods to reconstruct $P_{\rm in}(k)$ as a free function from CMB temperature anisotropy and LSS data (Figs. 1 and 2). The two methods are complementary to each other and give consistent results. We find that $P_{\rm in}(k)$ reconstructed from CMB data alone (*WMAP*, CBI, and ACBAR), or from CMB data together with LSS data (2dFGRS and PSCz), seems to indicate excess power for 0.002 Mpc$^{-1} \lesssim k \lesssim 0.03$ Mpc^{-1}, consistent with that found by the *WMAP* team (Peiris et al. 2003) but at a lower significance of $\sim$1 σ (Fig. 3). Note that the significance level deduced here is also low because we are reconstructing $P_{\rm in}(k)$ in a large number of bins. Neither a scale-invariant $P_{\rm in}(k)$ nor a power-law $P_{\rm in}(k)$ are ruled out by the current data.

We find that this apparent deviation of $P_{\rm in}(k)$ accompanies a slightly low Hubble constant (and correspondingly a slightly high Ω_m) and a nonvanishing $\tau_{\rm ri}$ (Table 1). However, the 1 σ error bars on our derived H_0-values overlap with the 1 σ error bar obtained by the *HST* Key Project (Freedman et al. 2001) and match well the H_0 determined using supernovae (Branch 1998). Note that because of parameter degeneracies, the H_0-values derived from CMB data represent *indirect* measurements, while the H_0-values derived from Cepheid distances (Freedman et al. 2001) or supernova data (Branch 1998) are *direct* measurements. It is also important to include the systematic uncertainty in local direct measurements of H_0 due to matter inhomogeneity in the universe (Wang, Spergel, & Turner 1998), as included in the error estimate of H_0 by Freedman et al. (2001). Clearly, more stringent independent measurements of H_0 can help tighten the constraints on $P_{\rm in}(k)$.

We have not included tensor contributions in our analysis, because the *WMAP* data are not yet constraining on the tensor perturbations from inflation. The inclusion of tensor contributions are expected to increase the deviation of $P_{\rm in}(k)$ from scale invariance.

Our results are consistent with those of Bridle et al. (2003) and Barger et al. (2003); both find that the $P_{\rm in}(k)$ derived from current data from *WMAP* (Bridle et al. 2003 included LSS data as well) are consistent with scale invariance. There are two basic differences between the analysis presented in § 4 of Bridle et al. (2003) and this paper. The sensitivity of their method to the feature around $k \sim 0.01$ Mpc^{-1} is reduced because their banding oversamples this region by a factor of 3. Also, Bridle et al. (2003) reconstructed $P_{\rm in}(k)$ using linear interpolation of amplitudes of $P_{\rm in}(k)$ at discrete k points, an approach pursued in Wang & Mathews (2002) and MW03a. This method is expected to lead to stronger correlations between adjacent $P_{\rm in}(k)$ amplitudes estimated from data. Since the different binning choices made by us and Bridle et al. (2003) lead to different correlations between the estimated parameters, they provide complementary and somewhat different information. Barger et al. (2003) used *WMAP* temperature data to constrain slow-roll inflationary models.

They find that $\tau_{\rm ri} = 0$ is preferred based on *WMAP* temperature data. We have found that taking $\tau_{\rm ri} = 0$ greatly diminishes the significance of any deviations of $P_{\rm in}(k)$ from scale invariance. We have chosen to take into consideration the implications of the *WMAP* polarization data by applying a Gaussian prior on $\tau_{\rm ri}$ based on the results of Kogut et al. (2003).

We note that Miller et al. (2002) have examined the pre-*WMAP* CMB temperature data in a nonparametric way to check whether the data can be better fitted by breaking away from our assumed cosmological model and to deduce the significance levels of the acoustic peaks in the C_l spectrum nonparametrically. This also helps test the robustness of the cosmological model.

We conclude that without making assumptions about the form of $P_{\rm in}(k)$, the $P_{\rm in}(k)$ derived from first-year *WMAP* data deviates from scale invariance (with a preferred scale at $k \sim 0.01$ Mpc^{-1}) only at a significance level of approximately 1 σ (Figs. 1 and 2). The simplest forms of $P_{\rm in}(k)$ (scale-invariant or power-law) are thus consistent with the data at present. The *WMAP* data in subsequent years, together with improved constraints from other independent cosmological probes, will allow us to place firmer constraints on very early universe physics.

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