THE ASTROPHYSICAL JOURNAL, 555:125–129, 2001 July 1

CONSTRAINTS ON Ω_B, Ω_m, AND h FROM MAXIMA AND BOOMERANG

T. PADMANABHAN
Inter-University Centre for Astronomy and Astrophysics, Post Bag 4, Pune 411 007, India; paddy@iucaa.ernet.in
AND
SHIV K. SETHI
Mehta Research Institute, Chhatnag Road, Jhusi, Allahabad 211 019, India; sethi@mri.ernet.in
Received 2000 October 21; accepted 2001 March 7

ABSTRACT

We analyze the BOOMERANG and MAXIMA results in the context of simplest inflationary universes: $\Omega_{\rm Total} = 1$, $n_s \simeq 1$. We attempt to constrain three other parameters—h, Ω_B, and Ω_m—from these observations. We show that (1) the data are consistent with the values of Ω_m and h inferred from other observations and (2) the value of $\Omega_B h^2$ is too high to be compatible with big bang nucleosynthesis observations at the 2 σ level for $n_s = 1$. We also include two cosmic background imager (CBI) band powers in our analysis. However, the inclusion of CBI band powers doesn't affect our conclusions.
Subject headings: cosmic microwave background — cosmology: observations — cosmology: theory

Precise determination of cosmic microwave background radiation (CMBR) anisotropies has long been expected to give accurate values of cosmological parameters (see, e.g., Bond, Efstathiou, & Tegmark 1997 and references therein). These cosmological parameters include parameters of the background Friedmann Robertson Walker model ($\Omega_{\rm Total}$, Ω_Λ, h, Ω_B, etc.), parameters that determine the formation of structure in the universe (σ_8, scalar spectral index n_s, etc.), and the parameters related to the reionization of the universe (the optical depth to the last scattering surface, τ, etc.).

While the future experiments *Microwave Anisotropy Probe* and *Planck*,[1] largely owing to their all-sky coverage, are expected to determine most of these parameters with a few percent accuracy (Jungeman et al. 1996; Zaldarriaga, Spergel, & Seljak 1997; Prunet, Sethi, & Bouchet 2000), recent observations have already begun to give important clues about some of these parameters (Miller et al. 1999; Mauskopf et al. 2000; Netterfield et al. 1997; for a summary of observation up to 1998 and parameter estimation from these observations, see Lineweaver & Barbosa 1998). Recent balloon experiments BOOMERANG and MAXIMA reported CMBR anisotropy measurements at angular scales between $\simeq 10°$ and $\simeq 10'$ (de Bernardis et al. 2000; Hanany et al. 2000). These experiments observed nearly 1% of the sky with angular resolution $\simeq 10'$. For both these experiments the cosmic variance was small enough (owing to the sky coverage) to determine precisely the position of the first Doppler peak ($l \simeq 200$) of the CMBR anisotropies. Both BOOMERANG and MAXIMA results gave strong evidence that $\Omega_{\rm Total} = 1$ (de Bernardis et al. 2000; Hanany et al. 2000), which was already indicated by other observations (Netterfield et al. 1997).

While the position of the first Doppler peak gives unambiguous evidence about the geometry of the universe, determination of other cosmological parameters is more difficult. This is because variation in several different parameters give the same change in measured anisotropies; e.g., the height of the first Doppler peak is nearly degenerate in Ω_B, h, Ω_Λ, and n_s. Some of this degeneracy can be lifted with the measurement of anisotropies at even smaller angular scales. BOOMERANG and MAXIMA probe with angular scales corresponding to multipoles $l_{\rm max} \simeq \{600, 700\}$, respectively, which is up to or beyond the expected position of the second Doppler peak. Although the results of these experiments have not been able to find the position of the second Doppler peak, accurate measurement of anisotropies at such angular scales is expected to break some of the degeneracy that measurements near the first Doppler peak alone cannot. Recent cosmic background imager (CBI) observations have, for the first time, revealed temperature anisotropies at $l \gtrsim 1000$ (Padin et al. 2001); the anisotropies at these angular scales are dominated by the damping of acoustic waves at the last scattering surface (White 2001). Therefore, it appears that temperature anisotropies originating from all the important physical processes at the last scattering surface have now been measured—the Sachs-Wolfe effect (*COBE* and Tenerife), acoustic oscillations (BOOMERANG and MAXIMA among other experiments), and damping of acoustic waves (CBI).

The BOOMERANG and MAXIMA data have been used to determine various cosmological parameters (Balbi et al. 2000; Jaffe et al. 2001; Lange et al. 2000; Tegmark & Zaldarriaga 2000; Bridle et al. 2001; Kinney, Melchiorri, & Riotto 2000). Combined with other independent measurements of cosmological parameters (e.g., measurement of $\Omega_B h^2$ from element abundance, measurement of h from nearby observations, or inference about the values of Ω_Λ and Ω_m from the Type Ia supernova [SN Ia] data, etc.), these data are expected to lead to a unique picture. However, owing to degeneracies in parameter estimation, the value of estimated parameters and their errors depend sensitively on various assumption related to the assumed allowed range of parameters, i.e., on the priors on the parameters.

In this paper, we perform a likelihood analysis on the band powers reported by the MAXIMA, BOOMERANG, and CBI experiments. However, we assume, as suggested by the simplest inflationary model, that the universe is spatially flat, i.e., $\Omega_{\rm Total} = 1$ and $n_s \simeq 1$, and then attempt to estimate three parameters—Ω_B, h, and Ω_Λ (or Ω_m)—assuming weak priors on their allowed values. In the next section, we

[1] For details see http://map.gsfc.nasa.gov and http://astro.estec.esa.nl/SA-general/Projects/Planck.

explain our choice of parameters and the method we use in brief. In the third section, we present and summarize our results.

1. COSMOLOGICAL PARAMETERS AND CMBR DATA

Most generic models of inflationary scenario give two unique predictions: $\Omega_{\rm Total} = 1$ and $n_s \simeq 1$ (see, e.g., Steinhardt 1995; Peebles 1993; Padmanabhan 1993). The first of these predications is confirmed by BOOMERANG and MAXIMA. The analysis of *COBE*–Differential Microwave Radiometer (DMR) data is consistent with $n_s \simeq 1$ (Bennett et al. 1996; Bunn & White 1997). Therefore, it is reasonable to believe that the current data are in good agreement with these predictions. We fix the value of these two parameters on the basis of these considerations and use $\Omega_{\rm Total} = 1$ and $n_s \simeq 1$. It should be pointed out that $\Omega_{\rm Total} = 1$ is a stricter requirement of inflation than $n_s = 1$; $n_s = 1$ only for exponential inflation. Most models of inflation give $0.9 \lesssim n_s \lesssim 1$ (see, e.g., Steinhardt 1995); we consider this range of n_s in this paper. Also note that we are not concerned with the origin of the values $\Omega_{\rm Total} = 1$ and $n_s = 1$. For example, these could arise merely from the requirement of scale invariance for the background universe (giving $\Omega_{\rm Total} = 1$) and the perturbations (giving $n_s = 1$) without invoking inflation—as was originally done by Harrison and Zeldovich, years before inflation was invented (Harrison 1970; Zeldovich 1972). But of course inflationary models made these parameter values fairly well accepted. Other parameters like Ω_Λ, h, and Ω_B cannot be fixed by theoretical considerations alone. In our analysis we assume a nonzero Ω_Λ because recent high-z SN Ia observations suggest a nonzero cosmological constant (Perlmutter et al. 1999; Riess et al. 1998). We do not consider CMBR anisotropies from tensor perturbations. Another important parameter is the optical depth from Thompson scattering to the last scattering surface, τ, after the reionization of the universe. The optical depth can be related to the redshift of reionization, $z_{\rm ion}$ as $\tau \simeq 0.04\Omega_B h(1 + z_{\rm ion})^{1.5}/\Omega_m^{0.5}$ (see, e.g., Padmanabhan 1993; Peebles 1993). Reionization of the universe can alter the primary CMBR anisotropies significantly (de Bernardis et al. 1997; Griffiths, Barbosa, & Liddle 1999). Present observations suggest that the universe is ionized up to $z \simeq 5$, which, for acceptable values of other cosmological parameters, gives $\tau \lesssim 0.02$; this value is too small to be of significance to CMBR anisotropies (see, e.g., Bond 1996). However, it is quite possible that $z_{\rm ion} \gtrsim 20$ (Ostriker & Gnedin 1996), in which case the optical depth to the last scattering surface can be $\gtrsim 0.2$. In view of this possibility we consider three values of the optical depth to the last scattering surface, $\tau = \{0, 0.2, 0.4\}$.

We fix the value of $\Omega_m + \Omega_\Lambda = \Omega_{\rm Total} = 1$ and compute the confidence levels on the best-fit values of Ω_B, h, and Ω_m. The range of parameters in which the minimum of χ^2 is searched is $0.01 \leq \Omega_B \leq 0.15$ (in steps of 0.00375), $0.4 \leq h \leq 1.1$ (in steps of 0.02), $0.1 \leq \Omega_m \leq 0.95$ (in steps of 0.03), and $0.9 \leq n_s \leq 1$. The normalization of the anisotropies is the fifth parameter in our analysis. For nonzero optical depth to the last scattering surface, τ is the sixth parameter.

The χ^2 for the model comparison with observations is given by

$$\chi^2 = \sum_{j=1}^{N} \sum_{i=1}^{N} (\mathscr{C}_l^{\rm obs} - \mathscr{C}_l^{\rm th})_i \mathscr{F}_{ij} (\mathscr{C}_l^{\rm obs} - \mathscr{C}_l^{\rm th})_j \ . \qquad (1)$$

Here $N = 28$ at 10 points from MAXIMA, 12 points from BOOMERANG, and 2 points from CBI. In addition, to fix the normalization of the anisotropies, we take three *COBE*-DMR band powers (Hinshaw et al. 1996; the band power for $21 \leq l \leq 40$ is excluded) and one Tenerife band power at $l \simeq 20$ (Gutiérrez et al. 1997); $\mathscr{C}_l^{\rm obs}$ are the measured band powers and $\mathscr{C}_l^{\rm th}$ are the theoretical band powers, while $\mathscr{F}_{ij}$ is the Fisher matrix for band powers. For MAXIMA and BOOMERANG, the Fisher matrix (or its inverse, the covariance matrix) has been released,[2] and we use it for our analysis. For other experiments the Fisher matrix is assumed to be diagonal with diagonal values corresponding the inverse of the square of the reported band-power errors. The cross-correlation coefficients between band powers are small as compared to the diagonal terms for both MAXIMA and BOOMERANG and make insignificant difference to our results. We take into account the reported calibration uncertainties of 10%, 4%, and 5% on the temperature for BOOMERANG, MAXIMA, and CBI, respectively. The theoretical band powers are calculated using the CMBR Boltzmann code CMBFAST (Seljak & Zaldarriaga 1996).

Equation (1) implicitly assumes that the likelihood function is Gaussian in band powers near its maximum. While this assumption is true in principle, in practice there can be significant deviation from Gaussianity near the maximum. Bond, Jaffe, & Knox (2000) advocate using another variable instead of band powers for doing the maximum likelihood analysis. We do not use it here. However, while quoting errors, we do not use the Fisher matrix approach for calculating errors on parameters, which can give meaningful results for only the Gaussian case. Instead, we directly give the confidence levels on $\Delta\chi^2$.

2. RESULTS

Our results are shown in Figures 1 and 2. For the $\tau = 0$ case, the χ^2 for 28 data points from BOOMERANG, MAXIMA, CBI, *COBE*, and Tenerife with five parameters (i.e., 23 degrees of freedom) is 23.9, which is an excellent fit. The best-fit values of the fitted parameters and 1 σ errors are $\Omega_B = 0.049^{+0.023}_{-0.0075}$, $h = 0.78^{+0.08}_{-0.14}$, and $\Omega_m = 0.34^{+0.26}_{-0.12}$. The smallest value of χ^2 is obtained for $n_s = 1$. The one-parameter best-fit values and errors are calculated by marginalizing over other parameters by integrating over them.

For nonzero τ, the best-fit χ^2 for the two cases are $\chi^2 = \{25.7, 31\}$ for $\tau = \{0.2, 0.4\}$. For 22 degrees of freedom (one less than the previous case because of nonzero value of τ), these fits are worse than the fit assuming zero optical depth. Although $\tau = 0.2$ is acceptable, the data disfavor the model with $\tau = 0.4$. For the $\tau = 0.2$ model, the best-fit values of the parameters and 1 σ errors are $\Omega_B = 0.056^{+0.018}_{-0.015}$, $h = 0.70^{+0.14}_{-0.8}$, and $\Omega_m = 0.41^{+0.19}_{-0.17}$; $n_s = 1$ minimizes the χ^2. The best-fit models along with BOOMERANG, MAXIMA, and CBI data points are shown in Figure 3.

The range of allowed h is in good agreement with the measurement of h from local observations, which suggests that $h = 0.72 \pm 0.08$ (Freedman et al. 2000). Recent SN Ia results suggest $\Omega_m = 0.28^{+0.09}_{-0.08}{}^{+0.05}_{-0.04}$ (1 σ) for the spatially flat models with non-zero cosmological constant (Perlmutter et al. 1999). This is within $\Delta\chi^2 = 1$ of the value

[2] For details see http://cfpa.berkeley.edu/maxima and http://www.physics.ucsb.edu/~boomerang/data/Nature00/b98.

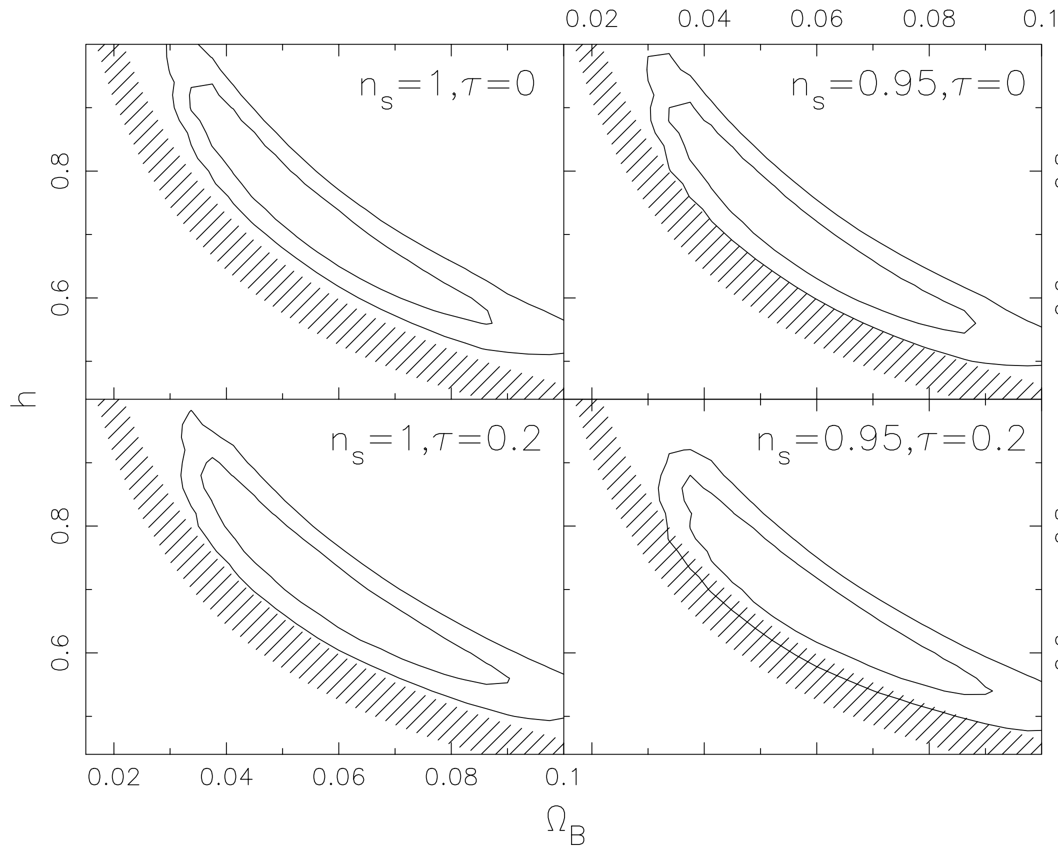

FIG. 1.—Contours correspond to allowed 1 and 2 σ regions by CMBR observations (see text for detail). The hatched region corresponds to the 95% region ($\simeq 2\ \sigma$) from primordial nucleosynthesis (Tytler et al. 2000). The values of n_s and τ are indicated in figure legends.

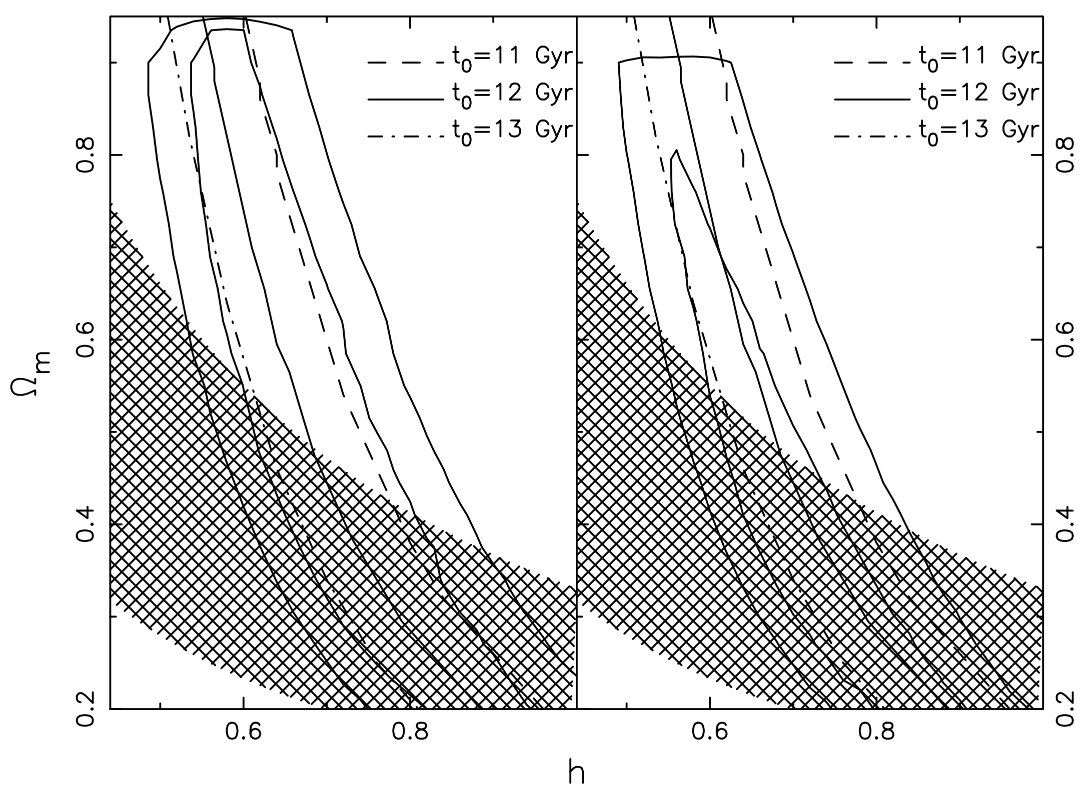

FIG. 2.—Contours correspond to allowed 1 and 2 σ regions by CMBR observations. The left and right panels correspond to $\tau = 0$ and $\tau = 0.2$, respectively. The cross-hatched regions correspond to constraints from the shape of galaxy clustering power spectrum (see text for details). Other curves show different ages of the universe t_0 (value indicated in the figure legend).

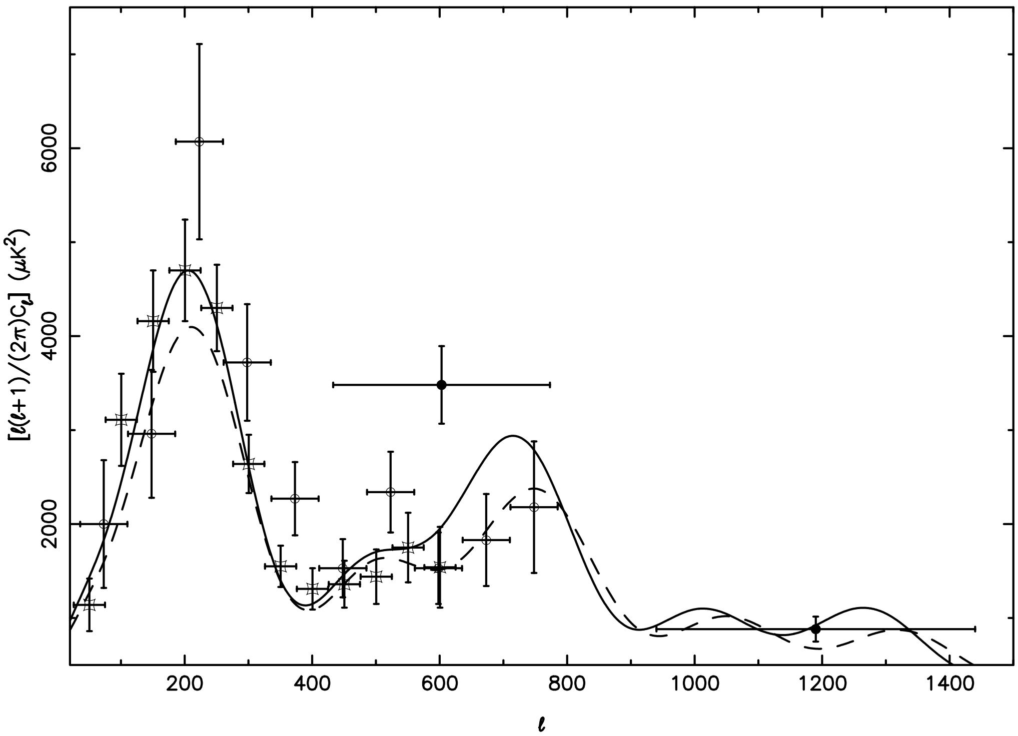

FIG. 3.—Best-fit models for $\tau = 0$ (*solid line*) and $\tau = 0.2$ (*dashed line*) are plotted along with BOOMERANG (*open polygons*), MAXIMA (*open circles*), and CBI (*solid circles*) data points. The errors shown do not include calibration errors.

inferred by our analysis. The $\Omega_m = 1$, $\Omega_\Lambda = 0$ model is within the 2 σ range of the best-fit value. This is due to the fact that CMBR observations are not sensitive to the values of either Ω_m or Ω_Λ but only to their sum.

In Figure 1 we show the confidence levels in the Ω_B-h plane for several values of n_s and τ. The other parameters are marginalized by integrating over them. The region bounded by the contours correspond to $\Delta\chi^2 \leq 2.3$ and $\Delta\chi^2 \leq 6.17$, which, for Gaussian errors, corresponds to 68% (1 σ) and 95.4% (2 σ) for two-parameter fits. We also show the 95% region allowed by the big bang nucleosynthesis (BBN) observations (for a recent review, see Tytler et al. 2000). As seen in Figure 1, the region allowed by CMBR observations is at variance with the predictions of the BBN for the $\tau = 0$, $n_s = 1$ model. This result agrees with earlier indications that $\Omega_B h^2$ is too large to be compatible with the BBN (Tegmark & Zaldarriaga 2000; Jaffe et al. 2001; Esposito et al. 2001). At smaller values of n_s the agreement between the BBN and CMBR constraints is seen to become better. The two 2 σ regions begin to overlap for $n_s \lesssim 0.95$. For $\tau = 0.2$, there is better agreement between CMBR and BBN results. This behavior is similar to the one obtained by decreasing the value of n_s and is expected because the CMBR anisotropies at large multipoles ($l \gtrsim 200$), which constrain Ω_B and h, decrease from both a decrease in n_s and also from a nonzero τ.

To quantify the disagreement of the CMBR results with the BBN results for the $\tau = 0$ case we also performed the following analysis: we fixed the value of $\Omega_B h^2$ to the range implied by the BBN and, assuming this prior, minimized the χ^2. The minimum is $\chi^2 \simeq 29$ with all the other parameters in the acceptable range; the best-fit value of $n_s = 0.9$. If we modify our prior by also fixing n_s at some value between 0.9 and 1, then the minimum χ^2 increases; for $n_s = 1$ the minimum $\chi^2 \simeq 33.3$. The goodness of fit for this value of χ^2 is $Q \simeq 0.045$, suggesting that the BBN value of $\Omega_B h^2$ is inconsistent with the CMBR data at 95% level for $n_s = 1$ and $\tau = 0$. We stress that a model with conventional value for $\Omega_B h^2$ from BBN is ruled out more strongly by the data than the cosmological model with $\Omega_m = 1$, $\Omega_\Lambda = 0$. In addition to the simplest inflationary models we consider in this paper, there are several alternative suggestions to reconcile the BBN constraints with the CMBR observations (Peebles, Seager, & Hu 2000; Kapilanghat & Turner 2001; Bouchet et al. 2000; Kanazawa et al. 2000; Durrer, Kunz, & Melchiorri 2000; Griffiths, Silk, & Zaroubi 2000).

The region corresponding to 1 and 2 σ in the Ω_m-h plane is shown in Figure 2. Our results show that the current CMBR observations favor a universe with an age between 11 and 13 Gyr. Although this is on the lower side of the expected age of the universe from estimated ages of globular clusters, etc., this is not in disagreement with those observations (for a recent status report, see Primack 2000). Another important constraint on the values of Ω_m and h comes from the shape of the power spectrum of galaxy clustering (see, e.g., Bond 1996). The shape parameter of the galaxy clustering: $\Gamma \simeq \Omega_m h \;\exp\{\Omega_B[1 + \Omega_m^{-1}(2h)^1/2] - 0.06\}$. Observations suggest that $\Gamma + (n_s - 1)/2 = 0.22^{+0.07}_{-0.04}{}^{+0.08}_{-0.06}$ (for more details see Bond & Jaffe 1999 and references therein). We plot this region in the Ω_m-h plane in Figure 2, assuming the best-fit values of Ω_B and n_s for estimating Γ. The CMBR anisotropy observations are in fair agreement with the galaxy clustering constraints.

It should be pointed out (also seen in Fig. 3) that the CBI point at $l \simeq 600$ is more than 2 σ above the MAXIMA and BOOMERANG points (Padin et al. 2001). However, including that point does not alter our results much, as compared to an analysis in which only BOOMERANG and MAXIMA points are included. This is largely because there is only one CBI point, as compared to nearly 10 from the other two experiments in that l range with comparable error bars. Therefore, the minimum of χ^2 is determined principally by the MAXIMA and BOOMERANG points.

In conclusion, recent BOOMERANG and MAXIMA observations, within the context of simplest inflationary models ($\Omega_{\rm Total} = 1$ and $n_s \simeq 1$), imply the following: (1) The values of cosmological parameters h and Ω_m are in agreement with values inferred from other observations. (2) The age of the universe is in the range 11 Gyr $\leq t_0 \leq$ 13 Gyr. (3) The value of $\Omega_m h$ is in agreement with the shape of the power spectrum of galaxy clustering, but (4) the value of $\Omega_B h^2$ is too large to be compatible with the BBN constraints at the 2 σ level for $n_s = 1$.

REFERENCES

Balbi, A., et al. 2000, ApJ, 545, L1
Bennett, C. L., et al. 1996, ApJ, 464, L1
Bond, J. R. 1996, in Les Houches, Session LX, Cosmology and Large Scale Structure, ed. R. Schaeffer (Amsterdam: Elsevier), 469
Bond, J. R., Efstathiou, G., & Tegmark, M. 1997, MNRAS, 291, L33
Bond, J. R., & Jaffe, A. H. 1999, Philos. Trans. R. Soc. London, 357, 57
Bond, J. R., Jaffe, A. H., & Knox, L. 2000, ApJ, 533, 19
Bouchet, F. R., Peter, P., Riazuelo, A., & Sakekkariadou, M. 2000, preprint (astro-ph/0005022)
Bridle, S. L., et al. 2001, MNRAS, 321, 333
Bunn, E. F., & White, M. 1997, ApJ, 480, 6
de Bernardis, P., et al. 1997, in Proc. XVI Moriond Astrophysics Meeting, Microwave Background Anisotropies, ed. F. R. Bouchet & R. Gispert (Gif-sur-Yvette: Éditions Frontières), 215
———. 2000, Nature, 404, 955
Durrer, R., Kunz, M., & Melchiorri, A. 2000, preprint (astro-ph/0010633)
Esposito, S., et al. 2001, Phys. Rev. D, 63, 043004
Freedman, W., et al. 2000, preprint (astro-ph/0012376)
Griffiths, L. Barbosa, D., & Liddle, A. 1999, MNRAS, 308, 854
Griffiths, L., Silk, J., & Zaroubi, S. 2000, preprint (astro-ph/0010571)
Gutiérrez, C. M., et al. 1997, ApJ, 480, L83
Hanany, S., et al. 2000, ApJ, 545, L5
Harrison, E. R. 1970, Phys. Rev. D, 1, 2726
Hinshaw, G., et al. 1996, ApJ, 464, L17
Jaffe, A. H., et al. 2001, Phys. Rev. Lett., 86, 3475
Jungman, G., Kamionkowski, M., Kosowsky, A., & Spergel, D. N. 1996, Phys. Rev. D, 54, 1332
Kanazawa, T., Kawasaki, M., Sugiyama, N., & Yanagida, T. 2000, preprint (astro-ph/0006445)
Kapilanghat, M., & Turner, M. S. 2001, Phys. Rev. Lett., 86, 385
Kinney, W. H., Melchiorri, A., & Riotto, A. 2000, preprint (astro-ph/0007375)
Lange, A. E., et al. 2000, preprint (astro-ph/0005004)
Lineweaver, C. H., & Barbosa, D. 1998, ApJ, 496, 624
Mauskopf, P., et al. 2000, ApJ, 536, L63
Miller, A. D., et al. 1999, ApJ, 524, L1
Netterfield, C. B., Devlin, M. J., Jarosik, N., Page, L., & Wollack, E. J. 1997, ApJ, 474, 47
Ostriker, J. P., & Gnedin, N. Y. 1996, ApJ, 472, L63
Padin, S., et al. 2001, ApJ, 549, L1
Padmanabhan, T. 1993, Structure Formation in the Universe (Cambridge: Cambridge Univ. Press)
Peebles, P. J. E. 1993, Principles of Physical Cosmology (Princeton: Princeton Univ. Press)
Peebles, P. J. E., Seager, S., & Hu, W. 2000, ApJ, 539, L1
Perlmutter, S., et al. 1999, ApJ, 517, 565
Primack, J. 2000, preprint (astro-ph/0007187)
Prunet, S., Sethi, S. K., & Bouchet, F. 2000, MNRAS, 314, 348
Riess, A., et al. 1998, AJ, 116, 1009
Seljak, U., & Zaldarriaga, M. 1996, ApJ, 469, 437
Steinhardt, P. J. 1995, Int. J. Mod. Phys. A, 10, 1091
Tegmark, M., & Zaldarriaga, M. 2000, preprint (astro-ph/0004393)
Tytler, D., O'Meara, J. M., Suzuki, N., & Lubin, D. 2000, Phys. Scr., 85, 12
White, M. 2001, preprint (astro-ph/0101086)
Zaldarriaga, M., Spergel, D. N., & Seljak, U. 1997, ApJ, 488, 1
Zeldovich, Ya. B. 1972, MNRAS, 160, 1