The Astrophysical Journal, 517:L63–L66, 1999 May 20

CARTOGRAPHY FOR MARTIAN TROJANS

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Received 1999 January 26; accepted 1999 March 12; published 1999 April 5

ABSTRACT

The last few months have seen the discovery of a second Martian Trojan (1998 VF31) as well as two further possible candidates (1998 QH56 and 1998 SD4). Together with the previously discovered Martian satellite 5261 Eureka, these are the only known possible solar system Trojan asteroids not associated with Jupiter. Here maps of the locations of the stable Trojan trajectories of Mars are presented. These are constructed by integrating an ensemble of in-plane and inclined orbits in the vicinity of the Martian Lagrangian points for between 25 and 60 million years. The survivors occupy a band of inclinations between 15° and 40° and longitudes between 240° and 330° at the L5 Lagrangian point. Around the L4 point, stable Trojans inhabit two bands of inclinations ($15° < i < 30°$ and $32° < i < 40°$) with longitudes restricted between 25° and 120°. Both 5261 Eureka and 1998 VF31 lie deep within one of the stable zones, which suggests that they may be of primordial origin. Around Mars, the number of such undiscovered primordial objects with sizes greater than 1 km may be as high as ∼50. The two candidates 1998 QH56 and 1998 SD4 are not presently on Trojan orbits and will enter the sphere of influence of Mars within half a million years.

Subject headings: minor planets, asteroids — planets and satellites: general — solar system: general

1. INTRODUCTION

For small planetary masses, the Lagrangian points L4 and L5 are stable in the restricted three-body problem comprising the Sun, planet, and massless asteroid (see, e.g., Danby 1988). However, the long-term survival of Trojans around the Lagrangian points of the planets in the presence of perturbations from the remainder of the solar system is a difficult and still unsolved problem (see, e.g., Érdi 1997). Jovian Trojan asteroids have been known since the early years of this century, while a number of Saturnian moons (e.g., Dione and Helene, Tethys and Calypso, and Tethys and Telesto) also form Trojan configurations with their parent planet. However, whether or not there exist Trojan-like bodies associated with the other planets has been a matter of both observational activity (see, e.g., Tombaugh 1961 and Kowal 1972) and intense theoretical speculation (see, e.g., Weissman & Wetherill 1974 and Mikkola & Innanen 1990, 1992). The answer to this problem came in 1990, with the discovery of 5261 Eureka, the first Trojan around Mars (see Mikkola et al. 1994 for details). The last few months of 1998 have seen further remarkable progress with the discovery of one certain Martian Trojan, namely, 1998 VF31, as well as two further candidates, namely, 1998 SD4 and 1998 QH56 (see the Minor Planet Electronic Circulars 1998-W04, 1998-R02, and 1998-S20 and the Minor Planet Circular 33085). The suggestion that 1998 QH56 and 1998 VF31 might be Martian Trojans was first made by G. V. Williams.

These recent discoveries raise very directly the following questions. Are there any more Martian Trojans? If so, where should the observational effort be concentrated? Of course, the first question can only be answered at the telescope, but the resolution of the second question is provided in this Letter. By integrating numerically an ensemble of inclined and in-plane orbits in the vicinity of the Martian Lagrangian points for 25 and 60 million years, respectively, the stable régimes are mapped out. On resimulating and sampling the ensemble of stable orbits, the probability density of Martian Trojans as a function of longitude and inclination can be readily obtained. If a comparatively puny body such as Mars possesses Trojans, the existence of such objects around the larger terrestrial planets also merits very serious attention. There are Trojan orbits associated with Venus and the Earth that survive for tens of millions of years (Tabachnik & Evans 1998). If objects populating such orbits exist, they must be small or else they would have been found by now.

2. MARTIAN TROJANS

Saha & Tremaine (1992, 1994) have taken the symplectic integrators developed by Wisdom & Holman (1991) and added individual planetary time steps in order to provide a fast code that is tailor-made for long numerical integrations of low-eccentricity orbits in a nearly Keplerian force field. In our simulations, the model of the solar system consists of the eight planets from Mercury to Neptune, together with test particles starting near the Lagrangian points. The effect of Pluto on the evolution of Martian Trojans is quite negligible. Of course, the Trojan test particles are perturbed by the Sun and planets but do not themselves exert any gravitational forces. The initial positions and velocities of the planets, as well as their masses, are provided by the JPL Planetary and Lunar Ephemerides DE405, and the starting epoch is JD 2,440,400.5 (1969 June 28). All our simulations include the most important post-Newtonian corrections as well as the effects of the Moon. Individual time steps are invaluable for this work, since orbital periods are much smaller in the inner solar system than in the outer solar system. For all the computations described in this Letter, the time step for Mercury is 14.27 days. The time steps of the planets are in the ratio 1:2:2:4:8:8:64:64 for Mercury moving outward, so that Neptune has a time step of 2.5 yr. The Trojan particles all have the same time step as Mercury. These values were chosen after some experimentation to ensure that the relative energy error has a peak amplitude of $\approx 10^{-6}$ over the tens of millions of years of integration time spans. After each time step, the Trojan test particles are examined to see whether their orbits have become hyperbolic or whether they have entered the planet's sphere of influence (defined as $r_s = a_p M_p^{2/5}$, where a_p and M_p are the semimajor axis and mass of the planet, respectively). If so, they are terminated. In methodology, our calculations are very similar to the magisterial work by Holman & Wisdom (1993) on the Trojan problem for the four giant planets. The earlier calculations of Mikkola & Innanen (1994, 1995) on the Trojans of Mars for time spans of between tens of thousands and 6 million years have also proved influential. Our integrations of Trojan orbits are pursued for durations ranging from 25 to 60 million years, the longest integration periods currently available. Nonetheless, the orbits have been followed for only a tiny fraction of the age of the solar system (∼4.5 Gyr), so it is wise to remain a little cautious about our results.

Figure 1 shows the results of our first experiment. Here the orbits of 1080 Trojan test particles around Mars are integrated for 25 million years. The initial inclinations of the test particles (with respect to the plane of the orbit of Mars) are spaced every 2° from 0° to 90°, and the initial longitudes (again with respect to Mars) are spaced every 15° from 0° to 360°. The starting semimajor axes and the eccentricities of the Trojans are the same as the parent planet. Only the test particles that survive until the end of the 25 million year integration are marked on Figure 1. The survivors occupy a band of inclinations between 10° and 40° and longitudes between 30° and 120° (the L4 Lagrangian point) or 240° and 330° (the L5 point). On the basis of 4 million year time span integrations, Mikkola & Innanen (1994) claim that stable Martian Trojans have inclinations between 15° and 30° and between 32° and 44° with respect to Jupiter's orbit. Our longer integrations seem to suggest a more complex picture. Mikkola & Innanen's instability strip between 30° and 32° can be detected in Figure 1, but only for objects near L4 with initial longitudes $\lesssim 60°$. In particular, this instability strip does not exist around L5, and here Trojans with starting inclinations $30° < i < 32°$ seem to be stable—as is also evidenced by the recent discovery of 1998 VF31. The instantaneous positions of the two certain Martian Trojans are marked on the figure, namely, 5261 Eureka (*square*) and 1998 VF31 (*asterisk*), as well as the two candidates 1998 QH56 (*triangle*) and 1998 SD4 (*diamond*). It is delightful to see that the two securely established Trojans lie within the stable zone, which was computed by Tabachnik & Evans (1998) before the discovery of 1998 VF31. In fact, they live deep within the heart of the zone, suggesting that they may even be primordial. The two candidates (1998 QH56 and 1998 SD4) lie closer to the rim. Let us finally note that Trojans starting off in or near the plane of the orbit of Mars are unstable. This has been confirmed by an extensive survey of in-plane Martian Trojans. On integrating 792 test particles with vanishing inclination but with a range of longitudes and semimajor axes, we found that all are unstable on timescales of 60 million years. Martian Trojans with low inclinations are not expected.

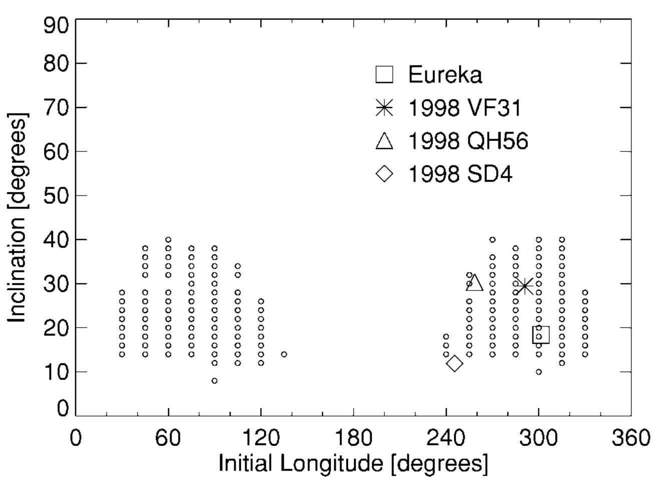

FIG. 1.—The stability zones of the inclined Trojans of Mars. The horizontal axis marks the longitude measured from Mars, and the vertical axis marks the inclination with respect to Mars of the starting positions of test particles. At the outset, the array of particles has inclinations spaced every 2° and longitudes spaced every 15°. The initial semimajor axes and eccentricities of the Trojans are the same as those of Mars. Only the particles that survive until the end of the 25 million year integration are marked on the figure; this provides a map of the stable regions. All the objects starting in-plane do not persist, and only the inclined Trojans are stable. The instantaneous positions of the two Martian Trojans, namely, 5261 Eureka (*square*) and 1998 VF31 (*asterisk*), as well as the asteroids 1998 QH56 (*triangle*) and 1998 SD4 (*diamond*) are also marked.

It is useful to an observer hoping to discover further Trojans to provide plots of the probability density. Accordingly, let us resimulate the stable zones with much greater resolution. This is accomplished by placing a total of 746 test particles every 1° in initial inclination and every 5° in initial longitude so as to span completely the stable regions. This ensemble of orbits is then integrated, and the orbital elements are sampled every 2.5 yr to provide the plots displayed in Figure 2. The upper panel shows the meshed surface of the probability density as a function of both inclination to the invariable plane and longitude with respect to the planet. The asymmetry between the two Lagrangian points is evident. The lower panels show the projections of the meshed surface onto the principal planes—in particular, for the inclination plot, we have shown the contribution at each Lagrangian point separately. There are a number of interesting conclusions to be drawn from the plots. First, as shown by the dotted line, the probability density is bimodal at L4. It possesses a flat maximum at inclinations between 15° and 30° and then falls sharply, before rising to a second maximum at 36°. At L5, all inclinations between 15° and 40° carry a significant probability, although the smaller inclinations in this band are most favored. It is within these inclination windows that the observational effort should be most concentrated. Second, the probability density is peaked at longitudes of ∼60° (L4) and ∼300° (L5). The most likely place to observe one of these Trojans is indeed at the classical locations of the Lagrangian points. This is not intuitively obvious since any individual Trojan is most likely to be seen at the turning points of its longitudinal libration. There are two reasons why this effect is not evident in our probability density plots. First, our figures refer to an ensemble of Trojans uniformly populating the stable zone. So the shape of the stable zone also plays an important role in controlling the position of the maximum of

TABLE 1

PROPERTIES OF TWO DEFINITE MARTIAN TROJANS AND TWO SUGGESTED CANDIDATES

Asteroid	a	e	i (deg)	λ (deg)	H	Diameter (km)
5261 Eureka ……	1.5235	0.0647	20.2802	301.4	16.1	2–4
1998 VF31 ……	1.5241	0.1005	31.2966	290.9	17.1	1–2
1998 QH56 …….	1.5506	0.0307	32.2222	258.5	17.9	1–1.5
1998 SD4 ………	1.5149	0.1254	13.6725	245.5	18.7	0.5–1.2

NOTE.—These properties include the instantaneous semimajor axis a, the eccentricity e, the inclination from the J2000 plane i, and the longitude measured from Mars λ. The epoch is JD 2,451,200.5 (1999 January 22). The magnitude H and the approximate diameter of the object (inferred using albedos of 0.05–0.25) are also given. Most of this information is abstracted from the Minor Planet Circulars 30250 and 33085 (Eureka and 1998 QH56) and the Minor Planet Electronic Circulars 1998-W04 and 1998-S20 (1998 VF31 and 1998 SD4).

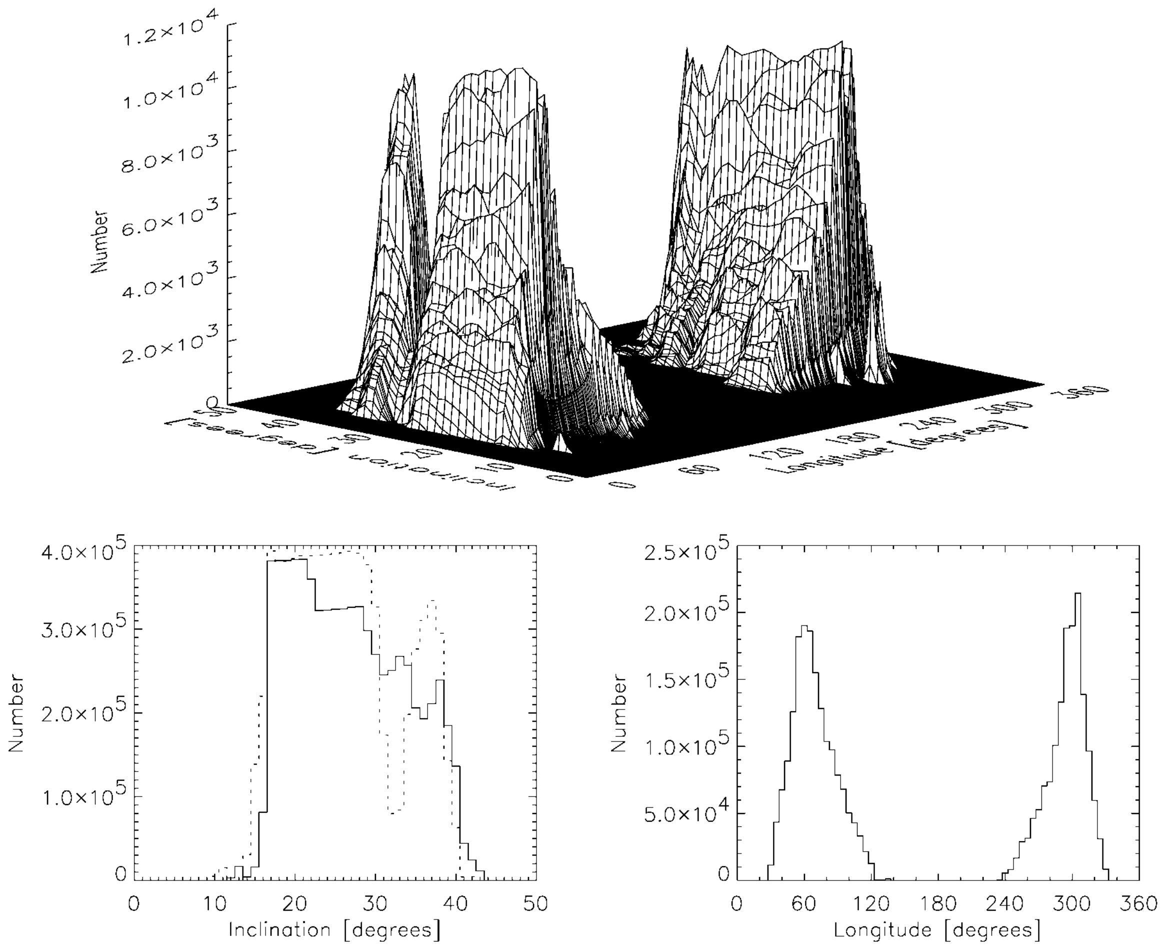

FIG. 2.—The most likely places to observe new Martian Trojans. These figures display the two-dimensional probability density as a function of the inclination with respect to the invariable plane and the longitude with respect to Mars (*upper panel*) together with the projections onto the principal planes (*lower panels*). They are constructed by resimulating the stable regions displayed in Fig. 1 at much greater resolution; 746 test particles are placed every 1° in inclination and every 5° in longitude so as to span the stable region, and the trajectories are sampled every 2.5 yr for 50,000 yr. The overall normalization of the probability density is arbitrary. In the inclination plots, the contributions from the Lagrangian points are separated: the dotted line refers to L4, and the solid lines refer to L5.

the probability density. Second, the positions of the Lagrangian points themselves are oscillating, and so the turning points of the longitudinal libration do not occur at the same locations, thus smearing out the enhancement effect.

Table 1 lists the orbital elements of the two secure Martian Trojans and the two candidates, as recorded by the Minor Planet Center. From the instantaneous elements, it is straightforward to simulate the trajectories of the objects. The amplitudes of libration in the orbital elements are listed in Table 2. Figure 3 shows the orbits plotted in the plane of longitude (with respect to Mars) versus semimajor axis. As the figures illustrate, both 5261 Eureka and 1998 VF31 are stable and maintain their tadpole character (see, e.g., Garfinkel 1977) for durations of 50 million years. Based on preliminary orbital elements, Mikkola et al. (1994) integrated the orbit of 5261 Eureka and found that its longitudinal libration was large, quoting 297° ± 26° as the typical range in the longitudinal angle. Our orbit of 5261 Eureka, based on the latest orbital elements, seems to show a smaller libration from 285° to 314°. The remaining two objects that have been suggested as Martian Trojans, 1998 QH56 and

TABLE 2

PROPERTIES OF ORBITS OF TWO CONFIRMED MARTIAN TROJANS[a]

Asteroid	Δa	Δe	Δi (deg)	$\Delta\lambda$ (deg)	Lagrangian Point	D (deg)
5261 Eureka	5.955×10^{-4}	0.08117	4.8874	29.47	$L5^{+14.3}_{-15.2}$	12.086
1998 VF31	1.386×10^{-3}	0.1128	4.9722	69.38	$L5^{+30.1}_{-39.2}$	47.523

NOTE.—This table gives the maximum variation during the entirety of the 50 million year integration time span in the semimajor axis Δa, in the eccentricity Δe, in the inclination Δi, and in the longitude measured from Mars $\Delta\lambda$. Both Trojans oscillate around the Lagrangian point L5, and the superscript and subscript indicate the extent of the angular libration. Part of this includes the oscillation of the Lagrangian point itself, so the final column D is the peak-to-peak angular libration measured from the Lagrangian point.

[a] Inferred from numerical integrations.

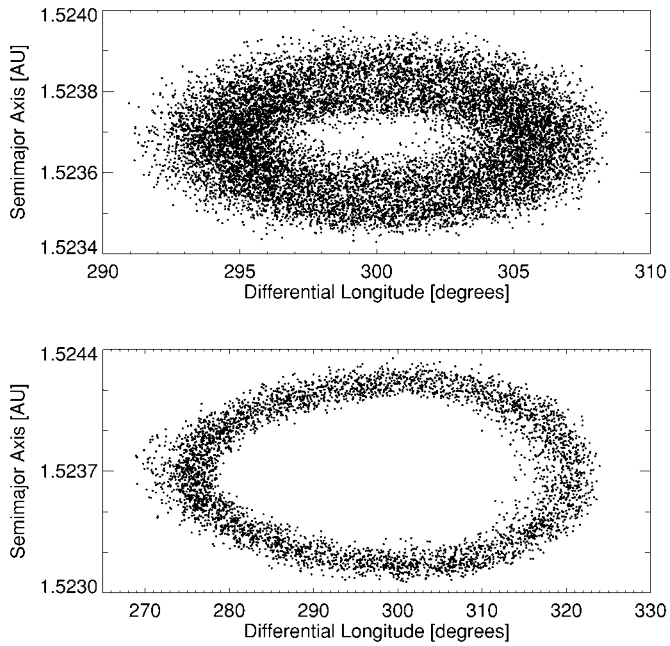

FIG. 3.—The plots of the longitude vs. the semimajor axis are shown for the orbits of 5261 Eureka (*upper panel*) 1998 VF31 (*lower panel*). The orbits are integrated for 50 million years and are sampled every 10,000 years.

1998 SD4, both enter the sphere of influence of Mars—in the former case after ~500,000 yr, in the latter case after ~100,000 yr. Although their orbits cross that of Mars, their eccentricities remain low, and their inclinations oscillate tightly about mean values until Mars's sphere of influence is entered. It is possible that these objects were once Trojans and have been ejected from the stable zones; a possibility that receives some support from their locations in Figure 1 at the fringes of the stable zones. Of course, another possibility is that they are ejected asteroids from the main belt.

The fact that both confirmed Martian Trojans lie deep within the stable zones in Figure 1 suggests that these objects may be primordial. If so, we can get a crude estimate of possible numbers by extrapolation from the number of main belt asteroids (cf. Holman 1997; Evans & Tabachnik 1999). The number of main belt asteroids N_{MB} is $N_{\mathrm{MB}} \lesssim \Sigma_{\mathrm{MB}} A_{\mathrm{MB}} f$, where A_{MB} is the area of the main belt, Σ_{MB} is the surface density of the protoplanetary disk, and f is the fraction of primordial objects that survive ejection (which we assume to be a universal constant). Let us take the main belt to be centered on 2.75 AU with a width of 1.5 AU. The belt of Martian Trojans is centered on 1.52 AU and has a width of $\lesssim$0.0025 AU. If the primordial surface density falls off inversely proportional to distance, then the number of Martian Trojans N_{MT} is

$$N_{\mathrm{MT}} \lesssim \left(\frac{2.75}{1.52}\right)\left(\frac{1.52 \times 0.0025}{2.75 \times 1.5}\right) N_{\mathrm{MB}} \approx 0.0016 N_{\mathrm{MB}}. \tag{1}$$

The number of known main belt asteroids with diameters $\gtrsim$1 km is $\gtrsim$40,000, which suggests that the number of Martian Trojans is $\gtrsim$50.

3. CONCLUSIONS

Motivated by the recent discovery of a new Mars Trojan (1998 VF31) as well as further possible candidates (1998 QH56 and 1998 SD4), this Letter has provided maps of the stable zones for Martian Trojans and estimates of the numbers of undiscovered objects. For Mars, the observational effort should be concentrated at inclinations satisfying $15° < i < 30°$ and $32° < i < 40°$ for the L4 Lagrangian point and between 15° and 40° for L5. These are the spots where the probability density is significant (see Fig. 2), although the lower inclinations in these bands are slightly more favored than the higher. Trojans in or close to the orbital plane of Mars are unstable. Crude estimates suggest that there may be as many as ~50 undiscovered Martian Trojans with sizes $\gtrsim$1 km. The orbits of 5261 Eureka and 1998 VF31 remain Trojan-like for durations of at least 50 million years. The other candidates, 1998 QH56 and 1998 SD4, are not currently Trojans, although it is conceivable that they once may have been. Both objects will probably enter the sphere of influence of Mars after $\lesssim$0.5 million years.

N. W. E. is supported by the Royal Society, while S. T. acknowledges financial help from the European Community. We wish to thank John Chambers, Luke Dones, Seppo Mikkola, Prasenjit Saha, and Scott Tremaine for many helpful comments and suggestions. We are also grateful for the remarkable service to the academic community provided by the Minor Planet Center. The anonymous referee helpfully provided improved orbital elements for the Trojan candidates for our integrations.

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