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THE 1:1 SUPERRESONANCE IN PLUTO'S MOTION

X.-S. Wan and T.-Y. Huang
Department of Astronomy, Nanjing University, 22 Hankou Lu, Nanjing, Jiangsu, 210093 China; and Chinese National Astronomical Observatories, of Beijing; xswan@nju.edu.cn, tyhuang@nju.edu.cn
and
K. A. Innanen
Department of Physics and Astronomy, York University, 4700 Keele Street, North York ONT M3J 1P3 Canada; kiminn@yorku.ca
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ABSTRACT

Three resonances, the 3:2 mean motion resonance, the Kozai resonance, and the 1:1 superresonance, are known to govern the stability of the motion of Pluto concurrently. In this work, we report an extensive numerical exploration of the 1:1 superresonance region in element space and the effects of the giant planets on this resonance. It is shown that the 1:1 superresonance region is narrower than those for the other two resonances and that it is a second-order resonance. We use the orbits of known Plutinos to investigate our result. None of these Plutinos is in the 1:1 superresonance. We also find that the Jovian planets, especially Jupiter, play important roles in concert in the superresonance.

Key words: celestial mechanics — Kuiper belt — planets and satellites: individual (Pluto)

1. INTRODUCTION

In the solar system, Pluto has always enjoyed a kind of "special status" as the result of its unique orbital configuration. By the remarkable concerted action of three known resonances, Pluto is protected from encounters with Neptune. The first dynamical protection mechanism was reported by Cohen & Hubbard (1965): Pluto is in a 3:2 mean motion resonance with Neptune so that the distance between the two planets can never reach its possible minimum in the vicinity of Pluto's perihelion. The second mechanism was identified by Williams & Benson (1971): Pluto's argument of perihelion, ω, librates about 90°, thus preventing Pluto from approaching the orbital planes of the other planets at its perihelion. The periods of the eccentricity, e, and the inclination, i, were found to be similar to that of ω, a result that is explainable by the Kozai mechanism (Kozai 1962). Williams & Benson (1971) also conjectured that the difference between the longitudes of the ascending nodes of Pluto and Neptune, $\Omega - \Omega_N$, is in resonance or near-resonance with the libration of ω. This was confirmed by Milani, Nobili, & Carpino (1989), who named it the 1:1 superresonance. Kinoshita & Nakai (1996) pointed out that when $\Omega - \Omega_N$ is 0°, ω is 90°, e reaches a minimum, and i reaches a maximum; in addition, when $\Omega - \Omega_N$ is 180°, ω is also 90°, e reaches a maximum, and i reaches a minimum. The concerted action of these three resonances ensures that Pluto and Neptune do not encounter each other.

Sussman & Wisdom (1988) numerically integrated the outer solar system for 845 million yr and found that the motion of Pluto is chaotic, with an e-folding (Lyapunov) time corresponding to about $10^{-7.3}$ yr^{-1}. This result does not imply some gross instability of Pluto's orbit, but rather it confirms the sensitivity of Pluto's trajectory to its initial coordinates and velocity. We report herein on a project to explore more thoroughly but qualitatively the regions of the resonances in parameter space, especially for the orbital elements. The resonance regions may thereby provide some clue to the evolutionary path of this planet and some information on the stability of its orbit. It is well known that many minor bodies, called Plutinos, are trapped in 3:2 resonance with Neptune. The corresponding resonance region should therefore not be narrow. By using a circular planar restricted three-body model, Malhotra (1996) showed theoretically that the range of the semimajor "radius" for the 3:2 resonance is about 0.6 AU. This value is quite close to the result from our numerical simulations. She went on to conjecture the possible migrations of planetary orbits in the outer solar system.

Our research focuses on the 1:1 superresonance. In § 2, the initial coordinates and velocities and the model for our numerical simulations are introduced. Methods for the numerical integration and methods for finding the periods of libration or circulation are also described in § 2. The range of the resonance zone is the main content of § 3. In § 4, we discuss the superresonance as a second-order resonance. As a check of our exploration of the resonance zone, in § 5 we apply our results to the so-called Plutinos. The role of the Jovian planets on the superresonance is discussed in § 6. Finally, in § 7, we provide a brief summary and discussion.

2. NUMERICAL EXPERIMENTS

The model in our simulation is composed of the Sun and the five outer planets, with the masses of the four inner planets added to the Sun. The masses and the initial values of the positions and velocities of the Sun and the planets are taken from DE234 (M. E. Standish 1993, private communication) with the associated epoch of 1969 June 28. Given in the heliocentric mean equatorial system of J2000.0, the initial coordinates and velocities have been transformed to the system of the heliocentric invariant plane. The origin of longitude for the latter system is chosen to assure that the node of the mean equator and the invariant plane have the same longitude in both systems. The initial orbital elements are listed in Table 1.

We have employed the modified symplectic method for solar system dynamics by Wisdom & Holman (1991). The Hamiltonian is divided into two parts, a Kepler and an interaction Hamiltonian. The former can be solved pre-

TABLE 1

ORBITAL ELEMENTS OF FIVE OUTER PLANETS

Planet	a (AU)	e	i (deg)	ω (deg)	Ω (deg)	M (deg)
Jupiter........	5.208	0.04727	0.329	56.58	317.93	174.29
Saturn	9.522	0.05405	0.932	329.73	124.56	302.91
Uranus	19.281	0.05138	1.028	219.76	312.53	10.53
Neptune......	30.176	0.004914	0.724	217.72	195.03	185.61
Pluto	39.775	0.2533	15.558	112.71	110.86	331.38

NOTE.—Data are for 1969 June 28 in the system of the heliocentric invariant plane of J2000.0.

cisely as several two-body problems; the latter has a small factor, μ, the ratio of the planetary masses to the solar mass. The relative energy error for the truncation of a sympletic method is about $\Delta = C(h/N)^{o+1}$, where h is the step size, o is the order, N is the number of function evaluations per step, and C is an error constant. As a result, C for the modified algorithm is smaller by a factor of the mass ratio, μ, than that for a general symplectic method. Consequently for the same energy error the step size can be enlarged by a factor of $\mu^{-1/(o+1)}$. Wisdom & Holman (1991) pointed out that the most efficient algorithm for qualitative exploration in solar system dynamics is the second-order modified symplectic method. This insight effectively makes long-period integrations more efficient. In addition, we have compared the second-order modified sympletic method with a twelve-step symmetric method (Quinlan & Tremaine 1990). After an 8 million year run, the relative error in the coordinates of Pluto reaches about 0.07, quite a large number. We then computed the corresponding orbital elements and found the main error to be in mean anomaly. It is well known that the main integration error is in in-track error (Huang & Innanen 1983). The errors in the semimajor axis, a, eccentricity, e, inclination, i, perihelion argument, ω, and the longitude of the ascending node, Ω, of Pluto are 0.15 AU, 0.03, 0°.03, 0°.8, and 0°.16, respectively. We found that the three resonances of Pluto are all well sustained for a 100 Myr run using the modified symplectic integrator.

Wisdom, Holman, & Touma (1996) proposed a symplectic corrector to remove spurious oscillation in energy and state variables in integrations by the modified symplectic integrator. The leading terms of correction for coordinate, x, and velocity, v, are

$$\Delta x = \frac{1}{24} h^2 F \Delta , \quad v = -\frac{1}{24} h^2 \frac{dF}{dt} , \tag{1}$$

where h is the step size and F is the perturbation acceleration (S. Mikkola 2000, private communication). Using the corrector will change some last decimal places in the final result, but it will not improve our calculation significantly. The errors in our integration are caused mainly by the relatively low order of the method.

Considering that this is a qualitative exploration, emphasizing the three resonances and the consequent need to do long computations, we decided to use the second-order modified symplectic integrator as the best available option. All the numerical experiments were performed in double precision, using FORTRAN 77 on a Sun OS with a step size of 365 days, and the output was stored every 15,000 steps. As our research agenda was focused on the boundaries of the stability regions, our calculations were thus not long enough to show the chaotic property of the orbits. It is for this reason that we have not included output giving Liapunov e-folding times.

As mentioned above, the 1:1 superresonance occurs between $\Omega - \Omega_N$ and ω, where the circulation period of $\Omega - \Omega_N$ is equal to the libration periods of ω. Next, we chose a criterion to define the edge of the resonance zone. The following rule was adopted: the 1:1 superresonance is considered broken when the phase difference between the two resonance arguments reaches π within the integration. As our integration period is 10^8 yr and both the periods of the resonance arguments are about 3.8×10^6 yr, the chosen criterion means that the difference of their periods reaches 8×10^4 yr, or 2% of the period.

To find the periods of the arguments $\Omega - \Omega_N$ and ω, we adopt simple rules. The circulation of $\Omega - \Omega_N$ is assumed to be $\phi_1 + (2\pi/T_1)t$ and the oscillation of ω, e, and i to be $A_0 + A_1 \sin\ [(2\pi t/T_1) + \phi_1]\{1 + A_2 \sin\ [(2\pi t/T_2) + \phi_2]\}$, where A_0, A_1, A_2, T_1, T_2, ϕ_1, and ϕ_2 are not necessarily the same for $\Omega - \Omega_N$, ω, e, and i. A program was written to find the best fit for these coefficients for all the numerical simulations by using the method of least squares. The periods of $\Omega - \Omega_N$, ω, e, and i for real Pluto in 100 Myr simulations are $(3.787 \times 10^6) \pm 128$, $(3.784 \times 10^6) \pm 271$, $(3.783 \times 10^6) \pm 396$, and $(3.783 \times 10^6) \pm 365$ yr, respectively. These values are consistent with the results obtained by Milani et al. (1989), who used a more complex model.

3. RESONANCE REGION IN ELEMENT SPACE

To find the boundary of the resonance region for the 1:1 superresonance in the space of the orbital elements, we have conducted a number of numerical experiments by changing the initial orbit elements little by little. Each experiment changes only one element, and the others keep the value of Pluto's orbit as shown in Table 1. The periods of libration or circulation were obtained by the method described in § 2. The fitting errors for the periods are less than 200 yr for $\Omega - \Omega_N$, 1500 yr for ω, and 2000 yr for i, so the relative errors of the periods we obtained are less than 10^{-3} to a good approximation.

Table 2 lists our main results: the boundaries of the 1:1 superresonance in element space. As a contrast, in this table we also list the corresponding boundaries of the 3:2 mean motion resonance and the Kozai resonance. These were obtained from our numerical experiments by a procedure similar to that described above. The value in the last decimal place in data in Table 2 displays the precision in our determination of the resonance boundary. Pluto's elements are also listed. It is clear that the mean motion resonance has the widest region, the Kozai resonance is next,

TABLE 2

UPPER AND LOWER LIMITS OF THE ORBITAL ELEMENTS FOR THE THREE RESONANCES

	Lower Limit				Upper Limit		
Orbital Elements	3:2	Kozai	Super	Pluto	Super	Kozai	3:2
a (AU)	39.39	39.52	39.57	39.78	39.82	39.86	40.02
e	0.065	0.222	0.230	0.253	0.270	0.296	0.434
i (deg)	0.0	7.4	10.9	15.6	22.9	49.8	144.0
ω (deg)	26.0	52.5	88.8	112.7	119.2	129.0	151.4
Ω (deg)	25.5	47.3	57.3	110.9	118.6	127.9	147.0
M (deg)	271.7	289.6	297.0	331.4	337.7	343.7	70.6

NOTE.— The epoch is the same as in Table 1.

and the superresonance has the narrowest zone. For example, the ranges of semimajor axis for the superresonance, the Kozai resonance, and the mean motion resonance are 0.25, 0.34, and 0.63 AU, respectively. It is interesting to compare these values with the result of Malhotra, who used a planar circular restricted three-body model. She found that the typical width of a Neptune resonance libration zone is about 0.6 AU, a value quite consistent with our results.

A. Milani (2000, private communication) pointed out to us that there should exist another resonance zone for the Kozai resonance around $\omega = 270°$ since it is a mirror image of $\omega = 90°$ with the invariant plane as the mirror. Table 2 lists only the resonance regions of the three resonances around Pluto. There should exist mirror images of the regions in Table 2, obtained by changing ω to $\omega + 180°$ and Ω to $\Omega + 180°$ at the same time. A resonance zone would not be perfectly symmetric to its mirror image since the inclinations of the Jovian planets are not exactly zero.

Table 2 shows that both the superresonance and the Kozai resonance exist only for an eccentricity near 0.25. It also shows that the superresonance zone is quite narrow. The width of this resonance zone for semimajor axis, eccentricity, and inclination is 0.25 AU, 0.040, and 12°, respectively. Pluto is not at the center of this resonance zone. Its semimajor axis, 39.78 AU, is about halfway to the upper limit.

It is evident that the 1:1 superresonance is very sensitive to a and e among the six elements. The width of the resonance zone for them is even smaller than the variation range of these elements during the time interval of our numerical integration. For example, Pluto's semimajor axis is in oscillation with an amplitude about 0.9 AU, much larger than the size of the resonance zone. This fact reminds us that our numerical result is valid only at the specified epoch we adopted, that of 1969 June 28.

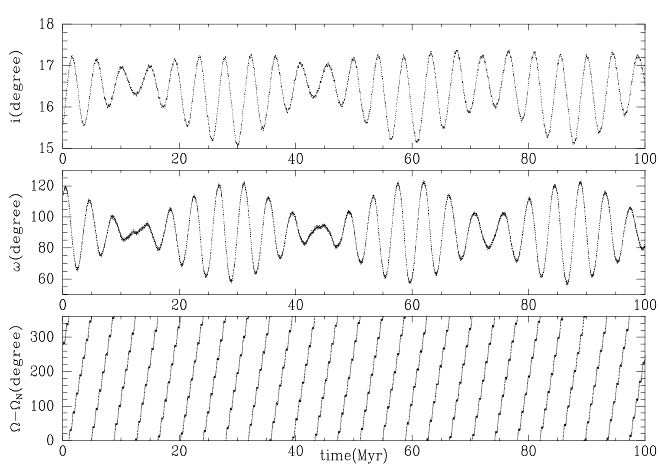

FIG. 1.—Variation of i, ω, and $\Omega - \Omega_N$ with time. Initial a is 39.56 AU, just below the superresonance zone. One sees that the period of ω switches from the period of i to that of $\Omega - \Omega_N$.

Figure 1 shows the variation of i, ω, and $\Omega - \Omega_N$ when a is 39.56 AU, just below the lower limit of the superresonance zone. It is observed that the amplitude of ω changes about 30° in about 15 Myr. The libration period of ω is also changing with amplitude, with a value close to the libration period of i, when the amplitude of ω is near 0°; however, it is almost the same as the circulation period of $\Omega - \Omega_N$ when the amplitude is large. When the initial position is further out of the resonance zone, the libration period of ω is still close to that of i, but then it differs more from the circulation period of $\Omega - \Omega_N$.

The above phenomenon can be also seen in Figure 2, where the periods T_ω, T_i, and $T_{\Omega-\Omega_N}$ are plotted as functions of the variation of orbital elements. The location of Pluto is marked by a vertical bar in Figure 2. From Figure 2, one notes the following: (1) all the periods change almost linearly with e, i, and ω but change nearly quadratically with a, Ω, and M; (2) T_ω stays close to T_i even in the boundary region but gradually moves away from $T_{\Omega-\Omega_N}$ near the boundary; and (3) the three periods remain together within the resonance zone when only i is changed. However, the three periods later have a sudden separation when i is out of the boundary. This sudden separation is not drawn in Figure 2 for the separation is so large that the point is out of the range of this figure. This kind of sudden change is quite typical at the boundary of resonance regions for all the elements.

From the above description, we have seen that the 1:1 superresonance region is the narrowest of the three resonance regions and is the easiest to break, compared with the other two resonances. This means that it is the least important protection mechanism for the orbital stability of Pluto. In fact, we have done a long integration, lasting for 100 Myr, with the initial condition outside the superresonance region but inside the 3:2 mean motion and Kozai resonance regions. We found this orbit to be stable.

4. 1:1 SUPERRESONANCE AS A SECOND-ORDER RESONANCE

The 1:1 superresonance occurs when the oscillation period of ω coincides with the circulation period of $\Omega - \Omega_N$.

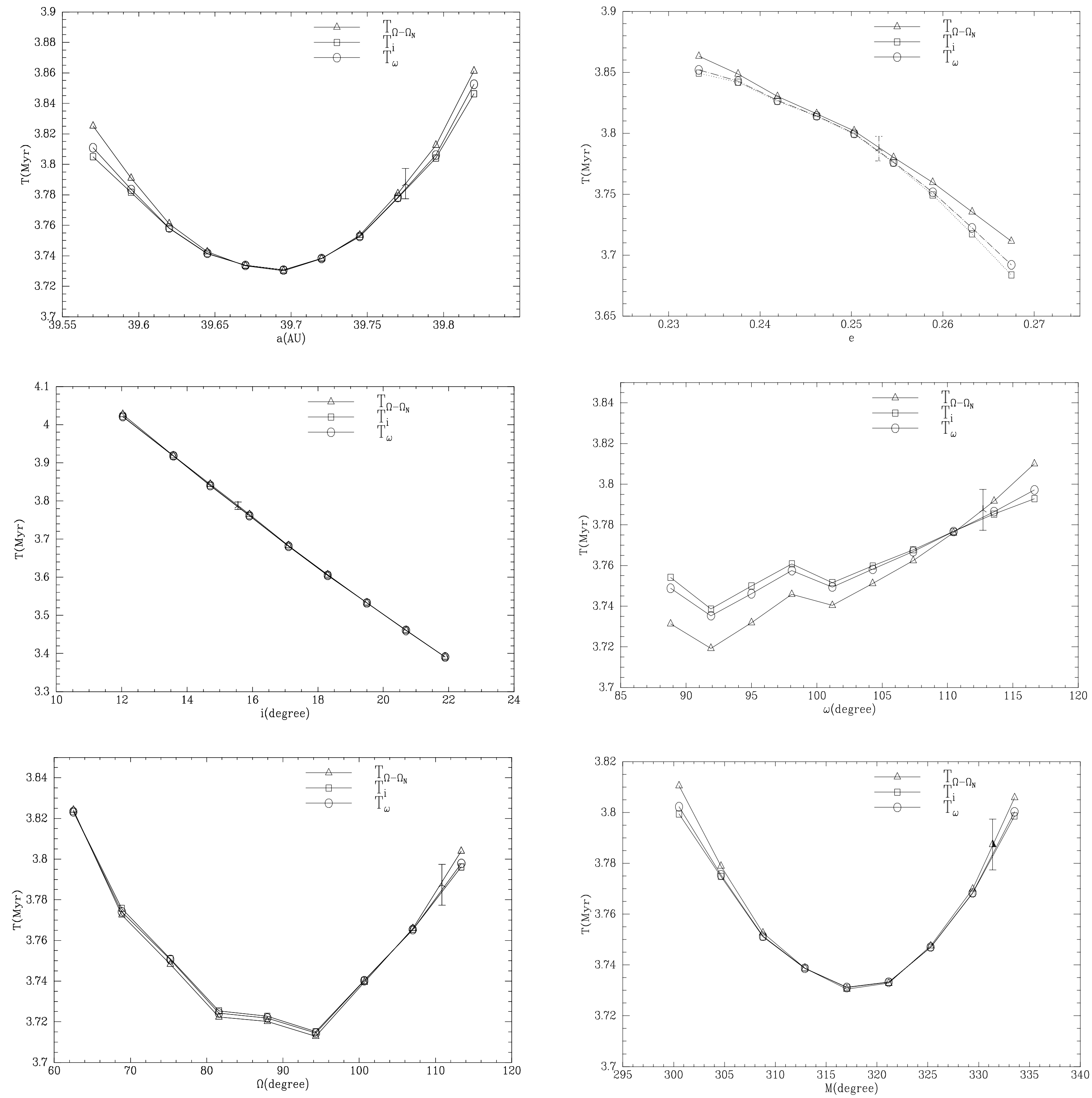

FIG. 2.—Periods of $\Omega - \Omega_N$, ω, and i with different initial orbital elements. *Left to right and top to bottom*: a, e, i, ω, Ω, and M.

This indicates that the superresonance is a second-order resonance (Lichtenberg & Lieberman 1990). In this section, we apply a theory of second-order resonance to discuss this resonance and estimate its critical argument.

Lichtenberg & Lieberman (1990) pointed out that the action-angle variables of a resonance can be written as

$$I = I_0 + I_1 \cos \phi, \quad \theta = \theta_0 + \theta_1 \sin \phi , \qquad (2)$$

where I_0 and θ_0 are constants. For a resonance without higher order resonances, I_1 and θ_1 are also constants. If ϕ is in resonance with another angle variable θ', I_1, and θ_1 can be expressed as

$$I_1 = (2JR)^{1/2}, \quad \theta_1 = (2J/R)^{1/2}, \quad \phi = \theta + \varphi , \qquad (3)$$

where J and φ are the action-angle variables for the second-order resonance and R is a constant parameter. It is evident that

$$J = J_0 + J_1 \cos \psi, \quad \varphi = \varphi_0 + \varphi_1 \sin \psi . \qquad (4)$$

If there are no further higher order resonances, in each right side of equations (4) ψ is in circulation and the other parameters are constants. In the above presentation, the critical argument for the first-order resonance is the angle variable, θ, and the critical argument for the second-order resonance is φ.

From equations (2)–(3) we get the final expression for I and θ as follows:

$$I = I_0 + (2JR)^{1/2} \cos \phi \ ,$$
$$\theta = \theta_0 + (2J/R)^{1/2} \sin \phi \ . \qquad (5)$$

In the current problem, the Kozai resonance is the first-order resonance and its critical argument, θ, is the element ω. Actually our ω is an instantaneous element and is not strictly one of the action-angle variables. An averaging procedure over ω for removing all the short-period oscillations and the 3:2 mean motion resonance period oscillation would result in the corresponding critical argument. We neglect this difference and just take ω as the critical argument of the Kozai resonance. In the same way θ' in equations (3) is now $\Omega - \Omega_N$, and the critical argument of the superresonance is $\phi - (\Omega - \Omega_N)$, where ϕ is the phase angle of ω oscillation. Finally, we have

$$e = e_0 + (e_1 + e_2 \cos \psi)^{1/2} \cos \phi \ ,$$
$$\omega = \omega_0 + (\omega_1 + \omega_2 \cos \psi)^{1/2} \sin \phi \ . \qquad (6)$$

Here e and ω are not canonical conjugate, but the form of equations (6) should be correct.

To use the data from our numerical simulation to evaluate the phase angle, ϕ, we set $\phi = v_1 t + \phi_0$ and $\psi = v_2 t + \psi_0$. A least-squares fit is then performed for e_0, e_1, e_2, ω_0, ω_1, ω_2, v_2, ψ_0, and ϕ_0, where we adopt the frequency of $\Omega - \Omega_N$, as in a previous fit (see § 1) as v_1. Then we use our numerical data and the previous fit results to determine the value of ϕ as a function of time from equations (6).

Figure 3*a* shows the critical argument of the superresonance, $\phi - (\Omega - \Omega_N)$, versus time for the case $e = 0.250$, the other elements having Pluto's values. We clearly see this angle to be in oscillation. For all the element sets that are deep in the resonance zone determined in § 3, the situation is similar. Difficulties arise when we move to the element sets that are near the boundary of the resonance zone, i.e., where the simple form of equations (6) does not work. In these cases we use the method suggested by Milani et al. (1989) to obtain ϕ. Figure 3*b*–3*d* shows the results using their method. Figure 3*b* shows the result for $e = 0.263$, near the boundary we determined in the last section. It shows that the critical argument is mostly in oscillation and that the form of its variation is distorted from a sine curve; that is, it has slow descent and more rapid ascension. At about 4×10^7 Myr the critical argument is in circulation within a very short interval. This phenomenon can also be seen in Figure 3*c*, occurring in a regular time interval of about 3.7×10^7 Myr. This figure is for $e = 0.270$, right on the

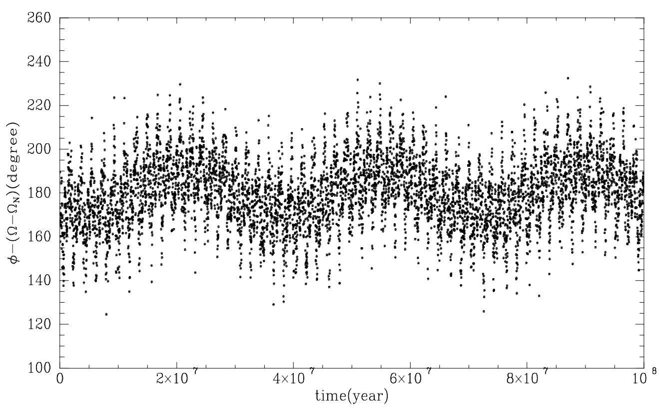

FIG. 3*a*

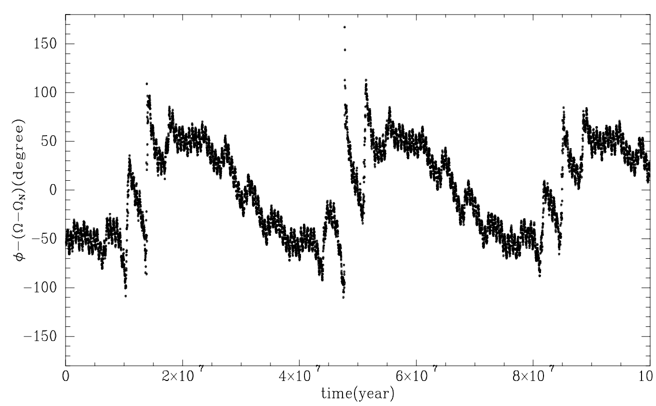

FIG. 3*b*

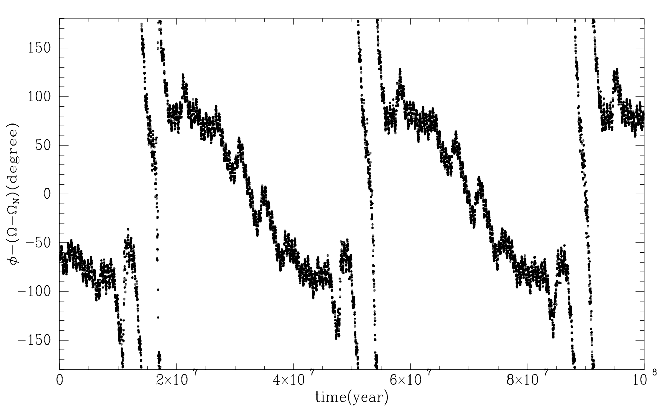

FIG. 3*c*

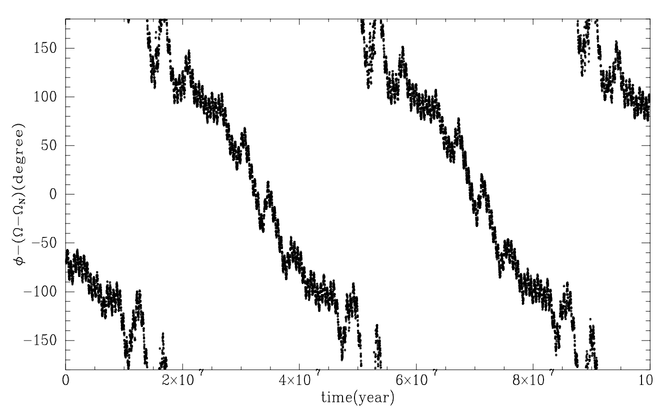

FIG. 3*d*

FIG. 3.—Variation of the critical argument of the superresonance with time. All the orbital elements are the same as that of Pluto except the eccentricity. (*a*) $e = 0.250$, inside the resonance zone. (*b*) $e = 0.263$, near the boundary of the resonance zone. (*c*) $e = 0.270$, on the boundary of the resonance zone. (*d*) $e = 0.274$, outside the boundary of the resonance zone.

boundary. The rapid descent of the critical argument is caused by the fact that ϕ has a temporary oscillation around 0°; thus the critical argument behaves just as $-(\Omega - \Omega_N)$, which is in circulation from west to east. This kind of phenomenon disappears in Figure 3*d*, which is for $e = 0.274$, just outside the boundary. It clearly shows that the critical argument is in circulation and the superresonance does not exist any more. One immediately conjectures that the superresonance is in its chaotic separatrix layer between $e = 0.263$ and $e = 0.270$. After a careful comparison between this numerical phenomenon and the theory (Lichtenberg & Lieberman 1990), we are convinced of this result.

The above presentation has shown that the 1:1 superresonance is a second-order resonance of the Kozai resonance and that the theory for a second-order resonance fits the data well. It would thus be the easiest among the three resonances to break. This also confirms that the resonance zone determined in § 3, i.e., by the behavior of the critical argument, is correct.

5. PLUTINOS

It is well known that a group of Kuiper belt objects called Plutinos share the 3:2 Neptune resonance with Pluto. One may use their real orbits to check against our resonance zone listed in Table 2.

The orbital elements of these trans-Neptunian objects have been obtained from the Minor Planet Circulars and Minor Planet Electronic Circulars. We selected the objects that have a semimajor axis around 39.5 AU. As they are in different initial epochs, the Sun and five outer planets were integrated from our epoch to the epoch of each Plutino. Next we integrated the system with each Plutino back to our epoch using a Cowell integrator. Then all the candidates were integrated together for a 100 Myr–long run by a twelfth-order symmetric integrator (Quinlan & Tremaine 1990). All the small bodies were treated as test particles during this integration. From this simulation, we selected those that were found to be in the 3:2 mean motion resonance with Neptune. Thirty-two such Plutinos were identified.

The resonance zone in the element space should be a six-dimensional region, but Table 2 lists only six intervals, which cross each other at the real Pluto. It is almost impossible for us to get the whole six-dimensional resonance zone by pure numerical integration, considering that a huge amount of CPU time is needed to fulfill this task. Nevertheless, we did complete a calculation to find the two-dimensional resonance region in the (a, e) plane, keeping the values of the other four elements the same as that of Pluto. The open circles in Figure 4 mark the edge of the superresonance region, as earlier calculated. The epoch is also at 1969 June 28. Pluto is marked on this figure with a filled circle. It is not at the center of the resonance region. We have marked the positions of all 32 Plutinos at the epoch of 1969 June 28 on Figure 4 with filled squares.

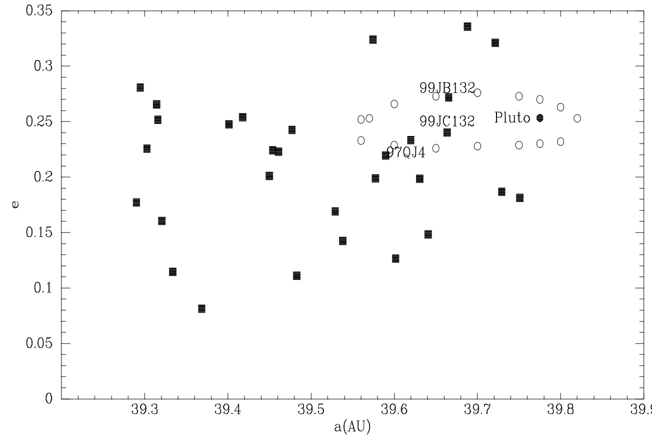

FIG. 4.—Region of the superresonance and the initial positions of Plutinos in the (a, e) plane, showing the edge of the resonance we calculated (*open circles*), Plutinos (*filled squares*), and Pluto (*filled circle*). Three Plutinos inside the resonance zone are labeled.

It can be seen that only three Plutinos are inside the resonance region. They are 1997QJ4, 1999JB132, and 1999JC132; the former two are very near the boundary of the region. On the other hand, we have found that there is no Plutino that is in 1:1 superresonance. We have therefore further investigated 1997QJ4, 1999JB132, and 1999JC132. Table 3 lists their elements at the epoch of 1969 June 28. For 1997QJ4, a and i are well inside the superresonance interval (see Table 2), e is at the edge, and ω is outside the superresonance interval but inside the Kozai resonance interval. It seems at first that it may be in the Kozai resonance but its Ω is far away from the Kozai resonance interval. Our numerical experiment thus shows that it is not in the Kozai resonance. Furthermore, it escapes after about 29 million yr. We did another 100 Myr run, in which Pluto's mass was set to zero and 1997QJ4 did not escape. This run proves that its escape is mainly because of the perturbation of Pluto.

Our numerical experiment shows that although it is not in the 1:1 superresonance, 1999JB132 stays in the Kozai resonance. We can see that the four elements of 1999JB132, a, e, i, and ω, are well inside the Kozai resonance interval and its Ω and M are out of the interval but not so far as for 1997QJ4. We believe that 1999JB132 is inside the Kozai resonance region but outside our resonance interval.

The perihelion argument, ω, of 1999JC132 is switching between circulation and oscillation but switches around 270° instead of 90° by our calculation. It should be at the boundary of the mirror image of the Kozai resonance zone listed in Table 2 (see § 3).

TABLE 3

ORBITAL ELEMENTS OF 1997QJ4, 1999JB132, AND 1999JC132 ON JUNE 28, 1969

	a (AU)	e	i (deg)	ω (deg)	Ω (deg)	M (deg)
1997QJ4	39.6199	0.2334	17.4334	86.5312	342.4243	270.8792
1999JB132......	39.6654	0.2719	13.5185	88.2016	218.3899	253.7382
1999JC132......	39.6636	0.2402	5.4341	295.5822	19.9260	241.9865

6. EFFECT OF THE JOVIAN PLANETS

Pluto's three resonances function mainly under Neptune's gravitational influence. What is the influence of the other three Jovian planets? To investigate, we proceeded in the following ad hoc way: we simply removed the other three Jovian planets one at a time from the model, maintaining self-consistency, and integrated for 100 Myr. Each time there is only one giant planet being removed. We found that the superresonance always disappeared when any Jovian planet is removed. In addition, we gradually decreased the mass of each Jovian planet to measure empirically its effect on the superresonance. One finds that the superresonance is present until 43% of the mass of Jupiter is left. For Saturn and Uranus these numbers are 39% and 37%, respectively. The decrease of the masses of the Jovian planets results in an increase of the libration period, T_ω, and a larger amplitude of libration, A_ω. This can be seen in Table 4. It should be noted that the larger the amplitude of the ω libration, the less stable Pluto's orbit would be.

Evidently Uranus has the greatest influence on the period and amplitude of the ω libration; these are about 5.69 Myr and 70°, respectively, near the edge of the superresonance. For Saturn, they are 5.05 Myr and 65°, respectively. The influence of Jupiter on the period and amplitude is not so strong, about 4.70 Myr and 25°, respectively, near the boundary. In other words, among the three Jovian planets, Uranus may be the main agency maintaining the Kozai resonance.[1] We also argue that, after Neptune, Jupiter is the most important body for maintaining the 1:1 superresonance. In summary, the other three giant planets play essential roles in concert.

7. SUMMARY AND DISCUSSION

We have studied the Plutonian multiple resonance region in element space. Numerical integrations have been performed to study the width of the superresonance zone. The result is shown in Table 2. This table and Figure 4 can be used to check whether a minor body is in resonance. The function of this check is limited as shown in § 5. The main limitation is caused by two factors: (1) We did not obtain the whole six-dimensional resonance region but only six resonance sections that are crossed at Pluto in this region, and (2) Pluto's gravitational perturbation was neglected during our exploration of this resonance region, although Pluto's gravitation may be important to the orbital evolution of Plutinos. In spite of these limitations, our results have still successfully been used to check the resonance status of Plutinos, as shown in § 5.

Our results apply at a special epoch, 1969 June 28. Our resonance interval is given by using osculating elements. We thought of using some kinds of proper elements. Because most of the orbits in our experiment are in the 1:1 superresonance, their elements, a, e, i, and ω, are in libration around an average value. Therefore we cannot distinguish these orbits clearly by averaging these elements with time. One could adopt proper elements as the elements after removing all their short-period oscillations. For this purpose a long run of numerical integration becomes necessary. Consequently it is better to use osculating elements to present our results.

Clearly, the 1:1 superresonance is a second-order resonance. It has the narrowest resonance zone and is the easiest to break, so it may be the most vulnerable protection mechanism for Pluto. We have not found one existing Plutino in this resonance. It should be pointed out that most Plutinos were discovered recently and only a small number of observations were used to determine their orbits. We have found that a few of the Plutinos do not have stable orbits with their recently published elements. More discoveries and further observations are very necessary to improve our understanding of these resonances.

From our numerical experiments that artificially removed or reduced the Jovian planets, we conclude that the superresonance is not a pure three-body phenomenon. Existence of the Jovian planets, especially Jupiter, appears to be an essential condition of the existence of the 1:1 superresonance. Finally, it is abundantly clear to us that Pluto's orbit remains an enigmatic and complicated subject, and our studies must be considered "work in progress." There clearly exist many possible avenues of future investigation.

[1] A. Milani wrote us: "It might be worth commenting that the close approaches between Pluto and Uranus are not controlled by the 3:2 Pluto-Neptune resonance because Uranus and Neptune are not in the 2:1 resonance. Thus, the Kozai resonance is the one effectively controlling the minimum approach distance between Pluto and Uranus: no wonder the period of the Kozai resonance is strongly affected by Uranus." We agree with him.

TABLE 4

VARIATIONS OF PERIOD AND AMPLITUDE OF ω LIBRATION AND CIRCULATION PERIOD OF $\Omega - \Omega_N$ WITH MASS DECREASE OF JOVIAN PLANETS

Variation	Mass Jupiter (%)	Mass Saturn (%)	Mass Uranus (%)	T_ω (Myr)	A_ω (deg)	$T_{\Omega-\Omega_N}$ (Myr)
1	50	100	100	4.6314	26.0	4.6394
2	45	100	100	4.6726	26.5	4.6982
3	43	100	100	4.6828	29.0	4.7043
4	100	50	100	4.4721	46.6	4.4700
5	100	40	100	5.0725	61.0	5.0732
6	100	39	100	5.1162	66.5	5.1127
7	100	100	50	5.2292	59.2	5.1975
8	100	100	40	5.6484	61.3	5.6081
9	100	100	39	5.6946	62.6	5.6491
10	100	100	38	5.7608	64.0	5.6894
11	100	100	37	5.7838	66.6	5.7211

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