

## THE MAXIMUM OPTICAL DEPTH TOWARD BULGE STARS FROM AXISYMMETRIC MODELS OF THE MILKY WAY

KONRAD KUIJKEN<sup>1</sup>

Kapteyn Instituut, PO Box 800, 9700 AV Groningen, The Netherlands

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### ABSTRACT

It has been known that recent microlensing results toward the bulge imply mass densities that are surprisingly high, given dynamical constraints on the Milky Way mass distribution. We derive the maximum optical depth toward the bulge that may be generated by axisymmetric structures in the Milky Way, and we show that observations are close to surpassing these limits. This result argues in favor of a bar as a source of significantly enhanced microlensing. Several of the bar models in the literature are discussed.

*Subject headings:* Galaxy: kinematics and dynamics — Galaxy: structure — gravitational lensing

### 1. INTRODUCTION

Large surveys for microlensing events in the Galaxy have grown into a serious tool for evaluating models of the mass distribution of the Milky Way, as is evident from the recent results of the MACHO (Alcock et al. 1997), OGLE (Udalski et al. 1994), EROS (Renault et al. 1996), and DUO (Alard, Mao, & Guibert 1995) surveys. Microlensing occurs whenever the light from a distant star passes sufficiently close to an intervening object (the “lens”) that its path is significantly deflected. The deflection results in a focusing of the source star’s rays, which may be detected as an apparent brightening of the source with a characteristic light curve. Typically, the brightening lasts for a number of days or months before the source and lens proper motions spoil the alignment. Both the optical depth  $\tau$ , which is the average fraction of the sources that is being amplified more than a factor of 1.34 at any given time, and the average duration of the individual events, depend on the distribution and masses of lensing objects between the observer and the sources (Paczynski 1986).

In this Letter we concentrate on microlensing toward the bulge of the Milky Way. Both the MACHO and OGLE teams find that the optical depth in this direction is rather higher than had been expected initially: the optical depths reported are in the range of  $3\text{--}6 \times 10^{-6}$ , roughly twice the expected value based on standard Galaxy models. Although initially this discrepancy was attributed to our Galaxy’s central bar (Paczynski et al. 1994; Zhao, Spergel, & Rich 1995; Zhao, Rich, & Spergel 1996), evidence for which had been accumulating for some time (e.g., Blitz & Spergel 1991; Weinberg 1992; for a review, see Kuijken 1996), the most recent models for the bar in fact do little to enhance the expected optical depth over that expected from axisymmetric models (Bissantz et al. 1996; Nikolaev & Weinberg 1997). It therefore seems opportune to return to the axisymmetric mass models once more.

In this Letter we derive a strong limit on axisymmetric models: a given optical depth toward the bulge comes with a minimum contribution to the Galactic circular speed at the Sun. It turns out that the current values for the optical depth toward the bulge are already close to implying a circular speed higher than that observed.

The only result from microlensing theory that is relevant for this discussion is the optical depth  $\tau$  to a source at distance  $L$ , due to a distribution of lenses with mass density  $\rho(D)$  at distance  $D$  down the line of sight:

$$\tau = \frac{4\pi G}{c^2} \int_0^L \rho(D) \frac{D(L-D)}{L} dD. \quad (1)$$

The term  $\tau$  is independent of the mass function  $f(m)$  of the lenses, although the rate and detectability of microlensing events are not: the survey teams correct for detection (in)efficiency in their analyses.

### 2. CAN AXISYMMETRIC MASS COMPONENTS EXPLAIN BULGE MICROLENSING?

Axisymmetric density distributions can be viewed as being constructed from concentric rings. We therefore start by considering the relation between the mass and the optical depth of a ring. We assume that the sources are on the Galactic minor axis, at a distance of  $R_0 = 8$  kpc, and not mixed spatially with the ring; these results are thus appropriate for the microlensing of the bulge clump giants (Alcock et al. 1997).

#### 2.1. Optical Depth from a Ring

Consider a ring of uniform surface density with radius  $R < R_0$  and radial width  $\Delta R \ll R$ . Its optical thickness toward bulge sources near the Galactic minor axis, at latitude  $b$  radians (assumed small), is (see eq. [1])

$$\tau_{\text{ring}} = \frac{4\pi G}{c^2} \frac{(R_0 - R)R}{R_0} \rho(z) \Delta R. \quad (2)$$

Here  $\rho$  is the density where the line of sight intersects the ring, at height  $z = (R_0 - R)b$  from the plane. Now, if the ring is axisymmetric with uniform surface density  $\Sigma$ , then its mass is

$$M_{\text{ring}} = 2\pi R \Delta R \Sigma = \frac{c^2 \tau_{\text{ring}} R_0 b}{2G} \left[ \frac{\Sigma}{z \rho(z)} \right] \quad (3)$$

(using  $R_0 - R = z/b$ ). The bracketed factor is dimensionless and depends only on the shape of the vertical density profile of the ring, not on the total mass of the ring. We will call it the

<sup>1</sup> Visiting Scientist, Department of Theoretical Physics, University of the Basque Country.

TABLE 1

MINIMUM VERTICAL SHAPE FACTOR  $F_z = \Sigma/(z\rho)$  FOR DIFFERENT VERTICAL DENSITY PROFILE SHAPES

$\rho(z)$	$F_z^{(\min)}$	Attained at $z =$
Constant to $z = a$ ....	2	$a$
$\exp(- z /a)$ .....	$2e$	$a$
$\text{sech}(z/a)$ .....	4.7	$1.2a$
$\text{sech}^2(z/a)$ .....	4.5	$0.8a$
$\exp(-z^2/a^2)$ .....	4.1	$0.7a$

vertical shape factor  $F_z$ . Its denominator tends to zero at small and large  $z$  values (provided that the ring has finite surface mass density); therefore, this geometrical factor has a minimum at some intermediate  $z$  value. Hence, equation (3) can be used to derive a lower limit on the mass of an axisymmetric ring of a given optical depth  $\tau$ .

The lowest vertical shape factor for monotonic  $\rho(|z|)$  is obtained when the density is constant up to a height  $a > z$ , and zero beyond. In this case  $\Sigma = 2a\rho$ , and  $F_z \geq 2$ . More realistic density profiles yield values considerably larger, however. Table 1 lists the lower limits to  $F_z$  for some different forms of  $\rho(z)$ . In light of those results, we adopt  $F_z \gtrsim 4.0$ .

The lower mass limit of equation (3) can be recast as a circular velocity at the solar circle: approximating the circular speed due to a ring by its monopole term (itself a lower limit), we obtain

$$v_{\text{circ}}^2 \geq \frac{GM_{\text{ring}}}{R_0} = \frac{c^2 \tau_{\text{ring}} b}{2} F_z \gtrsim 2.0 c^2 \tau_{\text{ring}} b. \quad (4)$$

Since the radius of the ring is no longer explicit in this equation, this limit applies equally to superpositions of rings, i.e., axisymmetric mass distributions. Scaling to observed parameters, we obtain

$$v_{\text{circ}}^2 \geq (210 \text{ km s}^{-1})^2 \left( \frac{\tau_{-6}}{4.0} \right) \left( \frac{b}{3.5} \right) \left( \frac{F_z^{(\min)}}{4.0} \right) \quad (5)$$

as the minimum circular speed generated at the solar position by axisymmetrically distributed matter responsible for an optical depth of  $\tau_{-6} \times 10^{-6}$  toward bulge sources at Galactic latitude  $b$ . A radially extended mass distribution, in order to lens optimally, needs to have  $F_z$  close to its minimum at all radii: this is achieved when the scale height of the mass is proportional to  $R_0 - R$ , i.e., tapers linearly to zero at the solar radius.

## 2.2. Specific Disk Models

We know more about the Milky Way rotation curve than its value at the solar position: we also know that the circular speed does not greatly exceed the local value at any point in the inner disk of the Galaxy. We now investigate how this extra constraint affects the maximum disk optical depth, by considering various models for the disk mass distribution and raising their mass until their rotation curve reaches  $200 \text{ km s}^{-1}$  at any point. We consider four functional forms for the disk surface mass density, whose rotation curves are shown in Figure 1. We will also compare these models to solar neighborhood-normalized models, where the surface mass density in stars is about  $35 M_{\odot} \text{ pc}^{-2}$  (Kuijken & Gilmore 1989). Note that the total surface density of matter within 1 kpc of the plane,  $71 M_{\odot} \text{ pc}^{-2}$  (Kuijken & Gilmore 1991) is consistent with such

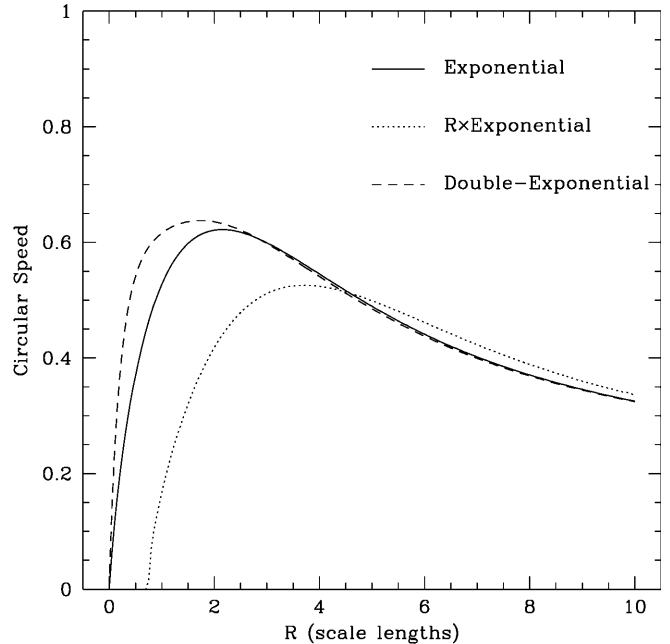


FIG. 1.—The rotation curves for the three types of disk models considered, in units of  $(GM/h)^{1/2}$ .

maximal disk models only for short ( $\lesssim 2.5$  kpc) scale length models (Sackett 1997).

The traditional disk model is a radially exponential disk,  $\Sigma(R) \propto \exp(-R/h)$ . We consider two vertical density profile shapes, a vertical exponential and a  $\text{sech}^2(z/a)$  “isothermal” model. If the scale height is small, then the rotation curve peaks at a radius of 2.2 scale lengths at the value  $0.62(GM/h)^{1/2}$  (Freeman 1970). Figure 2 shows the optical depth from such disks toward the Galactic minor axis at a latitude of  $b = 3.5^\circ$ , for various scale heights and lengths.

For comparison, we consider also two variations on the exponential disk: a less centrally concentrated model with  $\Sigma(R) \propto R \exp(-R/h)$  (e.g., Caldwell & Ostriker 1981) and a more concentrated one obtained by adding a second exponential disk of one-fourth the scale length, and one-eighth the mass of the main component. As may be seen from Figures 3 and 4, the maximum optical depth from such disks is comparable to the exponential disk.

Finally, we consider a Mestel disk (Mestel 1963), which has surface mass density  $\Sigma(R) = V^2/(2\pi GR)$  and whose circular speed is everywhere equal to  $V$ . Such a model microlenses more efficiently than an exponential disk since it can satisfy our rotation curve constraint at every radius simultaneously. The maximum optical depth from a Mestel disk toward  $b = 3.5^\circ$  turns out to be  $2.5 \times 10^{-6}$  for a vertically  $\text{sech}^2(z)$  mass distribution.

We summarize the results for the various disk models in Table 2. We give two numbers for each model: the maximal optical depth for each model assuming constant scale height, and the (higher) optical depth that can be achieved by tapering the disk scale height down to zero at the solar position, in proportion to  $(R_0 - R)$ .

## 3. DISCUSSION

The calculations presented above show that axisymmetric models of the Galaxy cannot avoid high circular velocities at

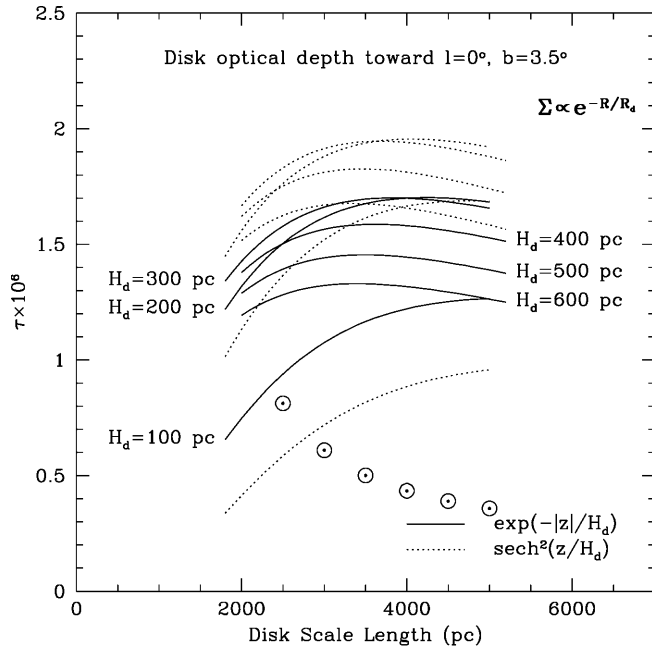


FIG. 2.—The optical depth toward sources at the Galactic minor axis from radially exponential disks with maximum circular speed  $200 \text{ km s}^{-1}$  and differing scale heights and lengths. The solid lines refer to vertically exponential disks, and the dotted ones to  $\text{sech}^2$  disks. Sun symbols show the optical depths from disks normalized to the “locally identified” stellar surface density of  $35 M_{\odot} \text{ pc}^{-2}$  at the solar position, vertically exponential with scale height 300 pc.

the Sun if the optical depth toward the Galactic minor axis is as high as is being reported. The current numbers are in fact so high as to exceed the maximal disk values derived in Table 2. They are also in conflict with the claimed detection of the dark halo toward the LMC (Alcock et al. 1996; Renault et al. 1996), since the small disk scale heights ( $\sim 300$  pc) required for bulge microlensing would produce virtually no microlensing toward the LMC. If indeed the optical depth is too high, then

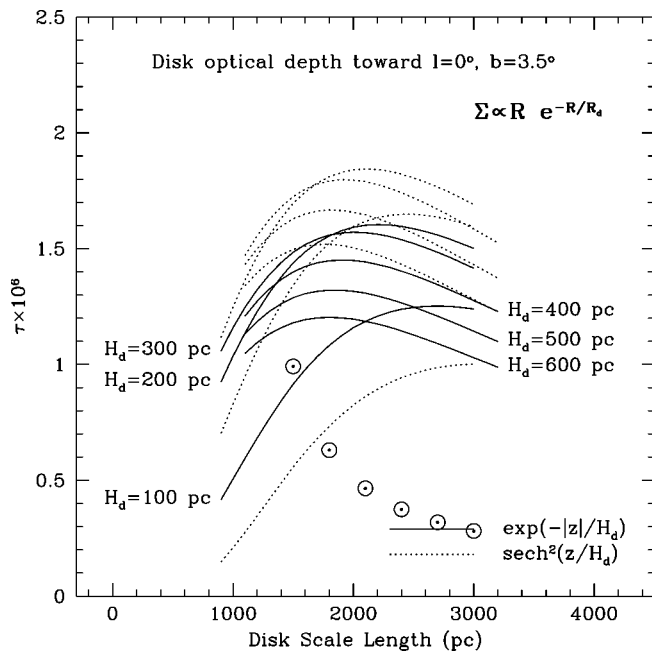


FIG. 3.—Same as Fig. 2, but with a central hole in the disk mass profile.

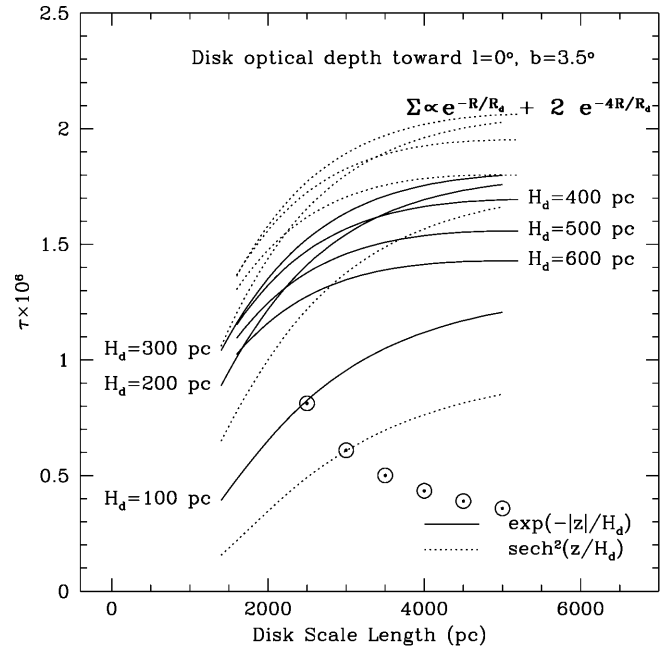


FIG. 4.—Same as Fig. 2, but with a more concentrated mass profile.

nonaxisymmetric models, particularly the Galactic bar, should be considered.

Several in-depth studies of microlensing by the central bulge/bar have been carried out already. While the recent investigations by the Basel/Oxford group (Binney, Gerhard, & Spergel 1996; Bissantz et al. 1996) and by Nikolaev & Weinberg (1997) conclude that the enhancement in optical depth over a standard bulge model (Kent 1992) is small, the earlier analysis by Zhao et al. (1995, 1996) did find a significant enhancement in  $\tau$ . These differences illustrate the difficulty in disentangling the structure of the inner Galaxy from our viewpoint in the disk, and the consequences of this uncertainty on the bar optical depth (Zhao & Mao 1996).

The Zhao et al. (1995, 1996) and Oxford studies are based on the COBE/DIRBE map of near-infrared emissions, the asymmetries of which are used to infer the bar's orientation and shape from perspective effects. Zhao et al. (1995, 1996) based their model on the fitting to the DIRBE data by Dwek et al. (1995), whose best-fit model is quite elongated, with major axis close to the line of sight. This model is therefore efficient at microlensing toward the bulge, since it places most of its mass density between us and the bulge. The Basel/Oxford group, on the other hand, use a Lucy deprojection of the emission to infer the bar properties. They find a much shorter

TABLE 2  
MAXIMUM OPTICAL DEPTH TOWARD STARS ON GALACTIC MINOR AXIS

Disk $\Sigma(R)$	$\tau_{\text{const}} \times 10^6$	$\tau_{\text{tapered}} \times 10^6$
$\propto \exp(-R/a)$ .....	2.0	2.6
$\propto R \exp(-R/a)$ .....	1.9	2.4
$\propto \exp(-R/a) + 2 \exp(-R/4a)$ .....	2.1	2.7
$\propto R^{-1}$ .....	2.5	3.3

NOTE.—Maximum optical depth toward stars on the Galactic minor axis at latitude  $b = 3.5^\circ$ , for various disk surface density profiles  $\Sigma(R)$ . All models have been normalized to have a peak circular speed of  $200 \text{ km s}^{-1}$  and have vertical density  $\propto \text{sech}^2(z)$ . The optical depths for models whose scale height is radially constant and tapered  $\propto (R_0 - R)$  are shown.

bar, which consequently generates less microlensing. Nikolaev & Weinberg (1997) derive their bar parameters from star counts of *IRAS* asymptotic giant branch sources. Their bar is considerably longer than the *COBE*-derived ones, but it is quite broad, providing only a modest enhancement in lensing over axisymmetric models.

It is difficult to prefer one of these models over another. The Zhao et al. (1995, 1996) model is a simple parametric fit to the *COBE* data, whereas the Basel/Oxford model is completely nonparametric. In fact, the Basel/Oxford inversion is puzzlingly robust: different starting conditions result in very similar solutions, in spite of the fact that a three-dimensional emissivity model is reconstructed out of a two-dimensional projected surface brightness map! One assumption made is, however, crucial: the models are taken to be eightfold symmetric, i.e., the bar is modeled as reflection symmetric in three orthogonal planes (it is this symmetry that allows the observed left-right differences in the map to be interpreted as perspective effects). In reality, even the most regular bars observed do not exhibit this symmetry: bars are generally somewhat twisted and often connect onto spiral arms joining at the ends. As Binney et al. 1996 themselves recognize, even a fourfold symmetry (reflection in Galactic plane and through the Galactic center) is insufficient to allow a stable inversion.

The Weinberg bar may be stronger than it appears in the analysis of *IRAS* sources, since a relatively small number of harmonics was used in modeling the star counts ( $m = 0, 2, 4$  azimuthal harmonics). It is therefore possible that the true density enhancement toward the bulge is higher than modeled. This bar is also clearly larger than the *COBE* models, suggesting that the Galaxy may well be a bar-within-a-bar galaxy, a type of system that is relatively common. In that case, the *COBE* models may need to take account of this extra foreground component.

#### 4. CONCLUSION

We have shown that if the mass responsible for the microlensing toward the Galactic bulge is distributed axisymmetrically, then the high optical depths  $\tau$  being reported may be translated into simple but significant lower limits on the circular speed at the solar position due to lensing matter:

$$v_c^2 \geq 2.0c^2\tau b, \quad (6)$$

where  $b$  is the Galactic latitude (in radians) of the sources. This limit evaluates to  $v_c \geq 210 \text{ km s}^{-1}$  for an optical depth of  $4 \times 10^{-6}$  at  $b = 3.5^\circ$ , very close to the actual rotation speed of  $200 \pm 20 \text{ km s}^{-1}$  (Merrifield 1992).

This limit is approached when the lensing matter has a scale height that is well matched to the height of the line of sight toward the stars being monitored; otherwise, even higher circular speeds are implied. More restricted, but realistic, models for the mass distribution fail to generate optical depths above  $2.5 \times 10^{-6}$  without requiring circular speeds in excess of  $200 \text{ km s}^{-1}$ .

This result comes close to ruling out purely axisymmetric models for the mass distribution in the inner Galaxy. Those models left require a very flattened distribution of the Galactic mass, with a thickness of about 300 pc, contrary to the conclusions drawn from the microlensing experiments toward the LMC, which favor a round dark matter distribution. A significant enhancement in  $\tau$  from a central bar appears to be a more plausible explanation.

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