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ON THE MAXIMUM MASS OF DIFFERENTIALLY ROTATING NEUTRON STARS

Thomas W. Baumgarte, Stuart L. Shapiro,[1] and Masaru Shibata[2]
Department of Physics, University of Illinois at Urbana-Champaign, Urbana, IL 61801
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ABSTRACT

We construct relativistic equilibrium models of differentially rotating neutron stars and show that they can support significantly more mass than their nonrotating or uniformly rotating counterparts. We dynamically evolve such "hypermassive" models in full general relativity and show that there do exist configurations that are dynamically stable against radial collapse and bar formation. Our results suggest that the remnant of binary neutron star coalescence may be temporarily stabilized by differential rotation, leading to delayed collapse and a delayed gravitational wave burst.

Subject headings: black hole physics — relativity — stars: neutron — stars: rotation

1. INTRODUCTION

One of the most important characteristics of a neutron star is its maximum allowed mass. The maximum mass is crucial for distinguishing between neutron stars and black holes in compact binaries and for determining the outcome of many astrophysical processes, including supernova collapse and the merger of binary neutron stars.

Observations of binary pulsars suggest that the individual stars in such systems have masses very close to 1.4 $M_\odot$ (Thorsett et al. 1993). If mass loss during the final binary coalescence can be neglected, the remnant of such a merger will then have a *rest* mass exceeding 3 $M_\odot$. If this mass is larger than the maximum allowed mass for neutron stars, then the merger will lead to prompt collapse to a black hole on a dynamical (millisecond) timescale. If, however, neutron stars can support such a high mass, at least temporarily, then the merger may result in a high-mass, quasi-equilibrium neutron star, which only later may collapse to a black hole. The two different outcomes may have important consequences for gravitational wave signals and, possibly, gamma-ray burst models.

The maximum mass of a cold, nonrotating, spherical neutron star is uniquely determined by the Tolman-Oppenheimer-Volkoff equations and depends only on the cold equation of state. For most recent equations of state, this maximum mass is in the range of 1.8–2.3 $M_\odot$ (Akmal, Pandharipande, & Ravenhall 1998), significantly smaller than the mass expected for the remnant of a binary neutron star merger.

Thermal pressure and uniform rotation can provide additional support and may stabilize slightly more massive stars. In this Letter, we point out that *differential* rotation can significantly increase the maximum allowed mass of neutron stars and may temporarily stabilize the remnant of binary neutron star mergers. Recent fully relativistic simulations of binary neutron star mergers (Shibata & Uryu 1999) show that merger remnants are indeed differentially rotating (as suggested by several Newtonian simulations) and that they may support masses much larger than the maximum allowed mass of spherical stars.

In the case of a head-on collision from infinite separation, thermal pressure alone may support the merged remnant of progenitors (Shapiro 1998). Thermal pressure is likely to have a much smaller effect for coalescence from the innermost stable circular orbit since shock heating on impact is less pronounced and will be dissipated by neutrino emission in ∼10 s.

Rotation can further increase the maximum allowed mass. The maximum mass of a *uniformly* rotating star is determined by the spin rate at which the fluid at the equator moves on a geodesic, and any further speedup would lead to mass shedding. This maximum mass can be determined numerically and is found to be at most ∼20% larger than the nonrotating value (e.g., Cook, Shapiro, & Teukolsky 1992 [hereafter CST], 1994, and references therein). It is therefore unlikely that uniform rotation could support the remnant of a binary neutron star merger. Rotating equilibrium configurations with rest masses exceeding the maximum rest mass of nonrotating stars constructed with the same equation of state are referred to as "supramassive" stars (CST).

The merger of a binary neutron star system, however, will not result in a uniformly rotating object, especially since the neutron stars are likely to be close to being irrotational before merger (Bildsten & Cutler 1992; Kochanek 1992). The remnant is likely to be *differentially* rotating (see Rasio & Shapiro 1999 for discussion and references; Shibata & Uryu 1999). The star's core may then rotate faster than the envelope, and it is easy to imagine that such a star could support a significantly larger mass than its uniformly rotating counterpart (see also Ostriker, Bodenheimer, & Lynden-Bell 1966, in which this effect was demonstrated for white dwarfs). We refer to equilibrium configurations with rest masses exceeding the maximum rest mass of a uniformly rotating star as "hypermassive" stars.

In contrast to the maximum mass of the nonrotating and uniformly rotating stars, the maximum mass of differentially rotating stars cannot be uniquely defined, since the value will depend on the chosen differential rotation law. In principle, one might even construct an extensive Keplerian disk around the equator of the star, possibly increasing the mass of the star by large amounts. Instead of constructing such extreme configurations, we seek to determine whether a reasonable degree of differential rotation can have a significant effect on the maximum mass of neutron stars.

Here we adopt a polytropic equation of state and a simple rotation law to explore the effects of differential rotation on the maximum mass. We construct relativistic equilibrium models and find that even for modest degrees of differential rotation the maximum mass increases significantly, easily surpassing

[1] Department of Astronomy and NCSA, University of Illinois at Urbana-Champaign, Urbana, IL 61801.
[2] Department of Earth and Space Science, Graduate School of Science, Osaka University, Toyonaka, Osaka 560-0043, Japan.

the likely remnant mass of a binary neutron star merger. We then evolve high-mass models dynamically in full general relativity and find that there do exist models that are dynamically stable against both radial collapse and bar formation. These are plausible candidates for binary neutron star remnants.

2. EQUILIBRIUM

We adopt a polytropic equation of state $P = K\rho_0^{1+1/n}$, where P is the pressure and ρ_0 is the rest-mass density. We take the polytropic constant K to be unity without loss of generality and choose the polytropic index $n = 1$.[3]

Relativistic equilibrium models of rotating stars have been constructed by several authors, including Butterworth & Ipser (1975), Friedman, Ipser, & Parker (1986), Komatsu, Eriguchi, & Hachisu (1989), CST, Bonazzola et al. (1993), and Stergioulas & Friedman (1995). A comparison between several different methods can be found in Nozawa et al. (1998). We use the numerical code developed by CST, which is based on the formalism of Komatsu et al. (1989).

Constructing differentially rotating neutron star models requires choosing a rotation law $F(\Omega) = u^t u_\phi$, where u^t and u_ϕ are components of the 4-velocity u^α and Ω is the angular velocity. For simplicity we follow CST and consider the rotation law $F(\Omega) = A^2(\Omega_c - \Omega)$, where Ω_c denotes the central angular velocity and where the parameter A has units of length. Expressing u^t and u_ϕ in terms of Ω and metric potentials yields equation (42) in CST or, in the Newtonian limit, $\Omega = \Omega_c/(1 + \hat{A}^{-2}\hat{r}^2 \sin^2\theta)$. Here we have rescaled A and r in terms of the equatorial radius R_e: $\hat{A} = A/R_e$ and $\hat{r} = r/R_e$. The parameter $\hat{A}$ is a measure of the degree of differential rotation and determines the length scale over which Ω changes. Since uniform rotation is recovered in the limit $\hat{A} \to \infty$, it is convenient to parametrize sequences by $\hat{A}^{-1}$.

We construct axisymmetric differentially rotating models using a modified version of the scheme adopted in CST. Instead of fixing the central density in the iteration scheme for each model, we fix the maximum density. This change allows us to construct higher mass models in some cases, since the central density does not always coincide with the maximum density and hence may not specify a model uniquely. Given a value of $\hat{A}$, we construct a sequence of models for each value of the maximum density by starting with a static, spherically symmetric star and then decreasing the ratio of the polar to equatorial radius, $R_{pe} = R_p/R_e$, in small increments. This sequence ends when we reach mass shedding (for large values of $\hat{A}$) or when the code fails to converge (indicating the termination of equilibrium solutions) or when $R_{pe} = 0$ (beyond which the star would become a toroid).

In Figure 1 we show the maximum mass values in each sequence as a function of the maximum value of the mass-energy density ϵ for different values of $\hat{A}$. Even for modest differential rotation, we can construct models with masses much higher than the maximum mass for static and uniformly rotating stars. Some of these models exceed the Kerr limit $J/M^2 > 1$, where J is the angular momentum and M is the total mass energy of the star.

[3] Since $K^{n/2}$ has units of length, all solutions scale according to $\bar{M} = K^{n/2}M$, $\bar{J} = K^n J$, $\bar{\Omega} = K^{-n/2}\Omega$, etc., where the barred quantities are physical quantities and the unbarred quantities are our dimensionless quantities corresponding to $K = 1$ (compare CST).

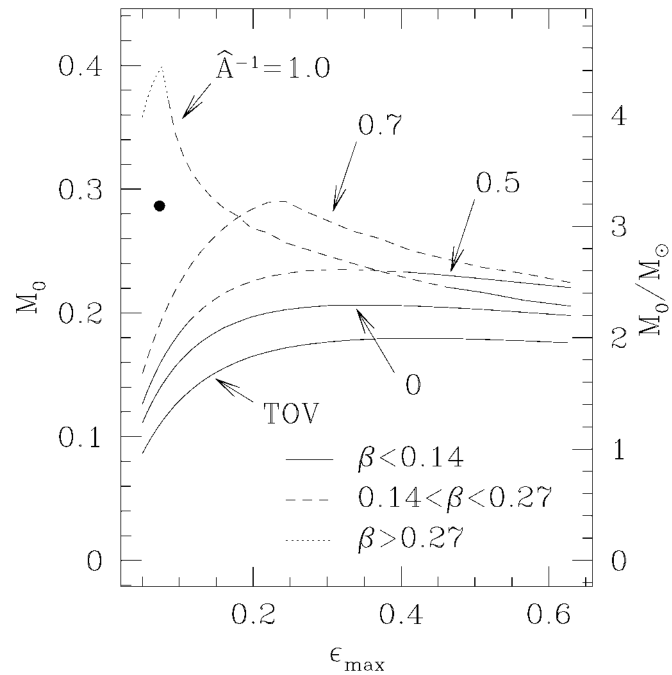

FIG. 1.—Maximum rest-mass configurations vs. maximum mass-energy density for differentially rotating $n = 1$ sequences specified by $\hat{A}^{-1}$. Values of $\beta = T/|W|$ are indicated. The mass-density relation for static equilibrium stars (TOV) is shown for comparison. Masses in solar masses are calculated by assuming that the maximum mass for nonrotating stars is 2 $M_\odot$. Note that even modest differential rotating may easily support $\gtrsim 3\ M_\odot$, the expected mass of binary neutron star merger remnants. We dynamically evolve the model marked with a dot and show that it is dynamically stable (see Fig. 2).

3. STABILITY

Nonrotating spherical stars are dynamically stable (unstable) against radial modes if $\partial M/\partial\epsilon_c > 0$ ($\partial M/\partial\epsilon_c < 0$), where ϵ_c is the central energy density. The same criterion can be applied to sequences of uniformly rotating stars of constant J to determine secular stability (Friedman, Ipser, & Sorkin 1988). Exact criteria do not exist for the dynamical stability of rotating stars; however, numerical simulations of uniformly rotating models suggest that the onset of dynamical stability is very close to the onset of secular instability (Shibata, Baumgarte, & Shapiro 1999).

As an indication of the stability of our models against nonaxisymmetric bar-mode formation, we have indicated values of the ratio of their kinetic energy T to potential energy W, $\beta \equiv T/|W|$, in Figure 1.[4] Newtonian stars develop bars on a dynamical timescale when $\beta \gtrsim \beta_{dyn} = 0.27$, while they develop bars on a *secular* timescale for $\beta \gtrsim \beta_{sec} = 0.14$ via gravitational radiation or viscosity (Chandrasekhar 1969, 1970; Houser, Centrella, & Smith 1994). For relativistic stars, β_{sec} for gravitational wave–driven bars is somewhat smaller than for Newtonian stars (Stergioulas & Friedman 1998), while β_{sec} for viscosity-driven bars is slightly larger (Bonazzola, Frieben, & Gourgoulhan 1996; Shapiro & Zane 1998).

To investigate the dynamical stability of our equilibrium models, we insert them as initial data in a dynamical simulation and evolve them in time. We employ a fully relativistic code that solves Einstein's equations coupled to hydrodynamics in three spatial dimensions plus time (Shibata 1999).

As a candidate for a dynamically stable star, we evolve the

[4] See CST for relativistic definitions of these quantities.

model with $\hat{A}^{-1} = 1.0$, $\epsilon_{\max} = 0.073$, and $R_{pe} = 0.3$ (marked with a dot in Fig. 1). This model has a rest mass about 60% higher than the maximum nonrotating rest mass, $\beta \sim 0.23$, $R_e/M \sim 5$, and $J/M^2 \sim 1$, and is plotted in Figure 2. The orbital period at the equator is about 3 times the orbital period at the center. We show contours at $t = 0$ and after 3.15 orbital periods at the center. Clearly, this model is *dynamically* stable against both quasi-radial collapse to a black hole and bar formation, even when small perturbations are included initially. This demonstrates that differentially rotating stars can stably support significantly higher masses than uniformly rotating stars for longer than a dynamical timescale. A more systematic study of the dynamical stability of differentially rotating neutron stars will be presented in a forthcoming paper (M. Shibata, T. W. Baumgarte, & S. L. Shapiro 1999, in preparation).

Dynamically stable differentially rotating neutron stars are subject to various *secular* instabilities. The timescale for gravitational wave–driven bar-mode formation can be estimated from

$$\tau_{\rm bar} \sim \left(\frac{M}{3\ M_\odot}\right)^{-3}\left(\frac{R}{15\ {\rm km}}\right)^{4}\left(\frac{\beta-\beta_{\rm sec}}{0.1}\right)^{-5}\ {\rm s} \qquad (1)$$

(Friedman & Schutz 1975), where the average radius R and mass M are scaled to values appropriate for a binary merger remnant. For $\beta \sim 0.2$, this yields timescales of 10 s. The final fate of bar-unstable stars is not known, except for incompressible Newtonian spheroids, where in the presence of gravitational radiation and viscosity they evolve to Jacobian or Dedekind ellipsoids (Chandrasekhar 1969; Miller 1974; Shapiro & Teukolsky 1983; Lai & Shapiro 1995). Gravitational waves may also drive an r-mode instability for arbitrarily small rotation rates (see, e.g., Lindblom, Owen, & Morsink 1998). For the hot remnants of binary neutron star mergers, however, these modes may be suppressed by bulk viscosity.

Magnetic braking and viscosity will eventually bring the star into uniform rotation. When a hypermassive star is driven to uniform rotation by viscosity or magnetic fields, it will undergo catastrophic collapse and/or mass loss. The lifetime of a hypermassive star is therefore set by these dissipative processes. If $J/M^2 > 1$, angular momentum must be dissipated either by radiation or mass loss before the star can form a Kerr black hole (see also Baumgarte & Shapiro 1998), which may produce a massive, hot, and thick disk around the newly formed black hole.

A frozen-in magnetic field will be wound up by differential rotation, which may create very strong toroidal fields. This process will generate Alfvén waves, which can redistribute and even carry off angular momentum. The timescale τ_B for this magnetic braking mechanism is related to the Alfvén speed $v_{\rm A} = B/(4\pi\rho)^{1/2}$ according to

$$\tau_B \sim \frac{R}{v_{\rm A}} \sim 10^2\left(\frac{B}{10^{12}\ {\rm G}}\right)^{-1}\left(\frac{R}{15\ {\rm km}}\right)^{-1/2}\left(\frac{M}{3\ M_\odot}\right)^{1/2}\ {\rm s}. \qquad (2)$$

Here B is the initial poloidal field along the gradient of Ω. Strong poloidal magnetic fields can increase the maximum allowed mass of neutron stars (Bocquet et al. 1995) and contribute to the dissipation of angular momentum by dipole radiation, but they are subject to a variety of MHD instabilities (e.g., Spruit 1999a, 1999b).

Since the fluid flow in differentially rotating equilibrium stars is divergence-free, the viscous timescale τ_V is determined by

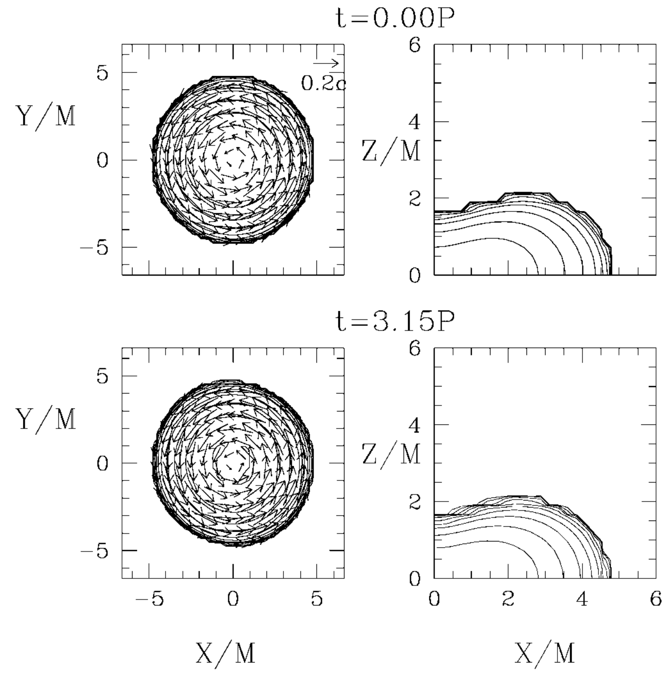

FIG. 2.—Snapshots of contours of the density $\rho_0 u^0(-g)^{1/2}$ of the hypermassive star marked with a dot in Fig. 1. We show contours of the initial data and after about 3 central orbital periods, both in the equatorial plane (*left*, including the velocity field u^i/u^t) and in a plane containing the z-axis of rotation (*right*). The density of successive contours decreases outward by $10^{0.3}$. The star has a rest mass of $M_0 = 0.29$, about 60% larger than the maximum nonrotating rest mass. The simulation shows that this model is dynamically stable against quasi-radial collapse and bar formation.

shear viscosity

$$\tau_V \sim \frac{\rho R^2}{4\eta} \sim 10^9\left(\frac{R}{15\ {\rm km}}\right)^{23/4}\left(\frac{T}{10^9\ {\rm K}}\right)^{2}\left(\frac{M}{3\ M_\odot}\right)^{-5/4}\ {\rm s}, \qquad (3)$$

where $\eta = 347\rho^{9/4}T^{-2}$ (cgs) (Cutler & Lindblom 1987). Molecular viscosity alone is likely to be less effective in bringing the star into uniform rotation than magnetic braking. Nascent neutron stars may also be subject to convective instabilities (e.g., Burrows 1987; Keil, Janka, & Müller 1996; Pons et al. 1999), but the role of convection in rotating magnetic stars is not well understood (see Tassoul 1978).

For weak magnetic fields and high values of β, the neutron star merger remnant is likely to develop a bar. The accompanying quasi-periodic gravitational wave signal may be observable by the new generation of gravitational wave laser interferometers under construction (Lai & Shapiro 1995; M. Shibata et al. 1999, in preparation).

For strong magnetic fields and small values of β, magnetic braking is likely to dominate the evolution of differentially rotating neutron stars and may alter the velocity profile within minutes. On this timescale, differential rotation will no longer be able to support hypermassive stars formed in binary merger. In the resulting delayed collapse, a brief secondary burst of gravitational waves will be emitted. The frequency of this secondary burst may be quite high,[5] but since the angular mo-

[5] The frequency of the fundamental quasi-normal mode of a Schwarzschild black hole is $\omega \sim 0.37M^{-1}$, which yields $f \sim 4$ kHz for $M = 3\ M_\odot$; the frequency of the axisymmetric mode is slightly higher for a Kerr black hole (Leaver 1985).

mentum parameter J/M^2 may be close to unity, the amplitude could be large enough to be observable by an advanced generation of gravitational wave detectors (Stark & Piran 1985). If the orbital parameters, including the masses and radii of the stars, can be determined during the inspiral and early merger phase and if the time of the initial coalescence can be inferred from the initial burst signal, then the measurement of this delay in the final collapse may provide an estimate for the strength of the wound-up magnetic field in the interior of the merged neutron star.

4. DISCUSSION

We find that the maximum mass of a differentially rotating star can be significantly higher than that of nonrotating or uniformly rotating stars, even for modest degrees of differential rotation. As an immediate consequence, it is possible that binary neutron star coalescence does not lead to a prompt black hole formation but that, instead, a differentially rotating, hypermassive quasi-equilibrium neutron star is formed. This has important consequences for the gravitational wave signal from such an event and possibly for the prospects of explaining gamma-ray bursts by binary neutron star mergers.

Pulsars are likely to be uniformly rotating, since magnetic braking and viscosity will bring any initially differentially rotating stars into uniform rotation. The well-established maximum masses of uniformly rotating neutron stars are therefore relevant for old neutron stars, including millisecond pulsars, while our much higher maximum masses may be relevant for nascent neutron stars in a transient phase in a supernova, following fallback, or in a merged binary.

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