THE DENSITY PROFILES OF THE DARK MATTER HALO ARE NOT UNIVERSAL

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ABSTRACT

We perform a series of high-resolution *N*-body simulations designed to examine the density profiles of dark matter halos. From 12 simulated halos ranging in mass from 2×10^{12} to 5×10^{14} h^{-1} $M_\odot$ (represented by ~1 million particles within the virial radius), we find a clear systematic correlation between the halo mass and the slope of the density profile at 1% of the virial radius, in addition to the variations of the slope among halos of similar mass. More specifically, the slope is ~ -1.5, -1.3, and -1.1 for galaxy-, group-, and cluster-mass halos, respectively. While we confirm the earlier simulation results that the inner slope is steeper than the *universal* profile originally proposed by Navarro, Frenk, & White, this mass dependence is inconsistent with several analytical arguments attempting to link the inner slope with the primordial index of the fluctuation spectrum. Thus, we conclude that the dark matter density profiles, especially in the inner region, are not universal.

Subject headings: cosmology: miscellaneous — galaxies: clusters: general — methods: numerical

1. INTRODUCTION

Can one recover (some aspects of) the initial conditions of the universe from the distribution of galaxies at $z \sim 0$? A conventional answer to this question is affirmative, *provided* that the effect of a spatial bias is well understood and/or that it does not significantly alter the interpretation of the observed distribution. This consensus underlies the tremendous effort in the past and at present to extract the cosmological implications from the existing and future galaxy redshift surveys. The two-point correlation function $\xi(r)$ is a good example supporting this idea; on large scales, it is trivially related to the primordial spectrum of mass fluctuations $P_i(k)$. Furthermore, the effective power-law index of the two-point correlation function on sufficiently small scales is related to the initial power-law index n_i of $P_i(k) \propto k^{n_i}$ as $\xi(r) \propto r^{-3(3+n_i)/(5+n_i)}$ (e.g., Peebles 1980; Suginohara et al. 1991; Suto 1993). In other words, the initial conditions of the universe are imprinted in the behavior of galaxies on small scales (again, apart from the effect of bias). This is why the phenomenological fitting formulae for the nonlinear power spectrum (Hamilton et al. 1991; Jain, Mo, & White 1995; Peacock & Dodds 1996; Ma 1998) turn out to be so successful. This fact, however, seems to be in conflict with the universal density profile proposed by Navarro, Frenk, & White (1996, 1997) for virialized dark matter halos. In their study, Navarro et al. (1997, hereafter NFW) selected halos that look to be virialized, and they found that the density profiles universally obey the NFW form $\rho(r) \propto r^{-1}(r + r_s)^{-2}$. It is yet unclear to which degree their results are affected by their selection criterion, which is not well defined. In general, different halos should have experienced different merging histories depending on their environment and mass. Thus, even if the halos do have a *universal* density profile *statistically* (i.e., after averaging over many realizations), it is also natural that individual halo profiles are intrinsically scattered around the universal profile (Jing 1999). This is definitely a matter of semantics to a certain extent; the most important finding of NFW is that such halo-to-halo variations are surprisingly small.

A universal density profile was also reported by Moore et al. (1999) on the basis of high-resolution simulations of one cluster-mass halo and four galaxy-mass halos, and they claim that the density profile $\rho(r) \propto r^{-1.5}$ in the most inner region.

In what follows, we will address the following quantitative and specific questions concerning the halo profile, especially its most inner region, using the high-resolution *N*-body simulations. Is the inner slope of the halo profile really described by $\rho(r) \propto r^{-1}$ or $\rho(r) \propto r^{-1.5}$ *universally* as NFW and Moore et al. (1999) claim? If not, does the slope vary among the different halos? Is there any systematic correlation between the slope and the mass of halos?

In fact, some of the above questions have been partially addressed previously with different approaches and methodologies (Fukushige & Makino 1997; Evans & Collett 1997; Moore et al. 1998; Syer & White 1998; Nusser & Sheth 1999; Jing 1999; Avila-Reese et al. 1999). In order to revisit those questions in a more systematic and unambiguous manner, we have developed a nested grid P³M *N*-body code that is designed for the current problem so as to ensure the required numerical resolution with the available computer resources. This enables us to simulate 12 realizations of halos in a low-density cold dark matter (LCDM) universe with $(0.5-1) \times 10^6$ particles in the mass range of $10^{12}-10^{15}$ $M_\odot$.

2. SIMULATION PROCEDURE

As Fukushige & Makino (1997) and later Moore et al. (1998) demonstrated, the inner profile of dark matter halos is substantially affected by the mass resolution of simulations. To ensure the required resolution (at least comparable to theirs), we adopt the following two-step procedure. A detailed description of the implementation and resolution test will be presented elsewhere.

First we select dark matter halos from our previous cosmological P³M *N*-body simulations with 256^3 particles in a $(100 \ h^{-1} \ \text{Mpc})^3$ cube (Jing & Suto 1998; Jing 1998). To be specific, we use one simulation of the LCDM model of $\Omega_0 = 0.3$, $\lambda_0 = 0.7$, $h = 0.7$, and $\sigma_8 = 1.0$ according to Kitayama & Suto (1997). The mass of the individual particle in this simulation is 7×10^9 $M_\odot$. The candidate halo catalog is created using the friend-of-friend grouping algorithm with the bonding length of 0.2 times the mean particle separation.

We choose 12 halos in total from the candidate catalog so that they have mass scales of clusters, groups, and galaxies (Table 1). Except for the mass range, the selection is random,

TABLE 1
PROPERTIES OF THE SIMULATED HALOS

Identification Number	M^{a}	$N_p{}^{\mathrm{b}}$	$r_{\mathrm{vir}}{}^{\mathrm{c}}$	$n_{\mathrm{eff}}{}^{\mathrm{d}}$
GX1	2.30×10^{12}	458,440	0.269	-2.47
GX2	5.31×10^{12}	840,244	0.356	-2.42
GX3	4.21×10^{12}	694,211	0.330	-2.44
GX4	5.60×10^{12}	1,029,895	0.363	-2.42
GR1	4.66×10^{13}	772,504	0.735	-2.31
GR2	4.68×10^{13}	907,489	0.736	-2.31
GR3	5.24×10^{13}	831,429	0.764	-2.30
GR4	5.12×10^{13}	901,518	0.758	-2.30
CL1	4.77×10^{14}	522,573	1.59	-2.12
CL2	3.36×10^{14}	839,901	1.42	-2.14
CL3	2.89×10^{14}	664,240	1.35	-2.16
CL4	3.17×10^{14}	898,782	1.39	-2.15

[a] Mass of the halo within its virial radius, in units of $h^{-1} M_{\odot}$.
[b] Number of particles within its virial radius.
[c] The virial radius, in units of h^{-1} Mpc.
[d] The effective slope of the linear power spectrum at the halo mass scale.

but we had to exclude about 40% of the halos of galactic mass from the original candidates since they have a neighboring halo with a much larger mass. We use the multiple-mass method to resimulate them. To minimize the contamination of the coarse particles on the halo properties within the virial radius at $z = 0$, r_{vir}, we trace back the particles within $3r_{\mathrm{vir}}$ of each halo to their initial conditions at redshift $z = 72$. This is more conservative than that adopted in previous studies and, in fact, turned out to be important for galactic mass halos. Note that we define r_{vir} such that the spherical overdensity inside is $\sim 18\pi^2\Omega_0^{0.4} \sim 110$ times the critical density, $\rho_{\mathrm{crit}}(z = 0)$.

Then we regenerate the initial distribution in the cubic volume enclosing these halo particles with a larger number of particles by adding a shorter wavelength perturbation to the identical initial fluctuation of the cosmological simulation. Next we group fine particles into coarse particles (consisting of at most eight fine particles) within the high-resolution region if they are not expected to enter the central halo region within $3r_{\mathrm{vir}}$. As a result, there are typically 2.2×10^6 simulation particles, $\sim 1.5 \times 10^6$ fine particles, and $\sim 0.7 \times 10^6$ coarse particles for each halo. Finally, about $(0.5–1) \times 10^6$ particles end up within the virial radius of each halo. Note that this number is significantly larger than those of NFW and is comparable to those of Fukushige & Makino (1997) and Moore et al. (1998, 1999). The contamination of the coarse particles, measured by the ratio of the mass of the coarse particles within the virial radius to the total virial mass, is small, about 10^{-4}, 10^{-3}, and 10^{-2} for cluster, group, and galactic halos, respectively.

We evolve the initial condition for the selected halo generated as above using a new code developed specifically for the present purpose. The code implements the nested-grid refinement feature in the original P^3M N-body code of Jing & Fang (1994). Our code implements a constant gravitational softening length in comoving coordinates, and we change its value at $z = 4, 3, 2,$ and 1 so that the proper softening length (about 3 times the Plummer softening length) becomes $0.004r_{\mathrm{vir}}$. Thus, our simulations effectively employ the constant softening length in proper coordinates at $z \leq 4$. The first refinement is placed so as to include all fine particles, and the particle-particle (PP) short range force is added to compensate for the larger softening of the particle-mesh (PM) force. When the CPU time of the PP computation exceeds twice the PM calculation as the clustering develops, a second refinement is placed around the center of the halo with the physical size about 1/3 of that of the first

refinement. The mesh size is fixed to 360^3 for the parent periodic mesh and for the two isolated refinements. The CPU time for each step is about 1.5 minutes at the beginning and increases to 5 minutes at the final epoch of the simulation on one vector processor of Fujitsu VPP300 (peak CPU speed of 1.6 Gflops). A typical run of 10^4 time steps, which satisfies the stability criteria (Couchman, Thomas, & Pearce 1995), takes 700 CPU hours to complete.

3. RESULTS

Figure 1 displays the snapshot of the 12 halos at $z = 0$. Clearly, all the halos are far from spherically symmetric and are surrounded by many substructures and merging clumps. This is qualitatively similar to that found by Moore et al. (1998, 1999) for their high-resolution halos in the $\Omega_0 = 1$ CDM model. The corresponding radial density profiles are plotted in Figure 2. The halo center is defined as the position of the particle that possesses the minimum potential among the particles within the sphere of radius r_{vir} around the center of mass of the fine particles. In spite of the existence of apparent subclumps (Fig. 1), the spherically averaged profiles are quite smooth and similar to each other, as first pointed out by NFW. The inner slope of the profiles, however, is generally steeper than the NFW value, -1, in agreement with the previous findings of Fukushige & Makino (1997) and Moore et al. (1998). We have fitted the profiles to $\rho(r) \propto r^{-\beta}(r + r_s)^{-3+\beta}$ with $\beta = 1.5$ (similar to that used by Moore et al. 1999; Fig. 2, *solid curves*) and $\beta = 1$ (NFW form; Fig. 2, *dotted curves*) for $0.01r_{200} \leq r \leq r_{200}$, where r_{200} is the radius within which the spherical overdensity is $200\rho_{\mathrm{crit}}(z = 0)$. The resulting concentration parameter c, defined as r_{200}/r_s, is plotted in the left panel of Figure 3. This is the most accurate determination of the concentration parameter for the LCDM model. There exists a significant scatter among c for similar mass (Jing 1999) and a clear systematic dependence on halo mass (NFW; Moore et al. 1999).

The most important result is that the density profiles of the four galactic halos are all well fitted by $\beta = 1.5$, but those of the cluster halos are better fitted to the NFW form $\beta = 1$. This is in contrast with Moore et al. (1999), who concluded that both galactic and cluster halos have the inner density profile $\rho(r) \propto r^{-1.5}$, despite that they considered one cluster-mass halo alone. In fact, our current samples can address this question in a more statistical manner. CL1 has significant substructures, and the other three are nearly in equilibrium. Interestingly, the density profiles of CL2 and CL3 are better fitted to the NFW form, and that of CL4 is in between the two forms. The density profiles of the group halos are in between the galactic and cluster halos, as expected. One is better fitted to the NFW form, whereas the other three follow the $\beta = 1.5$ form.

To examine this more quantitatively, we plot the inner slope fitted to a power law for $0.007 < r/r_{200} < 0.02$ as a function of the halo mass in the right panel of Figure 3. This figure indicates two important features: a significant scatter of the inner slope among the halos with similar masses and a clear systematic trend of the steeper profile for the smaller mass. For reference, we plot the predictions for the slope, $-3(3 + n)/(4 + n)$ by Hoffman & Shaham (1985) and $-3(3 + n)/(5 + n)$ by Syer & White (1998), using for n the effective power-law index n_{eff} of the linear power spectrum at the corresponding mass scale (Table 1). With a completely different methodology, Nusser & Sheth (1999) argue that the slope of the density profile within $\sim 0.01r_{\mathrm{vir}}$ is in between the above two values. On the basis of
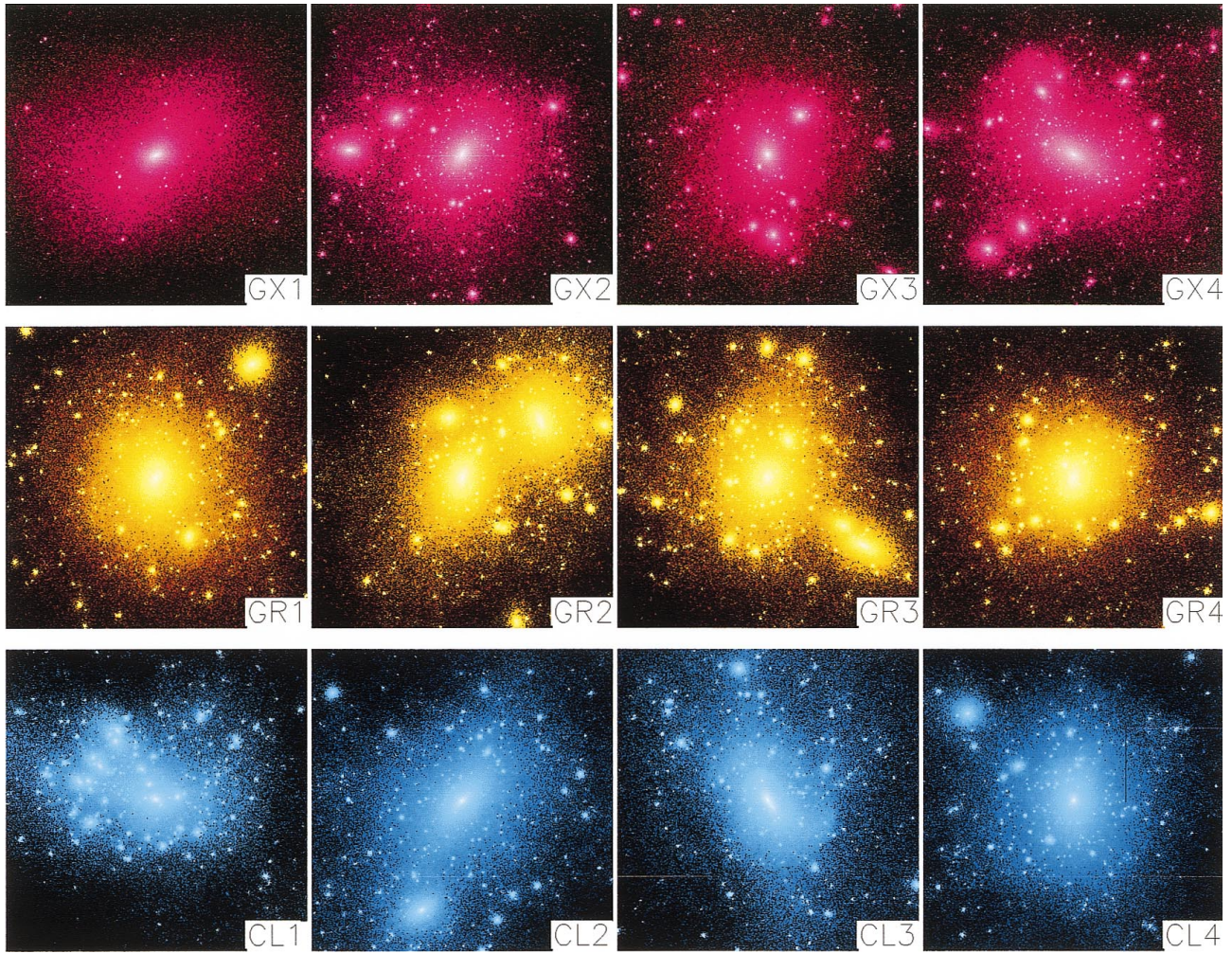
FIG. 1.—Snapshots of the simulated halos at $z = 0$. The top, middle, and bottom panels display the halos of galaxy, group, and cluster masses, respectively (see Table 1). The size of each panel corresponds to $2r_{\rm vir}$ of each halo.

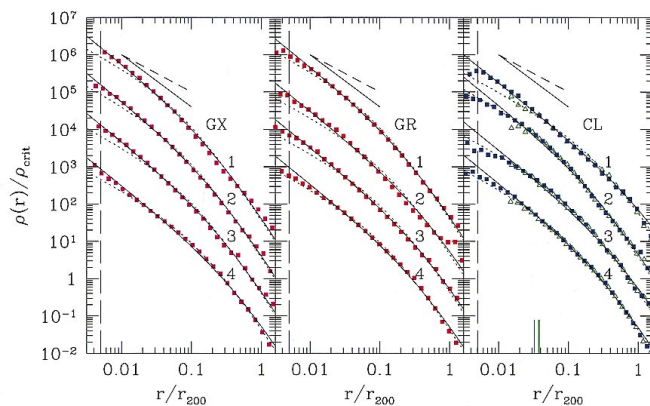the slope-mass relation that we discovered, we disagree with their interpretation; for the galactic halos, the analytical predictions could be brought into agreement with our simulation only if the effective slope were -2, which is much larger than the actual value of -2.5 on the scale.

FIG. 2.—Spherically averaged radial density profiles of the simulated halos of galaxy (*left*), group (*middle*), and cluster (*right*) masses. The solid and dotted curves represent fits of $\beta = 1.5$ and $\beta = 1$, respectively (see text for the fit form). For reference, we also show $\rho(r) \propto r^{-1}$ and $r^{-1.5}$ in dashed and solid lines. The vertical dashed lines indicate the force softening length, which corresponds to our resolution limit. The open triangles in the right panel show the results for the corresponding halos in the lower resolution cosmological simulation, and the long ticks at the bottom mark the force softening of the cosmological simulation. For illustrative purposes, the values of the halo densities are multiplied by 1, 10^{-1}, 10^{-2}, and 10^{-3} from top to bottom in each panel.
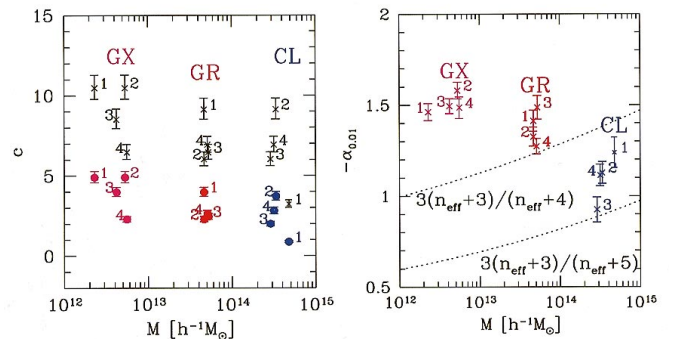
FIG. 3.—*Left*: The concentration parameters for each halo for the Moore et al. (1999) form (*filled circles*) and for the NFW form (*crosses*). The numbers labeling each symbol correspond to the halo identification in Table 1. *Right*: Power-law index of the inner region ($0.007 < r/r_{200} < 0.02$) as a function of the halo mass. The upper and lower dotted curves indicate the predictions of Hoffman & Shaham (1985) and Syer & White (1998), respectively.

We would like to emphasize that our results are robust against the numerical resolution for the following reasons. Since we have used the same time steps and the same force softening length in terms of r_{200}, the resolution effect, which is generally expected to make the inner slope of $\rho(r)$ shallower, should influence the result of the galactic halos more than that of cluster halos. In fact, this is opposite to what we found in the simulation. Furthermore, our high-resolution simulation results agree very well with those of the lower resolution cosmological simulations (*open triangles*) for the cluster halos on scales larger than their force softening length (the short thin lines at the bottom of Fig. 2). We have also repeated the simulations of several halos employing 8 times less particles and a 2 times larger softening length, and we made sure that the force softening length ($\sim 0.005 r_{200}$; the vertical dashed lines in Fig. 2) is a good indicator for the resolution limit.

4. CONCLUSION AND DISCUSSION

In this Letter, we have presented the results of the largest systematic study on the dark matter density profiles. This is the first study that simulates a dozen of the dark halos with about a million particles in a flat low-density CDM universe. This enables us to address the profile of the halos with unprecedented accuracy and statistical reliability. While qualitative aspects of our results are not inconsistent with those reported by Moore et al. (1999), our larger sample of halos provides convincing evidence that *the form* of the density profiles is not universal; instead, it depends on halo mass. Since the mass and formation epoch are linked in hierarchical models, the mass dependence may reflect an underlying link to the age of the halo. Older galactic halos more closely follow the $\beta = 1.5$ form, while younger cluster halos have shallower inner density profiles that are better fitted by the NFW form. Whether this difference represents secular evolution remains to be investigated in future experiments.

Our results are not fully expected with the existing analytical work. Although the analytical work (Syer & White 1998; Nusser & Sheth 1999; Lokas 1999) concluded that the inner profile should be steeper than -1, their interpretation and/or predicted mass dependence are different from our numerical results. This implies that while their arguments may cover some parts of the physical effects, they do not fully account for the intrinsically complicated nonlinear dynamical evolution of nonspherical self-gravitating systems.

We also note that the small-scale power that was missed in the original cosmological simulation has been added to the initial fluctuation of the halos. The fact that each halo has approximately the same number of particles means that more (smaller scale) power has been added to the low-mass halos than to the high-mass ones. It is as yet unclear how much effect this numerical systematics would have on the mass dependence of the inner slope found in this Letter, and we will investigate this question in future work.

In summary, the mass dependence of the inner profile indicates the difficulty in understanding the halo density profile from the cosmological initial conditions in a straightforward manner. Even if the density profiles of dark halos are not universal to the extent claimed by NFW, they definitely deserve further investigation from both numerical and analytical points of view.

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