

## CONSTRAINTS ON THE BINARY EVOLUTION FROM CHIRP MASS MEASUREMENTS

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### ABSTRACT

We estimate the observed distribution of chirp masses of compact object binaries for gravitational-wave detectors. The stellar binary evolution is modeled using the StarTrack population synthesis code. The distribution of the predicted “observed” chirp masses varies with the variation of the different parameters describing stellar binary evolution. We estimate the sensitivity of the observed distribution to the variation of these parameters, and we show which of the parameters can be constrained after observing 20, 100, and 500 compact object mergers. As a general feature of all our models, we find that the population of observed binaries is dominated by the double black hole mergers.

*Subject headings:* binaries: close — gravitational waves

### 1. INTRODUCTION

Compact object mergers are one of the most promising sources of gravitational waves for ground-based interferometric detectors like LIGO (Laser Interferometer Gravitational-wave Observatory; Abramovici et al. 1992) and VIRGO (Variability of Irradiance and Gravity Oscillations; Bradaschia et al. 1990). So far, most of the theoretical papers on the properties of these sources related to the gravitational-wave detections have concentrated on calculating the predicted rates (Narayan, Piran, & Shemi 1991; Phinney 1991; Kalogera et al. 2001). In this Letter, we wish to address another aspect of gravitational-wave detection, i.e., the distribution of observed masses of the compact objects.

Stellar mass compact object binaries will be detected during the in-spiral phase, while the consecutive merger and ring-down phases will most likely have lower signal-to-noise ratios. During the in-spiral phase, the motion of the binary components, and also the waveform, is governed by the chirp mass  $\mathcal{M} = (m_1 + m_2)^{-1/5} (m_1 m_2)^{3/5}$  (Peters & Matthews 1963). The waveform will depend on the individual masses of the binary components  $m_1$  and  $m_2$  when the post-Newtonian effects are taken into account. However, the analysis of the in-spiral phase alone shall not suffice to determine if a binary contained a neutron star (NS) or a black hole (BH) without the prior knowledge of the NS maximum mass. A careful modeling of the signal may yield the individual masses of the objects; however, the chirp mass will be the primary observable for the compact object mergers (Cutler & Flanagan 1994).

The theoretical estimates of BH masses depend on a number of parameters describing different stages of massive star evolution. There are several possibilities for constraining these parameters observationally.

One approach involves the detailed analysis of well-studied binaries containing BHs, i.e., deducing their past evolution. Such an analysis of the Galactic microquasar GRS 1915–105 (Belczynski & Bulik 2002) has shown that the winds in massive stars ( $M > 20 M_\odot$ ) are at least a factor of 2 smaller than the standard model predictions (see also Podsiadlowski, Rappaport, & Han 2003). Another type of constraint can be obtained from

a comparison of a number of different types of observed systems hosting compact objects. Lipunov, Postnov, & Prokhorov (1997c) have analyzed constraints that stem from the fact that only one system, like Cyg X-1, is observed in the Galaxy while no pulsar binary with a BH was yet detected. They found a lower limit on the ratio of the BH mass to the presupernova core mass in massive stars. With the use of observational constraints, the evolution of progenitors and masses of BHs and NSs were studied by other groups (e.g., Ergma & van den Heuvel 1998; Portegies Zwart & Yungelson 1998; Wellstein & Langer 1999), leading to somewhat different conclusions. These different conclusions are the result of a severely small sample of known double compact objects and the subjective choice of observations used for comparisons, combined with the fact that different theoretical models were used in different studies. A careful study of the constraints on the evolution of massive stars imposed by current observations of BH binaries using the StarTrack population synthesis code (Belczynski, Kalogera, & Bulik 2002) is currently under way (K. Belczynski et al. 2003, in preparation).

The aim of this Letter is to present the expected distributions of chirp masses observed by interferometric gravitational-wave observatories. We also address the question of whether or not such observations shall yield useful constraints on parameters describing stellar evolution. Although we could possibly eliminate some models based on current electromagnetic observations, we chose to present the whole set of calculations and to let the future gravitational-wave observations decide. We may yet learn that the things we will see in gravitational waves will look quite different from the things we see in the electromagnetic spectrum.

We use the StarTrack population synthesis code (Belczynski et al. 2002) to calculate the distributions of compact object binary masses, and we present these calculations in § 2. In § 3, we estimate the number of merger detections required to distinguish between different models of stellar binary evolution. Finally, § 4 contains conclusions and discussion.

### 2. DISTRIBUTION OF THE CHIRP MASSES

The StarTrack binary population synthesis code is described in detail in Belczynski et al. (2002). One of the important features of this code is that it makes it possible to study the parameters of a given property of the population of binaries, i.e., to estimate the dependence of the result on each of the parameters used to describe the stellar and binary evolution.

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TABLE 1  
DESCRIPTION OF DIFFERENT POPULATION SYNTHESIS MODELS FOR WHICH THE DISTRIBUTIONS  
OF MASS RATIOS HAVE BEEN FOUND

Model(s)	Description
A .....	Standard model described in Belczynski et al. 2002, but with $T_{\text{Hubble}} = 15$ Gyr
B1 .....	Zero kicks
B7, B11 .....	Single Maxwellian with $\sigma = 50, 500$ km s $^{-1}$
B13 .....	Paczynski 1990 kick with $V_k = 600$ km s $^{-1}$
C .....	No hypercritical accretion onto NS/BH in common envelopes
E1, E2, E3 .....	$\alpha_{\text{CE}}\lambda = 0.1, 0.5, 2$
F1, F2 .....	Mass fraction accreted: $f_a = 0.1, 1$
G1, G2 .....	Wind changed by $f_{\text{wind}} = 0.5, 2$
J .....	Primary mass: proportional to $M_1^{-2.35}$
L1, L2 .....	Angular momentum of material lost in mass transfer: $j = 0.5, 2.0$
M1, M2 .....	Initial mass ratio distribution: $\Phi(q) \propto q^{-2.7}, q^3$
O .....	Partial fallback for $5.0 M_{\odot} < M_{\text{CO}} < 14.0 M_{\odot}$
S .....	All systems formed in circular orbits
Z1, Z2 .....	Metallicity: $Z = 0.01, Z = 0.0001$

The models used in this Letter are listed in Table 1. We first use the standard model A results to present the intrinsic distribution of the chirp masses. This is shown in Figure 1. The distribution shows a clear peak at low chirp masses  $1 M_{\odot} < \mathcal{M} < 2 M_{\odot}$  that is due to the double NS systems. The mixed (BH-NS) systems populate the intermediate region, while the chirp masses of the BH-BH binaries extend up to above  $10 M_{\odot}$ .

In order to estimate the observed distribution of the chirp masses of compact objects, one has to take into account the sensitivity of the gravitational-wave detectors to signals from mergers of different binaries. The calculations of the signal-to-noise ratio (Finn & Chernoff 1993; Bonazzola & Marck 1994; Flanagan & Hughes 1998) show that the sampling distance in the first approximation is a function of the chirp mass only:  $D \propto \mathcal{M}^{5/6}$ . The additional corrections, which are due to the limited sensitivity window of the detectors, have been calculated by Flanagan & Hughes (1998) and amount to less than 10% for the binaries, with the total mass below  $18 M_{\odot}$  for the initial LIGO and less than that for the advanced LIGO. In this Letter,

we neglect these corrections. The distribution of the expected observed chirp masses can be calculated using the Monte Carlo method. We assume that the universe is uniformly filled with merging binaries, and for each merger we estimate the signal-to-noise ratio in the detector. We model the population of merging binaries by assuming a continuous star formation rate. The result is shown in the right panel of Figure 1. One can note that these distributions could also be obtained analytically by multiplying the distributions of Figure 1 by a volume proportional to  $\mathcal{M}^{5/2}$  and normalizing it. In this plot, the BH-BH systems are now the dominant contributors to the distribution. This is due to the fact that the sampling volume for the BH-BH binaries is more than 100 times larger than that for the NS-NS systems, which easily compensates for the lower merger rate of the BH-BH binaries.

### 3. EXPECTED OBSERVATIONS

Let us now address the following questions: Are the distributions of observed chirp masses expected in the framework of alternative models different? If so, are these differences

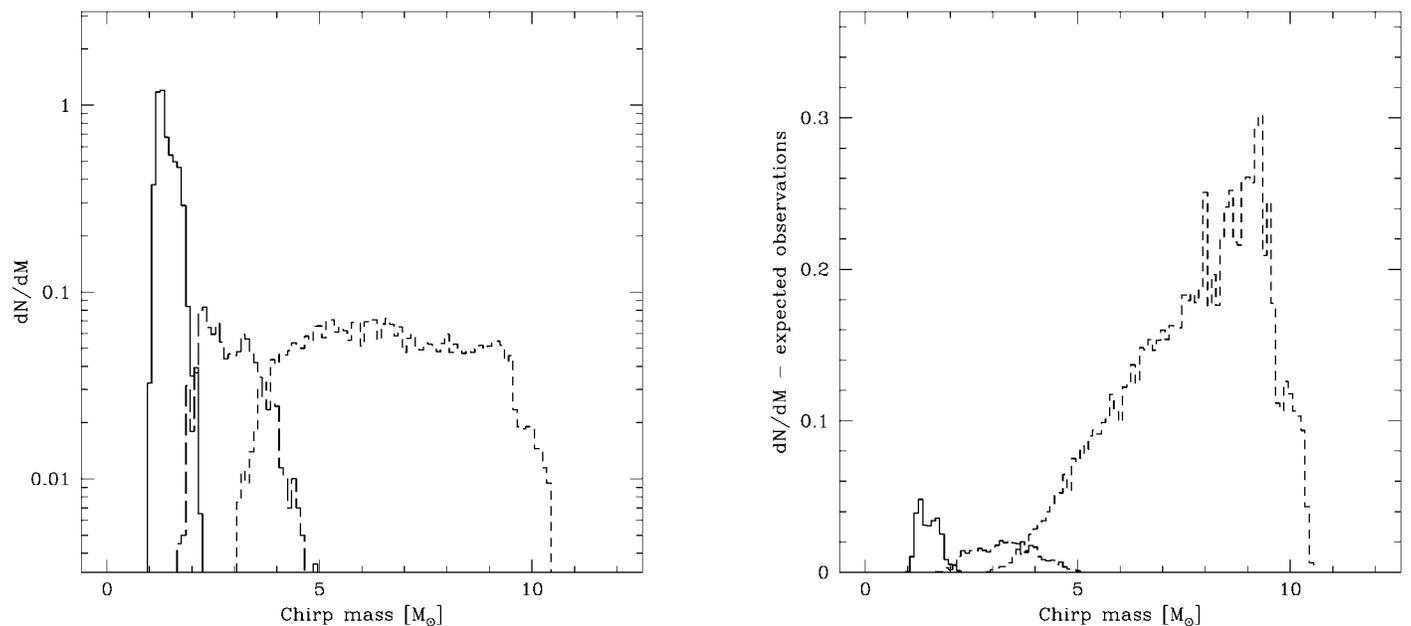


FIG. 1.—Intrinsic (galactic) distribution of the chirp masses in the framework of model A (*left panel*) and the distribution of the expected observations (*right panel*). The solid line corresponds to the NS-NS mergers, the short-dashed line represents the NS-BH mergers, and the dashed line stands for the BH-BH mergers. The sum of the three distributions in each panel is normalized to unity.

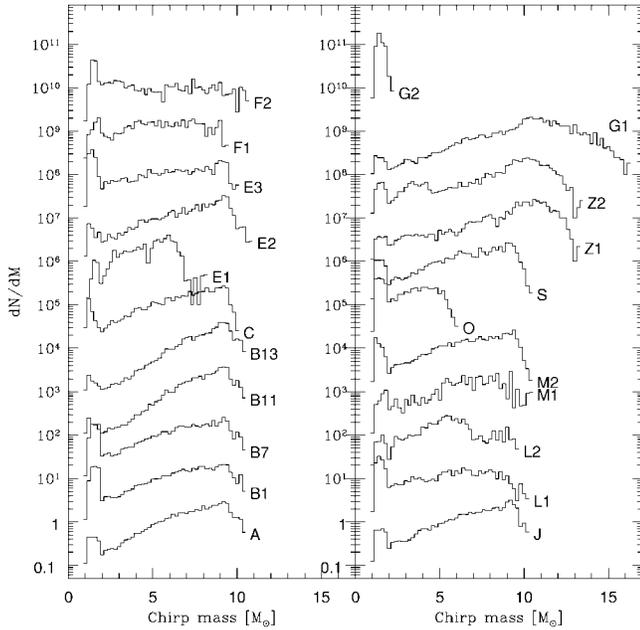


FIG. 2.—Distributions of the expected observed chirp masses in the framework of models listed in Table 1. For clarity, each distribution is shifted up by a factor of 10.

significant? We simulate the distributions of chirp masses in the expected observations with the binary populations obtained from the set of models of Table 1 in the same way as we have done above for the model A. We present the results in Figure 2. Different stellar evolution models lead to drastically different distributions of the chirp masses in the expected observations. Various parameters describing stellar evolution affect the distribution of observed chirp masses in several ways. Changing the kick velocity distribution (models B1, B7, B11, and B13) alters the ratio between the number of the NS binaries and BH binaries. Other models change the maximal masses of the BHs produced. This is especially clear in the case of models G1 and G2, where the stellar winds are varied by a factor of 2 upward (model G2) and downward (model G1). We note that the shapes of these distributions do not depend on the sensitivity of the detector.

In order to verify whether or not the differences between the distributions are significant, we turn to a simulation of a finite number of merger observations. We assume that the true stellar evolution goes through one of the models of Table 1. We then simulate the observations of a given number of mergers (we use 20, 100, and 500 mergers), and for each such simulated observation, using the Kolmogorov-Smirnov (K-S) test, we verify whether or not we can reject the hypothesis that the stellar evolution is described by model A. This allows us to test the sensitivity of the shape of the distribution of the expected chirp mass observations against the underlying model parameters describing stellar evolution. For each number of merger observations, we repeat this test 10,000 times in order to obtain a distribution of K-S test probabilities and to find the lowest probability that appeared in the top one percentile of this distribution. We can now set a detection confidence level, say at  $10^{-5}$ , and compare each probability with this value: if it is higher, we conclude that this particular model cannot be distinguished from model A with a given number of merger observations, while a smaller number means that this model can be distinguished and that some constraints can be imposed

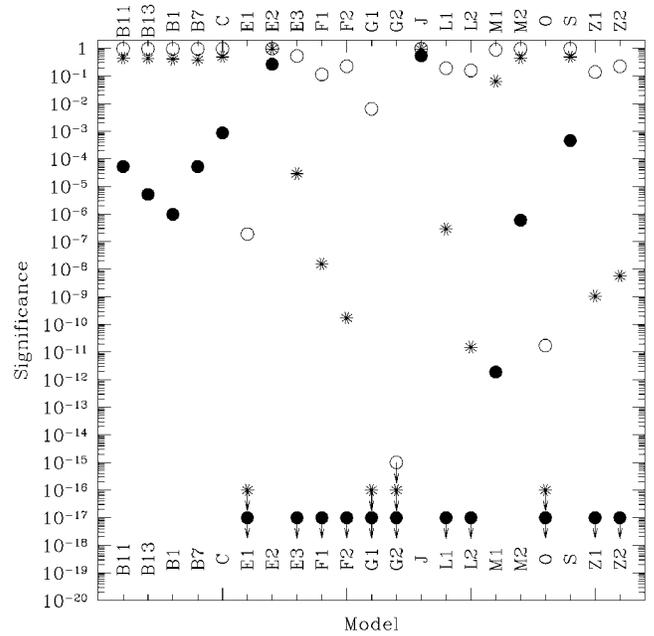


FIG. 3.—Significance of rejecting model A. The open circles correspond to observations of just 20 mergers, the asterisks correspond to observations 100 mergers, and the filled circles correspond to observations of 500 mergers. The symbols with an arrow denote the cases when the significance is off the scale.

on the particular parameter through which this model differs from model A. We present the results of the test in Figure 3.

Figure 3 presents a measure of the sensitivity of the expected observed distribution of chirp masses to the parameters describing stellar evolution. One can see from Figure 3 that even observations of a small number of mergers (the open circles correspond to 20 mergers) yield highly significant results for models E1, G2, and O. The reason for this is clear from Figure 2. These are the models for which the maximal chirp mass in the population is significantly lower than that for model A. Model G2's population (i.e., with increased stellar winds) contains hardly any BHs. In general, we see that these observations are very sensitive to the value of the maximum mass of stellar BHs in the population. Model G1 (with decreased stellar winds), which allows for the formation of BH binaries with chirp masses up to  $16 M_{\odot}$ , will stand out with less than a hundred merger observations.

With a larger number of merger observations (the asterisks in Fig. 3 correspond to 100 merger detections), more parameters can be constrained. Some constraints can be obtained for the value of the common-envelope efficiency  $\alpha_{\text{CE}}\lambda$  (models E1, E2, and E3; model E2 is similar to model A). Other parameters describing mass transfer events like the mass fraction accreted (models F1 and F2) and the amount of angular momentum lost (models L1 and L2) shall also be constrained. Moreover, the metallicity of the progenitor stars may also influence the observed distribution at a significant level (models Z1 and Z2).

Constraining the initial mass ratio distribution (models M1 and M2) will require an even higher number of merger detections: only for the case of the 500 observed mergers do the differences become significant. Models C (no hypercritical accretion onto a compact object), J (initial mass function slope), and S (systems circular initially) lead to very small differences in the observed distribution of chirp masses.

Models B1, B7, B11, and B13 (where the kick velocity distribution varies) begin to show significant differences with

a large number of observations only. Changing the kick velocity distribution strongly affects the absolute rates (Lipunov, Postnov, & Prokhorov 1997a; Belczyński & Bulik 1999) and the ratio of double NS mergers to the double BH mergers (Belczynski et al. 2002).

#### 4. CONCLUSIONS

We have used the stellar population synthesis models in order to simulate the distribution of observed chirp masses in the gravitational-wave detection of stellar mergers. We find that the population of observed mergers is dominated by the BH-BH binary mergers, as was suggested by Lipunov, Postnov, & Prokhorov (1997b). In most models, double BH mergers constitute more than 90% of the observed events. The exception is model G2, in which the formation of BHs is suppressed because of increased stellar winds. The shapes of the observed distributions of chirp masses vary considerably for different models of stellar binary evolution.

We simulate the observed distributions of chirp masses in the framework of various stellar evolution models and estimate the sensitivity with which these parameters can be estimated from a given sample of observed mergers. We find that there are a large number of parameters that can be constrained, given a set of measured chirp masses. The main and immediate constraints come from the fact that the observed population seems to be dominated by the highest mass BH binaries. Thus, even a small set of observations yields constraints on the maximal mass of merging BH binaries. A larger set of observations will lead to constraints on the evolution of high-mass binaries.

In our simulation, we use a simple statistical tool: the K-S test. Given a set of real observations with some measurements of individual masses of coalescing stars, one could use a more sensitive tool, like the maximum likelihood method. However,

even with such simple statistics as was used here, we can show the general properties of the expected observations and demonstrate the sensitivity of the observed distributions to different model parameters.

We note that consideration of the distribution of observed masses will lead to stricter constraints than consideration of just the observed rates. The theoretical calculation of rates involves estimating a number selection effects and calibrating them with other sources; this leads to several uncertainties. The calculation of the observed distribution of chirp masses is free from such uncertainties because a distribution is essentially equivalent to considering the ratios of a number of different type mergers, and all the normalization factors that enter into the rate estimates cancel out.

Finally, we have to mention several effects that have not been taken into account in this Letter. A more detailed calculation must include the consideration of the effects of lifetimes of binaries of different type. Belczynski et al. (2002) have shown that the typical lifetime of a double NS binary is much smaller than that of a BH binary. The effects that are due to changing the star formation rate with redshift will affect the observed population of merging binaries. When considering the advanced detectors that are sensitive to mergers at cosmological distances, one also needs to take into account the cosmological effects: the fact that the true quantity measured is the redshifted mass  $(1+z)M$ , and also the change of the observed volume with redshift (here we have assumed the Euclidean geometry). These issues will be considered in a separate paper.

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