

# MODEL-INDEPENDENT PRIMORDIAL POWER SPECTRUM FROM MAXIMA, BOOMERANG, AND DASI DATA

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Received 2000 November 19; accepted 2002 March 5

## ABSTRACT

A model-independent determination of the primordial power spectrum of matter density fluctuations could uniquely probe physics of the very early universe and provide powerful constraints on inflationary models. We parameterize the primordial power spectrum  $A_s^2(k)$  as an arbitrary function and deduce its binned amplitude from the cosmic microwave background (CMB) radiation anisotropy measurements of MAXIMA, BOOMERANG, and DASI. We find that for a flat universe with  $A_s^2(k) = 1$  (scale-invariant) for scales  $k < 0.001 h \text{ Mpc}^{-1}$ , the primordial power spectrum is marginally consistent with a scale-invariant Harrison-Zeldovich spectrum. However, we deduce a rise in power compared to a scale-invariant power spectrum for  $0.001 h \text{ Mpc}^{-1} \lesssim k \lesssim 0.01 h \text{ Mpc}^{-1}$ . Our results are consistent with large-scale structure data and seem to suggest that the current observational data allow for the possibility of unusual physics in the very early universe.

*Subject headings:* cosmological parameters — cosmology: theory — early universe

## 1. INTRODUCTION

The inflationary paradigm is currently the most plausible solution to the problems of standard cosmology. Inflation is consistent with all current observational data. However, we are still far from establishing a definitive model of inflation. There exists a broad range of inflationary models (e.g., Kolb 1997; Turner 1997), many of which appear consistent with observational data. In order to quantify what can be known about inflation, one desires model-independent measurements of the primordial power spectrum of matter density fluctuations, as these can provide unique and powerful constraints on inflationary models.

Although it has been conventional to take the primordial power spectrum to be a featureless power law in analyzing cosmological data, there are both theoretical and observational reasons to allow the primordial power spectrum to be a free function. That is, some inflationary models predict power spectra that are almost exactly scale invariant (Linde 1983) or are described by a power law with a spectral index less than 1 (Freese, Frieman, & Olinto 1990; La & Steinhardt 1991), while others predict power spectra with slowly varying spectral indices (Wang 1994) or with broken scale invariance (Holman et al. 1991a, 1991b; Randall, Soljagic, & Guth 1996; Adams, Ross, & Sarkar 1997; Lesgourgues, Polarski, & Starobinsky 1997; Lesgourgues 2000). The latter represents unusual physics in the very early universe. For example, inflation might occur in multiple stages in effective theories with two scalar fields (Holman et al. 1991a, 1991b) or in a succession of short bursts due to symmetry breaking during an era of inflation in supergravity models (Adams et al. 1997).

There is also tentative observational evidence for a peak in the power spectrum of galaxies at  $k \sim 0.05 \text{ Mpc}^{-1}$  (Einasto 1998; Baugh & Gaztanaga 1999; Retslaff et al. 1998; Broadhurst & Jaffe 2000; Gramann & Suhhonenko 1999; Gramann & Hütsi 2000). The simplest explanation for such a peak in the galaxy power spectrum is a new feature in the primordial power spectrum.

The cosmic microwave background (CMB) radiation anisotropies are signatures of primordial matter density fluctuations and gravity waves imprinted at the time when photons decoupled from matter. The large-scale structure in the distribution of galaxies is a direct consequence of the power spectrum of the primordial density fluctuations. Wang, Spergel, & Strauss (1999) have explored how one can use the upcoming CMB data from the *Microwave Anisotropy Probe* (MAP; Bennett et al. 1997)<sup>3</sup> and the large-scale structure data from the Sloan Digital Sky Survey (SDSS; e.g., Gunn & Weinberg 1995) to obtain a model-independent measurement of the primordial power spectrum and to simultaneously extract the cosmological parameters.

In this paper, we implement the concept of a model-independent measurement of the primordial power spectrum from Wang et al. (1999) to extract cosmological information from the CMB data from MAXIMA (Hanany et al. 2000), BOOMERANG (de Bernardis et al. 2000), and DASI (Halverson et al. 2002). We parameterize the primordial power spectrum as a continuous and arbitrary function determined by its amplitude at several wavenumbers [which are equally spaced in  $\log(k)$ ] via linear interpolation. We then measure these “binned” amplitudes from the CMB data of MAXIMA, BOOMERANG, and DASI.

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<sup>3</sup> See <http://map.gsfc.nasa.gov>.

## 2. MEASUREMENT OF THE PRIMORDIAL POWER SPECTRUM

We parameterize the primordial power spectrum as

$$A_s^2(k) = \begin{cases} \left(\frac{k_i - k}{k_i - k_{i-1}}\right)a_{i-1} + \left(\frac{k - k_{i-1}}{k_i - k_{i-1}}\right)a_i, & k_{i-1} < k \leq k_i, \quad i = 1 \dots n, \\ a_0 = 1, & k \leq k_0 = k_{\min}, \\ a_n, & k \geq k_n = k_{\max}, \end{cases} \quad (1)$$

where

$$k_i = \exp\left[\frac{i}{n} \ln\left(\frac{k_n}{k_0}\right) + \ln(k_0)\right] \quad \text{for } i = 0 \dots n.$$

We impose  $A_s^2(k) = 1$  for  $k \leq k_0 = k_{\min}$  for two reasons. First, on the largest scales, the CMB data are consistent with a scale-invariant primordial power spectrum (Smoot et al. 1992; Gorski et al. 1996), i.e.,  $A_s^2(k) = 1$ . Second, the bin amplitude of the primordial power spectrum on the largest scales is poorly constrained by the CMB data because of the effects of cosmic variance. We also assume that  $A_s^2(k) = a_n$  for  $k > k_n = k_{\max}$ , because  $k_{\max}$  is close to the scale corresponding to the angular resolution of MAXIMA and BOOMERANG. We choose  $k_{\min} = 0.001 h \text{ Mpc}^{-1}$  and  $k_{\max} = 0.1 h \text{ Mpc}^{-1}$ .

We perform the parameter estimation by computing the CMB angular power spectrum  $C_l(s)$  for a discrete set of cosmological parameters denoted as  $s$ . To save computational time and storage space, we have assumed a flat universe and limited our parameter search to a grid in six cosmological parameters  $\{H_0, \Omega_m, \Omega_b, a_1, a_2, a_3\}$ , with  $\Omega_\Lambda = 1 - \Omega_m$  and  $\tau_{ri} = 0$ . The  $a_i$  ( $i = 1-3$ ) parameterize the primordial power spectrum  $A_s^2(k)$  as in equation (1) with  $n = 3$ . Our assumption of a flat universe is consistent with all current observational data and is preferred by non-fine-tuned inflationary models. We use the *COBE* normalization for all the theoretical models.

Following the MAXIMA, BOOMERANG, and DASI teams (Jaffe et al. 2001; Lange et al. 2001; Pryke et al. 2002), we use an offset lognormal likelihood function to define a  $\chi^2$  goodness of fit (Bond, Jaffe, & Knox 2000). That is, for a given theoretical model with  $\mathcal{C}_l(s) \equiv l(l+1)C_l(s)/(2\pi)$ , we define

$$\chi^2(s) \equiv -2 \ln L = \chi_d^2 + \chi_{\text{cal}}^2 + \chi_{\text{beam}}^2, \quad (2)$$

$$\chi_d^2 = \sum_{i,j} (Z_i^t - Z_i^d) M_{ij}^Z (Z_j^t - Z_j^d), \quad (3)$$

$$\chi_{\text{cal}}^2 = \sum_{\alpha} \frac{(u_{\alpha} - 1)^2}{\sigma_{\alpha}^2}, \quad (4)$$

where

$$Z_i^d \equiv \ln(D_i + x_i), \quad (5)$$

$$Z_i^t \equiv \ln\left(u_{\alpha} \sum_{l_{\min,i}}^{l_{\max,i}} f_{il} \mathcal{C}_l g_l + x_i\right), \quad (6)$$

$$f_{il} = \frac{W_l^i / l}{\sum_{l_{\min,i}}^{l_{\max,i}} W_l^i / l}, \quad (7)$$

$$M_{ij}^Z = M_{ij}(D_i + x_i)(D_j + x_j). \quad (8)$$

In the above equation,  $D_i$  is the measured CMB band power in the  $i$ th bin,  $x_i$  is an offset that depends on experimental details, and  $W_l = B_l^2$  is the experimental window function (where  $B_l$  is the beam function). The weight matrix  $M_{ij}$  is given by the Fisher matrix  $F_{ij} = -\partial^2 L / \partial \mathcal{C}_i \partial \mathcal{C}_j$ . The calibration uncertainty is parametrized by  $u_{\alpha}$ , the factor of overall relative calibration, and has a dispersion of  $\sigma_{\alpha} = 0.08$  and  $0.2$  for MAXIMA-1 and BOOMERANG, respectively. We also consider beam uncertainties for BOOMERANG. Following Lange et al. (2001), we parameterize the beam uncertainty with  $\exp\{-(l+0.5)^2 [\Delta(\theta_s^2)]\}$ , with  $\langle \Delta(\theta_s^2) \rangle = 2\theta_{\text{FWHM}} \Delta\theta_{\text{FWHM}} = (572.0)^{-2}$ , where we have used  $\theta_{\text{FWHM}} = 12'.9$  and  $\Delta\theta_{\text{FWHM}} = 1'.4$ . Hence, we write

$$\chi_{\text{beam}}^2 = \sum_i \frac{\left(\sum_{l_{\min,i}}^{l_{\max,i}} f_{il} \mathcal{C}_l \exp[-g(l)\delta_{\text{beam}}] - \sum_{l_{\min,i}}^{l_{\max,i}} f_{il} \mathcal{C}_l\right)^2}{\left(\sum_{l_{\min,i}}^{l_{\max,i}} f_{il} \mathcal{C}_l \exp[-g(l)] - \sum_{l_{\min,i}}^{l_{\max,i}} f_{il} \mathcal{C}_l\right)^2}, \quad (9)$$

where we have defined  $\delta_{\text{beam}} \equiv 572.0^2 \Delta(\theta_s^2)$  and  $g(l) \equiv [(l+0.5)/572.0]^2$ .

We use the higher precision MAXIMA and BOOMERANG data as published in Lee et al. 2001 and Netterfield et al. 2001. The relevant experimental details (beam functions, covariance matrix, and lognormal offsets  $x_i$ ) have not yet been released. It has been shown that in the absence of the knowledge of  $x_i$ , Gaussian statistics describe the data better than a lognormal distribution with arbitrary  $x_i$  (Bond et al. 2000). Therefore, we replace equation (3) with

$$\chi^2(s) = \sum_i \frac{[\mathcal{C}_{\text{data}}^i - \mathcal{C}_{\text{BP}}^i(s)]^2}{\sigma_i^2}, \quad (10)$$

where  $\mathcal{C}_{\text{data}}^i$  are the experimental band powers with measurement errors  $\sigma_i$ , and

$$\mathcal{C}_{\text{BP}}^i(s) = \frac{\sum_{l_{\min,i}}^{l_{\max,i}} W_l \mathcal{C}_l(s) / l}{\sum_{l_{\min,i}}^{l_{\max,i}} W_l / l}, \quad (11)$$

$$W_l = B_l^2 = e^{-(0.425\theta_{\text{FWHM}} l)^2},$$

where  $\theta_{\text{FWHM}} = 10'$  and  $12'.9$  for MAXIMA and BOOMERANG, respectively. Equation (10) assumes symmetric error bars on  $\mathcal{C}_{\text{data}}^i$ . For the asymmetric errors given by MAXIMA, we take  $\sigma_i = \sigma_{i,-}$  for  $\mathcal{C}_{\text{data}}^i > \mathcal{C}_{\text{BP}}^i(s)$  and  $\sigma_i = \sigma_{i,+}$  otherwise.

The DASI team has released the experimental details (window functions  $f_{il}$ , lognormal offsets  $x_i$ , and covariance matrix  $V_{ij}$ ) together with their data.<sup>4</sup> This enables us to use lognormal statistics (eqs. [3]–[8]), with the weight matrix  $M_{ij}$  given by the inverse of the matrix  $N_{ij} = V_{ij} + \sigma_{\alpha} D_i D_j$ , where  $\sigma_{\alpha} = 0.08$  is the calibration uncertainty of DASI.

For a given set of cosmological parameters  $\{H_0, \Omega_m, \Omega_b, a_1, a_2, a_3\}$ , we marginalize the model over the calibration uncertainties of MAXIMA, BOOMERANG, and DASI and over the beam uncertainty of BOOMERANG. The beam uncertainty is only marginalized for BOOMERANG, since only this experiment appears to have significant beam uncertainty.

<sup>4</sup> See <http://astro.uchicago.edu/dasi>.

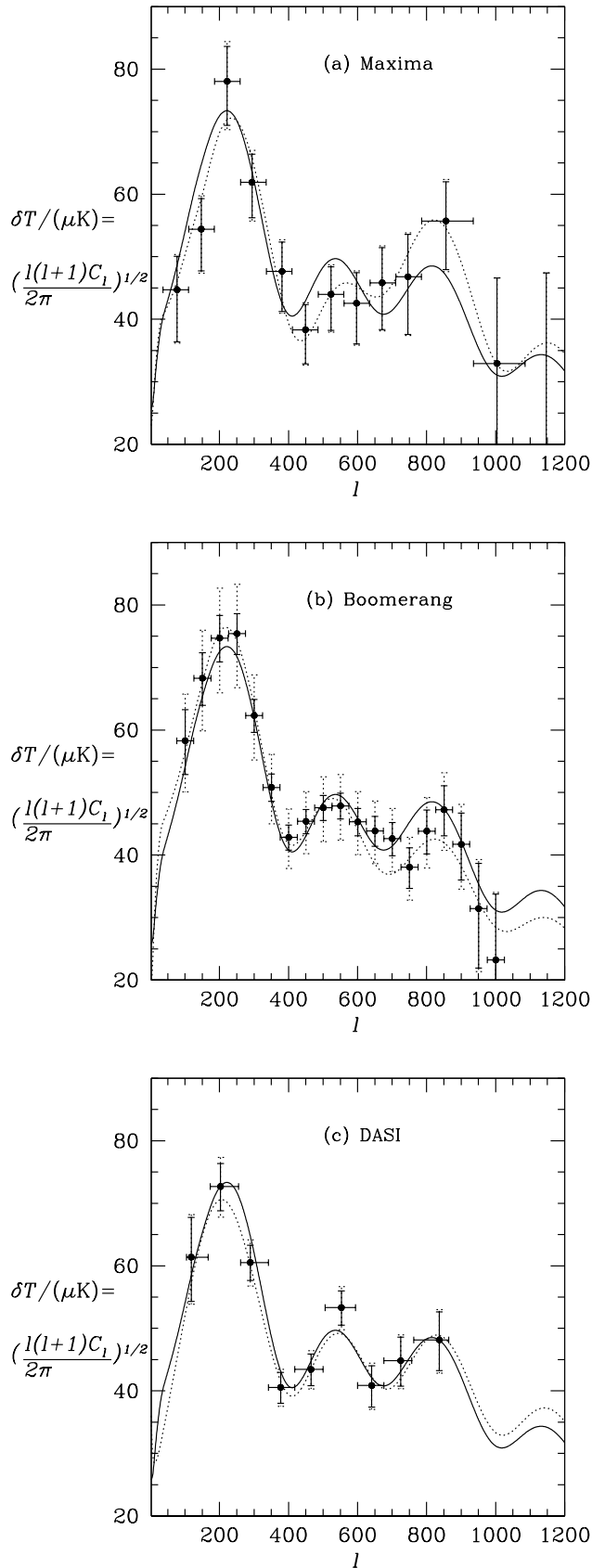


FIG. 1.—Data from (a) MAXIMA, (b) BOOMERANG, and (c) DASI, together with the best-fit models. In each figure, the solid curve is the best-fit model to the combined MAXIMA, BOOMERANG, and DASI data, while the dotted line is the best-fit model to the (a) MAXIMA, (b) BOOMERANG, and (c) DASI data separately. The dotted error bars are the errors on each data point including calibration uncertainty.

### 3. RESULTS

Since only five values of  $H_0 = 100 h \text{ km s}^{-1} \text{ Mpc}^{-1}$  ( $h = 0.5, 0.6, 0.7, 0.8, 0.9$ ) are computed for our grid, we present our results for the different values of  $h$  separately, instead of marginalizing over  $h$ .

Figures 1a–1c show the data from (1) MAXIMA, (2) BOOMERANG, and (3) DASI, together with the best-fit models. In each figure, the solid curve is the best-fit model to the combined MAXIMA, BOOMERANG, and DASI data, while the dotted line is the best-fit model to the MAXIMA (Fig. 1a), BOOMERANG (Fig. 1b), and DASI (Fig. 1c) data separately. The dotted error bars are the errors for each data point, including calibration uncertainty. We have combined the data from MAXIMA, BOOMERANG, and DASI assuming that the three experiments are independent of each other.

Table 1 shows the best-fit models to the combined MAXIMA, BOOMERANG, and DASI data.

Tables 2–4 show the best-fit models to the MAXIMA, BOOMERANG, and DASI data separately.

We constrain parameters individually by marginalizing over all other parameters. Figures 2a–2e show the likelihood functions for the combined MAXIMA, BOOMERANG, and DASI data for the set of parameters  $\{\Omega_m, \Omega_b, a_1, a_2, a_3\}$ , for  $h = 0.5$  (dotted line), 0.6 (solid line), 0.7 (dashed line), 0.8 (long-dashed line), and 0.9 (dot-dashed line). The likelihood functions have been derived using  $\chi^2$  values that resulted from multidimensional interpolations of the  $\chi^2$  values computed for the grid of models (e.g., Tegmark & Zaldarriaga 2000a).

Figure 3 shows the primordial power spectrum  $A_s^2(k)$  measured from the combined MAXIMA, BOOMERANG, and DASI data for  $h = 0.6$  (solid line) and 0.7 (dotted line). The  $\pm 1 \sigma$  errors are deduced from Figures 2c–2e.

Note that  $A_s^2(k) = 1$  corresponds to the scale-invariant Harrison-Zeldovich  $n_s = 1$  spectrum, with the primordial scalar power spectrum conventionally defined as  $k A_s^2(k) \propto k^{n_s}$ . Clearly, the primordial power spectrum is marginally consistent with a scale-invariant Harrison-Zeldovich spectrum. However, there is a rise in the power at  $0.001 h \text{ Mpc}^{-1} \lesssim k \lesssim 0.01 h \text{ Mpc}^{-1}$  compared to a scale-invariant power spectrum. Although this rise is not statistically convincing, it could be an indication of interesting physics worthy of future investigation.

### 4. DISCUSSION

Because of parameter degeneracies between  $h$  and  $\Omega_m$  and  $\Omega_b$ , the CMB data constrain  $h$  only weakly. Fortunately, there are a number of independent methods for constraining  $h$  (Freedman et al. 2001; Branch 1998; Kundic et al. 1997). The current combined CMB data from MAXIMA, BOOMERANG, and DASI marginally prefer  $h = 0.6$  (see Table 1).

Although the CMB data are sensitive to  $\Omega_m h^2$  and  $\Omega_b h^2$ , the likelihood curves for different values of  $h$  do not overlap (see Figs. 2a and 2b), since these curves correspond to different values of  $\Omega_\Lambda$  (we assume that  $\Omega_\Lambda = 1 - \Omega_m$ ). The likelihood curves for  $\Omega_b h^2$  are less sensitive to the value of  $h$  (see Fig. 2b). This is because varying  $\Omega_b$  has a much smaller effect on  $\Omega_\Lambda$  than varying  $\Omega_m$ .

Our measurement of the primordial power spectrum (see Fig. 3) is consistent with the large-scale structure data that

TABLE 1  
BEST-FIT MODELS TO THE COMBINED MAXIMA, BOOMERANG, AND DASI DATA

| $h$ | $\Omega_m$ | $\Omega_b$ | $a_1$ | $a_2$ | $a_3$ | $u_{MX}$ | $u_{BOOM}$ | $u_{DASI}$ | $\delta_{beam}$ | $\chi^2_{min}$ |
|-----|------------|------------|-------|-------|-------|----------|------------|------------|-----------------|----------------|
| 0.5 | 0.700      | 0.070      | 2.5   | 1.7   | 1.3   | 0.933    | 1.008      | 0.952      | 0.10            | 28.4500        |
| 0.6 | 0.425      | 0.055      | 2.0   | 1.2   | 1.0   | 0.974    | 1.056      | 0.987      | 0.05            | 28.2064        |
| 0.7 | 0.250      | 0.040      | 1.5   | 0.9   | 0.9   | 0.978    | 1.056      | 0.994      | 0.05            | 29.3004        |
| 0.8 | 0.200      | 0.040      | 1.0   | 0.8   | 1.0   | 0.958    | 1.048      | 0.971      | 0.10            | 29.8050        |
| 0.9 | 0.120      | 0.030      | 0.9   | 0.7   | 1.0   | 0.981    | 1.048      | 0.990      | 0.05            | 29.0687        |

TABLE 2  
BEST-FIT MODELS TO THE MAXIMA DATA

| $h$ | $\Omega_m$ | $\Omega_b$ | $a_1$ | $a_2$ | $a_3$ | $u_{MX}$ | $\chi^2_{min}$ |
|-----|------------|------------|-------|-------|-------|----------|----------------|
| 0.5 | 0.800      | 0.100      | 3.0   | 1.5   | 1.5   | 0.997    | 4.3313         |
| 0.6 | 0.600      | 0.100      | 2.5   | 1.0   | 1.5   | 0.997    | 3.2590         |
| 0.7 | 0.400      | 0.080      | 0.9   | 0.6   | 1.0   | 1.010    | 3.9188         |
| 0.8 | 0.300      | 0.070      | 1.0   | 0.6   | 1.2   | 0.994    | 4.5866         |
| 0.9 | 0.200      | 0.060      | 0.7   | 0.5   | 1.3   | 1.000    | 4.7581         |

seem to indicate a peak in the matter power spectrum at  $k \sim 0.05 \text{ Mpc}^{-1}$  (Einasto 1998; Baugh & Gaztanaga 1999; Retslaff et al. 1998; Broadhurst & Jaffe 2000; Gramann & Suhhonenko 1999; Gramann & Hütsi 2000). We note that the real-space power spectrum of the Point Source Catalog Redshift Survey, from 0.01 to  $300 h \text{ Mpc}^{-1}$  (Hamilton & Tegmark 2000), does not show this peak. The data from the Two-Degree Field Galaxy Redshift Survey (2df; Dalton et al. 2000) and SDSS (Gunn & Weinberg 1995) should help clarify any features in the large-scale structure power spectrum. However, if there is a feature at  $k \lesssim 0.002 h \text{ Mpc}^{-1}$  (corresponding to the characteristic length scale of the

SDSS; see Wang et al. 1999), satellite CMB data from *MAP* or *Planck* will be required to constrain such features in the primordial power spectrum (Wang et al. 1999).

A number of authors have used MAXIMA and BOOMERANG data to derive cosmological constraints (Abazajian, Fuller, & Patel 2000; Amendola 2001; Avelino et al. 2000; Balbi et al. 2000; Bento, Bertolami, & Silva 2001; Bouchet et al. 2002; Brax, Martin, & Riazuelo 2000; Bridle et al. 2001; Contaldi 2000; Durrer & Novosyadlyj 2001; Enqvist, Kurki-Suonio, & Valiviita 2000; Esposito et al. 2001; Griffiths, Silk, & Zaroubi 2001; Hannestad 2000; Hannestad & Scherrer 2001; Hu et al. 2001; Jaffe et al. 2001; Kanazawa et

TABLE 3  
BEST-FIT MODELS TO THE BOOMERANG DATA

| $h$ | $\Omega_m$ | $\Omega_b$ | $a_1$ | $a_2$ | $a_3$ | $u_{BOOM}$ | $\delta_{beam}$ | $\chi^2_{min}$ |
|-----|------------|------------|-------|-------|-------|------------|-----------------|----------------|
| 0.5 | 0.600      | 0.050      | 5.0   | 3.0   | 1.5   | 0.968      | -0.05           | 11.4684        |
| 0.6 | 0.375      | 0.050      | 2.5   | 1.5   | 1.0   | 0.968      | 0.00            | 11.7898        |
| 0.7 | 0.250      | 0.040      | 2.0   | 1.2   | 0.9   | 0.984      | 0.00            | 12.1868        |
| 0.8 | 0.150      | 0.030      | 1.5   | 0.9   | 0.8   | 1.016      | 0.00            | 12.4757        |
| 0.9 | 0.120      | 0.030      | 0.9   | 0.8   | 0.9   | 1.008      | 0.05            | 13.5643        |

TABLE 4  
BEST-FIT MODELS TO THE DASI DATA

| $h$ | $\Omega_m$ | $\Omega_b$ | $a_1$ | $a_2$ | $a_3$ | $u_{DASI}$ | $\chi^2_{min}$ |
|-----|------------|------------|-------|-------|-------|------------|----------------|
| 0.5 | 0.800      | 0.070      | 2.5   | 1.7   | 1.3   | 0.990      | 4.3187         |
| 0.6 | 0.450      | 0.050      | 4.0   | 1.7   | 1.5   | 1.010      | 3.3240         |
| 0.7 | 0.275      | 0.040      | 3.0   | 1.2   | 1.3   | 1.006      | 2.5608         |
| 0.8 | 0.200      | 0.040      | 2.0   | 0.9   | 1.3   | 0.994      | 2.6985         |
| 0.9 | 0.120      | 0.030      | 0.9   | 0.6   | 1.0   | 1.029      | 2.3055         |

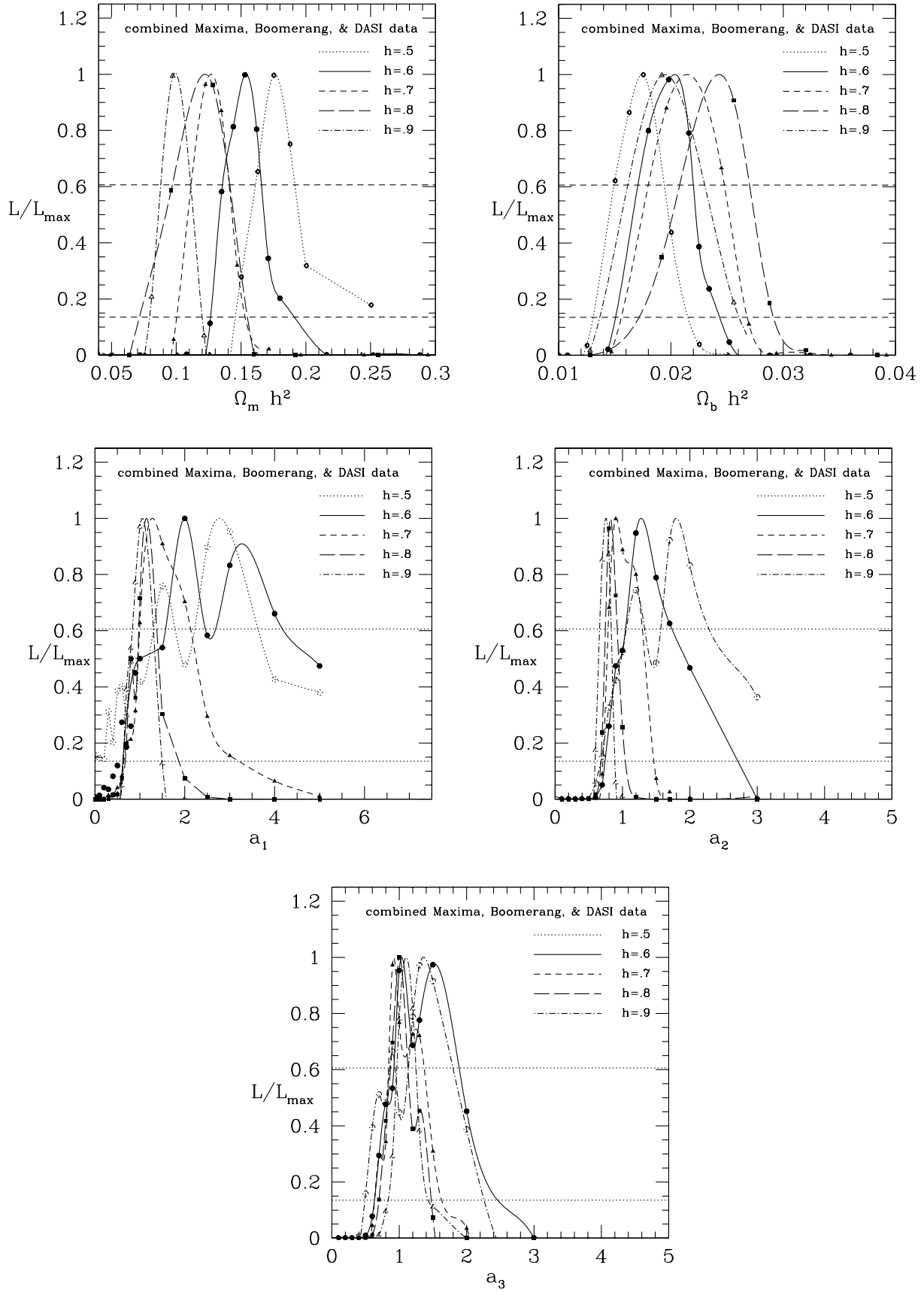


FIG. 2.—Likelihood functions for the combined MAXIMA, BOOMERANG, and DASI data for the set of parameters  $\{\Omega_m, \Omega_b, a_1, a_2, a_3\}$ , for  $h = 0.5$  (dotted line), 0.6 (solid line), 0.7 (dashed line), 0.8 (long-dashed line), and 0.9 (dot-dashed line).

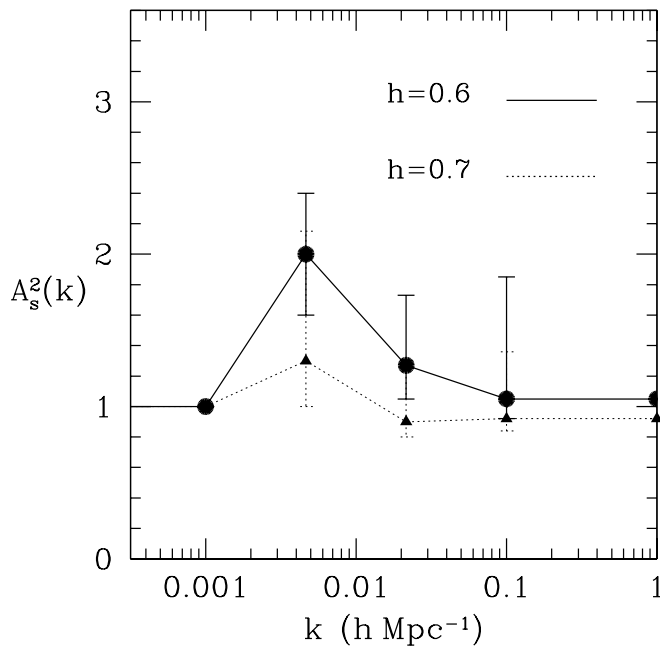


FIG. 3.—Primordial power spectrum  $A_s^2(k)$  measured from the combined MAXIMA, BOOMERANG, and DASI data for  $h = 0.6$  (solid line) and  $0.7$  (dotted line). The  $\pm 1\sigma$  errors are estimated from Figs. 2c–2e.

al. 2000; Kinney, Melchiorri, & Riotto 2001; Landau, Harari, & Zaldarriaga 2001; Lange et al. 2001; Lesgourgues & Peloso 2000; McGaugh 2000; Melchiorri & Griffiths 2001; Orito et al. 2001; Padmanabhan & Sethi 2001; Tegmark & Zaldarriaga 2000b; Tegmark, Zaldarriaga, & Hamilton 2001; White, Scott, & Pierpaoli 2000). Our work is unique in allowing the primordial power spectrum to be an arbitrary function, thus allowing the possibility for detecting features in the primordial power spectrum.

Our results seem to indicate that the current observational data do not rule out unusual physics (such as multiple-stage inflation) in the very early universe. The upcoming data from the CMB satellite missions *MAP* (Bennett et al. 1997) and *Planck* (De Zotti et al. 1999) and the large-scale structure data from 2df (Dalton et al. 2000) and SDSS (Gunn & Weinberg 1995) should allow for a more definitive measurement of the primordial power spectrum (Wang et al. 1999). These data will more precisely constrain the possibility for such complex physics in the very early universe.

We acknowledge the use of CMBFAST (Seljak & Zaldarriaga 1996) in computing the theoretical models. It is a pleasure for us to thank the referee for helpful suggestions. Work was supported in part by NSF CAREER grant AST 00-94335 at the University of Oklahoma and DOE grant DE-FG02-95ER40934 at the University of Notre Dame.

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