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SUPPRESSION OF FAST RECONNECTION BY MAGNETIC SHEAR

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ABSTRACT

Magnetic neutral sheets in weakly ionized interstellar gas are rapidly annihilated by ohmic diffusion. In this paper we extend the model to a sheared magnetic configuration, and show that the magnetic pressure associated with even a small nonzero field drastically reduces the reconnection rate of the reversing component.

Subject headings: conduction — diffusion — ISM: magnetic fields — MHD

1. MOTIVATION

Under conditions typical for the interstellar medium (ISM), the Lundquist number S—the ratio between the ohmic diffusion timescale and the Alfvén timescale—is of order $10^{15} < S < 10^{21}$. The large size of S has two implications. First, for most purposes the magnetic field can be regarded as perfectly frozen to the gas. Second, resistive processes can be important only in thin boundary layers separating regions in which the field is frozen in. The breakdown of the frozen field approximation in these layers leads to magnetic reconnection.

Reconnection is a hybrid process in the sense that it operates on timescales intermediate to the global diffusion timescale and the global dynamical timescale. The standard model of steady state reconnection (Parker 1957; Sweet 1958) predicts that the reconnection rate is faster than the ohmic diffusion rate by a factor of $\sim S^{1/2}$. However, given the large values of S in the ISM, this rate leads to reconnection timescales much larger than any typical dynamical time in galaxies. The standard time-dependent theory (Furth, Killeen, & Rosenbluth 1963) leads to a similar conclusion. Although it seems implausible that the topology of the Galactic magnetic field is rigorously invariant—and in fact, reconnection is necessary for the operation of a Galactic dynamo—the mechanisms by which rapid reconnection occurs in the ISM are still uncertain. Thus, the interstellar reconnection problem is of considerable importance.

Much of the material in the ISM is only weakly ionized, with the ion pressure generally much less than the magnetic pressure in regions of low ionization. If there is a null in the magnetic field, the plasma is accelerated down the magnetic pressure gradient, leading to the formation of a thin current sheet and establishing the conditions for rapid magnetic reconnection (Mestel 1966; Brandenburg & Zweibel 1994). Brandenburg & Zweibel (1995) and Heitsch & Zweibel (2003, hereafter HZ03) discussed one-dimensional steady-state reconnection within the current sheet model. Fast reconnection in one dimension is possible because recombination is an efficient flow sink, eliminating the ions brought into the reconnection region and preventing the buildup of ion pressure (see also Vishniac & Lazarian 1999). These authors showed that fast reconnection, i.e., reconnection independent of S, is possible in the (cold) ISM and in protoplanetary disks, although the ohmic heating rate is so high that the process may actually be self-limiting.

This fast reconnection mechanism is based on the existence of a magnetic null, and thus is highly specialized. It is qualitatively clear that the pressure force exerted by an additional component of field, perpendicular to the plane defined by the flow and the original, reversing field and without a null itself, should inhibit current sheet formation and dissipation (Zweibel & Brandenburg 1997). The purpose of this paper is to quantify these effects by deriving the maximum strength of this additional field component such that fast reconnection can still proceed. We find that for practical purposes the vertical field must vanish at the outer boundary in order to permit fast reconnection. Otherwise, the field dissipates at the (slow) ohmic diffusion rate in our one-dimensional model, and we have to rely on outflows in two-dimensional geometry to accelerate reconnection. In the limit of a strong vertical field, this leads to the slow reconnection rate discussed in Zweibel (1989).

In § 2 we formulate the problem. In § 3 we develop a simple analytical model that leads to an upper bound on the vertical field permitted in fast reconnection. In § 4 we briefly review the numerical method developed for HZ03 and recast the problem in a form suitable for use. In § 5 we discuss the results and show that they are consistent with the analytical predictions. Section 6 is a summary and discussion.

2. DESCRIPTION OF THE PROBLEM

As in HZ03, we consider the reconnection problem in a weakly ionized gas (Fig. 1). We assume a reversal in $B_x(z)$ at $z = 0$. When the charged and neutral matter are strongly coupled, regions of magnetic field reversal can achieve force balance, with neutral pressure compensating for the deficit in magnetic pressure. Reducing the coupling between neutrals and ions leads to ambipolar diffusion, in which case the neutrals lag behind the field lines. Neutral pressure support is lost, and the ion pressure—which is much smaller than the neutral pressure—is overwhelmed by the magnetic pressure, so that plasma flows toward the neutral sheet, transporting the field lines with it. The magnetic field gradients steepen around the null plane, leading to high current densities and ohmic dissipation. An important simplification of the Parker-Sweet problem arises because the flux of ions is

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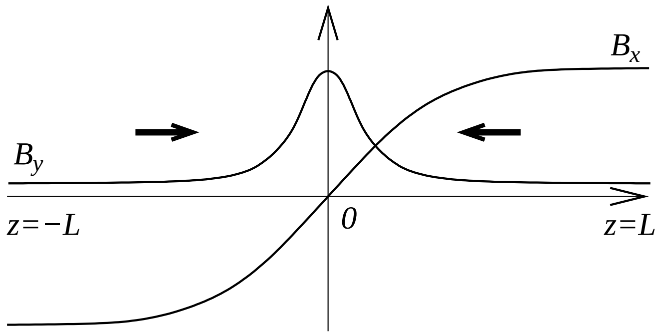

FIG. 1.—Schematic configuration for steady state reconnection as described in § 2. Plasma and magnetic field are accelerated inward by the gradient in B_x^2 and advected toward the plane $z=0$. The plasma is removed by recombination, while the magnitude of B_y is limited by resistive diffusion.

not conserved. Recombination represents a sink, thus eliminating the need for an outflow and making a steady state possible in one dimension. We are interested in finding the scaling of the steady state inflow speed, or equivalently the electric field, with the ohmic diffusivity λ_Ω.

We look for steady state configurations in which magnetic field and plasma are advected toward the plane $z=0$ by a one-dimensional ion flow $\hat{z}u(z)$. We specify both components of the magnetic field at the outer boundaries $z=\pm L$. The inflow velocity $u(\pm L)$ is a measure of the reconnection rate, and our goal is to determine its dependence on $\boldsymbol{B}(\pm L)$.

We assume that B_x is antisymmetric in z while B_y is symmetric. Thus, B_y takes over the role of ion pressure in HZ03. Since we assume a recombination time short enough to maintain ionization equilibrium (see HZ03, § 2.3), the ion pressure is negligible, which in turn helps us to determine the effect of $B_y>0$. We assume the layer width L is small enough that the neutrals remain at rest, with constant density.[3] The ion density is then also constant, and the ion pressure gradient can be ignored. Our numerical integrations have verified that the flow remains subsonic, so the Reynolds stress can be neglected. Under these conditions, u_z is determined solely by balancing acceleration down the magnetic pressure gradient against frictional drag by the neutrals

$$u_z = \frac{-1}{8\pi\rho_i\nu_{in}}\frac{\partial}{\partial z}(B_x^2+B_y^2) \ , \tag{1}$$

where ν_{in} is the ion-neutral collision frequency. The electric field $\boldsymbol{E}$ is given by

$$c\boldsymbol{E} = -\boldsymbol{u}\times\boldsymbol{B} + \lambda_\Omega\nabla\times\boldsymbol{B} \ , \tag{2}$$

where λ_Ω, the magnetic diffusivity, is related to the electrical conductivity σ by $\lambda_\Omega = c^2/(4\pi\sigma)$. We need the x and y components of $\boldsymbol{E}$:

$$cE_x = u_zB_y - \lambda_\Omega\frac{\partial}{\partial z}B_y \ , \tag{3}$$

$$cE_y = -u_zB_x + \lambda_\Omega\frac{\partial}{\partial z}B_x \ . \tag{4}$$

[3] The layer considered here is actually a sublayer of a larger region in which the neutral density has adjusted to compensate for the spatially varying magnetic pressure. On sufficiently small scales, however, the ions and neutrals decouple.

With the geometry and spatial structure assumed here, $\boldsymbol{E}$ must be constant in a steady state.

It is useful to express the magnetic field components in units of $B_0\equiv B_x(L)$ as $b_{x,y}\equiv B_{x,y}/B_0$. We introduce the ambipolar diffusivity $\lambda_{\rm AD}\equiv B_0^2/(4\pi\rho_i\nu_{in})$ and the characteristic timescales $\tau_{\rm AD}=L^2/\lambda_{\rm AD}$, $\tau_\Omega=L^2/\lambda_\Omega$. We express the coordinate z in units of L and measure E_x and E_y in terms of the inverse timescales $\kappa_{x,y}\equiv cE_{x,y}/LB_0$. With these definitions, we can rewrite equations (3) and (4) as

$$\kappa_y = \frac{b_x}{2\tau_{\rm AD}}\frac{\partial}{\partial z}(b_x^2+b_y^2) + \frac{1}{\tau_\Omega}\frac{\partial}{\partial z}b_x \ , \tag{5}$$

$$\kappa_x = -\frac{b_y}{2\tau_{\rm AD}}\frac{\partial}{\partial z}(b_x^2+b_y^2) - \frac{1}{\tau_\Omega}\frac{\partial}{\partial z}b_y \ . \tag{6}$$

Equations (5) and (6) constitute a system of two first-order ordinary differential equations with two free parameters, κ_x and κ_y. We can specify both components of $\boldsymbol{B}$ at the outer boundary $z=1$, and also impose conditions on $\boldsymbol{B}$ at $z=0$, if we regard κ_x and κ_y as eigenvalues. We take as boundary conditions on b_x and b_y

$$b_x(1)=1 \ , \qquad b_x(0)=0 \ , \tag{7}$$

$$b_y(1)=\text{given} \ , \qquad \frac{\partial}{\partial z}b_y(0)=0 \ . \tag{8}$$

The second condition on b_y follows from symmetry considerations.

Equations (6) and (8) imply $\kappa_x\equiv 0$. Since the resistive contribution to $\boldsymbol{E}$ is presumably small at the outer boundary of the layer, κ_y is essentially a measure of the inflow speed, or equivalently, the reconnection rate.

3. PREDICTIONS

Equations (5) and (6) are nonlinear, and cannot be solved analytically. We can, however, use them to estimate the behavior of the scaled vertical field b_y and to predict the reconnection rate.

Equation (3) admits the integral, expressed here entirely in dimensional units

$$B_y(0) = B_y(1)\exp\left[-\int_0^L\frac{u(z)}{\lambda_\Omega}\,dz\right] \ . \tag{9}$$

Let us assume that $B_y\ll B_x$ and evaluate the integral in equation (9) for $B_y\equiv 0$. In this case, there is an approximate analytical solution for u_z with limiting forms (see HZ03)

$$u_z = -\frac{L}{\tau_{\rm AD}}\left(\frac{\lambda_{\rm AD}}{3\lambda_\Omega}\right)^2 z \quad \text{for} \quad z<z_l \ , \tag{10}$$

$$u_z = -\frac{L}{\tau_{\rm AD}}z^{-1/3} \quad \text{for} \quad z>z_l \ , \tag{11}$$

where z_l marks the transition from resistively dominated ($z<z_l$) to inductively dominated ($z>z_l$), and is given by

$$z_l = \left(\frac{3\tau_{\rm AD}}{\tau_\Omega}\right)^{3/2} \ . \tag{12}$$

Using equations (10), (11), and (12) in equation (9) and expressing B_y in dimensionless form yields

$$b_y(0) = b_y(1)\ e^{\tau_\Omega/2\tau_{\rm AD}} \ . \tag{13}$$

TABLE 1
Model Parameters

Model	$b_y(1)$	$\tau_\Omega/\tau_{\rm AD}$
B3	10^{-3}	13.82
B4	10^{-4}	18.42
B5	10^{-5}	23.03
B6	10^{-6}	27.63

Notes.—Key to model names: B*i*, where $i = -\log b_y(1)$, and $b_y(1)$ gives the outer value of the additional field component according to eq. (14) or given $\tau_\Omega/\tau_{\rm AD}$. Physically realistic values for $\tau_\Omega/\tau_{\rm AD}$ in the ISM range around 10^8 or higher.

For the AD solution to remain valid, b_y must be dynamically unimportant, or $b_y(0) \ll b_x(1)$, resulting in the condition for $b_y(1)$

$$b_y(1) \ll e^{-\tau_\Omega/2\tau_{\rm AD}} \ . \tag{14}$$

Table 1 demonstrates just how small $b_y(1)$ must be in order to render the AD solution valid.

In order to solve for $b_y(0)$, we integrate equation (6) with $\kappa_x \equiv 0$. The result is

$$\frac{b_y^2(0)}{2} + \frac{\tau_{\rm AD}}{\tau_\Omega} \ln b_y(0) = \frac{1 + b_y^2(1)}{2} + \frac{\tau_{\rm AD}}{\tau_\Omega} \ln b_y(1) \ . \tag{15}$$

Rewriting equation (15) in terms of an expression for $b_y(0)$ yields

$$b_y(0) = b_y(1) \ \exp\left\{\frac{\tau_\Omega}{2\tau_{\rm AD}}\left[1 + b_y^2(1) - b_y^2(0)\right]\right\} \tag{16}$$

In the limit that $b_y^2(0)$ and $b_y^2(1) \ll 1$, equation (16) reverts to equation (13). Equation (16) can be solved by iteration. The results (Fig. 2) show that equation (13) is quite accurate for $b_y(1) \ll 1$, but does not predict the saturation of $b_y(0)$ as $b_y(1)$ increases.

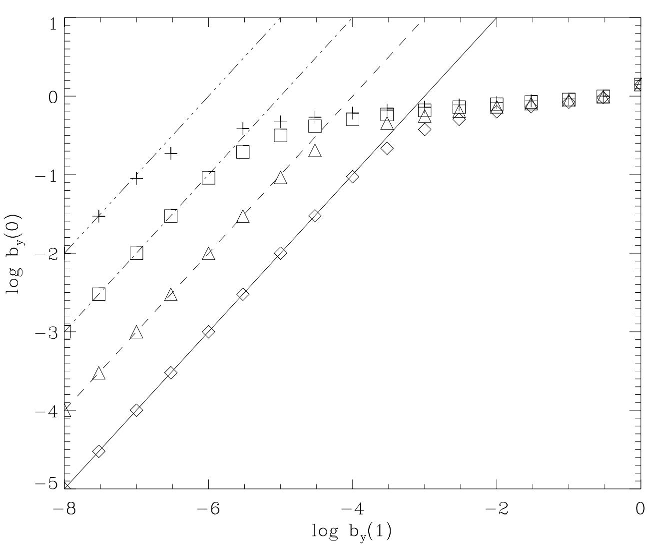

Fig. 2.—Central $b_y(0)$ in terms of outer field strength $b_y(1)$, by solving eq. (15) iteratively. *Diamonds*: B3; *triangles*: B4; etc. Note that the curves change slope at the locations where eq. (14) predicts that b_y becomes important. The overplotted lines show the values of $b_y(0)$ predicted by eq. (13).

We can estimate the dimensionless electric field κ_y in terms of the outer magnetic field strength b_y by replacing the derivatives in equation (5) with differences over the domain. The result is

$$\kappa_y \equiv \frac{cE}{LB_0} = \frac{1}{2\tau_{\rm AD}}\left[1 + b_y^2(1) - b_y^2(0)\right] + \frac{1}{\tau_\Omega} \ . \tag{17}$$

If $b_y \equiv 0$, equation (17) predicts $\kappa_y = (2\tau_{\rm AD})^{-1} + \tau_\Omega^{-1}$, while the exact solution for this case gives $\kappa_y = (3\tau_{\rm AD})^{-1} + \tau_\Omega^{-1}$. The discrepancy is the result of differencing rather than differentiating, but the answer is correct to order unity.

We estimate κ_y by substituting the solution of equation (15) for b_y. When b_y is small enough that equation (13) is valid,

$$\kappa_y \approx \frac{1 - b_y^2(1)(e^{\tau_\Omega/2\tau_{\rm AD}} - 1)}{2\tau_{\rm AD}} + \frac{1}{\tau_\Omega} \ . \tag{18}$$

Equation (18) predicts that the reconnection rate is significantly slowed for $b_y^2(1) \geq \exp(-\tau_\Omega/2\tau_{\rm AD})$, which is consistent with equation (14).

4. MODIFIED EQUATIONS AND NUMERICS

In HZ03 we described a relaxation method for solving the eigenvalue problem with adaptive mesh refinement on an initially exponential grid. This method requires expressing the derivative for each variable as a function only of the variables, not their derivatives. Equations (5) and (6) are not in suitable form for this technique. However, we can convert these equations to a system with the structure desired by introducing new dependent variables R and θ defined by

$$R\ e^{i\theta} = b_x + ib_y = R(\cos\theta + i\sin\theta) \ . \tag{19}$$

Subtracting $i\kappa_x$ from κ_y and using $\kappa_x \equiv 0$ leads to

$$\frac{d}{dz}R = \frac{\kappa_y\ \tau_\Omega\ \tau_{\rm AD}\ \cos\theta}{\tau_\Omega R^2 + \tau_{\rm AD}} \ , \tag{20}$$

$$\frac{d}{dz}\theta = -\frac{\kappa_y\ \tau_\Omega\ \sin\theta}{R} \ . \tag{21}$$

The boundary conditions corresponding to equations (7) and (8) are

$$R(1) = \sqrt{1 + b_y^2(1)} \ , \tag{22}$$

$$\theta(1) = \arctan\left[b_y(1)\right] \ , \tag{23}$$

$$\theta(0) = \frac{\pi}{2} \ . \tag{24}$$

Equations (20) and (21) are amenable to solution with the numerical machinery described in HZ03.

5. RESULTS

Figure 3 shows b_x and b_y over the whole domain for increasing τ_Ω, and $b_y(1) = 0.003$. For b_x, the linear resistive solution and the outer (inductive) solution are clearly distinguishable. Increasing τ_Ω lets the solutions converge to a purely resistive profile, so that the electric field—and thus the reconnection rate—is $E \propto \lambda_\Omega$, the diffusion solution. This proportionality we can clearly see in Figure 4.

We are interested in the threshold value of $b_y(1)$ below which the central $b_y(0)$ is unimportant and the electric field

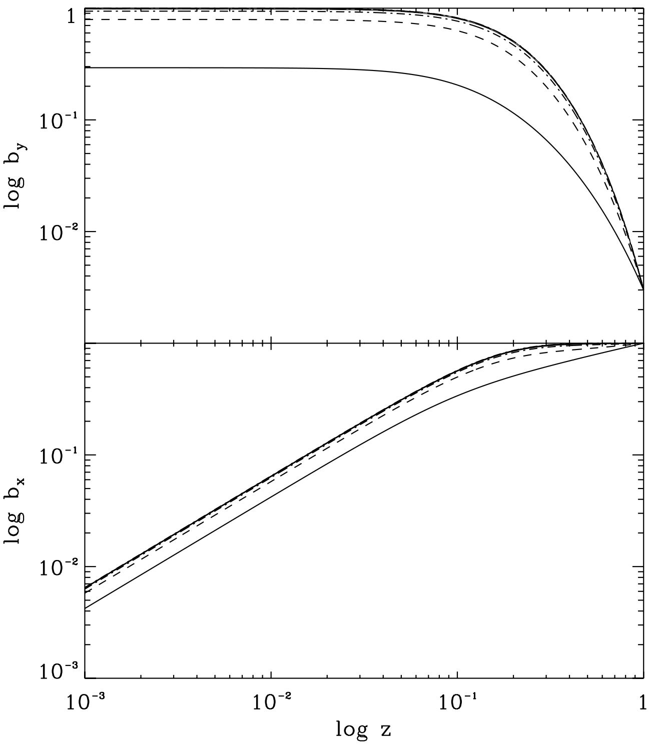

FIG. 3.—Magnetic field components $b_{x,y}(z)$ for various τ_Ω. The solutions converge to a resistive profile for increasing τ_Ω.

is given by the AD solution (see HZ03, eqs. [35] and [36])

$$\kappa_y = \frac{1}{3\tau_{\rm AD}} + \frac{1}{\tau_\Omega} \ . \tag{25}$$

This solution follows from equations (1)–(3) or equations (5) and (6) with $B_y \equiv 0$. Figure 5 shows κ_y against $b_y(1)$ for the models in Table 1. For a given τ_Ω we decrease $b_y(1)$ in order to check whether the prediction of equation (14)

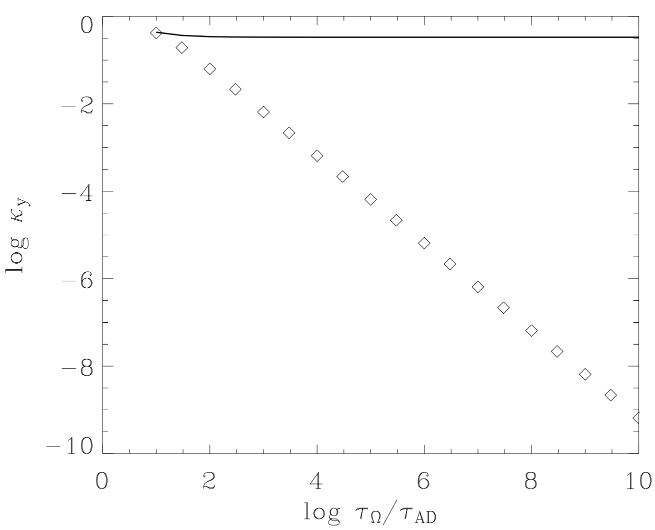

FIG. 4.—Electric field κ_y against $\tau_\Omega/\tau_{\rm AD}$. *Solid line*: prediction according to AD solution from HZ03, eq. (25); *diamonds*: numerical results for $b_y(1) = 0.003$. Electric field scales as τ_Ω^{-1}, indicating a pure diffusion solution.

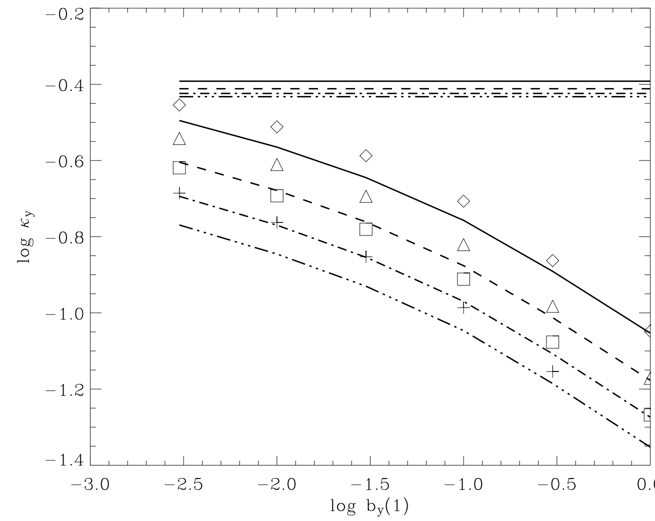

FIG. 5.—Electric field in terms of $b_y(1)$. *Diamonds*, *triangles*, *squares*, and *crosses*: Models B3–B6, respectively. The curves following the symbols denote the predictions of κ_y according to eq. (17). The lines of constant κ_y denote the AD solution as given by eq. (25).

holds. Only model B3 has nearly reached the AD solution (solid constant line and diamonds). The lines following the symbols denote the predicted κ_y according to equations (15) and (17).

In fact, none of the models could be run up to convergence with the pure neutral sheet solution, probably because the degree of the system drops from 2 to 1 when $b_y \equiv 0$. Nevertheless, we can see that the analytical prediction for κ_y is qualitatively correct. The agreement is quite good considering how crudely we arrived at the prediction.

6. SUMMARY

In a previous paper, HZ03, we showed that magnetic neutral sheets in cold, weakly ionized gases can merge extremely rapidly. Our results, which are in some respects an extension of Brandenburg & Zweibel (1995) and Vishniac & Lazarian (1999), rely on rapid recombination in the neutral layer, which limits the ion pressure.

In this paper, we extended the model to the more general case of a sheared magnetic field with a null in only one component. We found that even a small additional field component can prevent fast reconnection in one dimension. Fast reconnection is only possible for $B_y(L) < B_x(L) \exp(-\tau_\Omega/2\tau_{\rm AD})$. Since $\tau_\Omega/\tau_{\rm AD} > 10^8$ in the ISM, an additional nonzero field can prevent fast reconnection very efficiently.

Reconnection would, of course, be faster in two dimensions, with reconnection at an X-point and outflow away from it. In this geometry (see Parker 1957 and Sweet 1958), the magnitude of B_y is limited by the outflow. Previous analysis has shown that reconnection is slow if B_y is dominant (Zweibel 1999), but the possibility remains that a small but nonvanishing B_y is compatible with fast reconnection. Treatment of this case is beyond the scope of this paper.

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