# CONSTRAINING THE COSMIC ABUNDANCE OF STELLAR REMNANTS WITH MULTI-TeV GAMMA RAYS

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# ABSTRACT

If stellar remnants are cosmologically significant, the infrared flux from the remnant progenitors would contribute to the opacity of multi-TeV  $\gamma$ -rays. The multi-TeV  $\gamma$ -ray horizon is established to be at a redshift z > 0.034 by the observation of the blazar Mrk 501. By requiring that the optical depth due to  $\gamma \gamma \rightarrow e^+e^-$  be less than 1 for a source at z = 0.034, we limit the cosmological density of stellar remnants,  $\Omega_{\rm rm} \leq (2-4) \times 10^{-3} h_{70}^{-1} (h_{70})$  is the Hubble constant in units of 70 km s<sup>-1</sup> Mpc<sup>-1</sup>), and thus strongly constrain stellar remnants as a cosmologically significant source of dark matter.

Subject headings: diffuse radiation — stars: neutron — white dwarfs

### 1. INTRODUCTION

The constituents of the dark matter observed in galactic halos are, to date, unknown. Gravitational microlensing experiments, MACHO and EROS, have detected six to eight events (MACHOs) in the direction of the Magellanic Clouds (Alcock et al. 1997; Renault et al. 1997; Palanque-Delabrouille et al. 1998). The standard interpretation of the microlensing results is that 50% of the Galactic halo is composed of objects of mass roughly ~0.5  $M_{\odot}$  (Alcock et al. 1997). This interpretation has several problems. Hubble Deep Field star counts (Bahcall et al. 1994; Graff & Freese 1996a) and an extrapolation of parallax data (Dahn et al. 1995) suggest that faint stars (0.08–0.2  $M_{\odot}$ ) and brown dwarfs ( $\leq 0.08 \ M_{\odot}$ ) contribute negligibly to the Galactic halo (<1% of the halo mass: Graff & Freese 1996b; Mera, Chabrier, & Schaeffer 1996; Gould, Flynn, & Bahcall 1998). Gyuk, Evans, & Gates (1998) have also argued that the MACHO lenses cannot be a halo or spheroid population of brown dwarfs.

Stellar remnants are possible MACHO candidates; they have the right mass and can be dark enough, but they also have their problems. Overpollution by the remnant progenitors is difficult to avoid if the stellar remnants make up a significant fraction of the dark matter in galactic halos. If the MACHOs are stellar remnants, then (1) the additional mass density of the gas left over from the progenitors is significant and (2) even if the expelled progenitor mass is acceptable, virtually all of the baryons of the universe have been processed through the progenitors of the MACHOs. It is problematic to hide the gas and/ or metals ejected during the formation of the remnants (Gibson & Mould 1997; Fields, Freese, & Graff 1998). If carbon and nitrogen do not leave the stars, as suggested by Chabrier (1999), then the metal pollution is less severe, although helium is still very restrictive (Fields, Freese, & Graff 1999a). These problems of baryonic mass budget and chemical overproduction are particularly severe for higher mass progenitors that give rise to neutron stars and/or low-mass black holes (Venkatesan, Olinto, & Truran 1999).

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In this Letter, we constrain stellar remnant baryonic dark matter by examining the infrared radiation that is emitted during the evolution of the remnants' progenitors while on the mainsequence and during their subsequent red giant phase. This diffuse infrared background (DIRB) contributes to the opacity of multi-TeV  $\gamma$ -rays via pair creation  $\gamma \gamma \rightarrow e^+e^-$ . Too many infrared photons would prevent the TeV  $\gamma$ -rays from reaching us. Thus, observations of TeV  $\gamma$ -ray sources limit the DIRB (Nishikov 1962; Gould & Schréder 1966; Stecker, de Jager, & Salamon 1992; Dwek & Slavin 1994; Stanev & Franceschini 1998). Recent HEGRA observations of multi-TeV  $\gamma$ -rays from the blazar Mrk 501 (Aharonian et al. 1999) suggest that the universe is optically thin to 10 TeV  $\gamma$ -rays out to z = 0.034and thus limit (Stanev & Franceschini 1998; Funk et al. 1998) the DIRB. (Some evidence of absorption at ~20 TeV [Konopelko et al. 1999] may imply measurement of the DIRB, but see below.) We use this multi-TeV  $\gamma$ -ray horizon to constrain the DIRB expected from remnant halos. We show that  $\Omega_{\rm rm} h_{70} < 4 \times 10^{-3}$ , where  $h_{70}$  is the Hubble constant in units of 70 km s<sup>-1</sup> Mpc<sup>-1</sup> and  $\Omega_{\rm rm}$  is the density of stellar remnants in units of the critical density  $\rho_c = 3H^2/8\pi G$ .

#### 2. MEASURING THE DIRB WITH MULTI-TeV GAMMA RAYS

The universe appears to be optically thin to multi-TeV  $\gamma$ rays out to a redshift of (at least)  $z \sim 0.03$  based on the detection of two blazars, Mrk 421 (z = 0.031) and Mrk 501 (z = 0.034). Neglecting self-absorption, the dominant source of opacity for multi-TeV  $\gamma$ -rays [energy E(z) = (1 + z)E] is pair creation of  $e^+e^-$  off diffuse background photons [energy  $\epsilon(z) = (1 + z)\epsilon$ ] with energy threshold

$$\epsilon_{\rm Th} = \frac{2m_e^2}{E(1 - \cos\theta)(1 + z)^2} = \frac{0.5}{E_{\rm TeV}(1 - \cos\theta)(1 + z)^2} \,\,\text{eV},\tag{1}$$

where  $\theta$  is the relative photon scattering angle in the rest frame of the microwave background,  $E_{\text{TeV}}$  is the observed source photon energy measured in TeV, and we have fixed  $\hbar = k =$ 

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TABLE 1 Stellar Parameters from Our Model

Initial Mass $(M_{\odot})$	Remnant $(M_{\odot})$	$\begin{array}{c} \left\langle T \right\rangle_{\mathrm{MS}} \\ (\mathrm{K}) \end{array}$	$\frac{\log{(E_{\rm MS})}}{({\rm ergs})}$	$\left< T \right>_{\rm HB}$ (K)	$\frac{\log{(E_{\rm HB})}}{({\rm ergs})}$
2	0.68 0.91	13260 19840	63.30 63.59	5424 12880	62.65 63.22
9	1.84	28940	64.04	19290	63.49

c = 1. The  $\gamma \gamma \rightarrow e^+e^-$  cross section is Bethe-Heitler:

$$\sigma_{\gamma\gamma}[E(z), \epsilon(z), \theta] = \frac{3\sigma_{\rm T}}{16} (1 - \beta^2) \\ \times \left[ 2\beta(\beta^2 - 2) + (3 - \beta^4) \ln\left(\frac{1 + \beta}{1 - \beta}\right) \right], \quad (2)$$

where  $\beta^2 \equiv 1 - (\epsilon_{\rm Th}/\epsilon)^2$  and  $\sigma_{\rm T} = 6.65 \times 10^{-25} \text{ cm}^2$  is the Thomson cross section.

The optical depth  $\tau_{\gamma\gamma}$  at observed source energy *E* out to redshift *z* due to a comoving background spectrum  $(1 + z)^3 n(\epsilon) d\epsilon$  is

$$\tau_{\gamma\gamma}(E,z) = \int_0^z dz \frac{dx}{dz} \int_{-1}^1 d\mu \frac{1-\mu}{2} \int_{\epsilon_{\rm Th}}^\infty d\epsilon (1+z)^3 n(\epsilon) \sigma_{\gamma\gamma}, \quad (3)$$

where  $\mu = \cos \theta$  and for a flat universe,  $(1 + z)dx/dz = H^{-1} = H_0^{-1} [\Omega(1 + z)^3 + \Omega_{\Lambda}]^{-1/2} \sim H_0^{-1} z \ll 1$ , with *H* the Hubble parameter,  $\Omega$  the present matter density, and  $\Omega_{\Lambda}$  the present cosmological constant energy density in units of  $\rho_c$ .

#### 3. THE DIRB PRODUCED BY REMNANT HALOS

In order to determine the DIRB produced by the progenitor stars of remnant MACHOs, we start with stellar models of intermediate mass (2–9  $M_{\odot}$ ) and low metallicity (Z = 10<sup>-4</sup>) generated specifically for this project. The models use the nuclear reaction cross sections of Bahcall, Bau, & Pinsonneault (1998). The equation of state is fully ionized in the interior, and the model uses the Saha equation at low temperatures (Guenther et al. 1992). We used the model opacities of Alexander & Ferguson (1994) for T < 10,000 K and OPAL opacities for higher temperatures (Iglesias & Rogers 1996). We assumed gray atmospheres, as is appropriate for these hightemperature models. Our models ran from zero-age main sequence to He core exhaustion. For comparison, Girardi et al. (1996) have also produced models for low-Z isochrones and find results that are very similar; the similarity suggests that our models are theoretically robust. Girardi et al. claim that the post-He core-burning lifetime of the star is of order 0.3% of the main-sequence lifetime and can thus be ignored here. Our models contain no convective overshoot and so conservatively underestimate the total light emitted by stars. Were we to follow the convective overshoot system of Girardi et al. (1996), we would expect stars to live approximately 20% longer, emit a total of 20% more light, and thus our limits would be 20% more restrictive. In addition to the  $Z = 10^{-4}$ models discussed above, we also examined models with Z = $10^{-8}$  and with zero metallicity. We found no substantive changes in the stellar models, with the following exception: for progenitors at the high-mass end (9  $M_{\odot}$ ), stars of the *extremely* low metallicity of  $Z = 10^{-8}$  gave similar results to the low-metallicity  $(Z = 10^{-4})$  models; however, stars of strictly zero metallicity actually never evolved off the main

sequence and produced about half as many infrared photons as the low-metallicity models. Although our limits would then be only half as severe, such a scenario in which all of the stars have not even the merest whiff of metallicity is probably academic.

Note that the mass range of our stellar models  $(2-9 M_{\odot})$  covers all the allowed white dwarf progenitors as well as some low-mass neutron star progenitors (remnant masses are taken from the models of van den Hoek & Groenewegen 1997). Stars below 2  $M_{\odot}$  would leave remnants so bright that they would have been detected (Graff, Laughlin, & Freese 1998). We do not consider progenitor masses larger than 9  $M_{\odot}$  in light of mass budget and chemical abundance problems, which are particularly severe for the highest mass progenitors that give rise to neutron stars and/or low-mass black holes.

We model the spectrum of light emitted by a star at a particular stage of stellar evolution as a blackbody spectral shape characterized by the effective temperature of the star. Due to the energy dependence of the cross section, more than 50% of the interactions of an incident  $\gamma$ -ray occur on background photons in the energy range  $(1 \pm 0.5)4m_e^2/E \sim (1 \pm 0.5)/E_{\text{TeV}}$  eV, where  $E_{\text{TeV}}$  is the  $\gamma$ -ray energy measured in TeV (~0.05–0.15 eV for a 10 TeV  $\gamma$ -ray). Unless there are features in the stellar photon spectrum that are much broader than  $\sim \epsilon_{\rm th}$ , the optical depth is fairly insensitive to the spectral shape and is largely determined by the energy density of background photons in this energy range. Any spectral features, even broad absorption bands, will have only a minimal effect on the optical depth. If anything, realistic spectra would show absorption at the ultraviolet end and reemission of the same energy as more photons at the infrared end (see further discussion of this point below). Thus, a blackbody spectrum conservatively underestimates the number of relevant photons produced by a star.

Stars emit almost all their total energy during two distinct stages of evolution, main sequence (MS) and helium core burning (HB). Thus, we approximate the total energy emitted by a progenitor as the sum of two blackbodies, each marked by an average effective temperature  $\langle T \rangle$  and a total emitted energy  $E_{\text{stage}}$ . These two quantities are determined by integrating over the stellar models as follows:

$$E_{\rm stage} = \int_{\rm stage} dt \, L(t) \tag{4}$$

$$\langle T \rangle = E_{\text{stage}}^{-1} \int_{\text{stage}} dt \, L(t)T(t),$$
 (5)

where T(t) is the effective temperature as a function of time and L(t) is the luminosity. We summarize the stellar parameters adopted in Table 1, where we have given the initial mass of the progenitor, the remnant mass, the average effective temperatures of the main-sequence  $\langle T \rangle_{\rm MS}$  and helium core-burning phases  $\langle T \rangle_{\rm HB}$ , and the log of the corresponding total emitted energies of the two phases.

The total number of photons of energy  $\epsilon$  emitted by a single star can thus be taken as the sum of the number produced by blackbody emission by the MS and red giant (RG) phases,

$$N(\epsilon) d\epsilon = \sum_{i \in \{\text{MS, RG}\}} E_{\text{tot,}i} \frac{15}{(\pi \langle T \rangle_i)^4} \frac{\epsilon^2}{\exp(\epsilon / \langle T \rangle_i) - 1} d\epsilon. \quad (6)$$

Note that this equation has been normalized by requiring that

$$n(\epsilon) = \Omega_{\rm rm} \rho_c m_{\rm rm}^{-1} \int dz \, N[\epsilon(1+z)]. \tag{7}$$

## 4. RESULTS

Over the energy range 1–10 TeV, the observed  $\gamma$ -ray spectrum of Mrk 501 is consistent with acceptable models of the source spectrum (Aharonian et al. 1999; Coppi & Aharonian 1999). That is,  $\tau_{\gamma\gamma} \lesssim 1$  out to z = 0.034 for E < 10 TeV (we comment below on the possibility of absorption above 10 TeV). We can then constrain the mass density of stellar remnants due to the infrared emission by their progenitors. For each progenitor mass in the range 2–9  $M_{\odot}$  and its resultant IR spectrum (calculated as a function of the formation redshift of the progenitor, which is a free parameter), we vary the  $\gamma$ -ray energy in the observed range 1–10 TeV and calculate the optical depth  $\tau_{\gamma\gamma}$ . Combining equations (3) and (7), we find a robust limit of

$$\Omega_{\rm rm} h_{70} < (2-4) \times 10^{-3}. \tag{8}$$

Although we do not expect a significant number of baryonic objects to form before  $z \sim 30$  (Tegmark et al. 1997), our limit applies to progenitor stars created at  $z \leq 60$ . One might worry that there might be decreased sensitivity to low-mass progenitors formed earlier than  $z \sim 30$ . Stars absorb photons at energies above the Balmer jump (3.4 eV); at z = 30, the Balmer jump is redshifted below 0.1 eV, the approximate energy of the IR background most likely to scatter with a 10 TeV  $\gamma$ -ray. Hence these photons are not available to scatter with the TeV  $\gamma$ -rays from Mrk 501. However, the universe is only 70 million years old at  $z \sim 30$ , and therefore these relatively long-lived low-mass progenitors emit most of their light at  $z \leq 30$  and would still contribute significantly to the  $\gamma$ -ray opacity today. More massive progenitors ( $\geq 4 M_{\odot}$ ) are hot enough in their main-sequence phase to emit most of their light at energies higher than the Balmer jump.

Observations of  $\gamma$ -rays from Mrk 501 extend out to 20 TeV, where there are indications of an absorption of  $\tau \sim 3$  (Aharonian et al. 1999). Photons with this high energy are primarily sensitive to IR photons with energy 0.2 eV, that is, to starlight emitted earlier than redshift  $z \ge 60$ . Hence we can place a weaker constraint than equation (8) on progenitors which emit most light at high redshift ( $z \ge 60$ ), since we must allow for the possibility that Mrk 501  $\gamma$ -rays are absorbed by the DIRB at 20 TeV. However, the measured absorption could just as well be due to light emitted within the blazar (Coppi & Aharonian 1999; Konopelko et al. 1999) and does not necessarily represent the detection of the IR background.

Since our limit depends on the number density of photons, which does not vary with redshift, and since we are sensitive to a large range of IR energies, our limit on  $\Omega_{\rm rm}$  is not sensitive to the redshift at which the stellar light is emitted for  $z \leq 30$ . In addition, the limit is relatively insensitive to the star formation rate and to the initial mass of the progenitor star (with

within a narrow range of redshifts ( $\delta z/z \le 1$ ). One possible way to evade our bound is dust. It may be that the progenitor stars are so enshrouded by dust that little of their light escapes, and if the dust is cool enough, that the light is reradiated away at too long a wavelength to interact with the TeV  $\gamma$ -rays. Such a situation is most likely at high redshifts, where the dust could absorb the UV photons that later (by  $z \sim 0.03$ ) would have been redshifted into the infrared (IR). However, at lower redshifts (z < 20), dust could actually make the limits stronger: dust could absorb UV photons and reradiate them in the IR, causing much more absorption in the DIRB. Dust absorption is very model dependent, depending on the type of dust and the dust geometry and temperature, and is beyond the scope of this work.

### 5. INTERPRETATION AND COMPARISON WITH OTHER LIMITS

Our bound,  $\Omega_{\rm rm} h_{70} < (2-4) \times 10^{-3}$ , constrains the cosmological abundance of stellar remnants. Our bound is conservative because we have underestimated the total light emitted by the progenitors of the remnants and because we have neglected other possible sources (e.g., galaxies themselves without remnant halos) of DIRB. Fields et al. (1998; 1999a) also placed a limit on stellar remnants by noting that their progenitors would overproduce carbon, nitrogen, and/or helium unless  $\Omega_{\rm rm} h_{70} < 3 \times 10^{-4}$ . However, if carbon and nitrogen do not leave the stars, as suggested by Chabrier (1999), then their overproduction is less severe and the above bound is weakened, although helium is still very restrictive and requires  $\Omega_{\rm rm} \lesssim 0.003$ . Our new limit in equation (8), based on the DIRB, is more robust than the carbon limit placed by Fields et al. (1998): although the carbon yields may be uncertain, intermediate-mass stars certainly do produce light. Our new limit applies to remnants with any initial mass function (between 2 and 9  $M_{\odot}$ ) and any star formation rate and is thus extremely model independent.

General constraints on all MACHOs, not just stellar remnants, have also been studied. If we assume that the Milky Way is not a special galaxy and that other, similar galaxies also have their coterie of halo MACHOs, then the universe should be filled with MACHOs. For example, Fields et al. (1998) examined the cosmic abundance of MACHOs and found that a simple extrapolation of the (supposed) Galactic population of MACHOs to cosmic scales gives a cosmic density  $\Omega_{\text{MACHO}} = (0.0051 - 0.024) f_{\text{gal}} h_{70}^{-1}$ . Here  $f_{\text{gal}}$  is the fraction of galaxies that contain MACHOs. If at least all spirals within 1 mag of the Milky Way Galaxy contain MACHOs, then  $f_{gal} > 0.17$ . Dalcanton et al. (1994) searched directly for a cosmological population of MACHOs by looking for a signal of amplification of continuum emission of OSOs at high redshift. The fact that they did not find such an amplification allowed them to constrain all compact objects (not just remnants) in the mass range  $0.1-10 M_{\odot}$  to  $\Omega_{\text{MACHO}} \leq 0.1$ . These two results are quite general since they apply to all possible MACHOs and not just stellar remnants. We can compare our new limit with the extrapolation of the Milky Way abundance of MACHOs placed by Fields et al. (1998). If all galaxies contain MACHOs in the same abundance as the Milky Way does, then the MACHOs cannot be stellar remnants.

Alternatively, it may be that  $\Omega_{MACHO} \ll 1$ , with the observed microlensing events due to chance protrusions of the Galaxy and/or the Large Magellanic Cloud (Sahu 1994; Gould 1995; Zhao 1998; Evans et al. 1998). Of course, it is impossible to place limits on the nature of the Milky Way halo based only on cosmological limits: the Milky Way could be the only galaxy in the universe to have MACHOs. Regardless, based on this work, we can say that there must be some galaxies whose halos are not dominated by remnants. Thus, we find that galactic dark

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matter probably does not consist of stellar remnants. Since baryonic dark matter can also not be hot gas, atomic hydrogen, snowballs (Hegyi & Olive 1986; Fukugita, Hogan, & Peebles 1999), dim stars, or substellar objects (see Fields, Freese, & Graff 1999b), we conclude that on galactic scales baryonic dark matter is insignificant.

We thank Julien Devriendt, Andy Gould, and Leo Stodolsky for helpful discussions. This work was supported at Ohio State by Department of Energy grant DE-AC02-76ER01545 and by the Department of Energy at the University of Michigan.

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