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MODELS OF METAL-POOR STARS WITH GRAVITATIONAL SETTLING AND RADIATIVE ACCELERATIONS. II. THE AGE OF THE OLDEST STARS

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ABSTRACT

Isochrones for ages between 12 and 18 Gyr have been derived from the evolutionary tracks presented in Paper I (Richard et al.) for masses from 0.5 to 1.0 $M_\odot$ and initial chemical abundances corresponding to (1) $Y = 0.2352$, $Z = 1.69 \times 10^{-4}$ ([Fe/H] $= -2.31$, [α/Fe] $= 0.3$) and (2) $Y = 0.2370$, $Z = 1.69 \times 10^{-3}$ ([Fe/H $= -1.31$, [α/Fe] $= 0.3$). These are the first models for Population II stars in which *both* gravitational settling and radiative accelerations have been taken into account. Allowance for these diffusive processes leads to a 10%–12% reduction in age at a given turnoff luminosity. However, in order for the diffusive models to satisfy the constraints from Li and Fe abundance data (see Paper I) and to reproduce the observed morphologies of globular cluster (GC) color-magnitude diagrams (CMDs) in a straightforward way, extra mixing just below the boundary of the convective envelope seems to be necessary. Indeed, when additional turbulent mixing is invoked, the resultant models are able to satisfy all of these constraints, as well as those provided by the CMDs of local subdwarfs, rather well. Moreover, they imply an age near 13.5 Gyr for M92, which is one of the most metal-deficient (and presumably one of the oldest) of the Galaxy's GCs, if the field subgiant HD 140283 is used to derive the cluster distance. Comparisons of field subdwarfs and subgiants with a recently published fiducial for M5 suggests that the cluster has [Fe/H] $\lesssim -1.4$, in conflict with some estimates based on high-resolution spectroscopy, if the metallicities of the field stars are to be trusted. In addition, an age of $\approx$11.5 Gyr is found for M5, irrespective of whether diffusive or nondiffusive isochrones are employed in the analysis. The implications of our results for the extragalactic distance scale and for the Hubble constant are briefly discussed in the context of the presently favored $\Omega_M \approx 0.35$, $\Omega_\Lambda \approx 0.65$ cosmological model.

Subject headings: convection — diffusion — globular clusters: general — stars: abundances — stars: evolution — stars: Population II

1. INTRODUCTION

Helioseismic studies (e.g., Christensen-Dalsgaard, Proffitt, & Thompson 1993; Bahcall et al. 1997; Guenther & Demarque 1997) have indicated a clear preference for solar models that take into account the gravitational settling of helium and heavier elements over those that neglect this physical process. In fact, radiative accelerations must also be treated; according to Turcotte et al. (1998), "radiative accelerations can be up to 40% of gravity below the solar convection zone and thus affect chemical element diffusion significantly." Furthermore, slow mixing just below the fully convective layers is apparently needed to explain the observed Li and Be abundances in the Sun (e.g., see Richard et al. 1996) and to further improve the agreement between the predicted sound speed profile and that inferred from solar oscillation data (Brun, Turck-Chièze, & Zahn 1999). As expected, all of these processes have important ramifications for the initial Y and Z values, as well as the value of the usual mixing-length parameter ($\alpha_{\rm MLT}$), that must be adopted in order to obtain a so-called standard solar model (see Richard et al. 2002, hereafter Paper I).

Models for Population II stars should be no less sophisticated; and indeed, it seems to be the case that the effects of settling (and radiative accelerations) on the surface chemistry of these objects are also moderated by some process. This is indicated by the near-constancy of the ^{7}Li abundance in field halo dwarfs with $T_{\rm eff} \gtrsim 5800$ K (the so-called Spite plateau; Spite & Spite 1982; also see Ryan, Norris, & Beers 1999), which is contrary to the predictions of diffusive models unless, for instance, enhanced mass loss rates (e.g., Swenson 1995) or extra mixing just below the convective envelope boundary (as defined by the Schwarzschild criterion) are assumed (see Paper I).

The close similarity of the iron abundances in near-turnoff, subgiant, and lower red giant branch (RGB) stars in NGC 6397 (Gratton et al. 2001; also see Castilho et al. 2000), NGC 6752 (Gratton et al. 2001), M71 (Ramirez et al. 2001), and M92 (see King et al. 1998) is also problematic for diffusive computations (see, e.g., Salaris, Groenewegen, & Weiss 2000; Paper I). In fact, of these four globular clusters (GCs), which are the only ones for which spectroscopically determined abundances have been obtained for near-turnoff stars to date, only M92 appears to show some variation of [Fe/H] with evolutionary state. King et al. have found that stars just past the turnoff in this cluster have lower [Fe/H] values, by $\approx$0.25 dex, than those determined by Sneden et al. (1991) for the brightest giants. While this result may be

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affected by systematic errors arising from the different analysis procedures used by the two groups, it is also possible that chemical abundance variations between the turnoff and giant branch become appreciable in only the lowest metallicity clusters; the lower the metal abundance, the thinner the convective envelopes of turnoff stars of a given age, and the more readily are their surface abundances expected to be modified by diffusive processes. (On the Zinn & West 1984 scale, M92, NGC 6397, NGC 6752, and M71 have [Fe/H] $= -2.24, -1.91, -1.54$, and -0.58, respectively.)

The M92 result clearly needs to be checked, especially as Gratton et al. (2001) find no difference in the iron abundances of turnoff and lower RGB stars in NGC 6397, which is only 0.3–0.4 dex more metal-rich than M92.[2] However, even if present indications of a small variation in [Fe/H] between turnoff and lower RGB stars in M92 are confirmed, they are inconsistent with the predictions of models that consider gravitational settling and radiative accelerations. As shown in Paper I, such computations imply that, for the metallicity of M92, the variation in [Fe/H] between the turnoff and lower RGB would be $\approx$0.4 dex at an age of 13.5 Gyr (with larger variations at younger ages). In fact, because of the growing importance of radiative accelerations at temperatures above $\approx$6000 K, some of the near-turnoff stars in M92 (i.e., stars with temperatures within 100–200 K of the turnoff) should have nearly the same [Fe/H] as lower RGB stars (see Paper I, § 4.2 and Fig. 14). A forthcoming study (O. Richard et al. 2002, in preparation [Paper III]) also shows that the variation of [Fe/H] in the vicinity of the turnoff depends strongly on the metallicity of the cluster. Be that as it may, diffusive models are able to satisfy current Li and Fe abundance constraints quite well if they assume a level of additional mixing below the convective envelope that is comparable to what is needed in AmFm stars (see Paper I; Richer, Michaud, & Turcotte 2000). (Interestingly, more extensive mixing seems to be needed to explain the solar Li abundance, although further work is required to establish whether or not the amount of extra mixing varies in a simple way with effective temperature or some other property of stars.)

The purpose of this investigation is to determine how well the models presented in Paper I are able to reproduce the color-magnitude diagrams (CMDs) of the most metal-deficient globular clusters and to evaluate their ages. The main focus of this study is M92, which is widely thought to be among the oldest of the Galaxy's globular clusters. However, M5 is also considered, mainly to add to the ongoing debate concerning the GC distance and [Fe/H] scales.

2. THE EVOLUTIONARY TRACKS AND ISOCHRONES

The stellar models were computed as described in Paper I (also see Turcotte et al. 1998). It suffices here to note that Rosseland opacities and radiative accelerations (see Richer et al. 1998 and references therein) were calculated self-consistently with the effects of the transport processes (namely, gravitational settling, thermal diffusion, radiative acceleration, and turbulent mixing). This has been made possible by the recent availability of the large OPAL database (Iglesias & Rogers 1996), which includes the atomic data needed to compute radiative accelerations and opacities throughout stellar interior models (also see Seaton 1993). The latter were included in the particle transport equations, leading to 56 coupled, nonlinear partial differential equations (see Burgers 1969) that were solved by standard methods.

The treatment of turbulence that has been employed is very similar to that described by Proffitt & Michaud (1991); by adjusting the free parameters in the adopted form of the turbulent diffusion coefficient, the variation of [Fe/H] along an isochrone can be limited to less than 0.1 dex, as required by observations, and a satisfactory match to the Li abundance data mentioned above can be obtained (see Paper I). It is also worth noting that the values of $\alpha_{\rm MLT}$ that have been assumed were determined from appropriate standard solar models. Finally, two choices for the initial chemical abundances have been considered: $Y = 0.2352$, $Z = 1.69 \times 10^{-4}$ and $Y = 0.2370$, $Z = 1.69 \times 10^{-3}$. These choices, which correspond to [Fe/H] $= -2.31$ and -1.31, respectively, with $[\alpha/\mathrm{Fe}] = 0.3$ in both instances, are identical with two of the cases for which grids of nondiffusive models were computed by VandenBerg et al. (2000). Reference may be made to the latter study for some justification of the selected Y and $[\alpha/\mathrm{Fe}]$ values.

It is always useful to intercompare the results from different stellar evolution programs to give one added confidence in the results that are reported, and Figure 1 shows that evolutionary tracks constructed using the Université de Montréal and the University of Victoria codes are in fine agreement when very similar (but not identical) physics is assumed. The small shift in luminosity at any age from the zero-age main sequence onward can likely be attributed to minor differences in the equation of state and/or the more sophisticated treatment of the opacity in the Montréal code (which evaluates, for instance, the changes in opacity that accompany the adjustments in the C, N, and O abundances owing to the operation of the CNO cycle). However, the small difference in the predicted mass-luminosity relation translates to a difference in the age of the test model at the base of the RGB amounting to only 1.2%. (Similar good agreement has been found between University of Victoria models and those computed by A.V. Sweigart; see VandenBerg et al. 2000.) Even the predicted effective temperatures along the tracks differ only slightly when the same atmospheric T-τ relation (Krishna Swamy 1966) and low-temperature opacities (see the VandenBerg et al. study) are adopted in the two codes. Given that the VandenBerg et al. models have been shown to satisfy a number of $T_{\rm eff}$ and color constraints (also see Bergbusch & VandenBerg 2001), the Montréal calculations will obviously fare comparably well in this regard.

Isochrones have been derived from the grids of evolutionary tracks presented in Paper I using the interpolation code described by Bergbusch & VandenBerg (1992). Examples of these isochrones for the nondiffusive and diffusive cases, where the latter include the effects of gravitational settling and radiative accelerations (but not turbulent mixing), are intercompared in the left panel of Figure 2. This plot is qualitatively very similar to those presented by, e.g., Stringfellow

[2] Although homogeneous data were obtained by Gratton et al., and internally consistent abundance analyses were performed, it is still possible that their results for stars in different evolutionary phases are affected by small systematic errors. Bergbusch & VandenBerg (2001) have argued, for instance, that there is an inconsistency in the metallicities derived by R. G. Gratton and his colleagues for bright GC giants, on the one hand, and local field subdwarfs, on the other, despite the use of similar reduction techniques in deriving them. This possibility is given added support in § 3.1 of this paper.

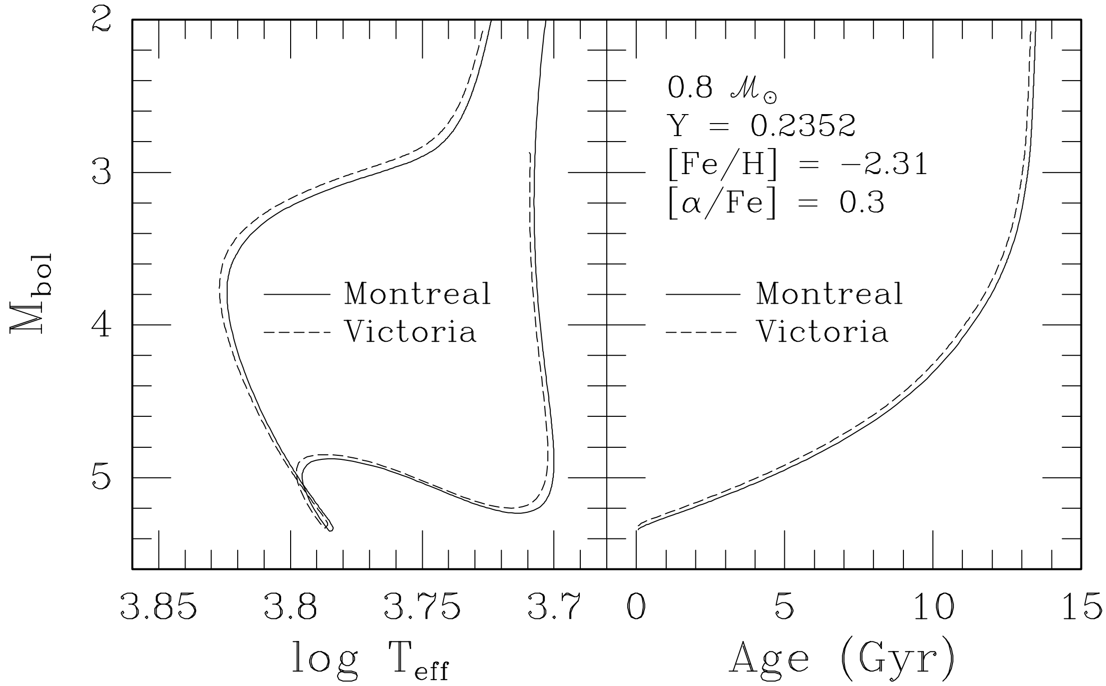


FIG. 1.—Comparison of evolutionary sequences computed using the Université de Montréal and University of Victoria codes for a star having the indicated parameters. As much as is practically possible, the same physics has been assumed. The small luminosity and temperature offsets between the two calculations (amounting to $\delta M_{\rm bol} \approx 0.025$ mag and $\delta \log T_{\rm eff} \approx 0.0025$, respectively) are probably due to minor differences in the respective opacity interpolation procedures and/or equation-of-state formulations.

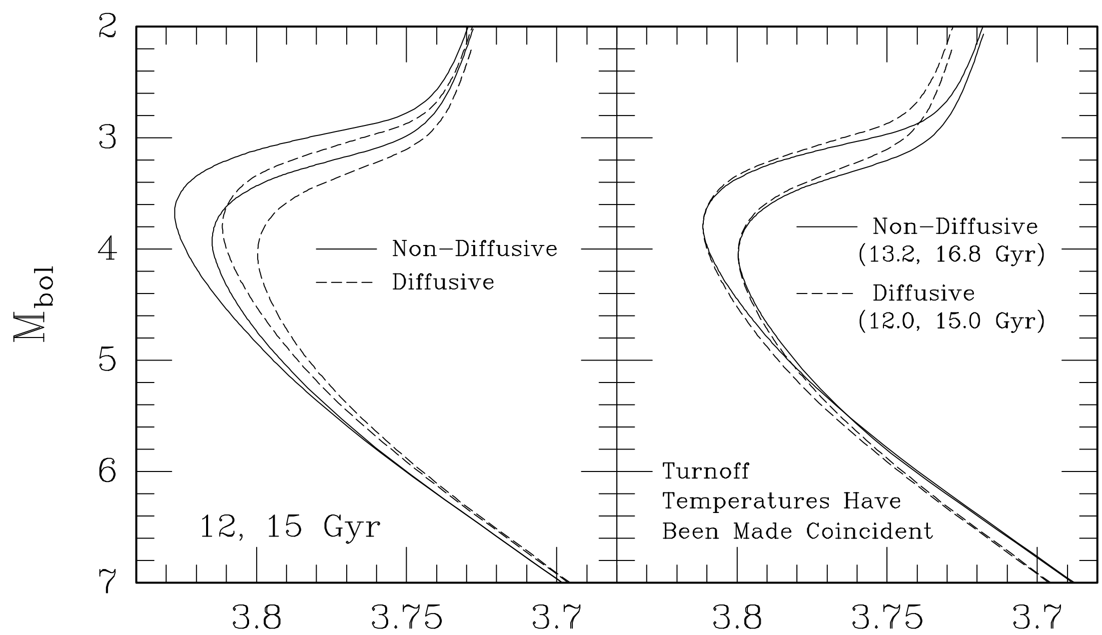


FIG. 2.—Comparison of nondiffusive and diffusive isochrones that have been derived from the corresponding grids of evolutionary tracks presented in Paper I (see Table 2 therein). These computations assume the Eddington T-τ relation for the structure of the atmospheric layers and initial chemical abundances corresponding to $[\mathrm{Fe/H}] = -2.31$ and $[\alpha/\mathrm{Fe}] = 0.3$. The left panel plots 12 and 15 Gyr isochrones with and without diffusion, whereas the right panel compares isochrones from the two grids that have essentially identical morphologies in the vicinity of the turnoff and, hence, the same turnoff magnitudes. To obtain the superposition of the turnoffs shown in the latter plot, the predicted temperatures along the 13.2 and 16.8 Gyr nondiffusive isochrones were adjusted in the horizontal direction by $\delta \log T_{\rm eff} = -0.0117$ and -0.0127, respectively.

et al. (1983) and Proffitt & VandenBerg (1991), who considered only the effects of helium settling on the evolution of Population II stars. At a fixed age, the additional physics causes a reduction in the turnoff temperature by $\delta \log T_{\rm eff} \approx 0.015$, as well as a $\sim$0.05 mag decrease in the turnoff luminosity ($L_{\rm TO}$).

The estimated change in $L_{\rm TO}$ is quite uncertain because of the great difficulty in determining precisely where the turnoff point is located. By definition, an isochrone is vertical at its turnoff point, and given that it remains nearly vertical for at least $\pm$0.15 mag on either side of the turnoff, $L_{\rm TO}$ is an ill-defined quantity. Indeed, using just the turnoff magnitude to gauge the effect of diffusion on ages is probably not the best approach. In our opinion, a more reliable way to evaluate the reduction in age caused by diffusion is to determine which of the nondiffusive isochrones provides the best match to a diffusive isochrone of a given age (or vice versa) in the near vicinity of the turnoff (i.e., within $\delta \log T_{\rm eff} \approx 0.02$), when both are shifted to the same turnoff temperature.

The results of such an exercise are illustrated in the right panel of Figure 2. This shows that the turnoff segments of diffusive isochrones for 12.0 and 15.0 Gyr are well matched by those of their nondiffusive counterparts for ages of 13.2 and 16.8 Gyr, respectively. (Note that the deviations between the solid and dashed curves are nearly symmetric about their respective turnoffs; this would not be obtained if the ages of the nondiffusive isochrones were varied by $\gtrsim\pm$0.2 Gyr.) We conclude that the predicted age at a given $L_{\rm TO}$ is reduced by $\approx$10% when diffusive processes are treated. (The size of the effect would be slightly larger on a color-M_V plane because the cooler turnoffs of the diffusive isochrones would imply increased bolometric corrections in a relative sense by 0.01–0.015 mag.) Apparently, the inclusion of radiative accelerations in our models does not have a large impact on turnoff ages as these results are fairly close to those obtained when only the settling of helium is considered (e.g., see Proffitt & VandenBerg 1991).

However, as discussed in § 1, the surface abundances of Li in field halo dwarfs (as well as in GC turnoff stars; see Molaro & Pasquini 1994) and the observed variation of [Fe/H] between the main sequence and lower RGB in globular clusters are contrary to the predictions of purely diffusive stellar models. One way to accommodate these observations, as described in Paper I, is to allow for turbulent mixing just below the convective envelope boundary. Figure 3 shows that the main effect of such mixing on computed isochrones is to shift their turnoffs to somewhat hotter temperatures (by $\delta \log T_{\rm eff} \lesssim 0.005$) without altering turnoff luminosities significantly. (Note that the "T" parameter distinguishing the two turbulence cases that have been plotted gives the $\log T$ value at which the turbulent diffusion coefficient has been normalized; reference should be made to Paper I for a detailed description of the assumed physics. Paper I also demonstrates that "T6.09" models are able to satisfy the chemical abundance constraints mentioned above quite well. Insofar as the isochrones are concerned, "T6.09" loci will clearly be barely distinguishable from those plotted in Fig. 3.)

3. THE SUBDWARF DISTANCE SCALE

Bergbusch & VandenBerg (2001) have shown that their nondiffusive isochrones agree very well with the locations of the best-observed local subdwarf standards ($\sigma_\pi/\pi < 0.07$, from *Hipparcos*) on both the ($\log T_{\rm eff}$, M_V) and $[(B-V)_0$,

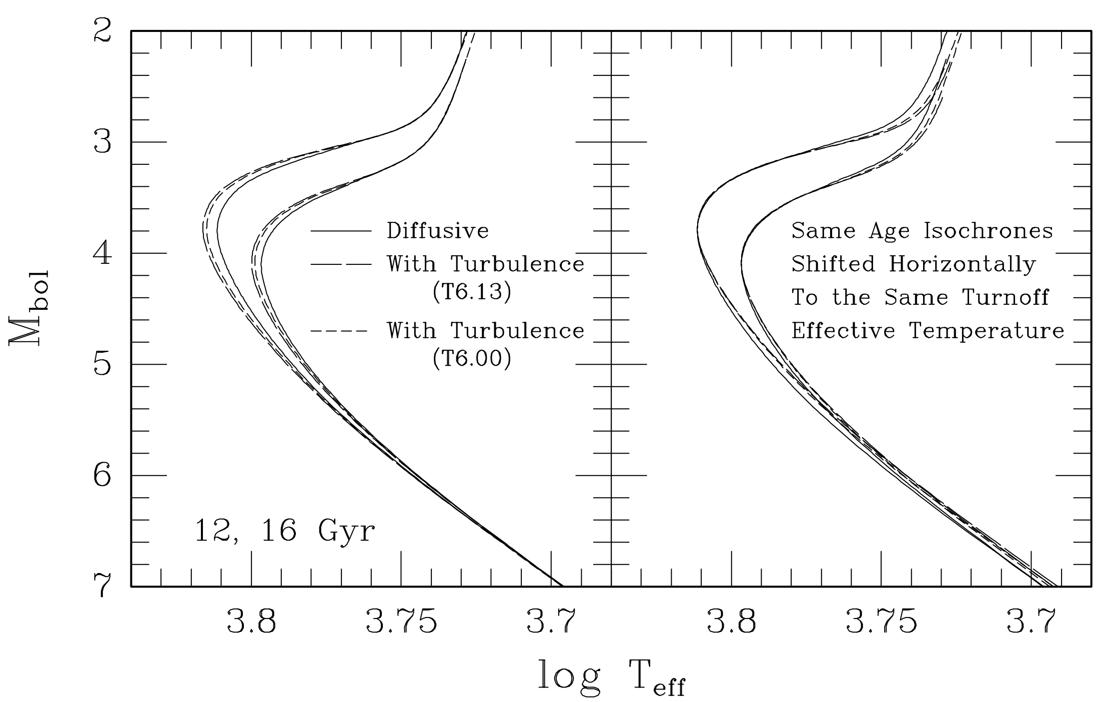


FIG. 3.—Left panel compares diffusive isochrones for the indicated ages that differ only in the amount of turbulent mixing that is assumed. They have been derived from the "diffusive" (without turbulence), "T6.0," and "T6.13" grids of evolutionary tracks presented in Paper I (see Table 2 therein). (The number in the names of the two turbulence cases gives the normalization $\log T$ value of the assumed turbulent diffusion coefficient; see Paper I for a detailed description.) The right panel plots the same isochrones but shifted to the same turnoff $T_{\rm eff}$ to show that the assumed turbulence does not affect predicted age vs. turnoff luminosity relations.

M_V] planes if their temperatures, Fe/H ratios, and intrinsic colors are as given by R. G. Gratton and colleagues (Gratton, Carretta, & Castelli 1996; Clementini et al. 1999; Carretta et al. 2000). To be more specific, they constructed a "monometallicity" subdwarf sequence consisting of nine stars fainter than $M_V = 5$ whose temperatures and $(B-V)_0$ colors were adjusted, at their observed M_V values (using isochrones only in a differential sense), to the values they would have if they all had [Fe/H] $= -2.31$. This reference metallicity was chosen because Bergbusch & VandenBerg also included three very metal-deficient subgiants in their sample; by choosing to compare the resultant subdwarf locus with [Fe/H] $= -2.31$ isochrones, the corrections that were applied to their intrinsic properties were minimized. The monometallicity subdwarf sequence obtained in this way was very tight, its position and morphology agreed well with theoretical predictions, and an age near 16 Gyr was inferred for the few subgiants in the sample.

However, most of the unevolved subdwarf calibrators used by Bergbusch & VandenBerg (2001) have [Fe/H] values between -1.2 and -1.5. These stars should provide especially good constraints on isochrones having metallicities within this range as no more than slight shifts to their observed properties would be needed to create a monometallicity sequence. In order to be able to conduct this test, grids of evolutionary tracks (see Paper I) and isochrones (this study) were computed for initial chemical compositions corresponding to [Fe/H] $= -1.31$, [α/Fe] $= 0.3$. The names and properties of the six subdwarfs that we have chosen to consider, along with the sources of the data, are given in Table 1. These stars are, in fact, the only ones in the sample of $\sim$60 Population II stars considered by Carretta et al. (2000) that have $M_V > 5$, $\sigma(M_V) \leq 0.15$, and $-1.5 <$ [Fe/H] < -1.2. Moreover, they include many of the same subdwarfs that have been used over the past 2–3 decades to determine the distances to GCs.

The $\delta(B-V)$ and $\delta T_{\rm eff}$ offsets that must be added to the subdwarf colors and temperatures to create a monometallicity sequence were determined from the Bergbusch & VandenBerg (2001) isochrones, which are provided for a fine spacing in [Fe/H]. It is a straightforward matter to interpolate in a suitable subset of them for a fixed age and the appropriate range in [Fe/H] to determine the *difference* in temperature and color, at the observed M_V value of a subdwarf, corresponding to the difference between its [Fe/H] and that metallicity for which a monometallicity sequence is to be constructed.[3] In fact, such interpolations have been carried out for two reference [Fe/H] values, specifically, -1.31 and -1.45. The first of these is appropriate for a comparison of the subdwarf locus with nondiffusive isochrones for [Fe/H] -1.31, since these models predict no variation in the surface heavy-element abundances as a function of evolutionary state. The second represents the mean [Fe/H] value along our "diffusion+turbulence" isochrones over the magnitude range encompassed by the subdwarfs; this mean value differs from the predicted iron abundances at the luminosities of the subdwarfs by $\lesssim \pm 0.03$ dex (see below), which is very small (certainly much smaller than the uncertainties in the subdwarf metallicities themselves). As a consequence, a monometallicity subdwarf sequence for [Fe/H] $= -1.45$ will not differ significantly from a subdwarf locus that is obtained by adjusting the properties of each star to reflect the precise difference between its metallicity and that of the isochrones at the star's luminosity (which is, in principle, what should be done). Note that the net effect of the additional physics is to cause a slow decrease in the surface [Fe/H] value with time and to limit the extent to which the gravitational settling of the metals depends on stellar mass, which explains why the variation of [Fe/H] with magnitude along an isochrone is quite small (see below).

As indicated in Table 1, the $\delta T_{\rm eff}$ offsets that have been determined in this way are all quite small, especially considering that the subdwarf temperatures themselves are probably uncertain by $\sim$100–150 K. The derived $\delta(B-V)$ offsets are also generally less than the uncertainties in the observed colors, which have been taken to be ± 0.02 mag (see Carretta et al. 2000). Hence, such small adjustments will be of little consequence, although we believe that they should be applied to the subdwarf data in order to obtain the best possible comparison between them and our isochrones in an absolute sense. The first such comparison is shown in Figure 4,

[3] Our calculations indicate, not unexpectedly, that these differences in $T_{\rm eff}$ and color are nearly independent of whether or not diffusive processes are treated. Moreover, they do not depend significantly on the assumed age of the isochrones that are used because the subdwarfs under consideration are so faint; at their M_V values, isochrones for a wide range in age are nearly coincident on both the ($\log T_{\rm eff}$, M_V) and [$(B-V)_0$, M_V] planes. This is illustrated in the two figures that are about to be discussed.

TABLE 1

SELECTED POPULATION II SUBDWARFS HAVING $-1.5 \leq$ [FE/H] ≤ -1.2

Name (1)	[Fe/H][a] (2)	M_V[a] (3)	$(B-V)_0$[a] (4)	$\delta(B-V)$ -1.31 (5)	$\delta(B-V)$ -1.45 (6)	$T_{\rm eff}$[b,c] (7)	$\delta T_{\rm eff}$ -1.31 (8)	$\delta T_{\rm eff}$ -1.45 (9)	$T_{\rm eff}$[d] (10)	$T_{\rm eff}$[e] (11)
HD 34328	−1.44	5.21 ± 0.15	0.478	+0.015	−0.001	5986	−44	+3	5725	...
HD 64090	−1.48	6.01 ± 0.06	0.614	+0.022	+0.003	5515	−50	−8	5300	5441
HD 103095........	−1.24	6.61 ± 0.02	0.752	−0.008	−0.023	5124	+22	+59	4950	5029
HD 134439........	−1.30	6.70 ± 0.08	0.767	−0.001	−0.016	5106	+3	+40	4850	4974
HD 134440........	−1.28	7.03 ± 0.11	0.845	−0.004	−0.017	4879	+9	+42	4750	4746
HD 188510........	−1.37	5.83 ± 0.10[f]	0.598	+0.008	−0.010	5628	−19	+23	5325	5564

[a] Carretta et al. 2000.
[b] Gratton et al. 1996.
[c] Clementini et al. 1999.
[d] Fulbright 2000.
[e] Alonso et al. 1996.
[f] Bergbusch & VandenBerg 2001; see footnote 7.

which reveals that there is very good agreement between the location and slope of the monometallicity subdwarf sequence for [Fe/H] = −1.31 and that of the nondiffusive isochrones for the same metallicity. (Note that the synthetic and observed CMDs have simply been overlaid on one another; no arbitrary shifts of any kind have been applied to the models.) If anything, one could argue that the predicted effective temperatures are slightly too hot and that the synthetic colors are just a little too blue (by about 0.015 dex).

In fact, were we to adopt the temperatures derived by Fulbright (2000) for the same subdwarfs, there would be a very substantial discrepancy between the isochrones and the subdwarfs on the observed plane. This is readily apparent by comparing the numbers in column (10) of Table 1, which gives Fulbright's $T_{\rm eff}$ estimates, and those listed in column (7), which contains the temperatures that we have adopted. The former are cooler than the latter by 223 K, on average, with the differences ranging from 129 K to 303 K. The main problem with such cool temperatures is that, if transformed to $B-V$ indices using the color transformations that we have employed (which are very similar to those presented by VandenBerg 1992), the isochrones would be considerably redder than the subdwarfs. (Obviously, Fig. 4 shows good consistency between the theoretical and observed planes, thereby offering strong support for the color-$T_{\rm eff}$ relations that we have used—*if* the hotter subdwarf temperature scale is the correct one.)

Nor is this problem easily avoided, as most alternative tables of synthetic colors predict redder $B-V$ indices than those used in this study (especially at cool temperatures). For instance, Bessell, Castelli, & Plez (1998) colors are redder by ≈0.04–0.05 mag at 5000 K (see the comparison given by VandenBerg et al. 2000, Table 8); using them would cause a mismatch between the predicted and observed colors of the subdwarfs by ∼0.04 mag if the $T_{\rm eff}$ estimates in column (7) of Table 1 were adopted, or by up to 0.1 mag if the Fulbright (2000) temperatures were assumed. However, the latter are really "excitation" temperatures, and they may not be good indicators of $T_{\rm eff}$. (The [Fe/H] values derived by Fulbright tend to be about 0.15 dex more metal-poor, on average, than the values listed in Table 1. Presumably this difference would be reduced if he adopted a hotter $T_{\rm eff}$ scale.)

Somewhat more worrisome is the fact that the temperatures derived for the subdwarfs by Alonso, Arribas, & Martinez-Roger (1996), using the highly regarded infrared flux method, tend to be cooler than the ones that we have adopted by about 100 K (compare cols. [11] and [7] in Table 1). Still, the same concern about color-$T_{\rm eff}$ relations applies, the estimated uncertainty of the Alonso et al. temperatures is ≈100 K, and there are other highly respected techniques, such as the fitting of Balmer line profiles (e.g., Fuhrmann, Axer, & Gehren 1994) that yield relatively warm temperatures. (The reader is referred to the paper by Gratton et al. 1996, who use the results of this and other methods to support their "new $T_{\rm eff}$ scale for the subdwarfs.") Undoubtedly, this issue will remain controversial for some time to come, but it seems to us that the Gratton et al. scale is at least as good as, and possibly better than, any other that has been proposed to date.[4]

[4] We note that when the boundary pressures for the stellar interior models are derived from model atmospheres for metal-poor stars, the resultant temperatures are higher than those obtained when the hydrostatic equation is integrated in conjunction with the Krishna Swamy (1966) T-τ relation (see VandenBerg et al. 2000). Thus, an even hotter $T_{\rm eff}$ scale than we have adopted is not out of the question. In this case, redder color transformations (perhaps close to those given by Bessell et al. 1998) would be required to achieve consistency between the theoretical and observed planes.

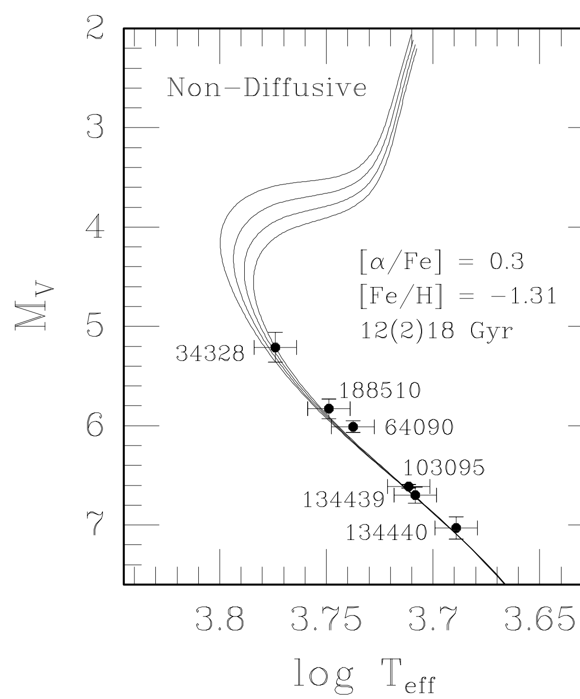


FIG. 4.—Comparison of nondiffusive isochrones with the [Fe/H] = −1.31 monometallicity subdwarf sequence consisting of the six stars whose HD numbers are given in the left panel. The uncertainties in M_V (see Table 1) correspond to the uncertainties in the parallax data (from *Hipparcos*); Lutz-Kelker corrections have been applied to the absolute magnitudes (see Carretta et al. 2000). The temperature uncertainty is (somewhat arbitrarily) taken to be $\sigma(\log T_{\rm eff}) = 0.01$, and the error bars on the $(B-V)_0$ values are from Carretta et al. (2000). To better reproduce the properties of the local calibrators in the right panel, the isochrones should be shifted redward by approximately 0.015 mag.

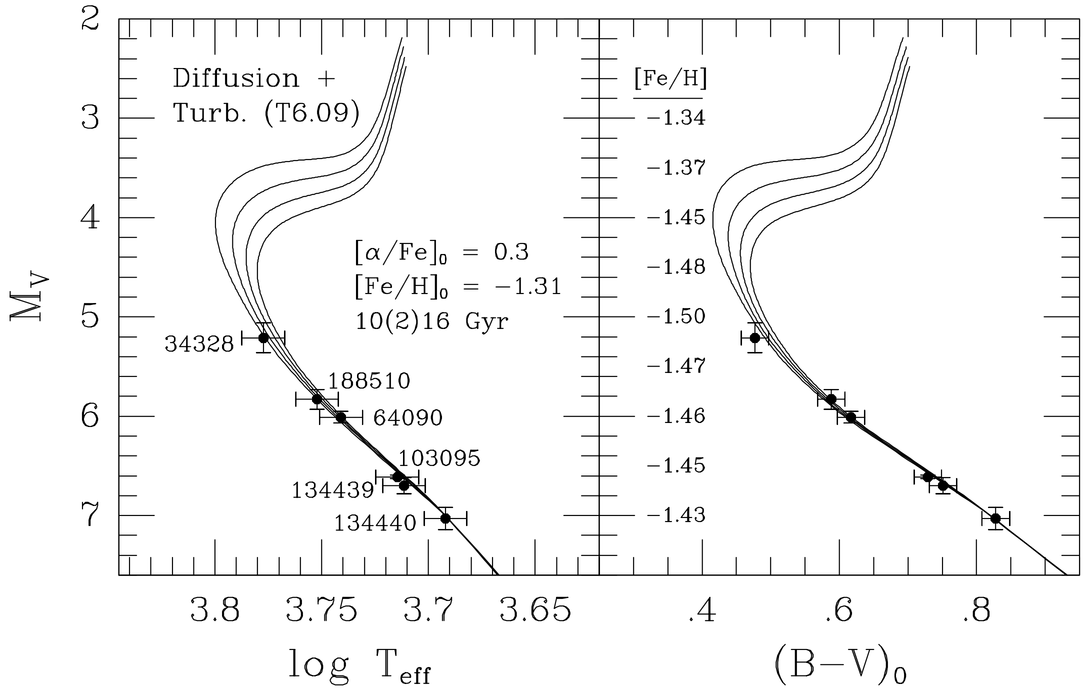


FIG. 5.—Same as Fig. 4, except that "T6.09" isochrones (see Paper I, Table 2) are compared with a subdwarf sequence for [Fe/H] $= -1.45$, which is the appropriate comparison in this case. The predicted variation of [Fe/H] with M_V along these isochrones is approximately as given adjacent to the ordinate of the right panel (see the text). Note that, in contrast to Fig. 4, the isochrones are able to reproduce the properties of the subdwarf calibrators on the CMD without any adjustment of the synthetic colors.

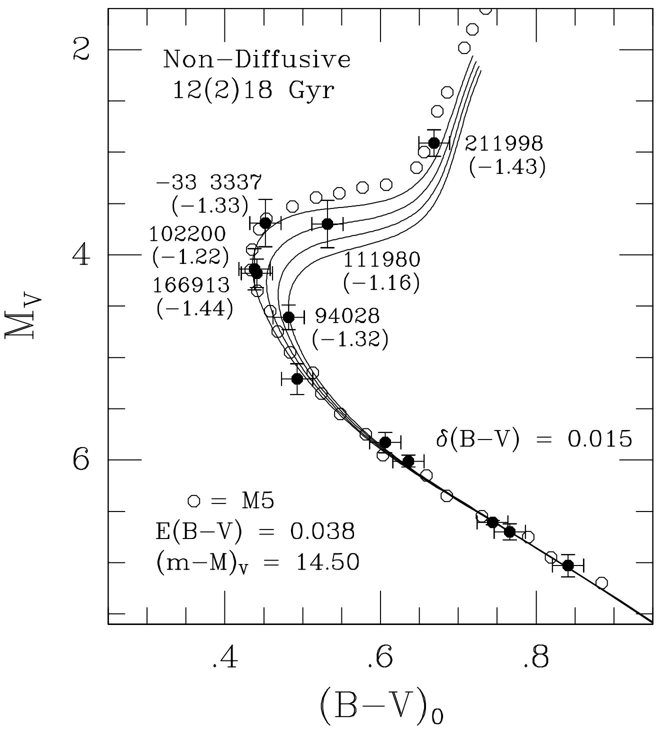
Figure 5 shows that isochrones that allow for settling, radiative accelerations, and turbulent mixing are able to match the location and slope of the subdwarf sequence just as well as the nondiffusive models. In this case, if anything the isochrones appear to be slightly too cool, but the discrepancies are clearly very much smaller than the uncertainties associated with the temperatures of the subdwarfs. The comparison between theory and observation in the right panel can hardly be improved upon (given the small sample of stars). Note that, although the stellar models had [Fe/H] $= -1.31$ throughout their interiors (including the surface layers) at a time $t = 0$, the chemical transport processes have conspired to give the variation with M_V in the surface iron abundance at $t = 14$ Gyr that is indicated along the ordinate of the right panel. Such variations, which were taken into account in the transformation of the isochrones to the $[(B-V)_0, M-V]$ plane, are predicted to be slightly larger or smaller by a few hundredths of a dex for younger or older ages, respectively. (Isochrones that neglect turbulent mixing altogether predict a 1 dex variation in [Fe/H] over the same range in magnitude.) The main conclusion to be drawn from Figures 4 and 5 is that the isochrones satisfy the subdwarf constraint quite well.

Of course, the close correspondence of *both* sets of models with the subdwarfs arises because (1) the predicted location of the lower main sequence is not greatly affected by diffusive processes and (2) the isochrones must be compared with monometallicity subdwarf sequences for slightly different [Fe/H] values (for the reasons discussed above). To be able to distinguish between the two grids, it is necessary to extend the subdwarf locus to just past the turnoff or to consider the CMD of a globular cluster having [Fe/H] ≈ -1.3, since it is in the vicinity of the turnoff that chemical transport processes have their largest effects (recall Fig. 2). In fact, given the substantial uncertainties in GC metallicities (see Rutledge, Hesser, & Stetson 1997), a comparison of field and cluster turnoff stars may well be crucial to the analysis. This is illustrated below using M5 as an instructive example.

3.1. *Evolved* [Fe/H] ~ -1.3 *Field Stars and the M5 CMD*

Figures 6 and 7 compare the CMD locations of several somewhat evolved field halo stars (with *Hipparcos* parallaxes) and the fiducial sequence for M5 (from Sandquist et al. 1996) with those of our nondiffusive and diffusive isochrones, respectively.[5] Note that the nondiffusive isochrones have been shifted to the right by $\delta(B-V) = 0.015$ mag in order to match the locations of the calibrating subdwarfs (those fainter than $M_V = 5$; recall Fig. 4); no such adjustment to the colors of the models was required to produce Figure 7 (as shown previously in Fig. 5). In each case, the distance modulus of M5 was derived by performing a main-sequence fit of the cluster fiducial to the isochrones/subdwarfs on the assumption of an $E(B-V)$ value based on the Schlegel, Finkbeiner, & Davis (1998) reddening maps.

[5] In the extensive study of GC ages by VandenBerg (2000), the ridge line for M5 was derived from unpublished *BV* observations kindly provided by P. B. Stetson. That ridge line does not agree particularly well with the fiducial sequence reported by Sandquist et al., in that the former has a bluer turnoff and RGB than the latter by 0.03–0.04 mag, with only minor differences along the lower main sequence. Thus, the two loci vary significantly in a systematic sense. We have chosen to use the Sandquist et al. observations, which imply a turnoff color in good agreement with that derived by Richer & Fahlman (1987), mainly because they provide a more favorable comparison with both the local subdwarfs and our isochrones. If the Stetson et al. fiducial sequence is the more correct one, then we suspect that it will be difficult to obtain similar good consistency with our models unless M5 has [Fe/H] < -1.4. This may, however, be the case (see VandenBerg 2000 for some discussion of this possibility).



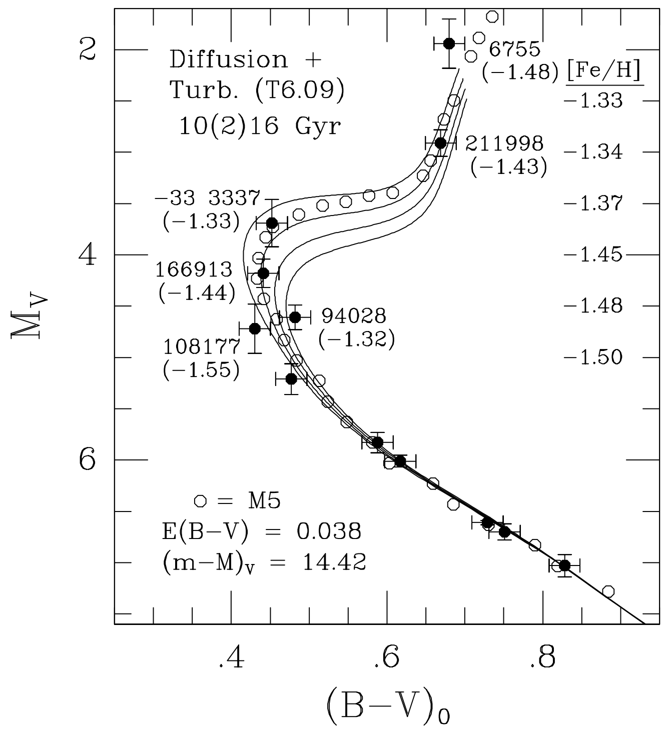


FIG. 6.—Main-sequence fit of the unsmoothed fiducial for M5 (*open circles*) by Sandquist et al. (1996) to the same nondiffusive isochrones that were plotted in Fig. 4, after the latter have been adjusted to the red by 0.015 mag (see Fig. 4) in order to match the monometallicity subdwarf sequence for [Fe/H] $= -1.31$ (*filled circles fainter than* $M_V = 5$). The adopted reddening (Schlegel et al. 1998) and derived distance modulus are noted in the lower left-hand corner. Also plotted are the locations of several somewhat evolved field halo stars, identified by their HD or CD numbers and having [Fe/H] values within ± 0.15 dex of that of the isochrones (as noted). (Their positions have *not* been adjusted to a common metallicity; see § 3.1 for relevant discussion.)

FIG. 7.—Same as Fig. 6, except that the same diffusive isochrones that were plotted in Fig. 5 are overlaid on a monometallicity subdwarf sequence for [Fe/H] $= -1.45$ (*filled circles fainter than* $M_V = 5$), which is the appropriate comparison in this case. The [Fe/H] values along these isochrones, which vary with M_V approximately as given along the right edge of the plot, may be compared with the metallicities of the field stars brighter than $M_V = 5$ to permit an assessment of the level of agreement between theory and observation. (To be consistent with the adopted criterion that the field stars have metal abundances within δ[Fe/H] $= \pm 0.15$ dex of the isochrone [Fe/H] value, which varies with M_V in this case, the sample of field stars plotted here differs somewhat from that considered in the previous figure.)

This implicitly assumes that the globular cluster has exactly the same chemical abundances as the models and that the GC metallicity scale is close to the one given by Zinn & West (1984), who found [Fe/H] $= -1.40$ for M5. Such fits are inappropriate if M5's iron abundance is as high as [Fe/H] $= -1.11$, as determined by Carretta & Gratton (1997) using high-resolution spectroscopy. However, as discussed below, the metallicity that has been derived by Carretta & Gratton, based on a number of the brightest giants, seems to be inconsistent with their [Fe/H] estimates for field subdwarfs and subgiants, judging from the difficulty of reconciling the latter with their counterparts in M5 on the $[(B-V)_0, M_V]$ plane. (For further arguments in support of the Zinn-West [Fe/H] scale over that advocated by Carretta & Gratton, see Bergbusch & VandenBerg 2001.)

The field halo stars identified in Figures 6 and 7 include *all* of the stars given in the $\sigma(\pi)/\pi < 0.12$ sample considered by Carretta et al. (2000) that have $1.5 < M_V < 5.0$ and [Fe/H] values within ± 0.15 dex of the isochrones (at the same M_V, in the case of the nondiffusive models). No color adjustments have been applied to correct for the difference in metallicity between the isochrones and these stars, whose [Fe/H] values are given below their HD or CD numbers. However, one can easily visualize that, in Figure 6, for instance, HD 166913 and HD 211998 would sit on slightly redder isochrones if their [Fe/H] values were increased to -1.31 (assuming similar ages and evolutionary states). In the case of HD 111980, which provides a particularly important constraint on stellar ages by virtue of the fact that it lies approximately midway between the turnoff and lower RGB, a star of the same age and intrinsic $B-V$ color, but with [Fe/H] $= -1.31$, will be slightly brighter.

However, given the small sample of field Population II stars with accurate distances, some apparent difficulties (like HD 102200 and HD 166913 having essentially identical locations on the CMD, despite a 0.22 dex difference in [Fe/H]—an age effect?), and the many uncertainties that play a role in such comparisons with models, one cannot argue compellingly in support of either the diffusive or nondiffusive isochrones. To be sure, the models that treat diffusion and radiative acceleration provide a slightly better match to the unevolved subdwarf calibrators and to the observed difference in color between the turnoff and lower RGB in M5. However, with small modifications to the adopted color-$T_{\rm eff}$ relations and to the treatment of convection (by, e.g., varying $\alpha_{\rm MLT}$), the nondiffusive models could be made to fare equally well.

Interestingly, both sets of models imply similar ages for M5: 11–12 Gyr, judging from the relative location of its turnoff with respect to those of the isochrones. This follows from the fact that main-sequence fits of the observed CMD to the isochrones/subdwarfs yield distance moduli that differ by nearly 0.1 mag in the two cases. This difference arises because nondiffusive models predict no variation in the

[Fe/H] value between the main sequence and lower RGB, whereas a $\gtrsim$0.13 dex variation between these two phases is predicted by the diffusive isochrones. If [Fe/H] $= -1.31$ is a good estimate of the metallicity of M5 giants and if the main-sequence fitting technique is used to derive the cluster distance, then a larger distance modulus will be found if the cluster fiducial is fitted to a monometallicity subdwarf sequence that has been constructed for [Fe/H] $= -1.31$ (as appropriate for nondiffusive models) than to a subdwarf sequence for [Fe/H] ≈ -1.45 (to be consistent with the diffusive isochrones). As far as the metallicity of M5 is concerned, if it were as high as [Fe/H] ≈ -1.1 (Carretta & Gratton 1997), then the inferred distance modulus from a main-sequence fit would have to be increased by $\approx$0.07 mag, per 0.1 dex increase in [Fe/H], according to Bergbusch & VandenBerg (2001) isochrones. In this case, one would have to explain why the lower RGB fiducial of M5 would be approximately centered between the field stars HD 6755 and HD 211998, which have [Fe/H] values within $\pm$0.03 dex of -1.45. This provides yet a further argument (beyond those given by Bergbusch & VandenBerg and VandenBerg 2000) in support of something close to the Zinn & West (1984) metallicity scale for the GCs.

4. THE AGE OF M92

There are no faint [Fe/H] < -2 subdwarfs whose properties are sufficiently well known that they can be used to set *tight* constraints on the main-sequence positions of isochrones for very metal-deficient stars. Most of the other calibrators with $\sigma(\pi)/\pi \lesssim 0.1$ that are often considered (besides those used in the above analysis) are either too metal-rich (like HD 25329 and HD 145417, which have [Fe/H] ≈ -1.65 according to Carretta et al. 2000) or are too evolved (like HD 84937 and HD 140283, which are both post-turnoff stars). However, as shown by VandenBerg et al. (2000), theoretical models are able to reproduce the observed CMD locations of subdwarfs more metal-poor than [Fe/H] ~ -1.3 reasonably well. Indeed, when one forms a monometallicity subdwarf sequence for [Fe/H] $= -2.3$, using all of the best-observed field stars having [Fe/H] < -1.2, the resultant locus shows very encouraging agreement with isochrones for that same metallicity (see Bergbusch & VandenBerg 2001).

Moreover, as emphasized by VandenBerg (1999 and Grundahl et al. 2000), HD 140283 provides an especially important constraint on the ages of field stars with [Fe/H] $\lesssim -2.3$ because of its auspicious location in the H-R diagram (approximately halfway between the turnoff and lower RGB). Given that predicted subgiant branches are not steeply sloped, small errors in the model temperatures or colors will not alter the inferred age of this object by more than $\approx\pm$1 Gyr. Obviously, HD 140283 may also be used to set the distance to a globular cluster that has a similar metallicity (such as M92) if it is assumed that the field subgiant does not differ in any significant way from cluster subgiants at the same $(B-V)_0$. Grundahl et al. found that HD 140283 implies $(m-M)_V \lesssim 14.60$ for M92, as well as an age $\gtrsim$16 Gyr (assuming the nondiffusive isochrones by Bergbusch & VandenBerg 2001).

Since the models used in this study have been shown to reproduce the properties of local subdwarfs having [Fe/H] ≈ -1.3 quite well (i.e., they satisfy this zero point), it is reasonable to expect that our models for other metallicities will be not be seriously in error either (i.e., that they are correct in a differential sense as well). (This supposition is supported by the work of VandenBerg et al. 2000 and Bergbusch & VandenBerg 2001.) In fact, if we use HD 140283 to derive the distance to M92, we can make an assessment of how well the relevant isochrones match the observed CMD and thereby test the accuracy of our models. M92 is obviously a key target cluster because it is one of the most metal-poor, and presumably one of the oldest, GCs in the Galaxy, and the age obtained for it on the basis of models that treat diffusive processes will provide what should be *the best constraint on the age of the universe to date*.

We assume that the properties of HD 140283 are as tabulated by Carretta et al. (2000, their Table 2): specifically, $M_V = 3.32 \pm 0.12$, $(B-V)_0 = 0.463 \pm 0.019$, and [Fe/H] $= -2.40$. They adopted a reddening for this star equal to $E(B-V) = 0.024$ mag, which agrees well with the value that is obtained from Strömgren photometry (Schuster & Nissen 1989), and applied a Lutz-Kelker correction (Lutz & Kelker 1973). For M92, we have chosen to use the fiducial sequence derived by Stetson & Harris (1988) and to assume that $E(B-V) = 0.023$ (Schlegel et al. 1998). As most estimates of the cluster iron abundance (see Sneden et al. 1991; Carretta & Gratton 1997; Zinn & West 1984) are 0.1–0.25 dex larger than that of the field subgiant (as noted above; also see Grundahl et al. 2000), the subgiants in M92 should be intrinsically slightly fainter than HD 140283 if they all have the same age. The models that we have computed for comparison with these data assume initial metal abundances corresponding to [Fe/H] $= -2.31$ and $[\alpha/\mathrm{Fe}] = 0.3$ (see the Sneden et al. study; also see Carney 1996).[6]

As illustrated in Figure 8, the fit of the M92 CMD to HD 140283 implies a distance modulus for the former that is within a few hundredths of a magnitude of $(m-M)_V = 14.60$. Furthermore, it is apparent that the isochrones provide a good match to the morphology of the cluster turnoff, without the need for any adjustment in color of the diffusive models and only a very small color offset (as noted) in the case of the nondiffusive calculations. This offers encouraging support for our isochrones in both an absolute and relative sense. The most noticeable discrepancies between theory and observations occur along the lower main sequence, which could indicate a small problem with, for instance, the observed fiducial, the assumed chemical abundances (of either HD 140283 or M92), or the adopted color-T_{eff} relations. (For instance, a redward adjustment of the predicted colors, or a blueward shift of the cluster fiducial, by as little as 0.02 mag at $M_V \gtrsim 5.5$ would remove most of the apparent difficulties.) It is also evident that neither set of models provides a " perfect " match to the morphology of the post-turnoff stars in M92.

However, such discrepancies are well within the uncertainties of the models. Simply by choosing to use the

[6] It is possible that very metal-deficient stars have high O abundances ([O/Fe] $\gtrsim$1 at [Fe/H] $\lesssim -2$, according to, e.g., Boesgaard et al. 1999, Israelian et al. 2001), but such results have been called into question by Asplund & Garcia Perez (2001), among others. Depending on the further progress that is made toward a resolution of this issue in the coming months, the consequences of such high O/Fe ratios for synthetic CMDs and GC ages may be explored in a follow-up paper. In the meantime, interested readers are encouraged to refer to VandenBerg & Bell (2001) for a recent study of the effects of high oxygen abundances on synthetic CMDs and GC ages.

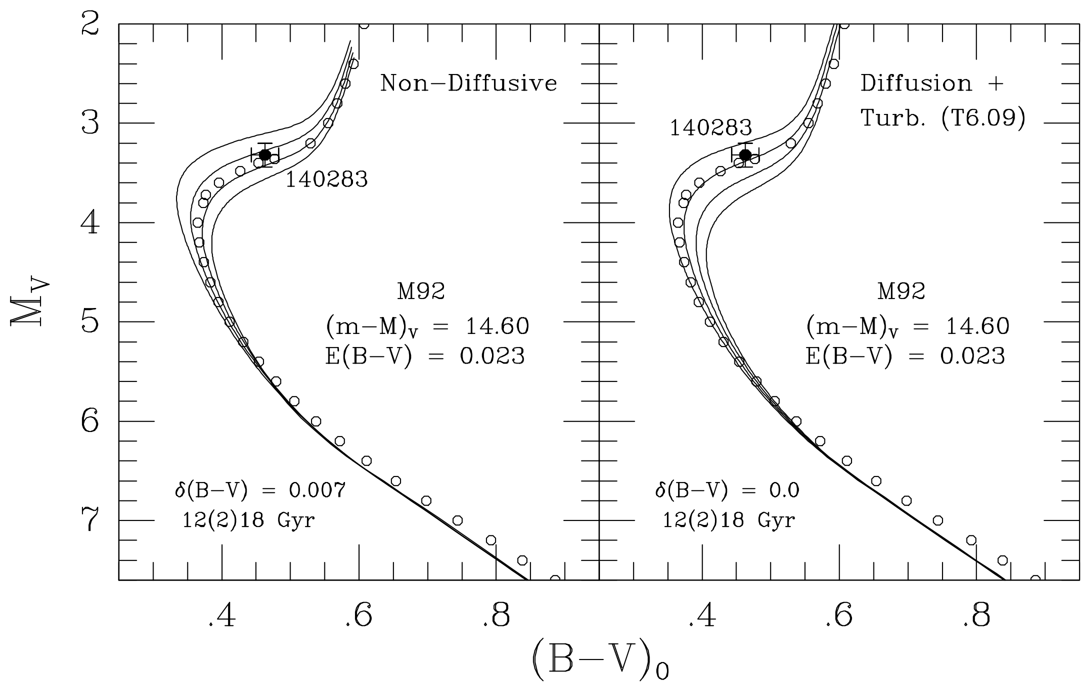


FIG. 8.—Fit of the M92 fiducial (Stetson & Harris 1988) to the field subgiant HD 140283; both have very similar metallicities and reddenings (see § 4). Nondiffusive and diffusive (" T6.09 ") isochrones have been overlaid onto the observed CMDs and slightly adjusted in color, as necessary, in order to determine which isochrone has the same turnoff magnitude as the cluster.

Krishna Swamy (1966) T-τ relation to describe the atmospheric structure (instead of the gray relation), along with the higher value of $\alpha_{\rm MLT}$ that is required by an appropriate standard solar model and a slight upward revision to the assumed cluster distance, a more satisfactory fit to the M92 CMD can be produced. This is shown in Figure 9, which gives the best match of the diffusive isochrones to the M92 fiducial that we have obtained in our limited exploration of parameter space.[7] It is, of course, highly improbable that an empirical solar T-τ relation is appropriate for stars having significantly different metallicities and evolutionary states; presumably, errors in the low-temperature opacities or in the treatment of convection (or in many other factors, including our understanding of the basic properties of observed stars) have conspired in such a way that stellar models based on this treatment of the atmosphere appear to be favored. The main point of these remarks is that there is sufficient enough leeway in the theory that minor deviations between synthetic and observed CMDs should not be taken too seriously.

The main result contained in Figure 9 is that the predicted age of M92 is 13.5 Gyr according to the most advanced models that have been constructed to date for Population II stars (when the cluster distance is set using HD 140283). This estimate is ~2 Gyr less than that which is obtained on the basis of nondiffusive isochrones (see Fig. 8; Grundahl et al. 2000), showing the importance of chemical transport processes. Worthy of emphasis is the fact that sufficient turbulence has been invoked just below the convective envelope to reduce the variation in the surface iron abundance

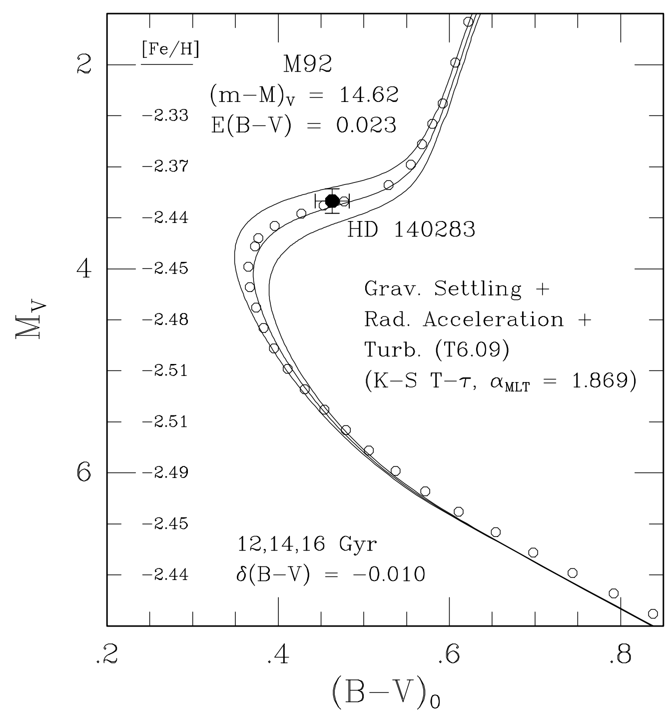


FIG. 9.—Similar to the right panel in Fig. 8, except that isochrones based on the " T6.09KS " tracks from Paper I are compared with the observations (see § 4). The predicted variation in [Fe/H] as a function of M_V along the 14 Gyr isochrone is indicated along the ordinate.

[7] We have not bothered to compute a nondiffusive grid of tracks under the same assumptions, but note that VandenBerg et al. (2000) derived the boundary conditions for their nondiffusive models on the basis of the Krishna-Swamy T-τ relation and a very similar value of $\alpha_{\rm MLT}$. Those calculations are able to reproduce observed CMDs exceedingly well (see VandenBerg 2000), including that of M92 (Grundahl et al. 2000). Moreover, as shown in § 2, evolutionary tracks computed using the University of Victoria and Montréal codes agree very well when similar physics is assumed.

along the diffusive isochrones to below detectable levels. As indicated by the column of numbers just to the right of the ordinate, which gives the predicted variation of [Fe/H] with M_V along the 14 Gyr isochrone, at 0.5 mag intervals the difference in [Fe/H] between turnoff and lower RGB stars in M92 can be as small as $\approx$0.1 dex. (It should also be appreciated that the surface Li abundance in the same models hardly varies over the entire range in $T_{\rm eff}$ that encompasses the Spite plateau, and thus our computations satisfy this critical observational constraint as well; for a complete discussion, see Paper I.)

5. SUMMARY AND DISCUSSION

The evolutionary tracks presented in Paper I, and the isochrones that we have derived from them in this study, are the first ones for Population II stars in which gravitational settling, thermal diffusion, radiative accelerations, and turbulence are treated in a fully self-consistent way. These calculations satisfy a number of constraints to within current theoretical and observational uncertainties, including (1) the measured Li abundances in field halo stars (see Paper I), (2) the lack of a perceptible variation in [Fe/H] across the subgiant branches in GCs, (3) the properties of local subdwarfs having $[\rm Fe/H] = -1.35 \pm 0.15$ and $\sigma(\pi)/\pi < 0.07$ from *Hipparcos* (if their temperatures and metallicities are close to the values given by R.G. Gratton and colleagues), and (4) the morphologies of globular cluster CMDs. Moreover, they predict somewhat younger ages at a given turnoff luminosity (by $\approx$10%) than nondiffusive models for the same initial chemical composition so that our best estimate of the age of M92, which is one of the most metal-poor GCs in the Galaxy and likely one of the oldest, is 13.5 Gyr.

Importantly, our analysis favors the *short* distance scale for the globulars. Although $(m-M)_V \approx 14.6$ is obtained for M92 when the cluster subgiants are fitted to the well-observed field subgiant HD 140283, the same or slightly smaller modulus (depending on the assumed metallicity of M92) would be obtained were we to perform a main-sequence fit of the cluster fiducial to our isochrones. This is normally considered to be a risky procedure, but we have no reason to distrust the accuracy of our models in either an absolute or relative sense; the isochrones for $[\rm Fe/H] = -1.3$ require no adjustment in color whatsoever to match a monometallicity subdwarf sequence for the same metallicity, and those for $[\rm Fe/H] = -2.3$ are nearly coincident with the M92 main-sequence fiducial when the cluster distance is based on HD 140283. In fact, an apparent distance modulus within $\approx\pm0.1$ dex of 14.6 has been obtained in many investigations over the years (e.g., Stetson & Harris 1988; Bolte & Hogan 1995; VandenBerg, Bolte, & Stetson 1996; Pont et al. 1998).

The significantly higher distance moduli (and much younger ages) that have been derived for M92 by, e.g., Reid (1997), Reid & Gizis (1998), Gratton et al. (1997), and, to a lesser extent, by Carretta et al. (2000), are inconsistent with the HD 140283 contraint. If M92 is as distant as these workers have suggested, then either HD 140283 is much older than any globular cluster, which seems unlikely, or our understanding of the basic properties of the field subgiant is somehow faulty—which also seems unlikely (although not impossible), because HD 140283 is an extremely well-studied star. We suspect (see § 4; Bergbusch & VandenBerg 2001) that part of the problem with the aforementioned studies is that they assumed an [Fe/H] scale for the GCs that is too high (especially at metal abundances similar to those of M3 and M5). Given that there are no main-sequence calibrators having $[\rm Fe/H] < -2$ whose properties are known to the same precision as those listed in Table 1, there seems to be little choice at the present time but to rely on HD 140283 and the presumed accuracy of isochrones in a *differential* sense to derive the distance to M92.

In fact, there is a lot of support for relatively *short* distances, including most direct determinations of the absolute magnitudes of metal-poor RR Lyrae stars (e.g., Fernley et al. 1998; Gould & Popowski 1998) and the fits of white dwarf sequences in globular clusters to their counterparts in the solar neighborhood (Renzini et al. 1996; Zoccali et al. 2001). Although the latter investigations involved GCs that are more metal-rich than M92, all three of the favored *standard candles* (i.e., subdwarfs, RR Lyrae variables, and white dwarfs) seem to give reasonably consistent results (see VandenBerg et al. 2000 and VandenBerg 2000 for more extensive discussions of this issue). This is not the case if the high luminosities that have been derived for low-metallicity RR Lyraes by, e.g., Sandage (1993), McNamara (1997), and Kovács & Walker (1999), are correct. To reiterate, HD 140283 and our analysis of local subdwarfs suggest that most of the first studies to make use of *Hipparcos* results (Reid 1997; Gratton et al. 1997) probably overestimated GC distance moduli (and, consequently, the intrinsic luminosities of their RR Lyrae populations) by $\gtrsim$0.25 mag. However, there are still many mysteries that need to be understood before we can claim that a consensus has been reached.

We estimate the uncertainty in the M92 distance modulus to be $\approx\pm0.12$ mag, based on the error in the absolute magnitude of HD 140283 and on the results of previous studies by Bergbusch & VandenBerg (2001; see, in particular, their § 4), VandenBerg (2000), VandenBerg et al. (2000), Grundahl et al. (2000), and Pont et al. (1998). As an error bar of $\pm$1–1.5 Gyr can be attributed to the uncertainties in basic stellar physics (see, e.g., Chaboyer et al. 1998), we believe that estimates of the age of M92 less than 10.5 Gyr or greater than 16.5 Gyr are excluded at the 95% confidence level. It is useful to examine how well our age estimate (13.5 Gyr, with a 2 σ uncertainty of $\pm$3 Gyr) fits into the $\Omega_M \approx 0.35$, $\Omega_\Lambda \approx 0.65$ cosmological model that seems to be the preferred description of the universe as the result of studies of distant Type Ia supernovae (e.g., Perlmutter et al. 1999) and the recent BOOMERANG and MAXIMA-1 experiments (e.g., Balbi et al. 2000; Schwarschild 2001). In a flat universe with a nonzero cosmological constant (Λ), time (t) is a function of redshift (z), the matter density parameter ($\Omega_M = 1 - \Omega_\Lambda$, in this particular case), and the Hubble constant (H_0) according to (see Tayler 1986)

$$t = \frac{2}{3H_0(1-\Omega_M)^{1/2}} \sinh^{-1} \sqrt{\frac{(1-\Omega_M)}{\Omega_M(1+z)^3}} . \qquad (1)$$

The age of the universe (t_0) corresponding to particular values of H_0 and Ω_M is then obtained by setting $z = 0$. If we equate t with GC ages and z with the formation redshift of globular clusters, $z_{\rm GC}$, then the dependence of t on H_0 and $z_{\rm GC}$ is as given in the upper panel of Figure 10. Our best estimate of the age of M92 and its (1 σ) uncertainty are indicated by the thick solid curve and the "hatched" region, respectively. The lower panel plots t-t_0 as a function of $z_{\rm GC}$

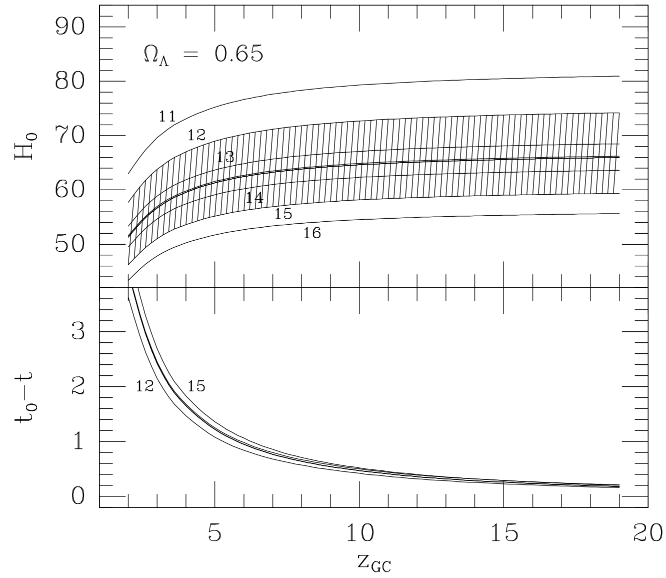


FIG. 10.—Upper panel illustrates the relationship between GC ages (in the range 11–16 Gyr), the Hubble constant H_0 (in km s^{-1} Mpc^{-1}), and the formation redshift of the globulars z_{GC}, on the assumption of a universe having $\Omega_M = 0.35$ and $\Omega_\Lambda = 0.65$. The lower panel shows, for the same cosmological model, how the elapsed time between the Big Bang and the formation of GCs varies as a function of z_{GC} and cluster ages from $t = 12$ to 15 Gyr (t_0 represents the age of the universe). The thick solid curve and "hatched" region (*upper panel*) indicate our best estimate of the age of M92 (13.5 Gyr, with a 1 σ uncertainty of $\pm$1.5 Gyr).

for GC ages between 12 and 15 Gyr, with the thick solid curve again representing the derived age of M92. Thus, for instance, if the most metal-poor globular clusters formed at a redshift of 5–6, then one would conclude that the universe is older than they are by $\approx$1 Gyr—in which case the age of the universe is 14.5 Gyr. This may be compared with the recent determination of $t_0 = 14.0 \pm 0.5$ Gyr by Knox, Christensen, & Skordis (2001) from the measured anisotropy in the cosmic microwave background (assuming $\Omega_{total} = 1$), which is clearly in superb agreement with our result. (It is evident in Fig. 10 that the age difference rises steeply at lower z_{GC} and asymptotically approaches zero at very high redshift.)

Of particular interest in Figure 10 is the implication of the M92 age that we have derived for H_0. According to Freedman et al. (2001), the final result from the *Hubble Space Telescope* (*HST*) Key Project to determine the Hubble constant is $H_0 = 72 \pm 8$ km s^{-1} Mpc^{-1}. While it is very encouraging that the 1 σ error bars overlap, the agreement is actually less close than Figure 10 suggests—because our age estimate is based on a *short* distance scale. The Freedman et al. result rests on the assumption that the true distance modulus of the Large Magellanic Cloud is $\mu = 18.50$ (for which there is considerable support; see, e.g., van den Bergh 1999; Walker 1999). However, in their study of three old, metal-poor GCs in the LMC, Johnson et al. (1999) obtained $\mu_{LMC} = 18.46 \pm 0.09$ if the true M92 modulus is 14.61. (Very similar conclusions were reached by Carretta et al. 2000.) This is greater than our estimate by 0.06 mag, assuming $A_V = 3.1E(B-V)$, which implies that our results are consistent with $\mu_{LMC} = 18.40$.

Even this may be too high. The Cepheid-based distance to the galaxy NGC 4258 is appreciably larger than that derived by Miyoshi et al. (1995) from the orbits of water masers about a central black hole (i.e., using a more direct, geometrical approach). Newman et al. (2001) have noted that, if the maser distance is correct and the Cepheids in NGC 4258 are used to recalibrate the extragalactic distance scale, then $\mu_{LMC} = 18.31$ and the Freedman et al. (2001) estimate of H_0 would have to be increased by $\sim$10%. In further support of such a revision, the luminosities of red clump stars imply $\mu_{LMC} \lesssim 18.30$ (Popowski 2000; Udalski 2000), in good agreement with the values obtained from analyses of the eclipsing LMC binary HV 2274 (Guinan et al. 1998; Ribas et al. 2000; Nelson et al. 2000) and from a main-sequence fit to NGC 1866 (Walker et al. 2001). A modulus near 18.3 has also been found by Luri et al. (1998) from their consideration of both *Hipparcos* RR Lyrae and Classical Cepheid data, and the first application of the infrared surface brightness technique to a Cepheid in the LMC has yielded $\mu = 18.42 \pm 0.10$ (Gieren et al. 2000).

It is beyond the scope of this paper to review the many determinations of the LMC distance that have been made in recent years, including those studies that favor the *long* distance scale (e.g., Feast & Catchpole 1997; Panagia 1998; Sandage, Bell, & Tripicco 1999; Sakai, Zaritsky, & Kennicutt 2000). We have chosen to highlight just a few investigations (in the previous paragraph) because it is important to appreciate that there is substantial, if not compelling, support for an LMC modulus in the range 18.3–18.4 and that our results (also see VandenBerg 2000) add to the weight of the evidence in favor of such estimates. In this case, the Key Project result should be revised upward to 75–79 km s^{-1} Mpc^{-1}, making it only marginally compatible with the age that we have derived for M92 (see Fig. 10).

However, many have found from Type Ia supernovae (e.g., Suntzeff et al. 1999; Saha et al. 1999; Jha et al. 1999) and a whole host of other methods (see, e.g., Kundić et al. 1997; Keeton & Kochanek 1997; Hughes & Birkinshaw 1998; Paturel et al. 1998; and especially Sandage 1999 and references therein) that the global value of the Hubble constant is 65 km s^{-1} Mpc^{-1} or less. Indeed, Sandage has consistently argued that when observational selection biases are properly taken into account, low values of H_0 are invariably

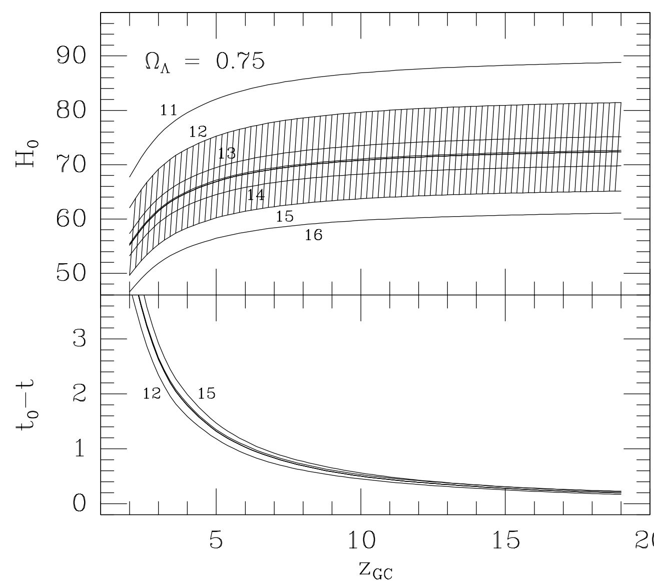


FIG. 11.—Same as Fig. 10, but $\Omega_M = 0.25$ and $\Omega_\Lambda = 0.75$ are assumed

obtained. As the investigations involving Cepheids that he has carried out also derive the zero point of the *P*-*L* relation assuming $\mu_{\rm LMC} = 18.50$, we would argue that his estimates of H_0, which have typically been in the range 55–60 km s^{-1} Mpc^{-1}, should be increased by 5%–10% to be consistent with the reduced LMC distance that our work favors. It is clear from Figure 10 that, with such an adjustment, there would be excellent consistency between the resultant value of H_0 and the M92 age constraint (in the context of an $\Omega_M = 0.35$, $\Omega_\Lambda = 0.65$ universe).

It may turn out that the particular value of Ω_Λ that we have considered is too high or too low, in which case, lower or higher values of H_0, respectively, would be needed to obtain similar consistency with GC ages. For instance, as shown in Figure 11, which is identical to the plot just discussed except that $\Omega_\Lambda = 0.75$ is assumed, $H_0 = 68$ km s^{-1} Mpc^{-1} is obtained if, say, $t = 13.5$ Gyr and $z_{\rm GC} = 6$. (This may be compared with the $\mu_{\rm LMC}$ adjusted value of 75–79 km s^{-1} Mpc^{-1} from the *HST* Key Project consortium.) It is also possible, of course, that our estimate of the age of M92 is still not right. However, the stellar models that we have employed are already quite advanced and will probably not change significantly in the foreseeable future, the distance scale that we favor has a lot of independent support, and the properties of M92 and HD 140283 represent current best estimates. Hence, the cosmological constraint that has been derived in this study warrants serious consideration.

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