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NATURE OF FAULT PLANES IN SOLID NEUTRON STAR MATTER

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ABSTRACT

The properties of tectonic earthquake sources are compared with those deduced here for fault planes in solid neutron star matter. Neutron star matter, not being absolutely stable and with isotropic pressure several orders of magnitude greater than its shear modulus, cannot exhibit brittle fracture at any temperature or magnetic field strength. This conclusion is significant for current theories of pulsar glitches and of the anomalous X-ray pulsars and soft gamma repeaters.

Subject headings: pulsars: general — stars: neutron — X-rays: stars

1. INTRODUCTION

It is widely assumed that brittle fractures caused by Maxwell or other stresses in neutron star crusts are involved in a number of phenomena: for example, the soft gamma repeaters (SGRs; Thompson & Duncan 1995, 1996), the persistent emission of the anomalous X-ray pulsars (AXPs) (Thompson et al. 2000; see, however, Heyl & Hernquist 1997 for a thermal emission model assuming neutron stars with thin accreted hydrogen or helium envelopes), and large pulsar glitches (Ruderman, Zhu, & Chen 1998). In current theories of the AXP and SGR sources, brittle fractures, propagating with a velocity of the order of the shear-wave velocity c_s, generate shear waves, which in turn couple with magnetospheric Alfvén modes. At angular frequencies $\omega \approx 10^4$–10^5 rad s^{-1}, the coupling is thought to be an efficient mechanism for energy transfer to the magnetosphere, as shown by Blaes et al. (1989). Statistical comparisons of SGR burst properties with those of terrestrial earthquakes are not inconsistent with the brittle fracture assumption (Hurley et al. 1994; Cheng et al. 1996; Gogus et al. 1999). However, the purpose of this paper is to note that elementary deductions of the properties of neutron star fault planes show that brittle fracture is not possible.

2. NEUTRON STAR FAULT PLANES

Changes in Maxwell stress or in the distribution of stress caused by the Magnus force on neutron superfluid vortices in the crust are equivalent to energy differences that seem large, for example, of the order of 10^{40} ergs in a Vela pulsar glitch. But they are actually small in comparison to potential energy changes associated with departures from equilibrium neutron star chemical composition. This is expressed in the relation between stability and stratification for neutron star matter (Reisenegger & Goldreich 1992; see also Jones 2002), which constrains the movement of matter, bounded by any fault plane that may be formed, to an almost exactly spherical equipotential surface. Acceptance of this is a severe limit on possible crust movement. In general, the movement associated with a given stress-energy change would be expected to involve shear planes with area tending to the minimum possible and so mostly perpendicular to the neutron star surface and extending through the crust from the surface to the boundary with the liquid core of the star. (As an exception, it is possible that small-scale changes in Maxwell stress might be confined to densities below the neutron-drip threshold $\rho_{\rm nd} = 4.3 \times 10^{11}$ g cm^{-3}.) The rotation of a circular cylinder of solid crust under Maxwell stress, by plastic flow or brittle fracture on the cylindrical fault surface delimiting it, is an elementary and possibly unique case (see Thompson et al. 2000). More usually, as in the glitch model of Ruderman et al. (1998), movement of an element of crust toward the equator must be accompanied by a complex backflow to the pole in order to satisfy the stability and stratification relation. The principal movement in these cases is an in-plane shear whose general form is shown in Figure 1 by the displacement of a series of constant surfaces in Lagrangian coordinates that intersect a local element of the fault plane xz. Crack propagation is in the fault plane in the direction of the x-axis. In the neutron star case, the local z-axis is perpendicular to the stellar surface. The components σ_{ij} of the stress tensor act on the surfaces of the volume elements shown on opposite sides of the fault plane. (We assume an isotropic elastic medium in which, before fracture, stress propagates perfectly across the plane.)

In the brittle fracture of a terrestrial earthquake, the stress falls at the instant of failure from $f = \sigma_{xy}$ to a much smaller value (zero in the ideal case). Stress energy is largely converted to kinetic energy so that strain relaxation $\Delta\epsilon$ occurs with an acceleration $\dot{v}$ such that shear waves are generated efficiently (see Kostrov & Das 1988 concerning the definition of a tectonic earthquake source). The mechanical properties of terrestrial matter, as a function of depth, have been tabulated, for example, in Kaye & Laby (1986). At a typical earthquake focus depth of 15 km, the pressure $P \approx 10^{-2}\mu$, where μ is the shear modulus. Crack propagation is possible because P is not so large that it inhibits void formation behind the tip. At greater depths, the increases in T/T_m, where T_m is the melting temperature, and in P/μ both inhibit the void formation that is necessary for crack propagation. Hence a transition to plastic stress response is observed at approximately 20 km depth (see Scholz 1990, pp. 35 and 179) because crystalline dislocation glide, which is not pressure-inhibited, becomes the only possible failure mechanism. Tectonic earthquake sources are not found in the plastic region. The deep-focus events, which occur in subduction zones at depths up to 500 km, are probably

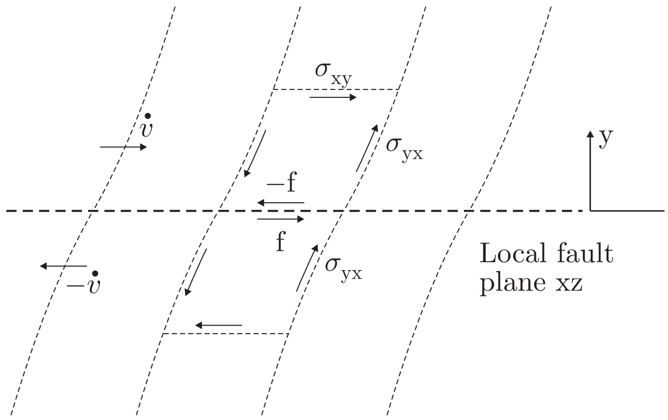

FIG. 1.—For in-plane shear, with displacement shown by a series of constant surfaces in Lagrangian coordinates, the components of the stress tensor acting on volume elements on opposite sides of the $y = 0$ fault plane are $f = \sigma_{xy}$ before failure. In ideal brittle failure, $f = 0$ immediately afterward and strain relaxation occurs with acceleration $\dot{v}$ such that stress energy is efficiently converted to shear waves. The crack propagates in the x-direction within the fault plane. For neutron star matter, f can exhibit no sudden decrease because no sufficiently long-lived void is formed.

intermittently running polymorphic phase transitions, following the original idea of Bridgman (1945). Link, Franco, & Epstein (1998) have noted that analogous phase transitions might occur in neutron star matter. That metastability exists within a volume large enough for observable effect is, in the Earth, a direct result of the continuous subduction process itself. But in neutron star matter, subduction is not consistent with the stratification and stability constraint referred to above, and there is no obvious alternative way in which significant metastable volumes could be formed.

The contrast with neutron star matter is extreme because the latter is not absolutely stable, being in equilibrium only at finite pressure. Those isotropic components of the stress tensor derived from the electrons and from Coulomb interaction (the Coulomb-electron partial pressure P_{Ce} that excludes the neutron partial pressure) are 1 or 2 orders of magnitude larger than the shear modulus, and this largely determines the structure of defects such as monovacancies (Jones 1999). The way in which the nearest neighbors to a monovacancy site relax by displacement considerably reduces the monovacancy formation enthalpy, for example, to 13 MeV at a matter density of 8×10^{13} g cm^{-3} in the neutron-drip region. Such relaxation would not be possible for nuclei on the surface of a hypothetical void whose linear dimension is of the order of several times the mean internuclear separation a and is also of the order of, or greater than, the relativistic electron screening length. These nuclei could not be in stable equilibrium and would move with velocities of the order of $\hbar/Ma$, where M is the nuclear mass, which are within an order of magnitude of c_s. The void would therefore be unstable against dissociation to monovacancies with a lifetime, in neutron star matter, of the order of 10^{-19} s. (This conclusion should be valid for any likely value of the mean nuclear charge Z because the relativistic electron screening length $\propto aZ^{-1/3}$ is only a slowly varying function of that variable.) Thus we can assert that nuclei at a fault plane may have different, probably less, small-distance order than exists elsewhere but almost the same nearest-neighbor separations. Reference to the theory of crack stability (Anderson 1995; see also Landau & Lifshitz 1970) shows that in brittle fracture, the important properties of the stress distribution of an unstable crack are the zero on the fault plane and the singularity at its tip (in the ideal case) that enable it to propagate into regions of much lower stress. These do not exist in neutron stars because of the impossibility of forming a sufficiently long-lived void.

Link et al. (1998) observed, in a footnote, that pressures of the order of the elastic moduli are associated, in terrestrial matter, with plastic rather than brittle stress response, but they did not proceed further with this question. Much earlier, plastic deformation of neutron star matter had been considered by Smoluchowski & Welch (1970) in the long-time limit of the logarithmically time-dependent stress-strain relation, in which the strain rate is proportional to T/t. The large time t in this context is that for pulsar spin-down, and the question is whether stress developing over such times can be relieved by continuous plastic flow. This differs from the present paper, which is concerned primarily with relatively short times and with the propagation velocity of structural failure.

Previous formation enthalpy calculations (Jones 2001) have shown that an amorphous heterogeneous solid phase is formed in the crust and is likely to persist as the star cools. The mean square deviation $\overline{(\Delta Z)^2}$ from the average nuclear charge is large compared with unity. The system is analogous to a multicomponent amorphous alloy in having some local order, which vanishes with increasing length scales but differs in not being absolutely stable. Laboratory experiments on systems of this kind with absolute stability show that shear stress produces faults in the form of localized shear bands (see, for example, Xing, Eckert, & Schultz 1999, who also give a typical stress-strain relation). These are thin layers of inhomogeneous plastic flow, formed sequentially, which permit local strain relaxation through dissipation and the transfer of stress energy to neighboring regions. Their structure is analogous to that of a viscous layer being deformed adiabatically by the relative motion of parallel plates. A system such as solid neutron star matter, amorphous and not absolutely stable, must deform similarly under shear stress at any density either above or below $\rho_{\rm nd}$. The process is not analogous to brittle fracture because most of the local stress energy, instead of being converted to kinetic energy, is either dissipated or transferred to neighboring regions.

Solid phases with low-dimensional structures may exist between the region of spherical nuclei, considered above, and the liquid core. Their elastic constants have been calculated by Pethick & Potekhin (1998). For the same reasons as in the spherical nuclei phase, brittle fracture is not possible, but the mode of failure under stress is not known. The orientation of these structures relative to the local stellar surface is a significant but unknown variable.

A very elementary model satisfying the stability and stratification constraint is that of a cylinder with axis perpendicular to the stellar surface and able to rotate under Maxwell stress, by plastic flow on the cylindrical fault surface delimiting it (see also Thompson et al. 2000). A very simple equation of motion consistent with the laboratory stress-strain relation can be obtained by assuming a plastic flow threshold f_0 and an effective Maxwell stress component $f(1 - s/s_0)$, decreasing with slip distance $s \ll s_0$, where s_0 and f are constants. If some unspecified stress transfer at

$t = 0$ changes f discontinuously so that $\Delta f = f - f_0 > 0$, the equation of motion is

$$\frac{1}{4}\rho d\ddot{s} = \Delta f - f\frac{s}{s_0}\ , \tag{1}$$

where ρ is the matter density and d is the cylinder radius. The total slip distance is $2s_p$, where $s_p = s_0\Delta f/f$, and the slip velocity is $v = \dot{s} = \tilde{\omega}s_p \sin(\tilde{\omega}t)$. The energy available for transfer to shear waves is only a fraction $\Delta f/2f_0$ of that dissipated in the plastic flow, and it is anticipated that, in real physical systems, this will always be 1 or more orders of magnitude smaller than unity. Moreover, the angular velocity of the motion is too small for efficient coupling with shear waves. The angular velocity $\tilde{\omega} = (4f/\rho ds_0)^{1/2}$ can be reexpressed by noting that $f \approx f_0$ and by assuming a threshold $f_0 \approx 10^{-2}\ \mu$, where μ is the shear modulus (see, for example, the stress-strain relation observed by Xing et al. 1999). It becomes $\tilde{\omega} \approx 0.2(c_s^2/ds_0)^{1/2}$ and, as shown by its dependency on c_s, is greatest at large ρ. Although the system is amorphous its shear modulus, for present purposes, will not differ much from the standard *bcc* lattice expression. For $\rho = 10^{14}$ g cm^{-3}, we can therefore assume a shear modulus $\mu = 10^{30}$ ergs cm^{-3}. The depth of the neutron-drip crust could be assumed as the natural scale length for variation of the Maxwell stress except that Hall drift might lead to the formation of strong small-scale variation (see Hollerbach & Rüdiger 2002, who also give references to earlier work on this problem). For the purposes of estimation, a small value $s_0 = 10^4$ cm is assumed, identical to the cylinder radius $d = 10^4$ cm. These values give $\tilde{\omega} = 2 \times 10^3$ rad s^{-1}, small compared with those values, 10^4–10^5 rad s^{-1}, giving efficient coupling of shear and Alfvén waves. The factors involved in slip are mechanical and do not depend on the size of the Maxwell stress components but on the extent to which they deviate from values that would give an equilibrium in a completely liquid star (the effective component with parameters f and s_0 in the model). The mode of failure is determined by the nature of the solid, and a solid with $P_{Ce} \gg \mu$ that is far from being absolutely stable does not exhibit brittle fracture at any magnetic flux density.

In principle, adiabatic heating on the fault surface could lead to a localized solid-liquid phase transition and a reduction in viscosity by many orders of magnitude, so giving slip velocities much larger than $\tilde{\omega}s_p$. An order-of-magnitude estimate of the condition that slip and adiabatic heating should increase the temperature to T_m on the fault surface is given by $f_0 s_p \approx CT_m[(\kappa/\tilde{\omega}C)]^{1/2}$, where κ is the thermal conductivity and C is the specific heat. At a typical density of 8×10^{13} g cm^{-3}, $T_m = 5.8 \times 10^9$ K for the lattice parameters given by Negele & Vautherin (1973). The specific heat is almost entirely that of normal neutrons; $C = 4.9 \times 10^{20}$ ergs cm^{-3} K^{-1} for an effective mass of $0.8m_n$ (see Pines & Nozières 1966). The thermal conductivity $\kappa = 1 \times 10^{20}$ ergs cm^{-1} s^{-1} K^{-1} has been estimated from the electron conductivity results given by Gnedin, Yakovlev, & Potekhin (2001). For $f_0 \approx 10^{28}$ ergs cm^{-3}, this condition shows that a local slip distance exceeding $s_p \approx 3$ cm would be required to increase the fault surface temperature to T_m. Thus a slip of this order of magnitude could produce melting and subsequent slip velocities much larger than $\tilde{\omega}s_p$. But the assumption of a single shear band in the form of a cylindrical fault surface is artificial and unlikely to be realized in physical systems. The general case is of more complex movements, with backflow to satisfy the stability and stratification constraint. Plastic flow must be of complex form, stress transfer producing many distinct shear bands with small slip distances not satisfying the melting condition. In such flow, the velocity $v \approx c_s\Delta\epsilon$ and acceleration $\dot{v} \approx \omega v$ necessary for efficient shear wave generation at angular velocity ω are not reached.

3. CONCLUSIONS

Our conclusion is that the strain relaxation conditions necessary for the generation of shear waves are not present in neutron stars. This need not have a great impact on theories of the pulsar glitch phenomena. In the theory of Ruderman et al. (1998), the movements caused by Maxwell stress and by spin-down of the neutron superfluid in the core can be many orders of magnitude slower than those associated with brittle fracture and yet remain consistent with the observed upper limit on the spin-up time for Vela glitches. But the hypothesis that a plastic-brittle transition, on cooling to internal temperature $T \approx 10^{-1}T_m$ at ages between 10^3 and 10^4 yr, is responsible for the absence of large glitches in very young pulsars is not consistent with the properties of neutron star matter fault planes deduced here. These may also be useful in determining the mechanism for energy release in AXP and SGR sources (Thompson & Duncan 1996; Thompson et al. 2000). These authors consider a critical internal magnetic flux density $B_\mu = (4\pi\mu)^{1/2}$ above which Maxwell tensor components exceed the shear modulus. They assume that brittle fractures occur at $B < B_\mu$ and are the source of Alfvén wave generation (Blaes et al. 1989) leading to both quiescent and burst emission, while only plastic flow is possible for $B > B_\mu$. But such a transition cannot occur, and our conclusion that brittle fracture is not possible under any conditions within a neutron star indicates that other mechanisms (see, for example, Thompson, Lyutikov, & Kulkarni 2002; Mereghetti et al. 2002) need to be considered in more detail.

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