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LIGHT-CONE EFFECT ON HIGHER ORDER CLUSTERING IN REDSHIFT SURVEYS

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ABSTRACT

We have evaluated a systematic effect on counts-in-cells analysis of deep, wide-field galaxy catalogs induced by the evolution of clustering within the survey volume. A multiplicative correction factor is explicitly presented, which can be applied after the higher order correlation functions have been extracted in the usual way, without taking into account the evolution. The general theory of this effect combined with the *Ansatz* describing the nonlinear evolution of clustering in simulations enables us to estimate the magnitude of the correction factor in different cosmologies. In a series of numerical calculations assuming an array of cold dark matter models, it is found that, as long as galaxies are unbiased tracers of underlying density field, the effect is relatively small ($\simeq 10\%$) for the shallow surveys ($z < 0.2$), while it becomes significant (of order unity) in deep surveys ($z \sim 1$). Depending on the scales of interest, the required correction is comparable to or smaller than the expected errors of ongoing wide-field galaxy surveys such as the Sloan Digital Sky Survey and the 2 Degree Field Survey. Therefore, at present, the effect has to be taken into account for high-precision measurements at very small scales only, while in future deep surveys it amounts to a significant correction.

Subject heading: cosmology: theory — dark matter — galaxies: distances and redshifts — large-scale structure of universe

1. INTRODUCTION

Cosmological observations are necessarily carried out on a null hypersurface or a light cone. At low redshifts ($z < 0.1$), this can be regarded as providing information on the constant-time hypersurface ($z = 0$), which is a quite conventional implicit approximation underlying cosmological studies using the galaxy redshift surveys. When the depth of the survey volume exceeds $z \sim 0.1$, however, this approximation breaks down, and one should simultaneously take account of the intrinsic evolution of galaxy clustering, and the light-cone effect in addition to any other selection effect, in interpreting the data. This is indeed the case for the ongoing wide-field surveys of galaxies including the 2 Degree Field Survey (2dF) and the Sloan Digital Sky Survey (SDSS).

To our knowledge, the first quantitative consideration of the light-cone effect has been made by Nakamura, Matsubara, & Suto (1998), who derived the systematic bias in the estimate of $\beta \approx \Omega_0^{0.6}/b$ from magnitude-limited surveys of galaxies combining the cosmological redshift distortion effect (Matsubara & Suto 1996) and the evolution of galaxy clustering within the survey volume. In this paper we examine the light-cone effect on higher order statistics of galaxy clustering, considering counts-in-cells analysis specifically.

Let us consider first the higher order statistics on the idealistic constant-time hypersurface. Denote the volume-averaged N-th-order correlation functions at a redshift z by $\bar{\xi}_N(R; z)$, where R is the comoving smoothing length, and introduce the normalized higher order moments $S_N(R; z) \equiv \bar{\xi}_N(R; z)/[\bar{\xi}_2(R; z)]^{N-1}$. The hierarchical clustering *Ansatz* states that $S_N(R; z)$ is constant and independent of the scale R. This is a good approximation in nonlinear regimes, although small but definite scale dependence is clearly detected from N-body experiments (Lahav et al. 1993; Suto 1993; Matsubara & Suto 1994; Suto & Matsubara 1994; Jing & Börner 1997). In addition, perturbation theory predicts that $\bar{\xi}_N(R; z)$ evolves in proportion to $[\bar{\xi}_2(R; z)]^{N-1}$, and therefore $S_N(R; z)$ is independent of time, i.e., it is constant with respect to z.

The next section describes the general theory of the light-cone effect on $S_N(R; z)$ defined above. Using the *Ansatz* by Jain, Mo, & White (1995, hereafter JMW), § 3 evaluates the appropriate correction in an array of cold dark matter (CDM) models. Finally, § 4 summarizes the results and discusses the implications for redshift surveys.

2. OBSERVING THE HIGHER ORDER MOMENTS ON THE LIGHT CONE

It is difficult to estimate $\bar{\xi}_N(R; z)$ observationally, since z is changing over the volume of the galaxy sample. While in principle one could measure the N-point functions on $z =$ constant surfaces, in practice this would result in a diminished volume and thus a significant increase in the errors. Instead it is more practical to extract the following N-th-order correlation functions averaged over the volumes on the light cone:

$$\bar{\bar{\xi}}_N(R; < z_{\rm max}) \equiv \frac{\int_0^{z_{\rm max}} z^2\, dz\, w(z)\, \bar{\xi}_N(R; z)}{\int_0^{z_{\rm max}} z^2\, dz\, w(z)}\,. \tag{1}$$

In the above expression, we assume that the observation is performed with the fixed solid angle, and the sampling cells for the

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TABLE 1
CDM MODEL PARAMETERS

Model	Ω_0	λ_0	h	σ_8
SCDM (standard CDM)	1.0	0.0	0.5	0.6
OCDM (open CDM)	0.45	0.0	0.7	0.8
LCDM (λ-CDM)	0.3	0.7	0.7	1.0

analysis are placed randomly in the z-coordinate with $w(z)$ being their weighting function. If the cells were located randomly in the comoving coordinates, the volume element $z^2\,dz$ should have been replaced by $d_A(z;\,\Omega_0,\lambda_0)^2 c|dt/dz|dz$ (d_A is the angular diameter distance; see Nakamura et al. 1998), and thus the procedure itself becomes dependent on adopted values of Ω_0 and λ_0. In principle $w(z)$ is an arbitrary function, and should be determined so as to maximize the signal-to-noise ratio given the selection function of individual observation. By $z_{\max}$ we denote the redshift corresponding to the depth of the survey. For a volume-limited sample, for instance, it is natural to set $w(z) = 1$ and $z_{\max}$ as the maximum redshift of the sample. Similarly, we define the (observable) higher order moments averaged over the light cone as

$$\overline{S_N}(R;<z_{\max}) \equiv \frac{\bar{\xi}_N(R;<z_{\max})}{[\bar{\xi}_2(R;<z_{\max})]^{N-1}}\,. \tag{2}$$

It is useful to introduce the function $G(z)$, which describes the evolution of the averaged two-point correlation function:

$$\bar{\xi}(R;\,z) = G(z)\bar{\xi}(R;\,0). \tag{3}$$

In the linear regime, $G(z)$ is equivalent to $[D(z)/D(0)]^2$, where $D(z) = D(z;\,\Omega_0,\lambda_0)$ is the linear growth rate. Although the above relation (3) does not hold exactly in the nonlinear regime, several approximation formulae are derived in the literature that empirically describe the evolution by allowing $G(z)$ to depend on the scale R (see § 3 for details).

Once we accept the evolution law (eq. [3]), equation (2) is explicitly written as

$$\overline{S_N}(R;<z_{\max}) = \frac{\int_0^{z_{\max}} z^2\,dz\,w(z)S_N(R;\,z)\{G(z)\}^{N-1}[\int_0^{z_{\max}} z^2\,dz\,w(z)]^{N-2}}{[\int_0^{z_{\max}} z^2\,dz\,w(z)G(z)]^{N-1}}\,. \tag{4}$$

If $z_{\max} \ll 1$, the above expression is expanded in terms of $z_{\max}$ as follows:

$$\begin{aligned}\overline{S_N}(R;<z_{\max}) &= S_N(R;\,0) + \frac{3}{4}S_N'(0)z_{\max}\\ &+ \left[\frac{3}{160}(N-1)(N-2)S_N(0)G'(0)^2 + \frac{3}{80}(N-1)S_N'(0)G'(0) + \frac{3}{10}S_N''(0)\right]z_{\max}^2\\ &+O(z_{\max}^3),\end{aligned} \tag{5}$$

where $S_N'(0)$ denotes $\partial S_N(R;\,z)/\partial z|_{z=0}$ and so on. The above expansion up to $O(z_{\max}^2)$ is valid as long as the weighting function is well approximated up to the same order:

$$w(z_{\max}) = w(0) + w'(0)z_{\max} + \frac{1}{2}w''(0)z_{\max}^2. \tag{6}$$

In other words, $z_{\max}$ should be set to be smaller than the effective window size of $w(z_{\max})$.

It is interesting to note that up to $O(z_{\max}^2)$ equation (5) is independent of $w(z)$, and that the $O(z_{\max})$ term is determined only by $S_N'(0)$ independently of $G(z)$. Since $S_N'(0)$ is expected to vanish in linear theory (Fry 1984; Goroff et al. 1986; Bernardeau 1992), and is shown to be relatively small even in quasi- and fully nonlinear regimes (Bouchet et al. 1992; Lahav et al. 1993; Colombi, Bouchet, & Hernquist 1995; Szapudi et al. 1997), equation (5) implies that the light-cone effect is very small for the 2dF and SDSS galaxy redshift surveys ($z_{\max} < 0.2$). It should be noted, however, that if galaxies are biased relative to the mass density field, $S_N'(0)$ may not necessarily be small. So any signal proportional to $z_{\max}$ provides a clear indication of the time-dependent biasing of galaxies (see, e.g., Fry 1996; Mo & White 1996; Mo, Jing, & White 1997).

3. EVALUATING THE LIGHT-CONE EFFECT: AN EXAMPLE

In order to evaluate the effect of observational average on the light cone, we assume that $S_N(R;\,z)$ does not evolve with z, i.e.,

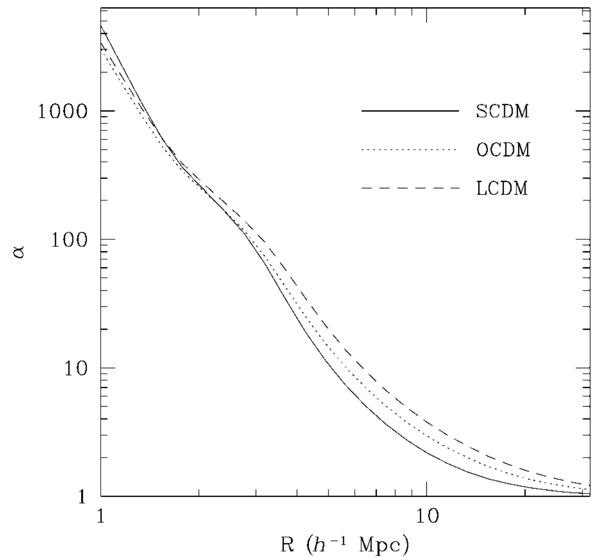

FIG. 1.—$\alpha(R)$ is plotted against $\log_{10} R(1\ h^{-1}$ Mpc) for SCDM (*solid line*), OCDM (*dotted line*), and LCDM (*dashed line*) summarized in Table 1.

$S_N(R; z) = S_N(R; 0)$. As described above, this is a reasonable approximation as long as galaxies are unbiased tracers of the underlying density field. If we introduce the measure of the light-cone effect:

$$\Delta_N(R; < z_{\rm max}) \equiv \frac{\overline{S_N}(R; < z_{\rm max})}{S_N(R; 0)} - 1, \tag{7}$$

equations (4) and (5) with $S_N(R; z) = S_N(R; 0)$ reduce to

$$\Delta_N(z_{\rm max}) = \frac{3}{160}(N-1)(N-2)G'(0)^2 z_{\rm max}^2 + O(z_{\rm max}^3). \tag{8}$$

Note that $(1 + \Delta_N)$ can be regarded as a correction factor as well, if one measures values of S_N *without* considering the evolution of clustering. This constitutes a simple and practical method, which we propose for future measurements, when compensation for the light-cone effect is required.

To evaluate equation (4), we need a model for $G(z)$. For this purpose, we adopt the *Ansatz* originally put forward by Hamilton et al. (1991) and improved later by Peacock & Dodds (1994, 1996) and JMW. To be specific, we apply the fitting formula by JMW which relates the evolved two-point correlation function $\xi_E(R; z)$ to its linear counterpart $\xi_L(R_0; z)$ as follows:

$$\bar{\xi}_E(R; z) = B(n)F[\bar{\xi}_L(R_0; z)/B(n)], \tag{9}$$

$$F(x) = \frac{x + 0.45x^2 - 0.02x^5 + 0.05x^6}{1 + 0.02x^3 + 0.003x^{9/2}}. \tag{10}$$

In the above equations, n denotes a power-law index of the power spectrum, $R_0 = [1 + \bar{\xi}_E(z, R)]^{1/3}R$, and $B(n) = [(3 + n)/3]^{0.8}$. JMW show that the above formula works reasonably well even for CDM models by replacing n by the effective spectral index evaluated at the scale which is just entering the nonlinear regime. In general the resulting n depends on z, which we neglect below for simplicity; for galaxy surveys, in which we are primarily interested, the z-dependence of n near $z = 0$ is expected to be very small.

The inverse of equation (9) is formally written as $\bar{\xi}_L(R_0; z) = B(n)F^{-1}[\bar{\xi}_E(R; z)/B(n)]$, and JMW's empirical fit to $F^{-1}(y)$ is

$$F^{-1}(y) = y\left(\frac{1 + 0.036y^{1.93} + 0.0001y^3}{1 + 1.75y - 0.0015y^{3.63} + 0.028y^4}\right)^{1/3}. \tag{11}$$

Then $\bar{\xi}_E(R; z)$ is expressed explicitly in terms of $\bar{\xi}_E(R; 0)$:

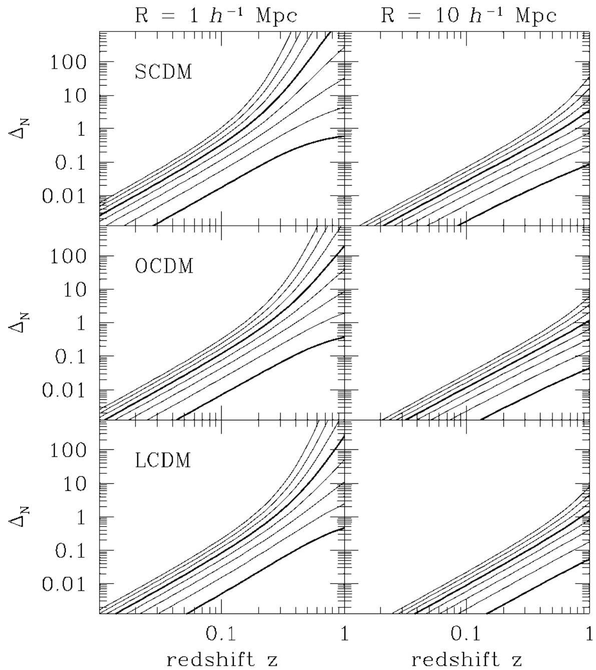

FIG. 2.—Values of $\log_{10} \Delta_N(R; z)$ are shown as functions of $\log_{10} z$ at $R = 1\ h^{-1}$ Mpc (*left-hand panels*) and $10\ h^{-1}$ Mpc (*right-hand panels*); SCDM (*top panels*), OCDM (*middle panels*), and LCDM (*bottom panels*). The family of curves display different orders from $N = 3$ to $N = 10$ monotonically upward; the curves for $N = 3$ and $N = 7$ are plotted with thick lines for orientation.

$$\bar{\xi}_E(R; z) = B(n)F\left\{\frac{D^2(z)}{D^2(0)}F^{-1}\left[\frac{\bar{\xi}_E(R, 0)}{B(n)}\right]\right\}. \tag{12}$$

Let us introduce a parameter $\alpha(R) \equiv F^{-1}[\bar{\xi}_E(R, 0)/B(n)]$, which characterizes the variance on a scale R at $z = 0$ and thus depends on Ω_0 and λ_0 through the shape of the fluctuation spectrum. Then the scale-dependent evolution factor $G(z) = G(R; z)$ in equation (3) is given by

$$G(R; z) \equiv \frac{\bar{\xi}_E(R; z)}{\bar{\xi}_E(R; 0)} = \frac{1}{F(\alpha)}F\left[\frac{D^2(z)}{D^2(0)}\alpha\right]. \tag{13}$$

For the convenience of z-expansion, we calculate the derivatives of the above quantity at $z = 0$:

$$\left.\frac{\partial G(R; z)}{\partial z}\right|_{z=0} = -2f_0\frac{\alpha F'(\alpha)}{F(\alpha)}, \tag{14}$$

$$\left.\frac{\partial^2 G(R; z)}{\partial z^2}\right|_{z=0} = 4f_0^2\frac{\alpha^2 F''(\alpha)}{F(\alpha)} + (2f_0^2 + 2f_0 q_0 + 3\Omega_0)\frac{\alpha F'(\alpha)}{F(\alpha)}, \tag{15}$$

where $f_0 = d\ln D/da|_{z=0}$, $q_0 = \Omega_0/2 - \lambda_0$. The above expressions indicate how the light-cone effect depends on Ω_0 and λ_0 at $z_{\max} \ll 1$. Note that they are involved in the $O(z_{\max}^2)$ term and thus do not contribute significantly at small z.

4. RESULTS AND CONCLUSIONS

Using equations (4) and (13) and assuming $S_N(z) = S_N(0)$, we can evaluate the evolutionary effect on $\overline{S_N}(R; < z_{\max})$ or $\Delta_N(< z_{\max})$. As examples, we consider three representative CDM models (Table 1) whose fluctuation amplitude σ_8 is normalized so as to reproduce the abundances of clusters of galaxies (e.g., Kitayama & Suto 1997; Kitayama, Sasaki, & Suto 1998). The results are displayed on a series of figures. Figure 1 shows how α is related to the comoving smoothing length R in these models; Figure 2 displays $\Delta_N(R; z)$ as a function of z; and, finally, Figure 3 plots $\Delta_N(R; z)$ against R.

The general appearance of the figures suggests that the light-cone effect is a fairly robust feature, although its details depend on the model. In all cases, standard CDM (SCDM) appears to give the strongest effect, while for open CDM (OCDM) and λ-

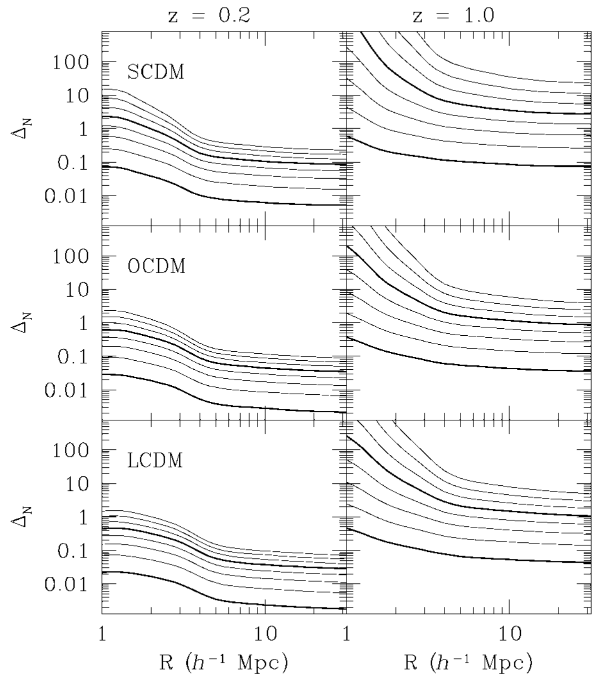

FIG. 3.—Values of $\log_{10} \Delta_N(R; z)$ are displayed as functions of $\log_{10} R$ at $z = 0.2$ (*left-hand panels*) and 1.0 (*right-hand panels*); SCDM (*top panels*), OCDM (*middle panels*), and LCDM (*bottom panels*). The family of curves is the same as for Fig. 2.

CDM (LCDM) it is slightly less pronounced. Nevertheless the difference is fairly small, and qualitatively all models behave similarly. Note also that the magnitude of the correction depends on the order N, and, in accord with intuition, it is monotonically increasing for higher order. As expected, the light-cone effect becomes larger as $z_{\max}$ increases, which can be seen in Figure 2. Although the correction is relatively small for shallow surveys with $z \lesssim 0.2$ samples, $\Delta_N(R; < z_{\max})$ becomes $\gtrsim$10% in nonlinear scales ($R \sim 1\ h^{-1}$ Mpc). In SCDM, for instance, $\Delta_N(R; < z_{\max})$ exceeds unity for $N \geq 6$ for the entire dynamic range plotted. Furthermore, Figure 3 indicates that even if the hierarchical *Ansatz* is correct, i.e., $S_N(R; z)$ is independent of R, the light-cone effect should generate apparent scale dependence, since the correction behaves differently at different scales at a given redshift.

The future SDSS will be able to measure the moments of the galaxy density field with unprecedented accuracy. Unless unforeseen systematics exists, it will determine them with less than a few percent error for $N \leq 3$ and 10% for $N = 4$ between 1 and 50 h^{-1} Mpc (see Colombi, Szapudi, & Szalay 1997 for details). According to Figures 2 and 3, the light-cone effect will be much smaller than these errors, or at most of the same order, depending on the scales and models. The correction could be potentially nonnegligible only at the smallest scales. A similar conclusion can probably be drawn about the 2dF survey. On the other hand, for future deep surveys, which should aim at smaller scales especially if carried out by the Next-Generation Space Telescope, our calculations will be of the utmost importance. According to Figure 3, the required correction can range from up to unity for S_3 through factors of a few for S_6 to factors of 100 for S_{10}.

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REFERENCES

Bernardeau, F. 1992, ApJ, 392, 1
Bouchet, F. R., Juszkiewicz, R., Colombi, S., & Pellat, R. 1992, ApJ, 394, L5
Colombi, S., Bouchet, F. R., & Hernquist, L. 1995, ApJ, 281, 301
Colombi, S., Szapudi, I., & Szalay, A. S. 1997, MNRAS, submitted
Fry, J. N. 1984, ApJ, 277, L5
———. 1996, ApJ, 461, L65
Goroff, M. H., Grinstein, B., Rey, S. J., & Wise, M. B. 1986, ApJ, 311, 6
Hamilton, A. J. S., Kumar, P., Lu, E., & Matthews, A. 1991, ApJ, 374, L1
Jain, B., Mo, H. J., & White, S. D. M. 1995, MNRAS, 276, L25 (JMW)
Jing, Y. P., & Börner, G. 1997, A&A, 318, 667
Kitayama, T., Sasaki, S., & Suto, Y. 1998, PASJ, in press (astro-ph/9708088)
Kitayama, T., & Suto, Y. 1997, ApJ, 490, 557
Lahav, O., Itoh, M., Inagaki, S., & Suto, Y. 1993, ApJ, 402, 387
Matsubara, T. & Suto, Y. 1994, ApJ, 420, 497
———. 1996, ApJ, 470, L1
Mo, H. J., Jing, Y. P., & White, S. D. M. 1997, MNRAS, 284, 189
Mo, H. J., & White, S. D. M. 1996, MNRAS, 282, 347
Nakamura, T. T., Matsubara, T., & Suto, Y. 1998, ApJ, 493, in press
Peacock, J. A., & Dodds, S. J. 1994, MNRAS, 267, 1020
———. 1996, MNRAS, 280, L19
Suto, Y. 1993, Prog. Theor. Phys., 90, 1173
Suto, Y., & Matsubara, T. 1994, ApJ, 420, 504
Szapudi, I., Quinn, T., Stadel, J., & Lake, G. 1997, in preparation