

THE EPOCH OF REIONIZATION IN MODELS WITH REDUCED SMALL-SCALE POWER

RACHEL S. SOMERVILLE,¹ JAMES S. BULLOCK,^{2,3} AND MARIO LIVIO¹

Received 2003 March 20; accepted 2003 April 21

ABSTRACT

Reducing the power on small scales relative to the “standard” Λ cold dark matter (Λ CDM) model alleviates a number of possible discrepancies with observations and is favored by the recent analysis of the *Wilkinson Microwave Anisotropy Probe* (*WMAP*) plus galaxy and $\text{Ly}\alpha$ forest data. Here we investigate the epoch of reionization in several models normalized to *WMAP* on large scales and with sufficiently reduced power on small scales to solve the halo concentration and substructure problems. These include a tilted model, the *WMAP* running-index model, and a warm dark matter model. We assume that the universe was reionized by stellar sources composed of a combination of supermassive ($\sim 200 M_\odot$) Population III stars and Population II stars with a “normal” initial mass function (IMF). We find that in all of these models, structure formation and hence reionization occurs late, certainly at redshifts below 10, and more probably at $z \lesssim 6$. This is inconsistent (at 2σ) with the determination of $z_{\text{reion}} \simeq 17$ from the *WMAP* temperature-polarization data and is only marginally consistent with Sloan Digital Sky Survey quasar observations. The tension between the galactic-scale observations, which favor low-power models, and the early reionization favored by *WMAP* can only be resolved if the efficiency of Population III star formation is dramatically higher than any current estimate or if there is an exotic population of ionizing sources such as miniquasars. Otherwise, we may have to live with the standard Λ CDM power spectrum and solve the small-scale problems in some other way.

Subject headings: cosmology: theory — galaxies: evolution — intergalactic medium

On-line material: color figures

1. INTRODUCTION

The first-year *Wilkinson Microwave Anisotropy Probe* (*WMAP*) satellite data have yielded a remarkable confirmation of what is coming to be regarded as the “standard model” of cosmology: a Hubble parameter $H_0 \simeq 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$, matter and vacuum densities consistent with a flat geometry $\Omega_m \simeq 1 - \Omega_\Lambda \simeq 0.3$, and a nearly scale-invariant primordial power spectrum, $n_s \simeq 1$, at least on large scales. There were some surprises, as we will discuss below, but in many ways the *WMAP* results highlight the marked success of standard Λ cold dark matter (Λ CDM) in reconciling diverse observations from the cosmic microwave background (CMB), to galaxy clusters, supernovae, and gravitational lensing. This remarkable agreement with data on large scales, however, brings into sharper focus several nagging discrepancies on small scales. Specifically, observed galaxy central densities appear to be significantly lower than the standard Λ CDM model predicts (Flores & Primack 1994; Moore 1994; Alam, Bullock, & Weinberg 2002; Swaters et al. 2003; McGaugh, Barker, & de Blok 2003; van den Bosch, Mo, & Yang 2003), and Λ CDM may also produce too much substructure compared with observed numbers of satellite galaxies in the Local Group (Klypin et al. 1999; Moore et al. 1999). A natural way to relieve these problems, while maintaining the large-scale success of the model, is to reduce the power on small scales relative to large, either by introducing a “tilt” in the primordial power

spectrum ($n_s < 1$), as expected in some variants of inflation (Alam et al. 2002; Zentner & Bullock 2002), or by resorting to warm dark matter (WDM) (Avila-Reese et al. 2001; Bode, Ostriker, & Turok 2001; Alam et al. 2002). In these models, collapse happens later, when the universe was less dense, and galaxy halos are naturally less centrally concentrated. Later collapse also alleviates the dwarf problem (Colín, Avila-Reese, & Valenzuela 2000; Knebe et al. 2002; Bullock & Zentner 2002; Zentner & Bullock 2003) and the “angular momentum catastrophe”—the ongoing problem with producing disk galaxies with sufficient specific angular momentum in hydrodynamic simulations within the Λ CDM framework (Sommer-Larsen & Dolgov 2001). Interestingly, one of the surprises of the *WMAP* analysis is that when combined with other small-scale data from the 2dF galaxy redshift survey and the $\text{Ly}\alpha$ forest, it favors (at $\sim 2\sigma$) a model in which the spectral index varies strongly as a function of wavenumber, $dn/d \ln k \simeq -0.03$ (Spergel et al. 2003), in precisely the manner needed to cure many of the problems on small scales (Zentner & Bullock 2003).

The second (and perhaps major) surprise from the *WMAP* report was the detection of a large amplitude signal in the temperature-polarization maps, indicating a large optical depth to Thomson scattering ($\tau = 0.17 \pm 0.04$). The straightforward interpretation of this result is that the universe became reionized at $z_{\text{reion}} = 17 \pm 5$ (Kogut et al. 2003), rather earlier than many had expected. Several workers have recently demonstrated that even within the standard Λ CDM framework, models of reionization require rather extreme assumptions in order to produce enough early star or quasar formation to reionize the universe by $z \sim 17$ (Wyithe & Loeb 2003b; Haiman & Holder 2003; Ciardi, Ferrara, & White 2003a; Sokasian et al. 2003; Cen 2003b). Put in the context of the small-scale crises facing the

¹ Space Telescope Science Institute, Baltimore, MD 21218; somerville@stsci.edu, mlivio@stsci.edu.

² Harvard-Smithsonian Center for Astrophysics, Cambridge, MA 02138; jbullock@cfa.harvard.edu.

³ Hubble Fellow.

standard model, and indeed, in view of the running-index model favored by the extended *WMAP* analysis, we are left with a paradox. The galactic-scale data favor low-power models precisely because they produce late structure formation, but the high optical depth measurement seems to favor early collapse instead. In fact, the late reionization implied by low-power models would more easily accommodate the simplest interpretation of the Gunn-Peterson troughs observed in several Sloan Digital Sky Survey (SDSS) quasars at $z \gtrsim 6$ (Fan et al. 2001; Becker et al. 2001) and the high temperature of the intergalactic medium (IGM) at $z \sim 4$ (Theuns et al. 2002; Hui & Haiman 2003). This tension has been anticipated for several years; for example, Barkana, Haiman, & Ostriker (2001) considered the constraints that could be placed on WDM based on knowledge of z_{reion} and suggested that *WMAP* would be able to provide this knowledge.

In this paper we set out to *quantify* these concerns for several classes of models with reduced small-scale power using a simple yet conservative analytic approach that is informed and motivated by numerical simulations. Our aim is to compute conservative upper limits on z_{reion} for several models that have small-scale power reduced to plausibly solve the galactic-scale difficulties, including the running spectral index model favored by *WMAP*. In what follows, we assume that reionization is due to stellar sources alone, as active galactic nuclei (AGN) are likely to be unimportant to reionization at high redshift unless there are exotic objects (such as “miniquasars”) that are completely disjoint from the known population (Schirber & Bullock 2003).

2. MODELS AND NORMALIZATION

In order to focus on the ramifications of reducing the small-scale power, we will fix the cosmological parameters in all of our models to canonical values: $\Omega_m = 1.0 - \Omega_\Lambda = 0.3$, $\Omega_b h^2 = 0.02$, and $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$. Additional parameters of each of the models are described in Table 1. In all cases, we assume that the primordial power spectrum takes the form $P(k) \propto k^{n_s}$, with a spectral index that can vary as function of scale as $n_s = n_s(k_0) + 0.5 \frac{dn}{d \ln k} \ln(k/k_0)$.⁴ In column (2) of Table 1, we list n_s , evaluated at $k_0 = k_{\text{WMAP}} \equiv 0.05 \text{ Mpc}^{-1}$, and in column (3), we give $dn/d \ln k$. We use the transfer function of Eisenstein & Hu (1999), which accounts for the effects of baryons. In order to make contact with previous work, our specific choice for normalizing standard Λ CDM

(with $n_s = 1.0$) is $\sigma_8 = 0.9$, where σ_8 (col. [4]) is the linear rms fluctuation amplitude of the power spectrum within spheres of radius $8 h^{-1} \text{ Mpc}$, evaluated at $z = 0$. This spectral index and normalization are very close to the best-fit parameters derived from the *WMAP* data alone, without combination with other data sets. In order to ensure consistency on large scales, we have chosen to fix the normalization of the rest of our models to match that of standard Λ CDM at $k = k_{\text{WMAP}}$.

The first of our models with reduced small-scale fluctuations, “TILT,” has $\sigma_8 = 0.75$ and modest tilt, $n_s = 0.95$, as expected in many simple models of inflation (see, e.g., Kinney 2003). The specific parameter choice is motivated by the work of Zentner & Bullock (2002, 2003), who concluded that this normalization/tilt is favored by galaxy rotation curve data and may also alleviate the dwarf satellite problem without the need for differential feedback. Interestingly, this parameter choice is also nearly identical to the best-fit power-law model in the *WMAP* joint analysis with other CMB data, the 2dF Galaxy Redshift Survey, and Ly α forest data (Spergel et al. 2003). The next model, “RSI,” is motivated by the same joint *WMAP* analysis but represents the case in which they have allowed a running spectral index. Their best fit has $n_s = 0.93$ and $dn/d \ln k = -0.03$. Our normalization for RSI, $\sigma_8 = 0.81$, is slightly below the quoted *WMAP* value (by $\sim 3.5\%$) because of the simple way we have elected to normalize on large scales, but this difference is small compared with the quoted uncertainty. This RSI case also does quite well in explaining the small-scale observations (Zentner & Bullock 2003).

Our final model is computed in the context of a WDM scenario in which the primordial power spectrum is scale-invariant, but free streaming damps fluctuations below a characteristic scale set by the WDM particle mass m_W ,

$$R_f = 0.2 (\Omega_W h^2)^{1/3} \left(\frac{m_W}{\text{keV}} \right)^{-4/3} \text{ Mpc} . \quad (1)$$

We calculate the WDM spectra assuming the same flat cosmology with $\Omega_W = \Omega_m = 0.3$ and use the approximate WDM transfer function given by Bardeen et al. (1986),

$$P(k) = \exp[-k R_f - (k R_f)^2] P_{\text{CDM}}(k) , \quad (2)$$

where $P_{\text{CDM}}(k)$ is the usual CDM power spectrum. We choose $m_W = 1.5 \text{ keV}$, or $R_f \simeq 0.027 \text{ Mpc}$, in order to suppress power below a mass scale of $\sim 10^{10} M_\odot$. This choice of WDM mass (with $n_s = 1.0$) significantly alleviates the usual small-scale problems (Eke, Navarro, & Steinmetz 2001; Alam et al. 2002; Zentner & Bullock 2003) yet is not ruled out by Ly α forest data (Narayanan et al. 2000) or by pre-*WMAP* constraints from reionization and early structure

TABLE 1
SUMMARY OF MODELS

Model (1)	$n(k_{\text{WMAP}})$ (2)	$dn/d \ln k$ (3)	σ_8 (4)	m_ν (keV) (5)	$z(n_\gamma = 2)$ (6)	$z(n_\gamma = 10)$ (7)
Λ CDM.....	1.0	0.0	0.90	0.0	13.8	9.0
TILT.....	0.95	0.0	0.75	0.0	9.7	6.0
RSI.....	0.93	-0.03	0.81	0.0	8.1	5.1
WDM.....	1.0	0.0	0.89	1.5	7.0	4.9

⁴ Note that our definition of n_s follows Spergel et al. (2003) and Hannestad et al. (2002) and differs by a factor of 2 from Kosowsky & Turner (1995).

formation (Barkana et al. 2001). We neglect the finite particle velocity dispersion, as well as associated phase-space restrictions, which are expected to have negligible consequences for a 1.5 keV model (Alam et al. 2002; Zentner 2003).

3. HALO COLLAPSE RATES

We show in Figure 1 the fraction of the total mass in the universe that is in collapsed halos in two different mass ranges, computed using Press-Schechter formalism (Press & Schechter 1974). In the top panel, we show the fraction of mass in halos with mass greater than $1.0 \times 10^6 h^{-1} M_\odot$ but with temperatures of less than 10^4 K. (Conversions between mass and temperature are the same as in Somerville & Primack 1999.) As discussed below, we expect gas in these halos (sometimes called “minihalos”) to be able to cool only via H_2 , and based on the results of hydrodynamic simulations by Yoshida et al. (2003), they may potentially harbor massive Population III stars. In the bottom panel, we show the mass fraction in halos with $T_{\text{vir}} > 10^4$ K, which should be able to cool via H I and form Population II stars with a normal Salpeter-like initial mass function (IMF). We see that all models with reduced small-scale power have dramatically decreased mass fractions relative to standard Λ CDM in halos of the scale expected to be eligible for star formation at high redshift. The TILT model shows the smallest decrease on all scales, while the WDM model shows the most dramatic decrease.

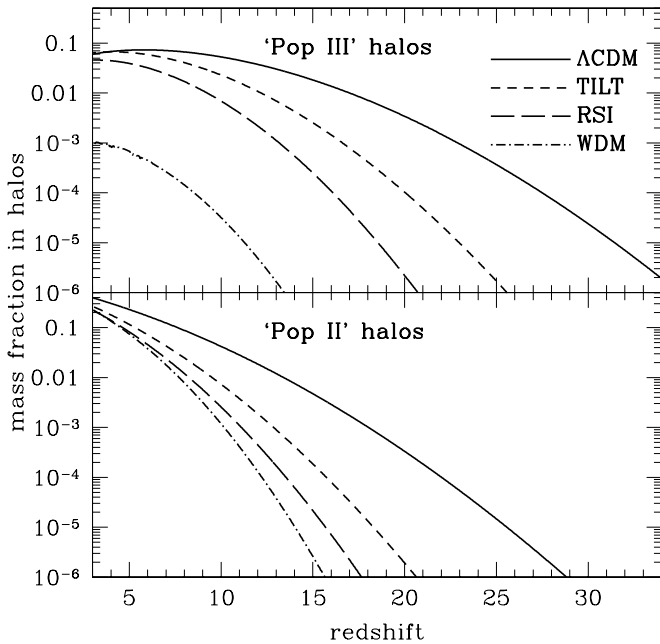


FIG. 1.—Fraction of mass in collapsed dark matter halos as a function of redshift for the four power spectra summarized in Table 1. The top panel shows the mass fraction in halos with mass greater than $1.0 \times 10^6 h^{-1} M_\odot$ but virial temperature $T_{\text{vir}} < 10^4$ K. In the absence of metals, gas in these halos can cool primarily by molecular hydrogen, and we associate them with possible sites of massive ($200 M_\odot$) Population III star formation. The bottom panel shows the mass fraction in halos with $T_{\text{vir}} > 10^4$ K. We associate these halos with H I cooling and Population II star formation with a “normal” IMF. The impact of reduced small-scale power on early star formation on both of these mass scales is dramatic. [See the electronic edition of the Journal for a color version of this figure.]

4. STAR FORMATION AND IONIZING PHOTONS

A basic requirement for star formation is the presence of cold dense gas. The two primary coolants in the early universe, before the production of heavy elements, are molecular and atomic hydrogen. Atomic hydrogen is inefficient at temperatures below $\sim 10^4$ K, implying that molecular hydrogen was probably the main coolant in the first halos to collapse at $z \sim 20$ –30, with temperatures of a few hundred kelvins. Recent theoretical work suggests that the first stars to form in these primordial halos may have been extremely massive, ~ 100 – $600 M_\odot$, but that the efficiency was rather low, with less than $\sim 1\%$ of the available gas converted to stars or star clusters (Abel et al. 2000, 2002; Bromm, Coppi, & Larson 2002).

The overall efficiency of early star formation is regulated in minihalos by a complex interplay between the destruction of H_2 by UV photons and its catalysis by X-rays (Machacek, Bryan, & Abel 2001, 2003; Ricotti, Gnedin, & Shull 2001, 2002; Cen 2003a) and in larger halos by photoevaporation and supernova feedback (Ciardi et al. 2000; Somerville 2002; Benson et al. 2002). The efficiency of production and mixing of heavy elements also determines the epoch at which the IGM becomes sufficiently polluted with metals to allow cooling to lower temperatures and fragmentation, leading to a shift from the formation of solely supermassive stars to a more normal Salpeter-like IMF (Bromm et al. 2001a). For a more detailed discussion of this scenario, see the excellent review by Loeb & Barkana (2001), as well as recent papers by Cen (2003a), Haiman & Holder (2003), and Wyithe & Loeb (2003a, 2003b).

We model the global star formation rate density (SFRD) by assuming that it is proportional to the rate at which gas collapses into halos in a given mass range,

$$\dot{\rho}_* = e_* \rho_b \frac{dF_h}{dt}(M > M_{\text{crit}}), \quad (3)$$

where $dF_h/dt(M > M_{\text{crit}})$ is the time derivative of the fraction of the total mass in collapsed halos with masses greater than M_{crit} , obtained from the halo mass function $dn_h/dM(M, z)$ given by the Press-Schechter model, and ρ_b is the mean density of baryons. We adopt $M_{\text{crit}} = M(T_{\text{vir}} = 10^4 \text{ K})$ and $e_*^{\text{II}} = 0.1$ for “Population II” halos and $e_*^{\text{III}} = 0.002$ and $M_{\text{crit}}^{\text{III}} = 1.0 \times 10^6 h^{-1} M_\odot$ for “Population III” halos. To avoid “double counting,” we also apply an upper mass cutoff on Population III halos of $M(T_{\text{vir}} = 10^4 \text{ K})$, although in practice this has little effect on our results. In Somerville & Livio (2003, hereafter SL03), we found that this simple prescription, with these parameter values, agrees well in the redshift range $3 \lesssim z \lesssim 30$ with more detailed semianalytic models and hydrodynamic simulations of Population II star formation including photoionization and supernova feedback (e.g., Somerville, Primack, & Faber 2001; Springel & Hernquist 2003) and also with the general analytic arguments outlined by Hernquist & Springel (2003). The global star formation rate predicted by this approach is also consistent with observational constraints at $3 \lesssim z \lesssim 6$ (SL03). Similarly, our Population III SFRD is in good agreement with the detailed numerical hydrodynamic simulations of Population III formation by Yoshida et al. (2003), as also shown explicitly in SL03. In both cases, the analytic recipe used here slightly *overestimates* the star formation rate relative to the more realistic simulations, thus leading to more optimistic results

for early reionization. As we do not know when the transition from solely supermassive Population III stars to Population II star formation will occur, we conservatively shut off the Population III mode at $z < 6$ (when the IGM is known to be significantly polluted with metals). Earlier shutoff times will only lead to later reionization, making our conclusions even stronger.

For Population II stars, we use the results of Leitherer et al. (1999) for the number of $\lambda < 912 \text{ \AA}$ photons produced by low-metallicity stars with a “bottom-light” Salpeter IMF. At ages less than about 3 million yr, these stars produce about $8.9 \times 10^{46} \text{ photons s}^{-1} M_{\odot}^{-1}$. For Population III, we assume that each star produces $1.6 \times 10^{48} \text{ photons s}^{-1} M_{\odot}^{-1}$ for a lifetime of 3 million yr (Bromm, Kudritzki, & Loeb 2001b). We note that, in the spirit of optimism that we have adopted throughout, these values are at the high end of the estimates of ionizing photon production and lifetime for both populations.

The cumulative number of ionizing photons per hydrogen atom in the universe n_{γ} is shown in Figure 2 for each of the four models summarized in Table 1. Many analytic and numerical studies have attempted to determine the critical value n_{γ}^{crit} needed to attain “overlap,” or reionization that is $\sim 99\%$ complete (Sokasian et al. 2003; Ciardi, Stoehr, & White 2003b; Razoumov et al. 2002; Haiman, Abel, & Madau 2001; Miralda-Escudé, Haehnelt, & Rees 2000; Gnedin 2000; Madau, Haardt, & Rees 1999). Typically, n_{γ}^{crit} is expected to be greater than unity, as not all photons escape from the galaxy, the IGM is clumpy, and ionized

hydrogen can recombine. We subsume the uncertainties associated with the escape fraction f_{esc} , the number of ionizations per UV photon f_{ion} , and the clumping factor C_{clump} into the parameter n_{γ}^{crit} . For reference, Sokasian et al. (2003) find that a gross budget of ionizing photons (i.e., before applying an escape fraction) $n_{\gamma} \sim 5\text{--}20$ is needed to achieve a volume-weighted ionization fraction of 99% in a recent high-resolution numerical simulation of reionization by Population II-like stellar sources. This range is consistent with other results from the literature. The clumping factors in numerical simulations are known to be potentially overestimated because of their limited numerical resolution (Haiman et al. 2001; Sokasian et al. 2003), and no numerical simulation to date has included the contribution from Population III stars, which are likely to be less clustered and to have high escape fractions. We therefore entertain the possibility that as few as ~ 2 ionizing photons per hydrogen atom may be able to do the job, although $n_{\gamma}^{\text{crit}} \sim 10$ is likely to be more realistic. The redshift at which $n_{\gamma} = n_{\gamma}^{\text{crit}} = 2$ or 10 in each of the models is shown in Figure 2 and recorded in Table 1. We regard these values as bracketing a generous but plausible range for the expected redshift of overlap in these models.

One can quickly see that it is difficult to achieve reionization earlier than $z \sim 10$ in any of the models with reduced small-scale power. In the RSI model favored by Spergel et al. (2003), $n_{\gamma}^{\text{crit}} = 2$ occurs at $z \sim 8$ —only slightly earlier than for the WDM model. Values of $n_{\gamma} \gtrsim 10\text{--}20$, corresponding to more typically favored values of the photon escape fraction ($f_{\text{esc}} \sim 0.2$), recombination rate, and clumping factor, are attained only at $z \sim 6\text{--}5$ in the models with reduced small-scale power—thus these models (particularly WDM and RSI) may not even be able to reionize the universe early enough for consistency with the SDSS quasar observations. As noted above, we have allowed massive Population III star formation to continue until $z = 6$, although we actually expect it to shut off much earlier. This would lead to even later reionization, as shown by the short-dashed lines in Figure 2, which show the contribution from Population II stars only.

4.1. Uncertainties and Extreme Parameter Values

Certainly it may be argued that the IMF and formation efficiency of early generations of stars are highly uncertain. We now consider how much we would have to vary the free parameters in our simple model to obtain reionization by $z \sim 17$ in the RSI model favored by the analysis of *WMAP* combined with other data (Spergel et al. 2003). These results are summarized in Table 2. In model RSI.x1, we increase the efficiency of star formation in Population II halos to 100% (corresponding to all baryons forming stars the moment they are within a halo) and the assumed mass of

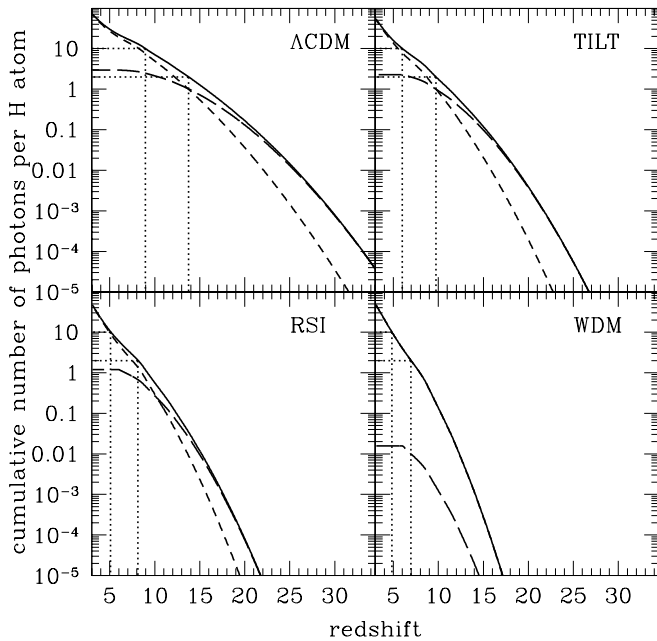


FIG. 2.—Cumulative number of hydrogen-ionizing photons per hydrogen atom produced, as a function of redshift, for the four initial power spectra. Long dashed lines show the contribution from Population III stars, short dashed lines show the contribution from Population II stars, and solid lines show the total. Straight dotted lines indicate the redshift at which two and 10 ionizing photons per atom have been produced and roughly bracket the range of redshifts in which the overlap of ionized regions is expected to occur. In all three models with reduced small-scale power, overlap occurs later than $z \sim 10$. This may be compared with the estimate of $z_{\text{reion}} = 17 \pm 5$ from the *WMAP* temperature-polarization results (Kogut et al. 2003). [See the electronic edition of the *Journal* for a color version of this figure.]

TABLE 2
RSI MODEL VARIANTS

Model	e_{*}^{II}	e_{*}^{III}	N_{γ}^{II}	$z(n_{\gamma} = 1)$	$z(n_{\gamma} = 2)$	$z(n_{\gamma} = 10)$
RSI.x1	1.0	0.006	$\times 1$	11.7	10.8	8.7
RSI.x2	1.0	1.0	$\times 1$	16.8	16.0	14.0
RSI.x3	0.1	0.002	$\times 2$	9.8	8.9	6.1
RSI.x4	0.1	0.002	$\times 10$	11.4	10.6	8.6
RSI.x5	0.1	0.002	N_{γ}^{III}	12.0	11.2	9.3

Population III stars to $600 M_{\odot}$, the maximum expected value suggested by Omukai & Palla (2003), although this is larger than the maximum value advocated by Abel et al. (2002). Reionization is then expected between $z \sim 10.8$ and 8.7 . Only if we assume 100% star formation efficiency in Population III halos as well (model RSI.x2) do we find that reionization could plausibly occur by $z \sim 16$ – 14 , in reasonable agreement with the *WMAP* TE results.

Alternatively, we can leave the star formation efficiency parameters at their fiducial values and vary the number of ionizing photons produced by each population. For example, the IMF of early Population II stars might be top-heavy because of the higher temperatures and pressures and lower metallicities at early times (Larson 1998). Moreover, Tumlinson, Shull, & Venkatesan (2003) find that zero-metallicity Population II stars with a Salpeter IMF produce only about 50% more hydrogen-ionizing photons than the low-metallicity Leitherer et al. (1999) models used here (see also Schaerer 2003). We find that increasing the number of ionizing photons per solar mass of Population II stars by a factor of 2–10 can only shift reionization to at best $z_{\text{reion}} \sim 10.6$ (models RSI.x3 and RSI.x4). Even with the extreme assumption that Population II stars produce as many ionizing photons per unit mass as supermassive Population III stars (about 20 times the fiducial value), we find it is unlikely that the universe could be reionized earlier than $z \sim 11$ in the context of the *WMAP* RSI model.

An additional uncertainty arises from the possible inaccuracy of the Press-Schechter model that forms the basis of our calculation. We do not think that this could produce a large error, since we have calibrated our star formation rate estimates against numerical simulations set within the standard Λ CDM framework (see § 4 and SL03). Moreover, Jang-Condell & Hernquist (2001) and Yoshida et al. (2003) found that the Press-Schechter model agrees well with N -body simulations in the mass and redshift range most relevant to our calculation. It is possible that the accuracy of the Press-Schechter model depends on the shape of the power spectrum, and this should be tested with N -body simulations. We find that the modified analytic mass functions proposed by Jenkins et al. (2001) and Sheth & Tormen (1999), which are a better approximation to N -body results at lower redshift, produce inferior agreement with the simulation results at very high redshift. In any case, if we adopt either of the modified mass function models, we find at most about a factor of 3 increase in the mass in collapsed halos at $z \sim 20$ in the RSI model—not enough to significantly alter our conclusions.

5. DISCUSSION AND CONCLUSIONS

We have investigated early structure formation in the Standard Λ CDM model and in three models with reduced small-scale power, each of which alleviates conflicts with observations on subgalactic scales at low redshift. All the models we considered are consistent with the *WMAP* data on large scales, and two of them (TILT and RSI) are favored by the combined analysis of *WMAP* and other CMB data with $\text{Ly}\alpha$ forest and 2dFGRS data (Spergel et al. 2003). We model star formation using a simple recipe that has been calibrated against more detailed semianalytic and numerical hydrodynamic simulations of Population II and massive Population III star formation (Somerville & Livio 2003; Springel & Hernquist 2003; Yoshida et al. 2003).

Using these fiducial parameter values, and assuming that a gross production of between 2 and 10 ionizing photons per hydrogen atom is needed to attain overlap, we estimate that the universe would become reionized between $z \sim 14$ and 9 with the standard Λ CDM power spectrum, in agreement with several other semianalytic and numerical studies of reionization (Gnedin 2000; Razoumov et al. 2002; Haiman & Holder 2003; Ciardi et al. 2003a; Sokasian et al. 2003). Using the same approach, we find that *none* of the models with reduced small-scale power can produce at least two ionizing photons per atom before $z \sim 9.7$, which is about 1.5σ lower than the epoch of reionization favored by Kogut et al. (2003) based on the *WMAP* TE measurement. More plausible values of $n_{\gamma} \sim 10$ are not attained until $z \lesssim 6$ in the models with reduced power on small scales. The WDM and RSI models may have difficulty reionizing the universe even by $z \gtrsim 6$, as required by observations of high-redshift quasars (Fan et al. 2001). Of the three models we considered, the model with a fixed tilt $n_s = 0.95$ (TILT) is the best candidate for obtaining a compromise between the requirements of early structure formation and observed galaxy central densities and substructure at low redshift. The RSI model favored by Spergel et al. (2003) is only marginally better than WDM in terms of the reionization constraints. It is also worth noting that any attempts to push reionization to higher redshift by *increasing* the small-scale power (e.g., by adopting a “red tilt” $n > 1$ as suggested by Cen 2003b) would further exacerbate the problems on subgalactic scales.

We find that varying the efficiency of star formation or the stellar IMF within a reasonable range of values cannot significantly change these conclusions. Only if the efficiency of star formation is pushed to an extreme upper limit (100% of baryons in halos with $M > 1 \times 10^6 h^{-1} M_{\odot}$ turn instantly into stars) can the RSI model plausibly reionize the universe by $z \sim 16$ – 14 . This result is in agreement with a similar analysis performed by Haiman & Holder (2003). If such extreme star formation were allowed to continue to lower redshift (even to $z \sim 4$), such a model would be in clear conflict with observations of galaxies and the IGM. Even stronger constraints will soon be obtained from direct observations of $z > 6$ galaxies with the *Hubble Space Telescope* and the planned *James Webb Space Telescope*.

We conclude that if we require that the universe was reionized at least by $z \sim 12$, within $\sim 1 \sigma$ of the *WMAP* result, then in the context of the current framework, this paradox can be resolved only by adopting one or more of the following: (1) the efficiency of Population III star formation is *much* higher than current theory suggests, (2) there is an additional population of ionizing sources (such as mini-quasars or hypernovae) at high redshift, (3) the power spectrum has some higher order feature that produces an upturn in power at masses just below the scale of dwarf galaxies, or (4) we must retain the scale-invariant $n = 1$ standard Λ CDM power spectrum and somehow solve the small-scale problems in another way. While none of these solutions is particularly attractive, we tend to favor scenario 4 from the context of reionization, although this has its own difficulties. If reionization indeed occurred early, and the small-scale problems are as robust as they appear, then there are significant gaps in our theoretical understanding of first light *and* the structure of galaxies on kiloparsec scales. The connection between the two certainly warrants more examination.

We acknowledge Lars Hernquist, Joel Primack, Naoki Yoshida, and Andrew Zentner for useful discussions. We also thank the anonymous referee for comments that improved this paper. J. S. B. would like to thank Rosemary Wyse for generous hospitality during his visit to The Johns Hopkins University, where a substantial fraction of this

work was completed. J. S. B. is provided for by NASA through Hubble Fellowship grant HF-01146.01-A from the Space Telescope Science Institute, which is operated by the Association of Universities for Research in Astronomy, Incorporated, under NASA contract NAS5-26555.

REFERENCES

- Abel, T., Bryan, G. L., & Norman, M. L. 2000, *ApJ*, 540, 39
 ———. 2002, *Science*, 295, 93
 Alam, S. M. K., Bullock, J. S., & Weinberg, D. H. 2002, *ApJ*, 572, 34
 Avila-Reese, V., Colín, P., Valenzuela, O., D'Ongia, E., & Firmani, C. 2001, *ApJ*, 559, 516
 Bardeen, J. M., Bond, J. R., Kaiser, N., & Szalay, A. S. 1986, *ApJ*, 304, 15
 Barkana, R., Haiman, Z., & Ostriker, J. P. 2001, *ApJ*, 558, 482
 Becker, R. H., et al. 2001, *AJ*, 122, 2850
 Benson, A. J., Lacey, C. G., Baugh, C. M., Cole, S., & Frenk, C. S. 2002, *MNRAS*, 333, 156
 Bode, P., Ostriker, J. P., & Turok, N. 2001, *ApJ*, 556, 93
 Bromm, V., Coppi, P. S., & Larson, R. B. 2002, *ApJ*, 564, 23
 Bromm, V., Ferrara, A., Coppi, P. S., & Larson, R. B. 2001a, *MNRAS*, 328, 969
 Bromm, V., Kudritzki, R. P., & Loeb, A. 2001b, *ApJ*, 552, 464
 Bullock, J., & Zentner, A. 2002, preprint (astro-ph/0207534)
 Cen, R. 2003a, *ApJ*, 591, 12
 ———. 2003b, *ApJ*, 591, L5
 Ciardi, B., Ferrara, A., Governato, F., & Jenkins, A. 2000, *MNRAS*, 314, 611
 Ciardi, B., Ferrara, A., & White, S. D. M. 2003a, *MNRAS*, submitted (astro-ph/0302451)
 Ciardi, B., Stoehr, F., & White, S. 2003b, *MNRAS*, submitted (astro-ph/0301293)
 Colín, P., Avila-Reese, V., & Valenzuela, O. 2000, *ApJ*, 542, 622
 Eisenstein, D. J., & Hu, W. 1999, *ApJ*, 511, 5
 Eke, V. R., Navarro, J. F., & Steinmetz, M. 2001, *ApJ*, 554, 114
 Fan, X., et al. 2001, *AJ*, 122, 2833
 Flores, R. A., & Primack, J. R. 1994, *ApJ*, 427, L1
 Gnedin, N. Y. 2000, *ApJ*, 535, 530
 Haiman, Z., Abel, T., & Madau, P. 2001, *ApJ*, 551, 599
 Haiman, Z., & Holder, G. 2003, preprint (astro-ph/0302403)
 Hannestad, S., Hansen, S. H., Villante, F. L., & Hamilton, A. J. S. 2002, *Astrophys. J.*, 17, 375
 Hernquist, L., & Springel, V. 2003, *MNRAS*, 341, 1253
 Hui, L., & Haiman, Z. 2003, *ApJ*, submitted (astro-ph/0302439)
 Jang-Condell, H., & Hernquist, L. 2001, *ApJ*, 548, 68
 Jenkins, A., Frenk, C. S., White, S. D. M., Colberg, J. M., Cole, S., Evrard, A. E., Couchman, H. M. P., & Yoshida, N. 2001, *MNRAS*, 321, 372
 Kinney, W. H. 2003, preprint (astro-ph/0301448)
 Klypin, A., Kravtsov, A. V., Valenzuela, O., & Prada, F. 1999, *ApJ*, 522, 82
 Knebe, A., Devriendt, J. E. G., Mahmood, A., & Silk, J. 2002, *MNRAS*, 329, 813
 Kogut, A., et al. 2003, *ApJ*, submitted (astro-ph/0302213)
 Kosowsky, A., & Turner, M. S. 1995, *Phys. Rev. D*, 52, 1739
 Larson, R. B. 1998, *MNRAS*, 301, 569
 Leitherer, C., et al. 1999, *ApJS*, 123, 3
 Loeb, A., & Barkana, R. 2001, *ARA&A*, 39, 19
 Machacek, M. E., Bryan, G. L., & Abel, T. 2001, *ApJ*, 548, 509
 ———. 2003, *MNRAS*, 338, 273
 Madau, P., Haardt, F., & Rees, M. J. 1999, *ApJ*, 514, 648
 McGaugh, S. S., Barker, M. K., & de Blok, W. J. G. 2003, *ApJ*, 584, 566
 Miralda-Escudé, J., Haehnelt, M., & Rees, M. J. 2000, *ApJ*, 530, 1
 Moore, B. 1994, *Nature*, 370, 629
 Moore, B., Ghigna, S., Governato, F., Lake, G., Quinn, T., Stadel, J., & Tozzi, P. 1999, *ApJ*, 524, L19
 Narayanan, V. K., Spergel, D. N., Davé, R., & Ma, C. 2000, *ApJ*, 543, L103
 Omukai, K., & Palla, F. 2003, *ApJ*, 589, 677
 Press, W. H., & Schechter, P. 1974, *ApJ*, 187, 425
 Razoumov, A. O., Norman, M. L., Abel, T., & Scott, D. 2002, *ApJ*, 572, 695
 Ricotti, M., Gnedin, N. Y., & Shull, J. M. 2001, *ApJ*, 560, 580
 ———. 2002, *ApJ*, 575, 49
 Schaerer, D. 2003, *A&A*, 397, 527
 Schirber, M., & Bullock, J. S. 2003, *ApJ*, 584, 110
 Sheth, R. K., & Tormen, G. 1999, *MNRAS*, 308, 119
 Sokasian, A., Abel, T., Hernquist, L., & Springel, V. 2003, preprint (astro-ph/0303098)
 Somerville, R. S. 2002, *ApJ*, 572, L23
 Somerville, R., & Livio, M. 2003, *ApJ*, 593, 611 (SL03)
 Somerville, R. S., & Primack, J. R. 1999, *MNRAS*, 310, 1087
 Somerville, R. S., Primack, J. R., & Faber, S. M. 2001, *MNRAS*, 320, 504
 Sommer-Larsen, J., & Dolgov, A. 2001, *ApJ*, 551, 608
 Spergel, D. N., et al. 2003, *ApJ*, submitted (astro-ph/0302209)
 Springel, V., & Hernquist, L. 2003, *MNRAS*, 339, 312
 Swaters, R. A., Madore, B. F., van den Bosch, F. C., & Balcells, M. 2003, *ApJ*, 583, 732
 Theuns, T., Schaye, J., Zaroubi, S., Kim, T., Tzanavaris, P., & Carswell, B. 2002, *ApJ*, 567, L103
 Tumlinson, J., Shull, J. M., & Venkatesan, A. 2003, *ApJ*, 584, 608
 van den Bosch, F. C., Mo, H. J., & Yang, X. 2003, *MNRAS*, submitted (astro-ph/0301104)
 Wyithe, S., & Loeb, A. 2003, *ApJ*, 586, 693
 ———. 2003, *ApJ*, submitted (astro-ph/0302297)
 Yoshida, N., Abel, T., Hernquist, L., & Sugiyama, N. 2003, *ApJ*, 592, 645
 Zentner, A. R. 2003, Ph.D. thesis, Ohio State Univ.
 Zentner, A. R., & Bullock, J. S. 2002, *Phys. Rev. D*, 66, 43003
 ———. 2003, *ApJ*, submitted (astro-ph/0304292)