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RELATIVISTIC EFFECTS IN THE PULSE PROFILE OF THE 2.5 MILLISECOND X-RAY PULSAR SAX J1808.4−3658

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ABSTRACT

I analyze the properties of the pulsed emission from the accreting millisecond pulsar SAX J1808.4−3658 in observations of its 1998 April outburst by the *Rossi X-Ray Timing Explorer*. Pulse phase spectroscopy shows that the emission evolves from a hard spectrum (power law with photon index of 2.39 ± 0.06) to a soft spectrum (index of 3.39 ± 0.24). This softening is also observable as a phase lag in the fundamental of low-energy photons with respect to high-energy photons. I show that this lag is roughly constant over 10 days of the outburst. I fit these data with a model in which the pulse emission is from a hot spot on the rotating neutron star and the flux as a function of phase is determined in a calculation which includes the effects of general relativity. The energy-dependent lags are very well described by this model. The harder spectra at earlier phases (i.e., as the spot approaches) are the result of larger Doppler-boosting factors that are important for this fast pulsar. Since this model is sensitive to the equatorial speed as an independent parameter and since the spin frequency is known, this offers us a new means of measuring the neutron star radius, which is notoriously difficult to determine.

Subject headings: accretion, accretion disks — black hole physics — stars: neutron — X-rays: stars

1. INTRODUCTION

Strong X-ray pulsations with a 2.49 ms period were discovered from SAX J1808.4−3658 in a 1998 April observation with the *Rossi X-Ray Timing Explorer* (*RXTE*; Wijnands & van der Klis 1998). The pulsar is a member of an accreting binary system with an orbital period of 2.01 hr and a low-mass companion (Chakrabarty & Morgan 1998). Although similar to other low-mass X-ray binaries in its timing and spectral properties (e.g., Wijnands & van der Klis 1999; Heindl & Smith 1998), SAX J1808.4−3658 is unique for its X-ray pulsations. As yet, no other such binary has shown coherent pulsations in its persistent flux. Searches have been conducted, and more are under way (e.g., Vaughan et al. 1994b).

As such, SAX J1808.4−3658 is the fastest known rotating accreting neutron star. If the pulsations are due to modulated emission from one hot spot on the neutron star surface, the 2.49 ms period corresponds to an equatorial speed of approximately 0.1c. With these high speeds, SAX J1808.4−3658 offers us an excellent system for studying relativistic effects.

One such effect may be the observed lag of low-energy photons relative to high-energy photons in the pulse discovered by Cui, Morgan, & Titarchuk (1998). Cui et al. (1998) suggest that the lags are due to Comptonization in a relatively cool surrounding medium. Alternatively, the lags may be the result of a relativistic effect: the high-energy photons are preferentially emitted at earlier phases because of Doppler boosting along the line of sight. This possibility was suggested for the similar lags in the 549 Hz oscillations in an X-ray burst of Aquila X-1 (Ford 1999), in which a simple model showed that the delays roughly match those expected.

In the following, I present new measurements of the pulsed emission from SAX J1808.4−3658. I show that the energy-dependent phase lags are equivalent to a hardening pulse profile. I model this behavior in terms of a hot spot on the neutron star, including relativistic effects.

2. OBSERVATIONS AND ANALYSIS

I have used publicly available data from the proportional counter array (PCA) on board *RXTE* in an "event" mode with high time resolution (122 μs) and high-energy resolution (64 channels). The observations occurred from 1998 April 10 to May 7, when the source was in outburst.

I generate folded light curves in each PCA channel. This is accomplished with the FASEBIN tool in FTOOLS version 4.2, which applies all known *RXTE* clock corrections and corrects photon arrival times to the solar system barycenter using the JPL DE-200 ephemeris, yielding a timing accuracy of several microseconds (much less than the phase binning used here). As a check, I have applied this method to Crab pulsar data, and the results are identical to Pravdo, Angelini, & Harding (1997). To produce pulse profiles in the neutron star rest frame, I use the SAX J1808.4−3658 orbital ephemeris found by Chakrabarty & Morgan (1998). An example folded light curve is shown in the top panels of Figure 1 for the observation of 1998 April 18 14:05:40–April 19 00:51:44 UTC.

To study the energy spectra at each phase bin, I take the rates at pulse minimum and subtract them from the rest of the data at other phases. This effectively accomplishes the background subtraction and eliminates the unpulsed emission, which we do not wish to consider. This approach is exact only to the extent that there is no emission from the cap in the phase minimum. The model described below suggests that this is the case since only one cap is used and since the Doppler-deboosting factors at pulse minimum are large. I generate detector-response matrices appropriate to the observation date and data mode using PCARSP v2.38, and I use XSPEC v.10.0 to fit model spectra.

I fit the spectrum here with a simple power-law function. Although the function itself is not meant to be a physical description, the power-law index provides a good measure of the spectral hardness. Fits in each phase bin have reduced χ^2 of 0.7–1.6. Including an interstellar absorption does not substantially affect these results. The power-law index clearly increases through the pulse phase (Fig. 1, *bottom panel*); i.e., the spectrum evolves from hard to soft. Note that in the following section, I invoke a thermal spectrum in the model for the emission in the rest frame of the cap. And, indeed, for a power-law emission spectrum, the model predicts no phase lags (see

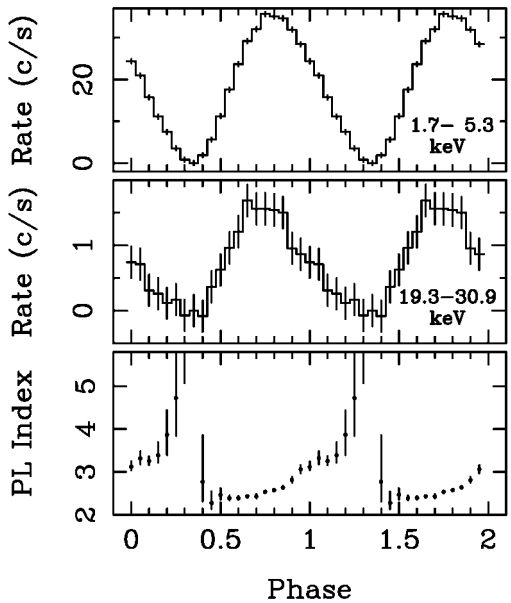

FIG. 1.—An example folded light curve of SAX J1808.4−3658 in a low-energy (*top*) and high-energy (*middle*) band and the index of a power law in the fit of the energy spectrum (*bottom*). The profile is repeated twice for clarity.

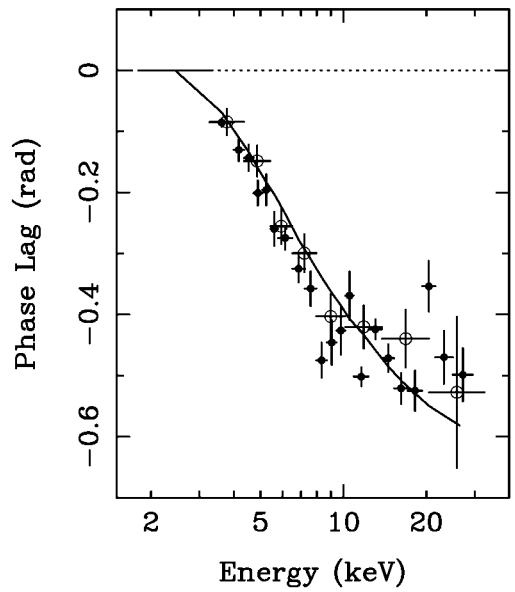

FIG. 2.—Phase delays in the fundamental of SAX J1808.4−3658 relative to the 1.83–3.27 keV band. A negative number indicates that high-energy photons precede low-energy photons. The filled circles indicate measurements from fitting the folded light curves; the open circles denote data from Fourier cross-correlation analysis. The line shows the model for parameters R/M = 5, M = 1.8 $M_\odot$, kT_0 = 0.6 keV, $v = 0.1c$, and $\beta = \gamma = 10^\circ$ as described in the text.

below). For a simple phenomenological model, however, the power law provides a better fit than does a blackbody untransformed for the relativistic effects, and so I have employed a power law here as a model-independent indicator of the spectral hardness. If one does use the simple blackbody, however, the same trends in the spectral parameters are present, i.e., the temperature decreases.

I also fit the profiles of the folded light curves in each energy channel using Fourier functions at the fundamental frequency and its harmonics. From these fits, I determine the phase lag in each channel relative to the fits in some baseline channel range. Results for the 1998 April 18 observation are shown in Figure 2 as filled circles. Note that negative numbers indicate that high-energy photons precede low-energy photons. The lags in the first harmonic are opposite in sign to the fundamental; i.e., low-energy photons precede high-energy photons in the first harmonic. No lags are measurable in the other harmonics.

Another way to measure energy-dependent phase lags is by Fourier cross-correlation analysis. This is the method used for SAX J1808.4−3658 by Cui et al. (1998) and for other timing signals as well (e.g., kilohertz quasi-periodic oscillations [QPOs]; Kaaret et al. 1999). For a description of the cross-correlation analysis, see, e.g., Vaughan et al. (1994a). From the PCA event mode data, I calculate Fourier spectra in various channel ranges with Nyquist frequencies of 2048 Hz and resolutions of 0.25 Hz. I then calculate cross spectra defined as $C(j) = X_1^*(j)X_2(j)$, where X are the complex Fourier coefficients for two energy bands at the pulsar frequency ν_j. The phase lag between the two energy bands is given by the argument of C. I measure all phase delays relative to the unbinned channels 5–8, i.e., 1.83–3.27 keV for five detector units in PCA gain epoch 3 (1996 April 15–1999 March 22). All of the power is in the 0.25 Hz–wide bin centered on the pulsar frequency, and I measure C in this one bin. The results for the 1998 April 18 observation are shown in Figure 2 as the open circles. The phase lags are consistent with those calculated from the light-curve fitting. It is not possible to measure lags in the much weaker harmonics from the cross-correlation spectra.

I have also calculated phase-lag spectra for other *RXTE* observations during the 1998 April outburst. These spectra are similar to the spectrum in Figure 2. To quantify the trends, I compute an average phase delay $\phi_{\rm avg}$ over all energies for each observation. I also fit a broken power-law function to each phase-delay spectrum: $\phi = E^{-\alpha}$ below a break energy E_b, and $\phi = \phi_{\rm max}$ above this energy. Figure 3 shows the quantities $\phi_{\rm avg}$, E_b, and $\phi_{\rm max}$ versus the time of each observation.

There is a clear connection between the results of the two analyses presented here. The phase-resolved spectroscopy shows that the spectrum softens and that, correspondingly, the peak of the pulse profile appears slightly earlier in phase for higher energies (Fig. 1). The method of measuring phase delays shows the same behavior: higher energy photons preferentially lead lower energy photons in the fundamental, and the magnitude of this phase delay increases with energy (Fig. 2). In the following, I discuss a model that can account for the phase delays/spectral softening measured here.

3. MODEL

I calculate the expected luminosity as a function of phase in a manner similar to Pechenick, Ftaclas, & Cohen (1983) and Strohmayer (1992), but I include Doppler effects (Chen & Shaham 1989) and time-of-flight delays (Ftaclas, Kearney, & Pechenick 1986). This treatment is based on a Schwarzschild metric, in which the photon trajectories are completely determined by the compactness, R/M. The predicted luminosity as a function of phase from the code employed here matches the results of Pechenick et al. (1983) and Chen & Shaham (1989) for the various choices of parameters. Braje, Romani, & Rauch

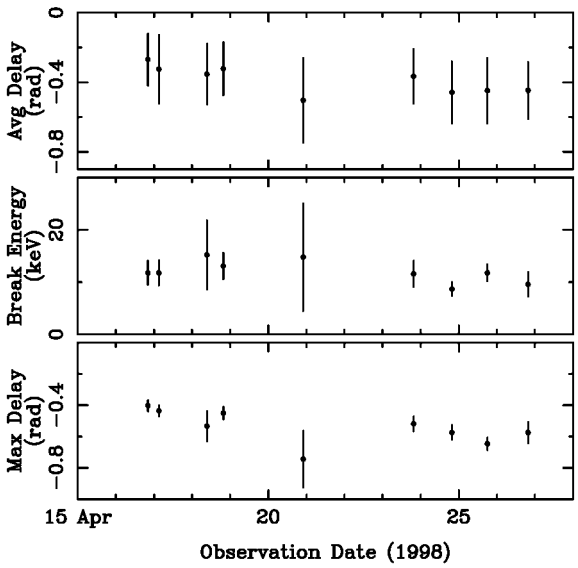

FIG. 3.—Parameters of the Fourier phase-delay spectra for each observation. The average phase delay over all energies (*top*), the break energy (*middle*), and the maximum phase delay (*bottom*) are plotted.

(2000) recently developed a model for pulse profiles using a slightly different approach.

The parameters in the model are the speed at the equator of the neutron star v, the mass of the neutron star M, the compactness R/M, the angular size of the cap α, the angle between the rotation axis and the cap center i, and the angle between the rotation axis and the line of sight γ. Another ingredient is the emission from the spot, which is taken here as isotropic and isothermal. The spectrum of energy emitted from the spot is another important input. Note that if the emitted spectrum is a power law, the observed spectrum will not evolve with phase since Doppler transformation preserves this spectral shape (see Chen & Shaham 1989). The intrinsic spectrum must therefore have some shape that transforms, in order to match the observed hardening and phase lags; I use a blackbody spectrum with temperature kT_0.

In the present model, there is only one cap on the neutron star. However, the predicted flux will be identical to a model in which there are two caps for a wide range of angles because for most angles only one cap is visible. Using two visible caps instead of one requires us to reduce v by a factor of 2, which would yield an equatorial speed that is likely too small given the neutron star radius.

The predicted phase lags are calculated from the model profile in count rates that are found by folding the calculated flux through the detector's response using table models in XSPEC. As with the data (see above), fits of these folded light curves yield the phase offsets as a function of energy.

A fit from the model is shown in Figure 2. This fit uses the following model parameters: $R/M = 5$, $M = 1.8\ M_\odot$, $kT_0 = 0.6$ keV, $v = 0.1c$, $i = \gamma = 10°$, and $\alpha = 10°$. The fit for this single set of parameters is good: $\chi^2 = 5.3$ from the Fourier cross-correlation data or a reduced χ^2 of 0.8 for all the parameters fixed.

For the particular parameters chosen here, the luminosity modulation at 20 keV is roughly 10%, even though the adopted geometry is nearly pole-on. In terms of pulse fractions, this yields a count rate rms fraction of roughly 6% at about 3 keV, decreasing with energy and depending somewhat on the assumed background. This agrees well with the observed rms fractions (Cui et al. 1998). As for the lags, the observed pulse fractions can also be accommodated by other parameter choices.

A full exploration of the parameter space of the model is beyond the scope of this Letter and is left to future work. However, the following trends in the phase lags are notable. The magnitude of the lags depends most sensitively on v and kT_0. Larger delays result from higher speeds because the pulses become increasingly asymmetric as a result of Doppler boosting. This asymmetry is also energy dependent, so there is a dependence on kT_0 as well (see Chen & Shaham 1989). The lags also depend somewhat on i and γ, especially at higher energies where there is the turnover noted above. The phase lags are less sensitive to R/M and α. I have tested the assumption of isotropic emission by using beaming factors of $f = 1$ (isotropic) and $f = \cos(\delta)$ and $f = \sin(\delta)$, where δ is the angle between the direction of the emitted photon and the normal to the surface. The phase lags depend only weakly on these beaming factors.

4. DISCUSSION

The pulsed emission from SAX J1808.4−3658 evolves through its phase from a relatively hard spectrum to a soft spectrum, as shown by the phase-resolved spectroscopy (Fig. 1). This evolution can also be thought of, and measured as, an energy-dependent phase lag in the fundamental (Fig. 2), i.e., higher energy photons emerging earlier in phase than lower energy photons.

I have applied a model to the data that consists of a hot spot on the rotating neutron star under a general relativistic treatment. The dominant effect accounting for energy-dependent delays is Doppler boosting, with the larger boosting factors at earlier phases giving harder spectra. The model fits the data quite well. The model also provides a stable mechanism for generating the phase delays, which meshes with the fact that the characteristic delays remain stable in time to within 25% (Fig. 3), even as there is a factor of 2 decrease in the X-ray flux, a possible tracer of accretion rate. As noted in Ford (1999), in addition to explaining the energy-dependent phase lags in SAX J1808.4−3658, this mechanism may also account for the lags in the burst oscillations of Aql X-1 (Ford 1999) and kilohertz QPOs (Vaughan et al. 1998; Kaaret et al. 1999). Phase-resolved spectroscopy has not yet been possible in these signals.

The model offers us a new means of measuring the neutron star mass and radius, which are notoriously difficult to determine in accreting binary systems. The radius is directly related to v, the equatorial speed ($v = \Omega_{\rm spin} R$). The fits also depend on the model parameters M and R/M, although the lag spectra are less sensitive to these quantities. The model could be independently constrained by future optical spectroscopy of the companion, which could provide information on M and the geometry angles α and i.

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