GAMMA-RAY BURST LIGHT CURVES—ANOTHER CLUE ON THE INNER ENGINE

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ABSTRACT

The nature of the inner engine that accelerates and collimates the relativistic flow at the cores of gamma-ray bursts (GRBs) is the most interesting current puzzle concerning GRBs. Numerical simulations have shown that the internal shocks' light curve reflects the activity of this inner engine. Using a simple analytic model, we clarify the relations between the observed gamma-ray light curve and the inner engine's activity and the dependence of the light curve on the inner engine's parameters. This simple model also explains the observed similarity between the observed distributions of pulses' widths and the interval between pulses, and the correlation between the width of a pulse and the duration of the preceding interval. Our analysis suggests that the variability in the wind's Lorentz factors arises because of a modulation of the mass injected into a constant energy flow.

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1. INTRODUCTION

According to the current fireball model, gamma-ray bursts (GRBs) are produced when a relativistic flow is slowed down via relativistic shocks. At the core of a GRB is a hidden inner engine that accelerates the flow. Since there are no direct observations of the inner engine, its nature is the most mysterious puzzle within the GRB phenomenon. GRB light curves provide the best clues on the nature of this inner engine. Using the variability seen in the majority of the light curves, Fenimore, Madras, & Nayakshin (1996) and Sari & Piran (1997) demonstrated that GRB shocks must be internal. These shocks require a continuous and variable inner engine that operates during the whole duration of the GRB and varies on the observed variability timescale.

Numerical simulations (Kobayashi, Piran, & Sari 1997, hereafter KPS97; Daigne & Mochkovitch 1998; Ramirez-Ruiz & Fenimore 2000; Spada, Panaitescu, & Mészáros 2000) involving different physical processes and different assumptions on the nature of the relativistic flow produced synthetic light curves. KPS97 revealed that the gamma-ray light curve reproduces the temporal activity of the "inner engine." Following simulations have shown this result. The goal of our analytic model presented here is to explain this result and to show the relationship between the behavior of the inner engine and the observed light curve.

The observed pulses width δt and the intervals between pulses Δt have similar distributions, and Δt is correlated with the consecutive δt (Nakar & Piran 2002, hereafter NP02; see also Quilligan et al. 2002). We show that in internal shocks with equal-energy shells (within the same burst), both δt and Δt reflect the initial separation between the shells, L. Therefore, both observational results arise naturally in this model. If, instead, the shells' masses are constant, Δt still reflects the shells' separation, but δt depends also on the distribution of the shells' Lorentz factors. In this case, the variance in γ wipes out both the δt - Δt similarity and the correlation. We confirm the analytic results using numerical simulations. These results suggest that the inner engine produces a variable Lorentz factor flow by modulating the mass of a constant energy flow. These results provide yet another strong support for the internal-shock model. They also give a new clue on the nature of the inner engine.

While calculating the pulse width, we assume that the cooling time is shorter than other physical timescales. This assumption holds for a large region of the parameter space (KPS97; Piran 1999; Wu & Fenimore 2000). It may break down for large radii (where the shells' densities are low) or for small radii (where the shells may be optically thick). Since the cooling time influences only the pulse width, the observed similarities and the correlation between the two provide further independent support for this assumption and an indication of the conditions within the emitting regions.

2. THE ANALYTIC MODEL

In our model, the inner engine emits relativistic shells that collide and produce the observed light curve. We make the following simplifying assumptions: (1) The shells are discrete and homogeneous. Each shell has a well-defined boundary and a well-defined γ . (2) The colliding shells merge into a single shell after the collision. The merged shell properties are obtained using energy and momentum conservation as described in KPS97. (3) Only efficient collisions produce an observable pulse. The efficiency ϵ is defined as the ratio between the postshock internal energy and the total energy. We consider only collisions with $\epsilon > 0.05$.

Under these assumptions, each shell (labeled *i*) is defined by four parameters: the ejection time t_i , the mass m_i , the Lorentz factor γ_i , and the shell's width l_i . We define $L_{i,j}$ as the interval between the rear end of the *i*th shell and the front of the *j*th shell. Note that $L_{i,i+1} \approx t_{i+1} - (t_i + l_i)$.¹

Consider, first, a single collision between two shells with widths l_1 and l_2 , a separation *L*, and ejection times $t_2 \approx t_1 + (l_1 + L)$. We define $\gamma_1 \equiv \gamma$ and $\gamma_2 \equiv a\gamma$ (a > 1). The collision efficiency depends strongly on *a* (Piran 1999); $\epsilon(a = 2) \approx 0.05$, and it decreases fast with decreasing *a*. Hence, we consider only collisions with a > 2.

The collision takes place at $R_s \approx \gamma^2 L [2a^2/(a^2 - 1)] \approx 2\gamma^2 L$. Note that as long as a > 2, R_s depends rather weakly on γ_2 . The emitted photons from the collision reach the observer

¹ Hereafter, we take c = 1. This relation is only approximate since the shell's velocity is almost (but not exactly) c.



FIG. 1.—Types of multiple collisions that result in two consecutive pulses. [See the electronic edition of the Journal for a color version of this figure.]

at

$$t_{\rm obs} \approx t_1 + l_1 + R_s / (2\gamma^2) \approx t_1 + l_1 + L \approx t_2.$$
 (1)

These photons are observed almost simultaneously with a hypothetical photon emitted from the inner engine at t_2 together with the faster shell. This result explains the numerical results of KPS97 (and others) that the observed light curves reproduce the activity of the inner engine.

Since we are interested in comparing the characteristics of two consecutive pulses, we identify the three types of collisions (from which the complete light curve can be produced) that produce two consecutive pulses (see Fig. 1): (1) two collisions between four consequent shells with $\gamma_2 = a\gamma_1$ and $\gamma_4 = b\gamma_3$, in which the collisions are between the first and second shells and between the third and fourth shells; (2) two collisions between three consequent shells with $\gamma_1 = \gamma_2/a = \gamma_3/b$, in which the two front shells collide and then the third shell collides with the merged one; and (3) same as the type II collisions, but here the rear shells collide first.

Type I collisions result in two observed pulses, at t_2 and at t_4 , separated by $\Delta t \approx t_4 - t_2 \approx l_2 + L_{2,3} + l_3 + L_{3,4}$. In type II collisions, the pulses are at t_2 and t_3 , and $\Delta t \approx t_3 - t_2 \approx l_2 + L_{2,3}$. In type III collisions, the last shell takes over the second one, and the first pulse is observed at t_3 . Then the merged shell

takes over the first one, releasing another pulse observed at $\sim t_3$.² Therefore, type III collisions result in a single wide pulse.

Detailed calculations (E. Nakar & T. Piran 2002, in preparation) show that these results are accurate up to an order of $a_{i,j}^{-2}$, where $a_{i,j} \equiv \gamma_i / \gamma_j$. This factor $a_{i,j}^{-2}$ is small for efficient collisions. These results depend weakly on the shell's mass distribution.

The relevant timescales that determine the pulse width are (Piran 1999) as follows: (1) the angular time t_{ang} , which results from the spherical geometry of the shells ($t_{ang} \approx R_c/2\gamma_{sh}^2$, (2) the hydrodynamic time t_{hyd} , which arises from the shell's width and the shock crossing time ($t_{hyd} \approx l_{c_{in}}$, where $l_{c_{in}}$ is the width of the inner shell at the time of the collision), and (3) the cooling time (either the cooling time of the emitting electrons in transparent shells or the radiation diffusion time in opaque shells). As stated in § 1, we assume that this time is shorter than t_{ang} and t_{hyd} . This assumption is well justified for most of the parameter space for synchrotron emission (KPS97; Wu & Fenimore 2000). Therefore, the pulse width is $\delta t \approx t_{ang} + t_{hyd} \sim t_{ang}$.³

Unlike the pulse's timing, the pulse's width depends strongly on the shells' masses. This follows from the strong dependence of the Lorentz factor of the shocked region, γ_{sh} , on the ratio of shells' masses. We examine two possible cases: equal-mass shells and equal-energy shells. Table 1 summarizes the intervals and the pulses' widths for the two different mass distributions for the three types of collisions.

3. NUMERICAL SIMULATION

The analytic model demonstrates that the properties of the light curve depend on the dominant type of collisions. In order to determine which collision type dominates, we performed numerical simulations. These simulations also verify the validity of some of the approximations used in the analytic toy model. All collisions are taken into account in the simulations, and we do not apply the efficiency constrain, a > 2.

Each simulation included 50 shells. Using the intuition gained by the analytic model, we choose a lognormal *L* distribution with $\mu(\log L) = -0.5$ and $\sigma(\log L) = 0.9$ (chosen in order to fit the observations). The initial shell's width is taken as a constant of 0.1 s. We obtain similar light curves for either the constant width or spreading shell model. The results presented are for a uniform Lorentz factor distribution ($\gamma_{\min} =$

² Detailed calculations show that the interval between these two pulses is shorter than the pulses' widths.

³ Usually, $t_{ang} \ge t_{hyd}$. Under extreme widening, $t_{hyd} \sim t_{ang}$.

TABLE 1 Δt and $\delta t_{1,2}$ for the Three Collisions Types

		Equal Mass		Equal Energy		
Type	Δt	δt_1	δt_2	δt_1	δt_2	
I	$l_2 + L_{2,3} + l_3 + L_{3,4}$	$l_{c_2} + \frac{L_{1,2}}{a}$	$l_{c_4} + \frac{L_{3,4}}{b}$	$l_{c_2} + L_{1,2}$	$l_{c_4} + L_{3, 4}$	
II	$l_2 + L_{2,3}$	$l_{c_2} + \frac{L_{1,2}}{a}$	$l_{c_3} + \frac{L_{1,2}}{b\sqrt{a}} + \frac{L_{2,3}\sqrt{a}}{b}$	$l_{c_2} + L_{1,2}$	$l_{c_3+}\frac{\sqrt{2}}{5}L_{1,2}+L_{2,3}$	
III	$\frac{L'_{1,2}}{(ab-1)}$	$l_{c_3} + \frac{L_{2,3}}{a}$	$>a\sqrt{\frac{a}{b}}L_{2,3} + \frac{L'_{1,2}\sqrt{ab}}{(ab-1)}$	No efficient collisions	No efficient collisions	

NOTE. $-l_i$ and l_{c_i} are the width at the ejection and the width at the collision ($l_i = l_{c_i}$ with no spreading). $L'_{1,2}$ is the separation between the first and second shells at the time that the second and third shells collide. The approximations are valid for $a_{i,i} > 2$.

TABLE 2						
Best-Fit Parameters of δt and Δt in the						
FOULL-ENERGY SIMULATIONS ^a						

	Δt (s)		δ <i>t</i> (s)	
PARAMETERS	μ	1 σ	μ	1 σ
Simulated Observed	1.4 1.3	0.6–3.4 0.5–3.1	1 1	0.5–2 0.5–2.2

^a Compared with the observed values (NP02). Note that this fit was achieved by tuning only $\mu(\log L)$ and $\sigma(\log L)$.

30, $\gamma_{\text{max}} = 2000$).⁴ The shell's mass is either constant (equal mass) or proportional to γ^{-1} (equal energy).

We identify the shells' collisions. Each collision produces a pulse (also the inefficient ones). The duration of a pulse is taken as $t_{ang} + t_{hyd}$. All the pulses have a fast-rise, slow-decay shape, with a ratio of 3 : 1 between the decay and the rise times. The area below a pulse is equal to its radiated energy. Using these pulses, we prepare a binned (64 ms time bins) light curve. We analyze this light curve using the Li & Fenimore (1996) peak finding algorithm, obtaining the observed pulses' timings and widths.

In both the equal-energy and equal-mass simulations, the number of observed pulses is between a third and a half of the total ejected shells. Efficient ($\epsilon > 0.05$) type I and type II collisions compose about 80% of the collisions. There are more type I collisions in the equal-mass model and more type II collisions in the equal-energy model. There are almost no efficient type III collisions in the equal-energy model. The efficiency in both models is about 20%–30%. This is, of course, the kinetic efficiency of the conversion of kinetic energy to internal energy. Since we do not simulate the emission process, we cannot determine what the ultimate gamma-ray production efficiency is. The efficiency decreases to 10% when the Lorentz factor is uniform between 100 and 1000. Most collisions takes place at radii larger than 10¹³ cm and smaller than 10^{15.5}, where the shells are transparent and the cooling time is short. This justifies our fast-cooling assumption.

Figure 2 illustrates the histograms of δt , Δt , and *L* as obtained by the simulations. In the equal-mass model (Fig. 2*a*), Δt reflects the *L* distribution, while δt is much shorter. In the equalenergy model (Fig. 2*b*), both distributions of Δt and δt reflect the *L* distribution, and both are consistent with the same lognormal distribution. By tuning only $\mu(\log L)$ and $\sigma(\log L)$, we obtain a perfect agreement between the δt and Δt distributions in the simulations and the observed ones (NP02). The best-fit parameters for the equal-energy simulations and the observations are described in Table 2.

The analytical results obtained in § 2 explain these results. The similarity between the interval distribution in both models' simulations follows from the weak dependence of the pulses' timings on the mass distribution. The similarity of the pulses' widths in the equal-energy model and the shells' initial separations, *L*, is explained by the analytical result $\delta t \propto L$. The deviation from a lognormal distribution and the short pulses found in the equal-mass model are explained by the analytical result $\delta t \propto L/a$.

NP02 find a correlation between an interval duration and the



FIG. 2.—Pulses' width, δt , intervals between pulses, Δt , and the separation between shells, *L*. (*a*) Equal-mass shells' simulation. (*b*) Equal-energy shells' simulation. [See the electronic edition of the Journal for a color version of this figure.]

consecutive pulse. In the equal-energy model, we find a highly significant correlation between the interval duration and the consecutive pulse. There is no significant correlation in the equal-mass model. This result is explained again by the equal-mass relation $\delta t \propto L/a$. Since the variations in *a* are larger than the variations in *L*, they wipe out the correlation.

4. DISCUSSION

KPS97 simulations of internal shocks have shown that the resulting light curves reflect the activity of the inner engine. This feature arises in all subsequent simulations. KPS97 show that this follows from the fact that the pulse timing is approximately equal to the ejection time of one of the colliding shells. We explain this feature (eq. [1]). Moreover, in most collisions (types I and II), the pulses are distinguishable, and each pulse reflects a single collision. The number of observed pulses is 30%–50% of the number of ejected shells. Therefore, the light curve reflects the emission time of one-third to one-half of the shells. The inner engine is slightly more variable than the observed light curve.

The observed similarity between the Δt and δt distributions is explained naturally in the equal-energy shells' model. Both parameters reflect the separation between the shells during their ejection. In the equal-mass shells' model, only Δt reflects the initial shells' separation, and therefore such a similarity is not expected. Our numerical simulations confirmed these predictions. Note that many of the simplifying assumptions can be relaxed with no significant change in the results. We will present a more detailed model and more elaborated simulations elsewhere (E. Nakar & T. Piran 2002, in preparation). The equalenergy simulations fitted the observations very well. These results imply that the inner engine most likely ejects equalenergy shells. These results provide more strong support for the internal-shock model. They also give one of the first clues on the nature of the inner engine.

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⁴ While the efficiency depends on this distribution, the light curves are similar for other Lorentz factor distributions, such as being uniform in the log or ranging between $\gamma_{\min} = 100$ and $\gamma_{\max} = 1000$.

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