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THE MODIFIED NEWTONIAN DYNAMICS PREDICTS AN ABSOLUTE MAXIMUM TO THE ACCELERATION PRODUCED BY "DARK HALOS"

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ABSTRACT

We have recently discovered that the modified Newtonian dynamics (MOND) implies some universal upper bound on the acceleration that can be produced by a "dark halo," which, in a Newtonian analysis, is assumed to account for the effects of MOND. Not surprisingly, the limit is on the order of the acceleration constant of the theory. This can be contrasted directly with the results of structure-formation simulations. The new limit is substantial and different from earlier MOND acceleration limits (discussed in connection with the MOND explanation of the Freeman law for galaxy disks and the Fish law for elliptical galaxies): it pertains to the "halo" and not to the observed galaxy; it is absolute and independent of further physical assumptions on the nature of the galactic system; and it applies at all radii, whereas the other limits apply only to the mean acceleration in the system.

Subject headings: galaxies: halos — galaxies: kinematics and dynamics — gravitation

1. INTRODUCTION

The acceleration constant of the modified Newtonian dynamics (MOND), a_0, appears in various predicted regularities pertinent to galaxies. For example, it appears as an upper cutoff to the mean surface density (or mean surface brightness, translated with M/L) of galaxies, as observed and formulated in the Freeman law for disks and in the Fish law for elliptical galaxies. We have now come across another such role for a_0 that had escaped our notice until recently: in spherical configurations, and in those relevant to the rotation-curve analysis of disk galaxies, the excess, $g_h \equiv g - g_{\mathrm{N}}$, of the MOND acceleration, g, over the Newtonian value for the same mass, g_{N}, is universally bounded from above by a value $g_{\max} = \eta a_0$, where η is on the order of 1. Thus, if we attribute effects of MOND to the presence of a fictitious dark halo, $g_{\max}$ is a universal upper bound to the acceleration produced by the "halo," in all systems and at all radii. If the halo is assumed to be quasi-spherical, this can be expressed as a statement on the accumulated (three-dimensional) surface density of the halo, which must obey the universal bound $M_h(r)/r^2 \le \eta a_0 G^{-1}$.

Inasmuch as MOND is successful in explaining the rotation curves of disk galaxies with reasonable stellar M/L values (Sanders 1996; Sanders & Verheijen 1998; de Blok & McGaugh 1998), we can deduce that, indeed, halo accelerations are bounded by $g_{\max}$. This is an important observation regardless of whether MOND entails new physics or whether it is just an economical way of describing dark halos. Newtonian, disk + dark-halo decompositions and rotation-curve fits are rather more flexible because they involve two added parameters for the halo, allowing one to maximize the contribution of the halo and minimizing that of the disk. But reasonable fits do give a maximum halo acceleration. For example, in the dark-halo best fits of Begeman, Broeils, & Sanders (1991), R. H. Sanders (1998, private communication) finds a maximum acceleration of $\sim 0.4 a_0$ for all the galaxies with reasonable fits.

In § 2, we derive this upper bound and explain the assumptions that go into the derivation. Then, in § 3, we compare this new limit with previous MOND limits on the acceleration in galactic systems.

2. DERIVATION OF THE UPPER BOUND

The absolute upper bound on g_h follows simply from the basic MOND relation between the acceleration g and the Newtonian acceleration g_{N}:

$$\mu(g/a_0)g = g_{\mathrm{N}}, \tag{1}$$

with $\mu(x)$ being the interpolating function of MOND. The validity of this relation constitutes part of the underlying assumptions (see below). The excess acceleration $g_h = g - g_{\mathrm{N}}$ can be written as a function of g:

$$g_h = g - g\mu(g/a_0). \tag{2}$$

Now g can take any (nonnegative) value, but, for all acceptable forms of $\mu(x)$, expression (2) has a maximum, which g_h can thus not exceed. Writing $x = g/a_0$ and $y = g_h/a_0$, $y(x) = x[1 - \mu(x)]$ is nonnegative and vanishes at $x = 0$. Thus, it has a global maximum if and only if it does not diverge at $x \to \infty$, i.e., if $\mu(x)$ approaches 1 at $x \to \infty$ (as it must do) no slower than x^{-1}. The parameter η defined above is just this maximum value of $y(x)$. There are solar system constraints on how slowly $\mu(x)$ can approach 1 in the Newtonian limit (Milgrom 1983). Such constraints practically exclude the possibility that $y(x)$ diverges at large x. Some examples are as follows: for $\mu(x) = x/(1 + x)$, the maximum, which is achieved in the Newtonian limit, is $\eta = 1$; for the often used $\mu(x) = x(1 + x^2)^{-1/2}$, $\eta = [(\sqrt{5} - 1)/2]^{5/2} \approx 0.3$; and for $\mu(x) = 1 - e^{-x}$, $\eta = e^{-1} \approx 0.37$ (we see that, in fact, η tends to be rather smaller than 1).

When is expression (1) valid? MOND may be viewed as either a modification of gravity or as a modification of inertia. "Mondified" gravity is described by the generalized Poisson equation discussed in Bekenstein & Milgrom (1984), which is of the form

$$\nabla \cdot [\mu(|\nabla\varphi|/a_0)\nabla\varphi] = 4\pi G\rho, \tag{3}$$

where φ is the (MOND) potential produced by the mass distribution ρ. For systems with one-dimensional symmetry (e.g., in spherically symmetric ones), equation (1) is exact in this theory. It was also shown to be a good approximation for the acceleration in the midplane of disk galaxies (Milgrom 1986; Brada & Milgrom 1995). An exact statement that can be made in this case for an arbitrary mass configuration is that the average value of $|\boldsymbol{g}_h|$ over an equipotential surface of the halo is bounded by $g_{\rm max}$. To see this, note that from equation (3),

$$\nabla\cdot\boldsymbol{g}_h = \nabla\cdot[\boldsymbol{g} - \mu(g/a_0)\boldsymbol{g}] \qquad (4)$$

(because $\nabla\cdot\boldsymbol{g}_{\rm N} = 4\pi G\rho = \nabla\cdot[\mu(g/a_0)\boldsymbol{g}]$). Let us take a Gauss integral for a volume bounded by an equipotential of $\varphi_h \equiv \varphi - \varphi_{\rm N}$. Because $\boldsymbol{g}_h$ is perpendicular to the surface, we have

$$\int [1 - \mu(g/a_0)]\boldsymbol{g}\cdot d\boldsymbol{s} = \int \boldsymbol{g}_h\cdot d\boldsymbol{s} = \int |\boldsymbol{g}_h| ds. \qquad (5)$$

Since we proved that $[1 - \mu(g/a_0)]g \le g_{\rm max}$, the left-hand side is bounded by $g_{\rm max}\int ds$, and so $\langle|\boldsymbol{g}_h|\rangle \equiv \int|\boldsymbol{g}_h|ds/\int ds \le g_{\rm max}$.

There is no concrete theory of mondified inertia yet, but, as was shown in Milgrom (1994), equation (1) is exact in all such theories for circular orbits in an axisymmetric potential. So our limit here would apply, in both versions of MOND, to the halo deduced from rotation-curve analysis.

3. COMPARISON WITH PREVIOUS MOND ACCELERATION LIMITS

The acceleration constant of MOND, a_0, has been found before to define a sort of limiting acceleration in two cases. The first case concerns self-gravitating spheres that are supported by random motions with constant tangential and radial velocity dispersions. The mean acceleration in all such spheres cannot exceed a certain value on the order of a_0 (Milgrom 1984). This was suggested as an explanation of the Fish law, by which the distribution of the central surface brightnesses in elliptical galaxies is sharply cut off above a certain value (which, assuming some typical M/L value, translates into a mean surface density $\Sigma \sim a_0G^{-1}$). The second instance concerns self-gravitating disks. In MOND, disks with a mean acceleration much larger than a_0 are in the Newtonian regime and are less stable than disks in the MOND regime, with mean accelerations smaller than a_0 (Milgrom 1989 and Brada & Milgrom 1998 and references therein). This was suggested as an explanation of the Freeman law in its revised form, whereby the distribution of central surface brightnesses of galactic disks is cut off above a certain value (see McGaugh 1996 for a recent review and further references).

The new limit we discuss here is different from those two in several important regards:

1. The previous limits concern the visible part of the galaxy, while the new limit pertains to the fictitious halo and thus lends itself to direct comparison with predictions of structure-formation simulations, which are rather vague as regards the visible galaxy. At the moment, such simulations are also equivocal on the exact structure of the halo itself. Different simulations start with different assumptions, and the effect of the visible galaxy on the halo is also poorly accounted for. Nonetheless, it may be easy to check for a specific structure-formation scenario, whether it predicts an absolute upper limit to the acceleration in halos of the order predicted by MOND. For example, the family of halos produced in the simulations of Navarro, Frenk, & White (1996) do not seem to have a maximum acceleration, with higher mass halos having higher accelerations exceeding a_0 (S. S. McGaugh 1998, private communication; R. H. Sanders 1998, private communication).
2. The new limit is "mathematical"; i.e., it does not make further assumptions on the physical nature of the galaxy. In contrast, the validity of the previous limits rests on additional assumptions. In the first example, quasi-isothermality and a nondegenerate, ideal-gas equation of state are assumed for the spherical system. The limit then applies neither to normal stars, which are not isothermal, nor to white dwarfs, whose equation of state is not that of an ideal gas. Indeed, these stars have mean accelerations much higher than a_0. In the second example, instability is relied on to cull out disks with high mean acceleration.
3. The former two acceleration limits apply to the mean acceleration in the system, while the new limit applies to the halo acceleration at all radii.

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