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ON THE THEORETICAL PERIOD-RADIUS RELATION OF CLASSICAL CEPHEIDS

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ABSTRACT

We present the results of a comprehensive theoretical investigation of the period-radius relation of classical Cepheids based on new sequences of full-amplitude, nonlinear, convective models constructed by adopting a wide range of both stellar masses and chemical compositions. In the period range $0.9 \leq \log P \leq 1.8$, very good agreement is found between theoretical predictions and current available data, whereas outside this range, at both shorter and longer periods, nonlinear radii attain intermediate values between empirical relations based on different Baade-Wesselink methods and photometric bandpasses.

Subject headings: Cepheids — galaxies: stellar content — stars: fundamental parameters — stars: oscillations

1. INTRODUCTION

The Baade-Wesselink (BW) method (Baade 1926; Wesselink 1946) has been receiving growing attention from the astronomical community since it allows the direct measurement of both radii and absolute magnitudes. Even though some physical assumptions of this method were questioned (Karp 1975; Gautschy 1987; Bono, Caputo, & Stellingwerf 1994; Butler, Bell, & Hindsley 1996), in the last few years, a paramount effort has been undertaken to improve its accuracy and consistency (Barnes & Evans 1976; Sollazzo et al. 1981; Laney & Stobie 1995, hereafter LS; Ripepi et al. 1997, hereafter RBMR). At the same time, Krockenberger, Sasselov, & Noyes (1997, hereafter KSN) have recently developed a new BW method, based on Fourier coefficients, for evaluating the uncertainty of mean stellar radii due to individual measurement errors.

Substantial improvements in the measurements of both Cepheid radii and distances were thoroughly discussed in several outstanding papers by LS and, more recently, by Laney (1997), Di Benedetto (1997), and Gieren, Fouqué, & Gómez (1997, hereafter GFG). Despite these ongoing observational efforts, theoretical investigations devoted to the Cepheid period-radius (PR) relation based on up-to-date evolutionary and pulsational models are lagging. In fact, LS, by comparing the PR relation derived for a sample of 49 Galactic Cepheids with Fernie's (1984) weighted mean theoretical PR relation, found that the slope of the empirical relation is steeper than the slope of the theoretical one and that BW radii are 12% smaller than the theoretical ones for a period equal to 10 days. On the other hand, RBMR, by adopting a new version of the CORS method (Sollazzo et al. 1981 and references therein), found, as expected (LS), that the slope of their PR relation is slightly shallower when compared with either empirical BW relations based on IR photometry or theoretical relations.

The reason why only a few investigations so far have been devoted to the evaluation of the mean theoretical PR relation is that its slope depends on the intrinsic width of the instability strip. The cool edge of the instability strip can be evaluated only by coupling the local conservation equations with a nonlocal and time-dependent equation for turbulent-convective motions (Stellingwerf 1982; Gehmeyr 1992). Theoretical PR relations available in the literature (Karp 1975; Cogan 1978) are based on radiative models and therefore cannot be considered "pure" theoretical relations. In fact, radiative models can fix only the location of the blue edge, whereas the temperature width of the instability strip is inferred from observational data. As a consequence, both the zero point and the slope of these "semitheoretical" PR relations depend on the completeness of the adopted sample and on the relations used for transforming the mean colors into mean effective temperatures. Moreover, Karp's and Cogan's relations have been derived by assuming that the width of the instability strip is constant when moving from short- to long-period Cepheids. However, this assumption is not supported by observational estimates; indeed, in a seminal investigation, Pel (1980) showed that the Cepheid instability region is not a rectangular-shaped but a wedge-shaped strip; i.e., the color range narrows toward short-period Cepheids. The main aim of this investigation is to establish the Cepheid PR relation on a genuine theoretical basis by adopting the mean radii and the periods predicted by full-amplitude, nonlinear, convective models and then to compare theoretical with empirical PR relations.

2. PULSATIONAL MODELS

Several sequences of envelope models were constructed by adopting four different stellar masses ($M/M_\odot$ = 5.0, 7.0, 9.0, 11.0) and two luminosity levels for each mass value. The luminosity levels were fixed according to the mass-luminosity (ML) relations predicted by both canonical (no overshooting) and noncanonical (mild overshooting, $\lambda_{\rm over} = 0.5$) evolutionary models. The former relation was chosen from the calculations of Castellani, Chieffi, & Straniero (1992), whereas the latter was fixed by increasing the canonical luminosity level by 0.25, i.e., $\log (L/L_\odot)({\rm NC}) = \log (L/L_\odot)({\rm C}) + 0.25$ (see, e.g., Chiosi et al. 1992 and Chiosi, Wood, & Capitanio 1993). Our investigation is also focused on the dependence of the PR relation on He and metal contents, and therefore calculations were performed by adopting three different chemical compositions that are representative of Cepheids in the Small ($Y = 0.25$, $Z = 0.004$) and Large ($Y = 0.25$, $Z = 0.008$) Magellanic Clouds and in the Galaxy ($Y = 0.28$, $Z = 0.02$). The models were arranged in sequences characterized by constant mass, luminosity, and chemical composition but by different values of the effective temperature ($4000 \leq T_e \leq 7000$ K). The physical and numerical assumptions adopted for performing the linear, nonadiabatic analysis, as well as the nonlinear, full-amplitude anal-

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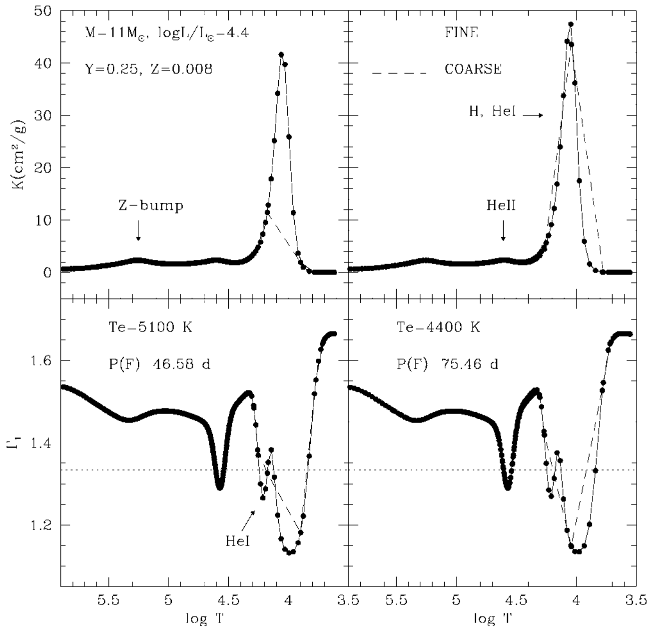

FIG. 1.—Opacity (*top*) and adiabatic exponent (*bottom*) as a function of the logarithmic temperature for two models located close to the blue (*left*) and red (*right*) edges of the instability strip. Solid and dashed lines refer to linear models with fine and coarse spatial resolutions in the H and He I ionization regions, respectively. The dotted lines plotted in the bottom panels display the edge between dynamically stable ($\Gamma_1 > 4/3$) and unstable ($\Gamma_1 \leq 4/3$) regions. The arrows mark the main features of the opacity and of the adiabatic exponent.

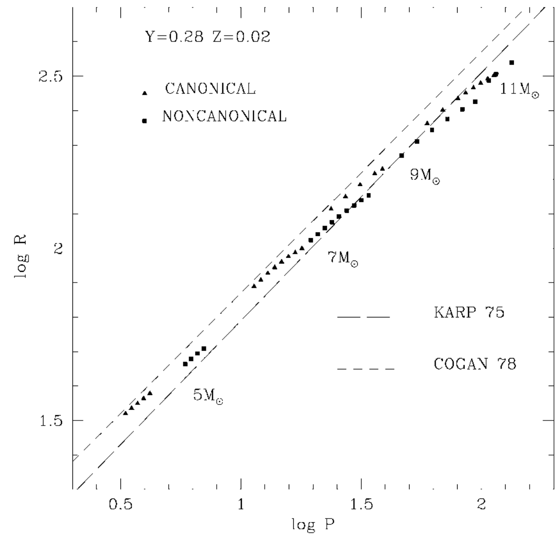

FIG. 2.—Comparison between different theoretical PR relations at solar metallicity. Triangles and squares show the nonlinear radii obtained by adopting a canonical and a noncanonical ML relation, respectively. The short-dashed line refers to the PR relation provided by Cogan (1978), while the long-dashed line refers to the PR relation provided by Karp (1975). Fernie's relation has not been plotted here since it is almost identical to Cogan's.

ysis, have already been the subjects of previous papers (Bono & Stellingwerf 1994; Bono & Marconi 1997; and references therein), and therefore they are not discussed further here. The theoretical framework we developed proved to be successful in reproducing the observational properties (amplitudes and modal stability) of Cepheids characterized by periods shorter than 40 days. This notwithstanding, we found that high-mass models ($M/M_\odot = 11.0$) present peculiarities in the nonlinear limit cycle stability. In fact, light and velocity curves display irregularities such as sharp bumps and sudden dips during both contraction and expansion phases. Moreover, pulsational properties undergo substantial changes over consecutive periods. A similar behavior was found by Christy (1975), who pointed out that both pulsation irregularities and very large amplitudes take place only in models with high radius/mass, period/radius, and period/luminosity ratios.

In order to investigate the intimate nature of this phenomenon, a detailed analysis of the dependence of the limiting amplitude behavior on physical and/or numerical assumptions was undertaken. We found that pulsation irregularities are caused by the coarse spatial resolution in the H and first He (He I) ionization regions. In fact, these layers, owing to their large back and forth motion, undergo a large excursion in both temperature and density over a full cycle. The coarse spatial resolution causes a sudden increase in the temperature and density gradients and, consequently, the formation and propagation of strong spurious shocks. This is a typical limit of the Lagrangian models when compared with the adaptive grid models. In order to solve this problem, we constructed a new sequence of linear models. The main differences between these equilibrium models and the standard ones are the following.

Standard models are constructed, following Stellingwerf (1975), by anchoring the opacity peak of the H ionization regions (HIR) and by locating a proper number of zones (20–30) between this peak and the surface layer. Instead of improving the accuracy by simply increasing the number of zones located above this peak, we developed a new method that, by means of a multiple iteration on the mass of the surface zone, ensures, via a secant method, a uniform sampling (ΔT = 500–650 K) of the layers located between the surface and the base of the H and He I ionization regions ($T \approx 2.1 \times 10^4$ K). The left- and the right-hand panels of Figure 1 show the opacity and the adiabatic exponent Γ_1 of two models located close to the blue and red edges, respectively, of the instability strip. The fine models show two substantial differences when compared with the coarse ones: (*a*) *Their adiabatic index attains smaller values and resolves the* He I *ionization zone*. The He I region is dynamically unstable; indeed, close to $\log T \approx 4.2$, the Γ_1 is smaller than 4/3. (*b*) *Their opacity peak attains larger values*. The differences are a factor of 4 in the blue models and on the order of 10% in the red models. The same differences were found by Gehmeyr (1992) in his comparison of two static RR Lyrae models constructed by adopting a Lagrangian and an adaptive grid code, respectively. It is hardly necessary to point out the role played by these changes in the instability and pulsation amplitudes of long-period Cepheids.

The nonlinear radii and periods discussed in this investigation were evaluated by adopting fine-zoning models. Figure 2 shows the comparison between the theoretical PR relations at solar metallicity provided by Cogan (1978) and Karp (1975) and our models constructed by adopting canonical (*triangles*) and noncanonical (*squares*) ML relations. The nonlinear radii of canonical Cepheids are larger than those of the noncanonical ones, with a difference ranging from 4% at $\log P \approx 0.6$ to 7% at $\log P \approx 2.0$. Interesting enough, our nonlinear radii are quite similar to the radii predicted by Cogan's relation in the range $0.6 \leq \log P \leq 1.2$, whereas toward longer periods they first attain values similar to those given by Karp's relation and then become systematically smaller than the radii predicted by the

TABLE 1

THEORETICAL PR RELATIONS ($\log R = \alpha + \beta \log P$)

Z[a]	α[b]	β[c]	r[d]
Canonical			
0.02 …….	1.188 ± 0.008[e]	0.655 ± 0.006	0.999
0.008 ……	1.192 ± 0.009	0.666 ± 0.007	0.998
0.004 ……	1.199 ± 0.010	0.670 ± 0.008	0.998
Noncanonical			
0.02 …….	1.174 ± 0.009	0.647 ± 0.006	0.999
0.008 ……	1.183 ± 0.009	0.653 ± 0.006	0.999
0.004 ……	1.183 ± 0.009	0.661 ± 0.006	0.998

[a] Metallicity.
[b] Zero points of the PR relations.
[c] Slopes of the PR relations.
[d] Correlation coefficients of the linear regression.
[e] The errors refer to the intrinsic dispersion.

quoted relations. This difference is mainly due to the proper location of red boundaries without invoking ad hoc assumptions and, more marginally, to new opacities. Table 1 summarizes the zero points and the slopes of the linear regression obtained by adopting different compositions and ML relations. An interesting result is that the average PR relations show a mild but nonnegligible dependence on metal content. In fact, for canonical radii, an increase in the metal content from $Z = 0.004$ to $Z = 0.008$ leads to a decrease that ranges from 2% at $\log P \approx 0.6$ to almost 4% at $\log P \approx 2.0$. An increase in both He and metal contents ($Y = 0.28$, $Z = 0.02$ vs. $Y = 0.25$, $Z = 0.004$) implies a decrease that ranges from 4% at $\log P \approx 0.6$ to 9% at $\log P \approx 2.0$. A similar outcome is obtained for noncanonical radii.

3. COMPARISON BETWEEN THEORY AND OBSERVATIONS

Figure 3 shows the empirical PR relations for Galactic Cepheids obtained by GFG (*short-dashed line*), Laney (1997; *long-dashed line*), and CORS (*dotted line*). Theory and observations were also compared by plotting canonical periods and radii of models with $Z = 0.008$ and $Z = 0.02$. We adopted two different compositions to account for the spread in metal content recently found by Fry & Carney (1997) among calibrating Galactic Cepheids. The comparison brings out two major results: (*a*) Theoretical predictions are, within the observational errors, in good agreement with average empirical PR relations obtained by adopting different methods and different photometric bands. In the period range $0.9 \leq \log P \leq 1.8$, observed and theoretical radii are almost identical. (*b*) Theoretical radii are systematically larger than the observed ones in the period range $0.4 \leq \log P \leq 0.6$, whereas they are smaller toward longer periods, i.e., $\log P > 1.8$. Both the paucity of long-period Cepheids detected in the Galaxy and the lack of a detailed analysis of the systematic errors involved in empirical PR relations based on different methods and/or photometric bands prevent a quantitative explanation of this discrepancy.

However, KSN have recently provided a thorough analysis of the uncertainty in the radius estimates introduced by individual measurement errors. Their results on the slope of the PR relation for Galactic Cepheids agree fairly well with those of RBMR, who adopted a CORS method that accounts for the loop performed by the variable in a color-color plane, the main advantage of this method being its independence from reddening corrections. Moreover, Di Benedetto (1997) obtained a very precise general PR relation by adopting both Galactic and Magellanic Cloud Cepheids for which high-precision photometric and spectroscopic data were available. In this method, the use of both magnitude and colors in evaluating stellar angular sizes ensures a marginal dependence of radii on both reddening and metallicity.

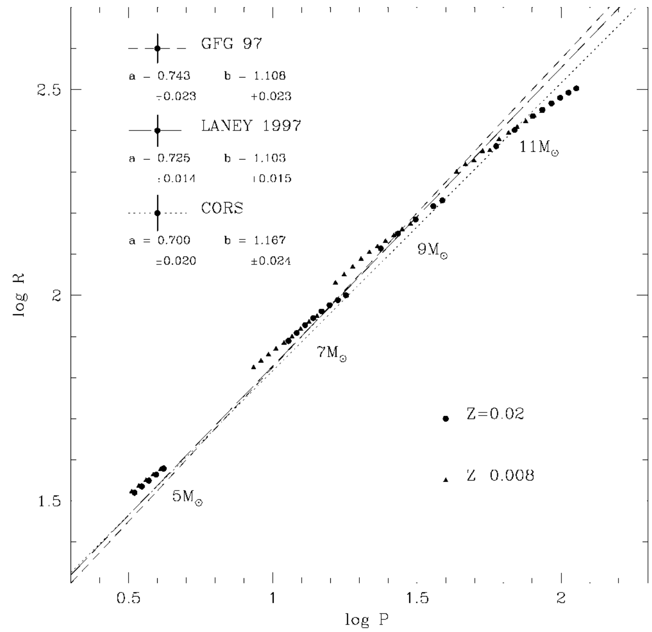

FIG. 3.—Comparison between current empirical PR relations for Galactic Cepheids and theoretical nonlinear radii obtained by adopting two different chemical compositions.

Figure 4 shows the last two empirical PR relations: the results obtained by KSN and the theoretical predictions for the three chemical compositions. The comparison discloses once again

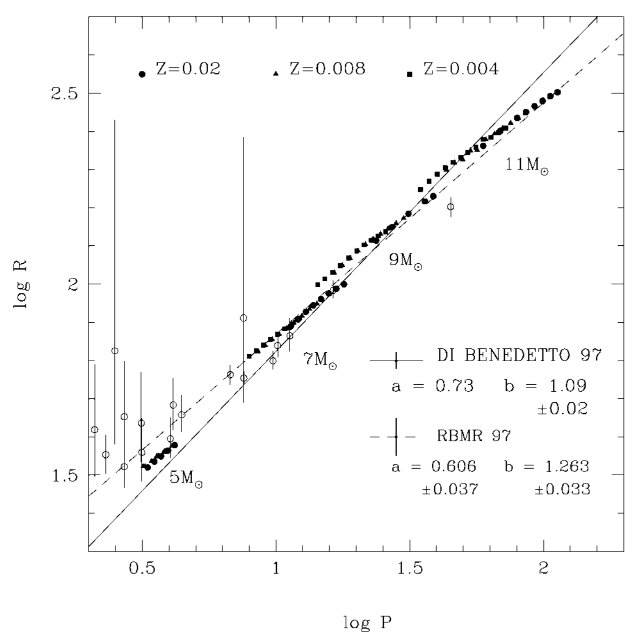

FIG. 4.—Comparison between different empirical PR relations and theoretical results. The solid and dashed lines show Di Benedetto's and RBMR's relations, respectively. The former is based on both Galactic and Magellanic Cloud Cepheids, whereas the latter is based on Galactic Cepheids only. Open circles refer to the mean radii for Galactic Cepheids obtained by KSN on the basis of visual magnitudes and $B - V$ colors.

a remarkable agreement between theory and observations. The major discrepancy is in the short-period range, in which theoretical radii are smaller than the radii obtained by KSN and RBMR and larger than the radii provided by Di Benedetto's relation. However, firm constraints on this observational discrepancy cannot be drawn since, as KSN clearly stated, the uncertainty in the mean radii is dominated by the error in the phase difference between color index and magnitude. Moving toward short-period Cepheids, this difference becomes smaller, and in turn the uncertainty becomes larger. This trend is reversed in the long-period range, and indeed for periods longer than 30 days, the radii obtained by KSN and RBMR are systematically smaller than the estimates of other authors, with theoretical radii being located once again between these two different estimates. This finding confirms the results obtained by LS concerning the systematic error that affects radius estimates; i.e., by neglecting the variation of the effective gravity over the pulsation cycle, we find that the radii based on optical bands systematically underestimate (overestimate) the radii of long- (short-) period Cepheids. Since Cepheid radii are proportional to the *p*-factor, i.e., the factor adopted for converting observed radial velocities into pulsational velocities, we suspect not only that this parameter is phase-dependent and that its value depends on both the BW method and the data sets adopted for estimating the radii (see, e.g., KSN) but also that it should attain smaller values in long-period Cepheids observed in the IR bands. In fact, the data in Figure 4 suggest that the dependence of *p* on period is stronger than that predicted by the Gieren, Barnes, & Moffett (1989) relation.

4. CONCLUSIONS

We developed a new theoretical scenario of the actual properties of classical Cepheids in the Galaxy and in the Magellanic Clouds. By adopting both radii and periods predicted by full-amplitude, nonlinear, convective models, we found that the use of two different ML relations based on canonical and noncanonical (mild overshooting) evolutionary models has a marginal effect on the PR relation; indeed, in the mean PR relation, the difference is on the order of 3%. At the same time, we also found that an increase in the metal content implies a decrease in the mean radius. This effect is not constant but increases when moving from short- to long-period Cepheids. In particular, a change in the chemical composition from $Y = 0.25$, $Z = 0.004$ to $Y = 0.28$, $Z = 0.02$ implies a decrease in the mean radius at $\log P \approx 2$ on the order of 9%. This result suggests that, within the current accuracy of both photometric and spectroscopic data, the dependence of the PR relation on metallicity could be detected and measured if a proper number of long-period variables ($P > 40$ days) are included in the sample.

Theoretical and empirical radii are found in very good agreement in the period range $0.9 \leq \log P \leq 1.8$ but present some discrepancies toward short- and long-period Cepheids. No firm conclusion was reached on the intimate nature of this discrepancy since current mean stellar radii estimated by adopting different BW methods, photometric bands, and data sets present a large scatter both at $\log P < 0.7$ and at $\log P > 1.8$. Comparison between theory and observations suggests that the value of the *p*-factor could change when moving from short- to long-period Cepheids. At the same time, the results of this investigation disclose a new approach for testing the internal accuracy and the consistency of the assumptions adopted by the different BW methods. In fact, observables predicted by nonlinear, convective models can be fed to the progeny of the BW method for assessing the intervening effects of systematic errors and/or of possible biases in the radius measurements.

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