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ON THE REALITY OF THE ACCELERATING UNIVERSE

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ABSTRACT

Two groups recently deduced the positive value for the cosmological constant, concluding at a high ($\geq$99%) confidence level that the universe should be accelerating. This conclusion followed from the statistical analysis of dozens of high-redshift supernovae. In this paper this conclusion is discussed. From the conservative frequentist's point of view, the validity of the null hypothesis of the zero cosmological constant is tested by the classical statistical χ^2 test for the 60 supernovae listed in Perlmutter et al. This sample contains 42 objects discovered in the frame of the Supernova Cosmology Project and 18 low-redshift objects detected earlier. Excluding the event SN 1997O, which is doubtlessly an outlier, one obtains the following result: the probability for seeing a worse χ^2—if the null hypothesis is true—is in the 5%–8% range, a value that does not indicate significant evidence against the null. Furthermore, if one excludes five possible outliers, as proposed by Perlmutter et al., then the sample of 54 supernovae is in excellent accordance with the null hypothesis. It also seems that supernovae from the High-z Supernova Search Team do not change the acceptance of the null hypothesis. This means that the rejection of the Einstein equations with zero cosmological constant—based on the supernova data alone—is still premature.

Subject headings: cosmology: miscellaneous — supernovae: general

1. INTRODUCTION

In recent years two independent groups (Perlmutter et al. 1999, hereafter P99; Riess et al. 2000, and references therein) concluded that the cosmological constant is positive with $\Omega_\Lambda \simeq 0.7$ and $\Omega_M \simeq 0.3$ (for a detailed review and references see, e.g., Riess 2000; for the latest developments, see Riess et al. 2001). As usual, Ω_M denotes the ratio of the density of the nonrelativistic matter in the universe to the critical density, $\Omega_\Lambda = \lambda c^2/(3H_0^2)$, where λ is the cosmological constant, c is the velocity of light, and H_0 is the Hubble constant. This conclusion was based purely on the data of the observations done in the Supernova Cosmology Project (P99 and references therein) and of the observations done by the High-z Supernova Search Team (Schmidt et al. 1998, Riess et al. 2000, and references therein). The universe should also be accelerating, because $\Omega_\Lambda > \Omega_M/2$ (see Riess 2000 for more details).

Both teams recognize that the supernovae at redshift $z \simeq (0.3\text{–}1.0)$ give on average a $\simeq$0.28 mag larger distance modulus than expected if $\Omega_M \simeq 0.3$ and $\Omega_\Lambda = 0$ (Riess 2000). This excess of distance modulus is so small and there are so many sources of uncertainties that extreme care is needed in drawing conclusions. This fact is, of course, clearly proclaimed by both teams. Therefore, additional careful analysis concerning the methods, statistics, errors, alternative explanations, etc., is required. Any new result—even of minimal technical importance—is highly desirable and should immediately be announced (R. Kirshner 2000, private communication). For example, Drell, Loredo, & Wasserman (2000) and Gott et al. (2001) gave smaller evidence for the nonzero cosmological constant.

In essence, this article is also such a contribution. It discusses one concrete question of the topic, namely, the probability of the rejection of the zero cosmological constant hypothesis. The discussion is done from a pure statistical point of view.

2. GENERAL CONSIDERATIONS

In Perlmutter et al. (1997, 1999), their analysis of the data gives the conclusion that $\Omega_\Lambda > 0$ holds with a 99% confidence level. In Riess et al. (2000) a higher than 99% confidence is deduced. Gott et al. (2001) deduced—from an earlier sample—that the confidence for $\Omega_\Lambda > 0$ is only 89.5% or smaller. All these statistical analyses followed the so-called Bayesian approach. The key idea of this approach is based on the procedure in which—even before there existed any measured data concerning a hypothesis—some preliminary degree of plausibility ("Bayesian prior") is assigned to the hypothesis (for more details about the Bayesian approach in astronomy, see Drell et al. 2000 and references therein).[1] In the case of the supernovae, the different confidence levels came from the different prior of the hypothesis $\Omega_\Lambda = 0$.

The author considers it is highly useful to provide an analysis of data from the frequentist's point of view, too. This approach proceeds classically and most conservatively. This means that, at the beginning, it is simply assumed that the Friedmann model (with either $\Omega_M > 1$ or $\Omega_M = 1$ or $0 < \Omega_M < 1$) *with zero cosmological constant* is the correct model. Then it is asked whether the observational data are in accordance with this model or not (for more details concerning this statistical approach, see any standard textbook of statistics, e.g., Trumpler & Weaver 1953; Kendall & Stuart 1976; from newer publications see, e.g., Feldman & Cousins 1998 and references therein).

The requirement of this analysis can be supported as follows. It is a standard knowledge that the Friedmann model with $\Omega_\Lambda = 0$ is based on two different assumptions: (1) grav-

[1] For different aspects of methods from the statistical point of view, see the Web site maintained by J. Berger at http://www.isds.duke.edu/~berger/papers/02-01.html.

itation is described by the Einstein equations with zero cosmological constant and (2) the universe has a symmetry defined by six linearly independent Killing vectors, and the character of this symmetry allows one to speak of a maximally symmetric three-dimensional submanifold (this assumption is called the cosmological principle; for more details, see Weinberg 1972, chap. 14.1).

These assumptions should be verified by observations, of course. In the verification of assumption 2 it is quite usual to proceed in the frame of the most conservative point of view. In addition, theoretically, even if the cosmological principle were rejected, it would not be clear which non-Friedmann model should then be used (for the survey of non-Friedmann models, see Kraśiński 1997). Simply, if the cosmological principle is not yet rejected unambiguously at a high significance, then the best option is to keep the cosmological principle as far as possible.

The author thinks that one should proceed similarly concerning assumption 1 as well. From the observational point of view, Drell et al. (2000) and Gott et al. (2001) suggest that one should remain careful in the final conclusions. In addition, from the theoretical point of view, even if the observations were rejecting assumption 1, one would be able to introduce *several different generalizations* of Einstein equations. For example, Gott et al. (2001) discusses both the usual generalization with cosmological constant but also the possibility with the time-variable "constant." This second possibility is identical to the introduction of a long-range scalar field coupled with the gravitation. In fact, there are many known similar theoretical attempts for other fields coupled with the gravitation (see Gott et al. 2001 and references therein). Add here that the author probed to introduce such long-range force defined by a pair of standard spin-2 fields (Mészáros 1987). This probe was proclaimed to be hopeless because of the unsolvable complications in the theory (Mészáros 1991). In any case, the introduction of the nonzero cosmological constant is not the only possible generalization of Einstein equations. Simply, the best choice in this case is to keep assumption 1 as far as possible.

The observations quite unambiguously suggest that $\Omega_M \geq 1$ is excluded, and, in addition, it may be assumed that $0.1 \leq \Omega_M \leq 0.5$ (Bahcall & Fan 1998). Hence, the null hypothesis will be the assumption of the correctness of the Friedmann model with $0.1 \leq \Omega_M \leq 0.5$ and $\Omega_\Lambda = 0$. The sample obtained from observations will be given by the 60 supernovae collected and discussed at P99. Hence, the precise purpose of this article is to test the following: does this sample alone reject the null hypothesis? This is studied in § 3. The remaining supernovae from the second project and also some other questions will be discussed in § 4. In § 5 the results of paper will be summarized.

3. THE χ^2 TEST

Let us turn to the data of 60 supernovae collected in Tables 1 and 2 of P99. Then one has to fit the $[m_B^{\rm eff},\ \log z]$ data pairs with the theoretical curves, in which $\Omega_\Lambda = 0$ holds *identically*. This means that there are only two independent parameters in these theoretical curves (H_0 and Ω_M). The procedure is a standard one and is described, e.g., by Press et al. (1992, chap. 15.1). One has to do three things: to determine the two best-fit parameters, to determine their allowed ranges, and to determine the goodness-of-fit due to the standard χ^2 test. Equation (15.1.5) of Press et al. (1992) takes the form

$$\chi^2 = \sum_{i=1}^{N} \left[\frac{m_{Bi}^{\rm eff} - m_B^{\rm eff}(z_i,\ H_0,\ \Omega_M,\ \Omega_\Lambda = 0)}{\sigma_i}\right]^2 , \quad (1)$$

where N is the number of supernovae in the sample, and σ_i is the uncertainty of the effective magnitude of ith supernova having measured the redshift z_i and corrected B-band magnitude $m_{Bi}^{\rm eff}$ ($i = 1,\ 2, \ldots,\ N$). The corrected B-band magnitude is given by

$$m_B^{\rm eff} = 25 + M_B + 5\log(c/H_0) + 5\log Q(z,\ \Omega_M,\ \Omega_\Lambda = 0) , \quad (2)$$

where M_B is the absolute magnitude, c/H_0 is in Mpc, and one has (Carroll, Press, & Turner 1992)

$$Q(z) = (2/\Omega_M^2)\left[2 + \Omega_M(z-1) - (2-\Omega_M)\sqrt{1+\Omega_M z}\right] . \quad (3)$$

This standard relation of cosmology is also obtainable directly (Mészáros & Mészáros 1996) without the integration of general equation presented by Carroll et al. (1992). The null hypothesis should then be rejected in the case when either the best-fit parameters are fully unphysical or the goodness-of-fit excludes the fit itself. In our case we will proceed in such a way that only the observationally allowed ranges of parameters will be considered—hence, if one obtains a good fit from the goodness-of-fit, then the fit is immediately acceptable.

In our case $N = 60$, and one may take in accordance with Perlmutter et al. (1997) $\tilde{M} = M - 5\log H_0 + 25 = -3.32$. A $\simeq$25% observational uncertainty in the value of H_0 (Gott et al. 2001; Freedman et al. 2001) gives maximally a $\simeq 5\log 1.25 = 0.48$ mag change in the value of $\tilde{M}$. This means that in the range $3.80 > -\tilde{M} > 2.84$, one should search for the best fits.

Using the measured redshifts, corrected effective magnitudes, and their uncertainties, one obtains the best fit for $\Omega_M = 0.1$, and for $\tilde{M} = 3.30$, namely, $\chi^2 = 107.1$. This value is the best fit for 58 degrees of freedom (dof).

Varying the free parameters in the allowed ranges, one obtains the following. If $\tilde{M} = -3.32$ and $\Omega_M = 0.1$, one has $\chi^2 = 108.0$. In fact, in all fits of this section the best fits for $\tilde{M}$ were practically always given by $\tilde{M} = -3.32$. The values of χ^2 obtained for $\tilde{M} = -3.32$ and for the best-fit values of $\tilde{M}$ gave practically the same significance levels—the differences were smaller than 1%, which is unimportant for the purpose of this paper. Therefore, in what follows, the value of $\tilde{M} = -3.32$ may always be taken as the best-fit value. In the case of parameter Ω_M the worst fit is obtained for $\Omega_M = 0.5$, namely, $\chi^2 = 129.6$. Between $\Omega_M = 0.5$ and $\Omega_M = 0.1$ the fitting is monotonously strengthening, if one goes toward the smaller values. Contrary to $\tilde{M}$, in the case of parameter Ω_M, the best-fit value is on the boundary of the allowed range of parameter.

The goodness-of-fit is given by the χ^2 probability function $P(\nu/2,\ \chi^2/2)$ (see Press et al. 1992, chap. 6.2) for $\nu = 58$ dof. Add here that the fast approximate probability of the goodness-of-fit is also obtainable without the calculation of this function directly from the table of the χ^2 distribution (see Trumpler & Weaver 1953, Table A5). One may use the fact that roughly for $\nu > 20$ dof the reduced χ^2/ν

distribution is practically not changing. For $\nu = 58$ and $\chi^2 = 108$ the significance level is between 1% and 0.1%. For $\Omega_M = 0.5$ the fit even is worse, and the significance level is around 0.1%.

For the sake of statistical precision, two notes must be added here. The first one concerns the degrees of freedom. In fact, $m_B^{\rm eff}$ itself is a corrected value in P99 and contains two additional parameters (see P99 for more details). Hence, the degree of freedom, as it seems, for 60 objects should be $\nu = 56$. In fit A of Table 3 in P99 this value is used. Furthermore, complications can arise from the fact that the best-fit value $\Omega_M = 0.1$ is a boundary value of the allowed range. This may cause some problems (for a discussion of this question see, for example, Protassov et al. 2002). In our case this may mean that, in essence, Ω_M should not be considered as a free parameter, but the value $\Omega_M = 0.1$ should be fixed immediately. In this case the degree of freedom should be increased by 1. All this means that there is an ambiguity in the concrete value of the degree of freedom. Fortunately, in our case, this problem is not essential: the significances are the same—the difference is smaller than 1%, once the degree of freedom is changed by 1. Therefore, in what follows, we may take $\nu = N - 2$ for N objects.

The second notes concerns the errors. Strictly speaking, one should also include the errors of $\log z$ into σ_i (see Press et al. 1992, chap. 15.3). But the errors in $\log z$—compared with the errors of magnitudes—are small (except for some low-redshift objects). In any case, these additional errors should decrease the significances of rejection, because they should decrease the value of χ^2. But this effect should also be unimportant here.

The approximate significance from the reduced χ^2/ν, the effect of errors in redshifts, the boundary value of Ω_M together with the ambiguity in the degree of freedom, and the choice $\tilde{M} = -3.32$ may cause a maximally 1%–3% uncertainty in the obtained significance. This impreciseness is inessential for our purpose.

For our purpose it is essential that the sample with $N = 60$ supernovae gives a fully wrong fit. *The null hypothesis for the whole sample should be rejected; the significance level is in the range 0.1%–3.0%, being enough to reject the null.*

Nevertheless, in coming to a final conclusion, care is still needed because of the following fact. An inspection of terms in $\chi^2 = 108$ for $\Omega_M = 0.1$ shows that in this sum a large amount, namely, 26.7, is contributed by one supernova, namely, by SN 1997O at $z = 0.374$. Hence, if this one single object were not considered in the sample, then the sample with $N = 59$ object would give only $\chi^2 = 81.3$ for $\nu = 57$ dof. This would already be an acceptable fit, because the rejection of null hypothesis would be occurring at a 6%–7% significance level. Taking into account the possible 1%–3% uncertainty, one may conclude that the usually requested less than 5% significance level, allowing the rejection, should not be reached.

Hence, we arrive at the surprising result: *The null hypothesis is rejected, but by one single object!*

There are three different arguments suggesting that SN 1997O should actually be removed from the sample. The first argument comes from general statistical considerations of outliers. It is never strange in statistics to remove an object from the sample if it is an "outlier." Generally, outliers are observations that are inconsistent with the remainder of the data set (a detailed discussion of outliers is given, e.g., by Jolliffe 1986, chap. 10.1). Looking into Figures 1 and 2 of P99, one immediately sees that SN 1997O is a good candidate for an outlier, because it is far above the magnitudes expected from the trend given by other objects. The object is "too faint."

The second argument follows from the text of P99. This article also discusses the question of outliers from the astrophysical point of view (different light curves, reddening in the host galaxy, etc.). Four supernovae, namely, SN 1992bo, SN 1992bp, SN 1994H, and SN 1997O, are proclaimed as "most significant outliers." Furthermore, there are two (SN 1996cg, SN 1996cn) that are also proposed to not be taken into the sample for different reasons. Hence, P99 also takes SN 1997O as an outlier. In addition, three or five objects are also proposed to be removed.

The third argument proceeds from the following consideration. Assume that no outliers are in the sample. Then the null hypothesis is rejected, and the generalization of Einstein equations is needed. There are several possibilities for this generalization. One of these is the nonzero cosmological constant. Then one should fit the whole sample with $N = 60$ with the theoretical curves allowing $\Omega_\Lambda \neq 0$. This was already done by P99 (Table 3, fit A); the value $\chi^2 = 98$ was obtained. But this is *again a wrong fit*, because for 58 dof one obtains a *rejection* at the significant level 1% (Trumpler & Weaver 1953). All this means that in this case *both* $\Omega_\Lambda = 0$ *and* $\Omega_\Lambda \neq 0$ *should be rejected.* Simply, the generalization with nonzero cosmological constant is also *not* acceptable. It is even questionable whether any theoretical curve—in the frame of cosmological principle—can fit this sample (see Weinberg 1972, chap. 14, for a general discussion of theoretical curves). Hence, the object SN 1997O *alone* should reject the Einstein equations with both zero and nonzero cosmological constant and, in addition, probably the cosmological principle itself.

The author means—in accordance with P99—that this object is a clear outlier and should be removed from the sample. All this means that the best way is to consider three different samples. The first one is the sample with $N = 59$ objects removing only SN 1997O. The second sample is the sample B of P99; the third one is the "primary sample" of P99 having $N = 54$ objects (sample C). P99 proposes to use this third sample as the best primary choice.

The first sample with $N = 59$ gives an acceptable fit; the rejection of the null hypothesis should be at the significance level 5%–8%. The second sample with $N = 56$ gives $\chi^2 = 68.3$ for $\Omega_M = 0.1$, which is again an acceptable fit for $\nu = 54$ dof. The null hypothesis is rejected at the 11% significance level. The primary sample with $N = 54$ gives $\chi^2 = 63.7$ for $\Omega_M = 0.1$. This value for $\nu = 52$ dof gives an excellent fit; the null hypothesis is rejected at the 28% significance level. In any case, $<5\%$ level is never reached.

4. DISCUSSION

Riess et al. (1998; see also Riess et al. 1999) discuss 10 additional high-redshift supernovae with $0.16 \leq z \leq 0.62$. Of course, the best solution would be to fit these objects together with the 60 objects of P99. Nevertheless, the errors in Riess et al. (1998) are listed in other ways than in P99. In addition—even while having a list of σ_i for any object obtained by the same manner—further complications can arise from the existence of outliers. (Clearly, the same criterion for an outlier should be required in such a "matched" sample. It is not clear how to define this criterion.) Simply matching all the possible observed supernovae into one single statistical sample leads to several technical problems, and the author—not being in the teams of two projects—is not able to solve this technical question. Therefore, the supernovae from the second team will be fitted separately.

Using equation (4) of Riess et al. (1998), in which $\Omega_\Lambda = 0$ and $\sigma_v = 0$, one may provide the fitting for 10 objects listed in Tables 5 and 6 of Riess et al. (1998). Taking the values of μ_0 and σ_{μ_0} from the last column of Table 5, and taking the possible values of free parameter H_0 (in units km s^{-1} Mpc^{-1}) between 55 and 90 (Freedman et al. 2001), one obtains the best fit for $H_0 = 57$ km s^{-1} Mpc^{-1}and $\Omega_M = 0.1$, namely, $\chi^2 = 9.03$. Taking the values of μ_0 and σ_{μ_0} from the last column of Table 6, one obtains the best fit for $H_0 = 78$ km s^{-1} Mpc^{-1} and $\Omega_M = 0.1$, namely, $\chi^2 = 7.7$. Both cases are excellent fits, because the significance level is around 40% and 50%, respectively, for 8 dof (Trumpler & Weaver 1953). Note that the change caused by Ω_M is weak. For example, in the first case for $\Omega_M = 0.5$ one still has $\chi^2 = 9.6$. The dependence on the change of H_0 is more essential, but in any case for $H_0 = 56$–59 km s^{-1} Mpc^{-1} acceptable fits are obtained. The choice $\sigma_v = 0$ is not a problem because eventual nonzero values further decrease the value of χ^2 and thus further strengthen the goodness of fits. The 10 supernovae from Riess et al. (1998) *alone* do not need a nonzero cosmological constant.

For the sake of completeness, the object SN 1997ff with $1.5 < z < 1.8$ should also be discussed (Riess et al. 2001). For this object the uncertainty at $\Delta(m - M)$ is so large ($\simeq$1 mag, as this is clear from Figs. 10 and 11 of Riess et al. 2001) that here the contribution for χ^2 should surely be smaller than 1. This object alone should even strengthen the acceptance of null hypothesis.

In discussing the results of article, the following must be made clear. Strictly speaking, this article does not claim that the introduction of a nonzero cosmological constant cannot be done. I only say that—purely from the most conservative statistical point of view and purely from the supernovae observational data *alone*—the assumption of zero cosmological is *not* rejected yet at a high enough significance level. The reality of the nonzero cosmological constant is not excluded; it remains an open problem yet. In any case, the different statistical methods—either from the Bayesian (Drell et al. 2000; Gott et al. 2001) or from the frequentist's point of view (this article)—still suggest that from the statistical point of view the "definite," "final," or "unambiguous" introduction of a nonzero cosmological term—based on the supernova data alone—is still premature. In fact, this is the key result of this article.

5. CONCLUSIONS

The results of paper can be summarized as follows:

1. The observational data of 60 supernovae—listed in P99—were reanalyzed from the conservative statistical point of view. The null hypothesis of a zero cosmological constant is not rejected by these data alone. The probability for seeing a worse χ^2—if the null hypothesis is true—is in the 5%–28% range, a value that does not indicate significant evidence against the null. If only one clear outlier is omitted, then this probability is 5%–8%; if further outliers—proposed by P99—are omitted, then this probability is 10%–28%. The value <5% is not reached.
2. The High-z Supernova Search Team data alone suggest that this conclusion further holds.
3. All this means that the introduction of a nonzero cosmological constant—based on the supernovae data alone—is still premature. The reality of the nonzero cosmological constant remains an open question.

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