γ -RAY BURST AFTERGLOW: CONFIRMING THE COSMOLOGICAL FIREBALL MODEL

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ABSTRACT

The recent detection of delayed X-ray and optical emission—"afterglow"—associated with γ -ray bursts (GRBs) supports models in which the bursts are produced by relativistic expanding blast waves—"fireballs"—at cosmological distances. The detection of absorption lines in the optical afterglow of the GRB of 1997 May 8 confirms that the sources lie at cosmological distance. We show here that the new features detected in GRB 970508 afterglow, radio emission 1 week following the burst and a 2 day increase in optical flux, are consistent with the blast wave model. The fireball optical depth at radio frequencies is much smaller than previously estimated, which accounts for the observed radio emission. The initial suppression of optical flux is consistent with that predicted from electron cooling. The combined radio and optical data imply that the fireball energy is ~10⁵² ergs and that the density of the medium into which the blast wave expands is ~1 cm⁻³, a value typical for gas within galaxies. We predict the time dependence of the radio flux and the absorption frequency, which constitute tests of the fireball model as described in this Letter.

Subject heading: gamma rays: bursts

1. INTRODUCTION

The origin of GRBs, bursts of 0.1–1 MeV photons lasting for a few seconds, remained unknown for over 20 years (Fishman & Meegan 1995), primarily because GRBs were not detected until this year at wave bands other than γ -rays. Phenomenological considerations based on γ -ray data were used to argue that the bursts are produced by relativistic expanding blast waves at cosmological distances (Paczyński 1986; Goodman 1986; Rees & Mészáros 1992; Piran 1996). The fireball model, which predicts delayed emission at wavelengths longer than γ-rays (Paczyński & Rhoads 1993; Katz 1994; Mészáros & Rees 1997; Vietri 1997), has gained support (Waxman 1997; Wijers, Rees, & Mészáros 1997) from the recent detection (van Paradijs et al. 1997) of delayed X-ray and optical emission associated with GRBs. The key new development is the availability from the BeppoSAX satellite of accurate positions for GRBs shortly after their detection.

GRB 970508 was detected by the BeppoSAX satellite (Piro, Scarsi, & Butler 1995) on 1997 May 8. The burst lasted for ~15 s, with γ -ray fluence ~3 × 10⁻⁶ ergs cm⁻² carried mainly by photons of energy ~0.5 MeV (Kouveliotou et al. 1997). Following the detection in γ -rays, X-ray (Piro et al. 1997), optical (Bond 1997; Djorgovski et al. 1997a, 1997b, 1997c; Mignoli et al. 1997; Chevalier & Ilovaisky 1997), and radio (Frail & Kulkarni 1997) emission varying on timescale of days was observed from the direction of the GRB. The location of the GRB is determined with accuracy better than $\sim 3'$, while the locations of the X-ray, optical, and radio sources are determined to better than 50", 1", and 1 mas, respectively. The fact that the locations of all sources coincide, and the unusual variability observed following the GRB, strongly suggest that the X-ray, optical, and radio emission are associated with the object producing the GRB. A spectrum of the optical transient, taken 2 days after the GRB, shows a set of absorption features, associated with Fe II and Mg II and shifted to long wavelength, implying that the absorbing system lies at a cosmological redshift z = 0.835 (Metzger et al. 1997). This sets a lower limit to the GRB redshift, $z \ge 0.835$, and to the energy emitted by the source in γ -rays (assuming isotropic emission), $E_{\gamma} \ge 10^{51}$ ergs. Intensive monitoring of GRB 970508 revealed two new

afterglow features: radio emission was observed 1 week following the burst (Frail & Kulkarni 1997) and the optical flux was observed to increase for 2 days following the burst (Djorgovski et al. 1997a, 1997b).

The early detection of radio emission, combined with optical data, appears to be inconsistent with published fireball model predictions (Paczyński & Rhoads 1993; Mészáros & Rees 1994). We show here, however, that the fireball optical depth at radio frequency is significantly smaller than previously estimated and that the same process responsible for the optical afterglow may also produce the observed radio emission. The combined radio and optical data are shown to be consistent with the fireball model and to provide information on the fireball parameters and on the ambient medium into which the blast wave expands. The suppression of optical flux at early times provides evidence for the predicted suppression due to electron cooling (Waxman 1997) and suggests that the GeV emission observed for several hours following several strong GRBs (Hurley et al. 1994) is produced by inverse-Compton scattering of afterglow X-ray photons.

Our goal is to present a qualitative analysis of the new afterglow features observed in GRB 970508, in order to identify the physical processes that are responsible for the observed behavior. We therefore adopt a simple approximate description of the fireball expansion that permits the derivation of the main results without complicated calculations. Our values for numerical parameters should be considered as order-of-magnitude estimates.

2. THE FIREBALL MODEL AND PREDICTED X-RAY, OPTICAL, AND RADIO AFTERGLOW

The underlying source which produces the initial explosion that drives the expanding fireball is unknown (although several plausible candidates have been proposed; see, e.g., Piran 1996 for review). However, the γ -ray observations suggest the following scenario for the emission of the observed γ -rays (Paczyński 1986; Goodman 1986; Rees & Mészáros 1992). A compact $r_0 \sim 10^7$ cm source releases an energy *E* comparable to that observed in γ -rays, $E \sim 10^{51}$ ergs, over a time T < 100 s. The large energy density in the source results in an optically thick plasma that expands and accelerates to relativistic velocity. After an initial acceleration phase, the fireball energy is converted to proton kinetic energy. A cold shell of thickness *cT* is formed and continues to expand with time-independent Lorentz factor $\gamma \sim 300$. The GRB is produced once the kinetic energy is dissipated at large radius, $r > 10^{13}$ cm, due to internal collisions within the ejecta (Paczyński & Xu 1994; Mészáros & Rees 1994) and radiated as γ -rays through synchrotron and possibly inverse-Compton emission of shock accelerated electrons.

Following internal collisions, which convert part of the energy to radiation and which result from variations in γ across the expanding shell, the fireball rapidly cools and continues to expand with approximately uniform Lorentz factor γ . As the cold shell expands it drives a relativistic shock (blast wave) into the surrounding gas, e.g., into the interstellar medium (ISM) gas if the explosion occurs within a galaxy. In what follows, we refer to the surrounding gas as "ISM gas," although the gas need not necessarily be interstellar. The shock propagates with Lorentz factor $\gamma_s = 2^{1/2}\gamma$, and behind it the (rest frame) number and energy densities of the shock-heated ISM are $n' = 4\gamma n$ and $e' = 4\gamma^2 nm_p c^2$, respectively, where *n* is the ISM number density ahead of the shock. The width of the shock-heated ISM shell is $r/4\gamma^2$, where *r* is the fireball radius.

At late times, the blast wave approaches a self-similar behavior. The expansion is well approximated by the self-similar solution for radii $r > r_c$, where (Waxman 1997)

$$r_c = \left(\frac{ET}{4\pi nm_p c}\right)^{1/4} = 2 \times 10^{16} \left(\frac{E_{52}T_{10}}{n_1}\right)^{1/4} \text{ cm.}$$
(1)

Here the fireball energy is $E = 10^{52}E_{52}$ ergs, the ISM density is $n = n_1 \text{ cm}^{-3}$, and $T = 10T_{10}$ s. We have chosen to give our numeric results using a fireball energy value that is somewhat higher than the typical energy observed in γ -rays, $E_{\gamma} \sim 10^{51}$ ergs, since the conversion of fireball energy to γ -rays is not expected to be 100% efficient. For $r \ge r_c$, the shell Lorentz factor is

$$\gamma = \left(\frac{r_c}{2Tc}\right)^{1/2} (r/r_c)^{-3/2} = 190 \left(\frac{E_{52}}{T_{10}^3 n_1}\right)^{1/8} (r/r_c)^{-3/2}.$$
 (2)

Photons emitted from the shell at radius r are seen by a distant observer at a time $t = r/2\gamma^2 c = T(r/r_c)^4$ after the GRB, with arrival time spread comparable to t. The main contribution to the time delay and spread is due to two effects. First, the radiation seen by a distant observer is emitted from a cone of the fireball around the source-observer line of sight, with an opening angle $\sim 1/\gamma$. Photons emitted from such a cone are spread over $t = r/2\gamma^2 c$. Second, as we show below, during most of the afterglow the synchrotron cooling time of electrons is larger than the fireball expansion time, $\sim r/\gamma c$ as measured in the fireball frame. Thus, electrons emit radiation over a time $\sim r/\gamma c$ in the fireball frame, corresponding to a time $\sim r/2\gamma^2 c$ as seen by a distant observer (Note that the delay due to the difference t_s between the shock propagation time to radius r and r/c, $t_s = r/16\gamma^2 c$, is not the main factor determining the time t at which radiation from radius r is observed).

The shock driven into the ISM continuously heats new gas and produces relativistic electrons that may produce the delayed radiation observed on timescales of days to months. In order to calculate the synchrotron emission from the heated ISM shell we need to determine the magnetic field and electron energy within the heated shell. We assume that the magnetic field energy density (in the shell rest frame) is a fraction ξ_B of the equipartition value, $B^2/8\pi = \xi_B e'$, and that the electrons carry a fraction ξ_e of the energy. Since the Lorentz factor associated with the thermal motion of protons in the shell rest frame is γ , this implies that the Lorentz factor of the random motion of a typical electron in the shell rest frame is $\gamma_{em} =$ $\xi_e \gamma m_p/m_e$. With these assumptions, and using equations (1) and (2), we find that the observed frequency of synchrotron emission from typical electrons, $\nu_m \simeq \gamma \gamma_{em}^2 eB'/2\pi m_e c$, is

$$\nu_m = 2 \times 10^{14} \left(\frac{1+z}{2}\right)^{1/2} (\xi_e/0.2)^2 (\xi_B/0.1)^{1/2} E_{52}^{1/2} t_{day}^{-3/2} \text{ Hz}, \quad (3)$$

where $t = 1t_{day}$ days, and z is the cosmological redshift of the burst. We have chosen to give numerical results using $\xi_e \sim \xi_B \sim 0.1$, since such values are typically required for the production of the GRB itself. We show below that the electron synchrotron cooling time is longer than the dynamical time, $t_d = r/\gamma c$, the time for significant fireball expansion. In this case, the observed intensity at ν_m is (Waxman 1997)

$$F_{\nu_m} = 1 \left(\frac{1+z}{2}\right)^{-1} \left[\frac{1-1/\sqrt{2}}{1-1/\sqrt{1+z}}\right]^2 n_1^{1/2} (\xi_B/0.1)^{1/2} E_{52} \text{ mJy.}$$
(4)

Here (and throughout the Letter) we assume a flat universe with Hubble constant $H_0 = 75$ km s⁻¹ Mpc⁻¹. The Jy flux unit is 1 Jy = 10^{-23} ergs cm⁻² s⁻¹ Hz⁻¹.

A natural prediction of the fireball model (see eqs. [3] and [4]) is optical emission at a level of 1 mJy at a delay of order 1 day following the GRB.

Emission at $\nu > \nu_m$ is produced by electrons with Lorentz factor higher than γ_{em} . If the electron distribution follows a power law, $dN_e/d\gamma_e \propto \gamma_e^{-p}$ for $\gamma_e > \gamma_{em}$, as expected for shock acceleration, then for $\nu > \nu_m$

$$F_{\nu} = F_{\nu_m} [\nu / \nu_m(t)]^{-\alpha}, \tag{5}$$

with $\alpha = (p - 1)/2$. Although p is expected to be similar for the GRB and for the afterglow, α during the afterglow is expected to be smaller by 1/2 due to increase in cooling time: the synchrotron cooling time must be short compared to the dynamical time during the GRB, resulting in $\alpha = p/2$, while it is long during the afterglow, giving $\alpha = (p - 1)/2$ (Waxman 1997). For the GRB $\alpha \sim 1$, implying $\alpha \sim 0.5$ for the afterglow.

The synchrotron emission of electrons with Lorentz factor γ_{em} is concentrated mainly at frequencies $\nu \sim \nu_m$. However, the emission extends to lower frequencies, with power radiated per unit frequency proportional to $(\nu/\nu_m)^{1/3}$. Thus, we expect the synchrotron flux to extend to $\nu < \nu_m$ following equation (5) with $\alpha = -1/3$. This implies that the flux at a fixed frequency ν increases with time as $t^{1/2}$ as long as $\nu < \nu_m$. We have so far assumed that the fireball optical depth to synchrotron absorption is small. This may not be the case for low frequencies. The synchrotron optical depth at ν_m is small, $\tau_m = 10^{-11}\xi_e^{-5}\xi_B^{-1/2}E_{51}^{-1/2}n_1t_{day}^{5/2}$. For $\nu > \nu_m$, $\tau \propto \nu^{-(p+4)/2}$, while $\tau \propto \nu^{-5/3}$ for $\nu < \nu_m$ due to the $\nu^{1/3}$ low-frequency synchrotron tail.

$$\nu_{A} = 1 \left(\frac{1+z}{2}\right)^{-1} (\xi_{e}/0.2)^{-1} (\xi_{B}/0.1)^{1/5} E_{52}^{1/5} n_{1}^{3/5} \text{ GHz}, \quad (6)$$

and the flux at $\nu_A < \nu \ll \nu_m$, $F_A = 2F_{\nu_m}(\nu/\nu_m)^{1/3}$, is

$$F_{\nu} = 0.3 \left(\frac{1+z}{2}\right)^{-7/6} \left[\frac{1-1/\sqrt{2}}{1-1/\sqrt{1+z}}\right]^2 (\xi_e/0.2)^{-2/3} (\xi_B/0.1)^{1/3} \times E_{52}^{5/6} n_1^{1/2} (t/1 \text{ week})^{1/2} (\nu/10 \text{ GHz})^{1/3} \text{mJy.}$$
(7)

Equations (6) and (7) indicate, that radio emission at a level of 1 mJy is expected at the ~ 10 GHz range on timescale of weeks. The emission should be suppressed below ~ 1 GHz due to high optical depth.

We note here, that a population of electrons with $\gamma_e \ll \gamma_{em}$ may also contribute to the flux and optical depth at $\nu \ll \nu_m$. Although in the model described in this Letter the electron distribution is dominated by electrons with Lorentz factor $\gamma_e \sim \gamma_{em}$, the distribution may extend to low energy, $\gamma_e \ll \gamma_{em}$, without significantly affecting the results (eqs. [3]–[5]), provided that low-energy electrons constitute a small fraction of the electron population. The distribution may extend, e.g., to $\gamma_e \ll \gamma_{em}$ as $dN_e/d\gamma_e \propto \gamma_e^{-p'}$, with $0 \le p' \ll 1$ or p' < 0 (for $p' \ge 1$ the fraction of electrons with $\gamma_e \sim \gamma_{em}$ is small). While the contribution of such low-energy electron population to the flux at $\nu < \nu_m$, $F_\nu \propto \nu^{(1-p')/2}$, is not large compared to that given in equation (7), their contribution to the optical depth at $\nu < \nu_m$, $\tau \propto \nu^{-(p'+4)/2}$, may result in self-absorption frequency significantly higher than equation (6). For p' = 0 we have $\nu_A = \nu_m \tau_m^{1/2}$, i.e.,

$$\nu_{A} = 4 \left(\frac{1+z}{2}\right)^{-3/4} (\xi_{e}/0.2)^{-1/2} (\xi_{B}/0.1)^{1/4}$$
$$\times E_{52}^{1/4} n_{1}^{1/2} (t/1 \text{ week})^{-1/4} \text{ GHz.}$$
(8)

Equation (8) implies that the existence of low-energy electron population does not change the conclusion that radio emission at a level of 1 mJy is expected at the ~10 GHz range on timescale of weeks. (Our estimate for v_A is significantly lower than previous estimates [e.g., Paczyński & Rhoads 1993; Mészáros & Rees 1994], since we have taken into account the fact that the electron distribution flattens below γ_{em} .)

3. GRB 970508

Let us now compare the fireball model predictions with observations of GRB 970508. The detected absorption lines imply that the GRB source redshift is $z \ge 0.835$, and the absence of Ly α lines imply z < 2.1 (Metzger et al. 1997). The absence of C IV absorption further implies z < 1.6 (N. Arav & D. Hogg 1997, private communication). We therefore adopt z = 1 as the GRB source redshift.

The radio flux detected 6 days after the GRB is well described by 0.2($\nu/3$ GHz) mJy in the range of 1–10 GHz (Frail & Kulkarni 1997). The ν^1 dependence, which is steeper than $\nu^{1/3}$ expected from the low-frequency tail of synchrotron emission in the absence of absorption, implies that the radio emission is partly absorbed, with stronger absorption at lower frequencies due to increase in optical depth. In our model, $F_{\nu} \propto \nu^{1/3}$ for $\nu/\nu_A \gg 1$, and $F_{\nu} \propto \nu^2$ for $\nu/\nu_A \ll 1$ (in the presence of low-energy electrons, $dN_e/d\gamma_e \propto \gamma_e^{-p'}$ with p' > -2/3 for

 $\gamma_e \ll \gamma_{em}$, the dependence of flux on frequency is stronger, $F_{\nu} \propto \nu^x$ with x > 2 for $\nu/\nu_a \ll 1$). Radio observations therefore indicate that 1 GHz $< \nu_A < 10$ GHz. The observed self-absorption frequency and the level of detected radio flux are in agreement with model predictions (eqs. [6]–[8]). The upper limit of 7 mJy at 6 day delay at 90 GHz (Shepherd, Metzger, & Kulkarni 1997) is also consistent with the model $\nu^{1/3}$ dependence at $\nu > \nu_A$, which predicts a flux at 90 GHz of 1 mJy. At delays shorter than 6 days, only upper limits to the flux at 1.4 GHz are available. The upper limits are comparable to the flux detected at 6 day delay, and therefore, while being consistent with the model, do not allow one to establish the transient nature of the radio emission or test the model prediction, that the flux should be increasing with time.

The *R*-band, $\nu_R = 4 \times 10^{14}$ Hz, observations at 2.0 to 5.5 day delay (Djorgovski et al. 1997a, 1997b, 1997c; Mignoli et al. 1997; Chevalier & Ilovaisky 1997) are well described by a power law, $F_R = 37 \ \mu Jy \ (t/2 \ day)^{-1.3}$. The power-law behavior is consistent with the model prediction (5). The approximately constant flux during 1 to 2 days delay indicate that the peak of the synchrotron emission v_m passed through the R band at a delay $t_R \sim 1$ day. This is consistent with the model prediction (3). (Note that extrapolation of the $t^{-1.3}$ behavior to t = 1 day results in a flux ~2 times higher than observed. However, since we do not expect a sharp peak with in the flux observed at a given frequency as $\nu_{\rm m}$ drops below this frequency, due to the spread in photon arrival times, the approximately constant flux observed near the peak on timescale comparable to the delay is consistent with the model). The normalization of the powerlaw fit to the *R*-band observations is \sim 3 times lower than implied by equations (4), (3), and (5). However, given the simple description of the fireball behavior adopted in this Letter, one should not draw conclusions based on this numerical discrepancy. The $t^{-1.3}$ decline of *R*-band flux implies, through equation (5), $\alpha \simeq 0.8$, or p = 2.6. The implied frequency dependence of the flux is consistent with observations at other optical bands, although the narrow frequency range does not allow accurate determination of α from the frequency dependence.

Consider next the optical (Bond 1997; Djorgovski et al. 1997a) detection at \sim 7 hr delay and the X-ray detection at 10 hr delay (Piro et al. 1997). At early times, t < 1 day, the Rband frequency is smaller than v_m , the frequency of radiation emitted by the typical fireball electrons, and the R-band flux should increase as $t^{1/2}$. Using the power-law fit to the observed *R*-band flux for t > 1 day, and a $t^{1/2}$ scaling for t < 1 day, the predicted flux at 7 hr delay is 50 μ Jy, significantly above the observed flux, $11.1 \pm 2.2 \mu$ Jy. This discrepancy may indicate the detection of the predicted (Waxman 1997) suppression of optical flux at early times due to rapid electron cooling. The ratio of synchrotron cooling time, $t_s = 6\pi m_e c/\sigma_{\hat{T}'e} B'^2$, to fire-ball deceleration time, $t_d \sim \gamma/\dot{\gamma}\gamma = 2r/3\gamma c$, is $t_s/t_d \sim 2(\xi_e/0.2)^{-1}(\xi_B/0.1)^{-1}E_{52}^{-1/2}t_{day}^{1/2}$. Thus, our assumption that the synchrotron cooling time is longer than the dynamical time is valid for $t \ge 1$ day. At a 7 hr delay the cooling time is comparable to the deceleration time, and the electrons cool on a dynamical timescale. At this time, synchrotron emission is significantly suppressed due to inverse-Compton emission (Waxman 1997), by which electrons lose a significant fraction of their energy to the production of photons of high energy, exceeding 1 GeV during the first hours following the burst. This may account for the delayed GeV emission observed in several strong bursts. Finally, the X-ray emission detected at ~10 hr delay (Piro et al. 1997), with a flux of ~0.03 μ Jy at 10¹⁸ Hz,

is consistent with the above model when the suppression due to electron cooling is taken into account.

4. CONCLUSIONS

The fireball model for GRBs is consistent with afterglow observation in X-ray, optical, and radio wave bands, provided the kinetic fireball energy is $E \sim 10^{52}$ ergs, the density into which the blast wave expands is $\sim 1 \text{ cm}^{-3}$, and provided that the fraction of energy carried by electrons and magnetic field is $\xi_e \sim \xi_B \sim 0.1$. The inferred kinetic energy is consistent with the observed γ -ray energy, $E_{\gamma} \sim 10^{51}$ ergs, and implies that the efficiency with which kinetic energy is converted to γ -rays is of order 10%. The density of the ambient medium is typical for interstellar gas, therefore suggesting that the explosions take place within galaxies. The requirement for significant energy to be carried by electrons and magnetic field is consistent with what is usually assumed for the production of the GRB itself. We have implicitly assumed that the fireball is spherical. However, our analysis is valid also for the case that the fireball is a jet, as long as the jet opening angle is larger than $1/\gamma$. The fireball Lorentz factor during the radio observations is not very

large, $\gamma = 4(E_{52}/n_1)^{1/8}(t/1 \text{ week})^{-3/8}$. This implies that the opening angle of the jet is not very small.

Our model predicts that the frequency, at which the optical depth for synchrotron absorption is unity, should change slowly with time, $v_A \propto t^{-x}$ with $0 \le x \le 1/4$, and that on timescale of weeks 1 GHz $< v_A < 10$ GHz (cf. eqs. [6]–[8]). The radio flux at $v > v_A$ should rise as $t^{1/2}$ following the GRB. These predictions can be tested with more frequent radio observations following future GRBs. (For GRB 970508 the observed self-absorption frequency and radio flux at 6 day delay are consistent with model predictions. However, at shorter delays only upper limits are available, which do not allow one to establish the transient nature of the source or test the above predictions.) Since the Lorentz factor after 1 week is not large, deviations from the simple scaling laws derived here may be observed at later times, due to the deceleration of the fireball to velocity, which is not highly relativistic.

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