THE ASTROPHYSICAL JOURNAL, 479:L51–L54, 1997 April 10
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DIFFERENTIAL ROTATION ENHANCED DISSIPATION OF TIDES IN THE PSR J0045−7319 BINARY

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Received 1996 September 20; accepted 1997 January 24

ABSTRACT

Recent observations of PSR J0045−7319, a radio pulsar in a close eccentric orbit with a massive B star companion, indicate that the system's orbital period is decreasing on a timescale of $\approx 5 \times 10^5$ yr. This is much shorter than the timescale of $\approx 10^9$ yr given by the standard theory of tidal dissipation in radiative stars. Observations also suggest that the B star is rotating rapidly, perhaps at nearly its breakup speed. We show that the dissipation of the dynamical tide in a star rotating in the same direction as the orbital motion of its companion (prograde rotation) with a speed greater than the orbital angular speed of the star at periastron (supersynchronous rotation) results in an increase in the orbital period of the binary system with time. Thus, if the magnitude of the rotation speed of the B star is supersynchronous, then the observed decrease in the orbital period requires the direction of the rotation of the B star to be retrograde. For subsynchronous prograde rotation of the B star, the energy in the dynamical tide, even if it is dissipated in one orbital period, is too small to account for the observed orbital evolution, unless the rotation speed is close to zero. Slow rotation of the B star is, however, ruled out by the observed apsidal motion of the system (Lai et al., Kaspi et al.). Thus, in order to explain both the observed apsidal motion and the orbital evolution of the PSR J0045−7319 binary, the B star must have retrograde rotation.

If the rotation in the interior of the B star is not synchronized, which we show is the case, then the work of Goldreich & Nicholson suggests that the B star should be rotating differentially, with the rotation speed of the outer layers close to the synchronous value. We show that the dissipation of the dynamical tide in such a differentially rotating B star is enhanced by almost 3 orders of magnitude, leading to an orbital evolution time for the PSR J0045−7319 binary that is consistent with the observations.

Subject headings: binaries: close — stars: oscillations — stars: rotation — stars: early-type — pulsars: individual (PSR J0045−7319)

1. INTRODUCTION

An unusual binary system was recently discovered in the Small Magellanic Cloud (SMC; McConnell et al. 1991; Kaspi et al. 1994). One member of this binary is a pulsar, PSR J0045−7319, with a spin period of 0.93 s; its companion is a main-sequence B star of mass $\sim 8.8\ M_\odot$ (Bell et al. 1995). The orbital period of this binary system is 51 days, the eccentricity is 0.808, and the separation between the stars at periastron is about 4 times the B star radius. Kaspi et al. (1996) report that the orbital period for this system is evolving on a timescale of $\sim 5 \times 10^5$ yr, whereas the standard theory for tidal dissipation predicts an evolution time of order 10^9 yr. Observations also indicate that the spin of the B star is not perpendicular to the orbital plane and that the component of the spin normal to the orbital plane is perhaps close to the breakup rotation speed (Lai, Bildsten, & Kaspi 1995; Kaspi et al. 1996).

It has been suggested (Lai 1996) that the rapid evolution of the orbital period of the SMC binary can be accounted for by the radiative dissipation of the dynamical tide, provided that the B star companion to PSR J0045−7319 has a rapid retrograde rotation rate. This is an interesting idea, and the observations do suggest that the B star is rapidly rotating, though the sign of the spin is unknown. We show, however, that as long as the B star is rotating rigidly, the dissipation of the dynamical tide is too inefficient to explain the observed evolution of the orbital period, regardless of the direction or the magnitude of the rotation. When the star has some differential rotation, so that the frequency of the tidally excited waves in the rest frame of the fluid near the surface of the star is smaller than in the deep interior, the wave dissipation increases dramatically, readily explaining the observations. A more detailed and rigorous presentation of this analysis will be given in a future paper (Kumar & Quataert 1997).

2. THE DYNAMICAL TIDE IN A RIGIDLY ROTATING STAR

The tidal force on a star in a highly eccentric binary system is sharply peaked at periastron, and so the frequency of the quadrupole tide is $\approx 2\Omega_p$, where $\Omega_p = \Omega_o(1 - e^2)^{1/2}/(1 - e)^2$ is the angular speed of the star at periastron, $\Omega_o = (GM_t/a^3)^{1/2}$ is the orbital frequency, e is the orbital eccentricity, M_t is the total mass of the binary system and a is the semimajor axis (for the SMC binary system: $e = 0.808$, $M_t \approx 10.2\ M_\odot$, $a \approx 126\ R_\odot$, $\Omega_o = 1.42 \times 10^{-6}$ rad s^{-1}, and $\Omega_p/2\pi = 3.61\ \mu$Hz). Thus, waves excited by the tidal forcing have frequencies $\approx 2\Omega_p$, as seen by an inertial observer. If the angular velocity of the B star perpendicular to the orbital plane is Ω_*, then the wave frequency in the local rest frame of the fluid is $\omega_{t*} \approx 2(\Omega_p - \Omega_*)$. The angular momentum luminosity in the dynamical tide (the angular momentum transported by waves across a sphere of radius r per unit of time) is

$$\dot{L}_{\rm tide} = 2\rho r^2 \omega_{t*} \xi_t^2(r) v_g, \tag{1}$$

where ξ_t^2 is the square of the transverse displacement (integrated over solid angle) associated with the dynamical tide, $v_g \sim r\omega_{t*}^2/(6^{1/2}{\rm N})$ is the group speed of quadrupole gravity

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waves in the limit of small wave frequencies ($\omega_{t*} \ll N$), and N is the Brunt-Väisäla frequency, which is the oscillation frequency of a fluid parcel displaced vertically in a stably stratified medium (cf. Unno et al. 1989). We note that the radial displacement amplitude of the dynamical tide is much smaller than its transverse displacement and therefore has been neglected in the above equation. It can be shown (Zahn 1975) that ξ_t is approximately equal to the transverse displacement associated with the equilibrium tide, $\xi_t^{(\mathrm{eq})}$, about a wavelength away from the convective core. Furthermore, it is easy to show that the squared transverse displacement of the equilibrium tide integrated over all angles is (Goldreich & Nicholson 1989)

$$|\xi_t^{(\mathrm{eq})}|^2 = \frac{3\pi}{10}\frac{G^2M^2}{R^6}\left[\frac{d}{dr}\left(\frac{r^4}{g}\right)\right]^2, \qquad (2)$$

where R is the distance between the two stars, $M \approx 1.4\ M_\odot$ is the mass of the neutron star, and g is the gravitational acceleration of the primary star. The total angular momentum in the dynamical tide is given by $L_{\mathrm{tide}} \approx T_{\mathrm{peri}}\dot{L}_{\mathrm{tide}}$, where $T_{\mathrm{peri}} \approx \pi/\Omega_p \approx 2$ days is the duration of the periastron encounter. The rate of change of the orbital energy due to the dissipation of the dynamical tide is $\dot{E}_{\mathrm{orb}} = -\Gamma E_{\mathrm{tide}}$, where Γ is the dissipation rate for the dynamical tide (calculated below) and E_{tide} is the energy in the dynamical tide, which can be shown to be equal to $L_{\mathrm{tide}}\Omega_p$, i.e.,

$$E_{\mathrm{tide}} \approx \frac{6\pi^2}{600^{1/2}}\left(\frac{M}{M_t}\right)^2\left[\frac{\omega_{t*}^3\Omega_o^4}{(1-e)^6}\right]\frac{r\rho}{N}\left[\frac{d}{dr}\left(\frac{r^4}{g}\right)\right]^2. \qquad (3)$$

We note that all quantities in the above equation that depend on r are evaluated about 1 wavelength away from the inner turning point of the wave. We take the radius of the B star to be 6.0 $R_\odot$; for the B star model provided to us by the Yale group, the radius of the convective core of the B star, where the gravity waves become evanescent, is about 0.23 R_*, the mass in the convective core is $\approx 3\ M_\odot$, and the density and Brunt-Väisäla frequency outside the core are 2 g cm^{-3} and 100 μHz, respectively. Substituting these into equation (3) we find that the total energy in the dynamical tide for a nonrotating star, so that $\omega_{t*}/2\pi \approx \Omega_p/\pi \approx 7.2$ μHz, is $\approx 10^{40}$ ergs. For retrograde rotation of the B star at a frequency of $\Omega_*/2\pi = -6.1$ μHz, or $\hat{\Omega}_* \equiv \Omega_*(R_*^3/GM_*)^{1/2} \approx -0.3$, the frequency of the tidally excited wave is $\omega_{t*}/2\pi \approx 19.4$ μHz and equation (3) implies that the energy in the tide is a factor of about 20 times larger than in the nonrotating star. This point was made by Lai (1996), who suggested that the enhanced energy in the dynamical tide could explain the observed evolution of the SMC system. Unfortunately, as we point out below, this is not tenable because the damping time for a wave with frequency ≈ 19.4 μHz is larger by a factor of about 100 than for a wave with frequency ≈ 7.2 μHz.

For prograde rotation the tidal frequency and energy decrease with increasing rotation rate so long as $\Omega_* < \Omega_p$ (see eq. [3]). The tidal energy does not actually vanish when $\Omega_* = \Omega_p$ because axisymmetric waves are still excited. For $\Omega_* > \Omega_p$ the tidal torque causes the angular momentum of the stellar spin to decrease (which is taken up by the orbit), and as a result the orbital energy and the orbital period increase with time. Equation (3) shows that $E_{\mathrm{tide}} < 0$, and so $\dot{E}_{\mathrm{orb}} > 0$, when $\Omega_* > \Omega_p$, which may seem paradoxical. We note that the tidal energy as seen in the rotating frame of the star is a positive definite quantity; the tidal energy as seen from an inertial frame (given in eq. [3]), however, also includes a contribution from the work done by the tidal force on the rotating fluid in the star, which can either increase or decrease the star's rotational kinetic energy. The negative tidal energy for $\Omega_* > \Omega_p$ corresponds to more energy being taken out of the rotation than is deposited in the dynamical tide. We emphasize that the tidal energy is positive so long as $\Omega_* < \Omega_p$, which obviously includes the case of arbitrary retrograde rotation, and is negative when $\Omega_* > \Omega_p$. Thus, if the rotation rate of the B star is greater than Ω_p, the observed period decrease of the PSR J0045−7319 binary implies that the direction of the rotation of the B star must be retrograde. We also note that the energy in the dynamical tide for prograde rotation of the B star with an angular rotation speed somewhat smaller than Ω_p, which is not ruled out by the observed apsidal motion, is about 10^{39} ergs. If all this energy is dissipated in one orbital period, the resulting orbital evolution time will be $\approx 4 \times 10^7$ yr, or about 2 orders of magnitude greater than the observed value. Thus, the apsidal motion and the orbital evolution observations together rule out both subsynchronous, as well as supersynchronous, prograde rotation of the B star.

A detailed calculation of the damping of gravity waves is given in Kumar & Quataert (1997). Here we present a crude estimate to point out that the wave damping increases very rapidly with decreasing wave frequency.

The local damping rate of a wave, $\gamma(r)$, is approximately equal to the inverse of the time it takes photons to diffuse across a wavelength times the ratio of the photon energy density to the total thermal energy density of the plasma. This can be written in the following convenient form (cf. Unno et al. 1989):

$$\gamma(r) \sim Fk_r^2/(\rho g), \qquad (4)$$

where F is the radiative flux of the star and $k_r \sim N[l(l+1)]^{1/2}/(r\omega_{t*})$ is the radial wave number for gravity waves ($l = 2$ for quadrupole waves). We see from the above expression that the wave damping increases with decreasing density in the star, and thus most of the contribution to the wave damping comes from near the upper turning point of the wave, which is located at a depth of $\sim R_*^2\omega_{t*}^2/(6g)$ beneath the stellar surface.[3] Integrating the local dissipation rate, weighted by the wave's energy density, yields the global wave damping rate. Since the energy density in the wave is inversely proportional to its group speed, we find that the global damping rate for the wave, in a star of polytropic index 2 (a value appropriate for the outer envelope of the Yale group's B star model we are using), scales as ω_{t*}^{-7}. This scaling applies only to waves with frequencies less than about 15 μHz for the SMC B star. Modes with higher frequencies have wavelengths of order the stellar radius, and their dissipation is not as concentrated near the outer turning point as it is for lower frequency modes. Thus, the global damping rate of high-frequency g-modes has a weaker dependence on mode frequency.[4] Using equation (4) and the parameters for the SMC B star given above, we

[3] The upper turning point for gravity waves of frequency ω and degree l is located at a radius r that is determined by solving the equation $l(l+1)/r^2 = \omega^2/c^2(r)$ (cf. Unno et al. 1989), where $c(r)$ is the sound speed. The sound speed near the surface of a star is given by $c^2(r) \equiv c^2(R_* - z) \approx zg$, and thus the depth ($z$) of the upper turning point measured from the stellar surface, for waves of $l = 2$, is approximately $\omega^2R_*^2/(6g)$.

[4] The transition from a strong (ω_{t*}^{-7}) to a weak dependence of mode dissipation on frequency is, of course, a gradual one, and so there is some arbitrariness in designating a boundary between the two regions.

estimate that the damping time for a low-order quadrupole gravity wave is ∼3000 yr, which is consistent with the results of a number of different nonadiabatic eigenvalue calculations (Kumar & Quataert 1997; Saio & Cox 1980).

The frequency of the dynamical tide in the nonrotating star is ≈7.2 μHz, while for rapid retrograde rotation ($\hat{\Omega}_* = -0.3$) the frequency is ≈19.4 μHz. Using the above scaling for the damping of the dynamical tide, we find that the ratio of the damping times in these two cases is $\approx 10^{-2}$. Since the rate of dissipation of the orbital energy is equal to the tidal energy times its damping rate, we see that even though the energy in the tide for rapid retrograde rotation is a factor of ∼20 larger than for no rotation, the rate of dissipation of the orbital energy is comparable for $\hat{\Omega}_* = -0.3$ and $\hat{\Omega}_* = 0$. Using the dissipation time for the dynamical tide in the retrograde rotating star of ∼3000 yr, the energy in the dynamical tide of $\sim 10^{41}$ ergs, and the orbital energy for the SMC binary, $E_{\rm orb} \approx 2 \times 10^{47}$ ergs, we find that the orbital period evolution time for the SMC binary is $\approx 10^9$ yr, or roughly 3 orders of magnitude larger than the observed value.

3. DIFFERENTIAL ROTATION ENHANCED DISSIPATION OF TIDES

It should be clear from this discussion that one way to get more rapid evolution of the orbit is to find a more efficient way of dissipating the energy in the dynamical tide. This would occur naturally, considering the sensitivity of wave dissipation on frequency, if the frequency of the tidally excited waves were smaller near the surface of the star, where the dissipation occurs. Waves with frequencies $\approx 2\Omega_p$, as seen in an inertial reference frame, are excited near the interface of the convective core and the radiative exterior, where the wavelength of the wave is the largest. Thus, in order to get a smaller wave frequency in the rest frame of the fluid near the surface of the star, we need the surface to be rotating at a frequency close to $\Omega_p/2\pi \approx 3.6$ μHz. We show below that this is in fact expected for the B star of the SMC binary.

For nearly circular orbits, the rotation of stars in close binary systems tends to synchronize with the orbital motion; that is, the rotation frequency of the star approaches the orbital frequency. For highly eccentric orbits, the tidal force is only appreciable near periastron, and so the rotation frequency of the star approaches Ω_p. This is called pseudosynchronization.

We estimate the relative timescales for spin pseudosynchronization and orbital circularization for the SMC system to determine if the B star should be rotating pseudosynchronously. The pseudosynchronization timescale due to the transfer of orbital angular momentum to the star is $L_*/\dot{L}_* = -(L_*/L_{\rm orb})(L_{\rm orb}/\dot{L}_{\rm orb})$, where $L_{\rm orb}/L_* \approx 4\hat{\Omega}_*^{-1}$ for the SMC binary and $\dot{L}_{\rm orb} \approx \dot{E}_{\rm orb}/\Omega_p$ (see above). Since $L_{\rm orb}\Omega_o/E_{\rm orb} \approx 1$, we find that $L_*/\dot{L}_* \approx 5\hat{\Omega}_* E_{\rm orb}/\dot{E}_{\rm orb}$ for the SMC binary system. It follows from the general equations for tidal evolution in the weak friction limit that a similar result holds for the equilibrium tide (Hut 1981). We conclude that so long as the B star in the SMC binary is rotating rapidly, both the equilibrium and the dynamical tides lead to timescales for spin pseudosynchronization that are comparable to the orbital circularization time. We therefore do not expect that the rotational speed of the B star's interior has changed appreciably, which seems consistent with the observations.

We do expect, however, that the rotation rate near the surface of the B star has changed significantly, leading to appreciable differential rotation in the B star. The physical reason for this follows from the seminal work of Goldreich & Nicholson (1989). Waves generated by tidal forces deposit their angular momentum at the place in the star where they are dissipated. Since the dissipation rate is largest near the surface of the star, and the moment of inertia of the surface region is a small fraction of the star's total moment of inertia, the outer region of the star tends to be pseudosynchronized on a much shorter timescale than the interior. For the retrograde rotating B star with $\hat{\Omega}_* \approx -0.3$, the frequency of the dynamical tide is ≈19 μHz; the outer turning point of this wave is at $r \approx 0.9\ R_*$ and so the rotation of the B star at $r \approx 0.9\ R_*$ should be pseudosynchronous. Those parts of the star lying above or significantly below this radius, however, experience little net tidal torque and thus can continue to rotate at the primordial rotation rate of the star provided that magnetic stresses and shear instabilities do not efficiently transport angular momentum to/from the pseudosynchronous layer. The optical linewidth measurements (cf. Bell et al. 1995) do not provide conclusive evidence for whether or not the surface of the star is rotating pseudosynchronously.

The frequency of the tidally excited gravity wave in the local rest frame of the star a distance r from the center is $\omega_{t*}(r) \approx 2[\Omega_p - \Omega_*(r)]$, and its wavenumber is $\sim 6^{1/2}N(r)/(r\omega_{t*})$, where $\Omega_*(r)$ is the local angular rotational velocity of the star normal to the orbital plane. Thus, as the wave approaches the pseudosynchronously rotating surface of the star, its frequency and wavelength approach zero and the energy it carries is entirely dissipated. The energy in the dynamical tide for retrograde stellar rotation at a frequency of approximately -6μHz ($\hat{\Omega}_* \approx -0.3$) is $\sim 10^{41}$ ergs (see eq. [3]). With a change in the orbital energy per orbit of $\Delta E_{\rm orb} \approx -E_{\rm tide} \approx -10^{41}$ ergs, and the orbital energy of 2×10^{47} ergs, we find that $P_{\rm orb}/\dot{P}_{\rm orb} \approx E_{\rm orb}/\dot{E}_{\rm orb} \approx 3 \times 10^5$ yr, which is in good agreement with the observations. Given the uncertainty in the mass and radius of the B star, the observed orbital evolution can be understood provided $\hat{\Omega}_*$ is between approximately −0.3 and 0. The observed apsidal motion, however, rules out $|\hat{\Omega}_*| \lesssim 0.1$ (Kaspi et al. 1996), and thus $\hat{\Omega}_* \lesssim -0.1$ is the only solution consistent with both the apsidal motion and the orbital evolution observations.

We note that prograde rotation of the B star at $\hat{\Omega}_* \approx 0.4$ gives an orbital evolution time of comparable magnitude, though with the opposite sign. This emphasizes that retrograde rotation is required not to explain the observed short timescale for orbital evolution but rather to get the correct sign for the time derivative of the orbital period. If the rotation of the star is taken to be prograde and subsynchronous, then in order to fit the observed apsidal motion Ω_* must be comparable to Ω_p. In this case, as we discussed in § 2, the energy in the dynamical tide is too small by about 2 orders of magnitude to explain the observed orbital evolution. For a prograde rotation rate close to zero, the very efficient dissipation mechanism for tidal energy described in this paper is sufficient to explain the orbital evolution, provided that the radius of the B star is $\approx 7\ R_\odot$ (instead of $\approx 6\ R_\odot$). Such a small rotation rate is, however, ruled out by the apsidal motion observations. Thus, in order to explain all of the available observations, the B star of the PSR J0045−7319 binary must have retrograde rotation.

It is easy to show that the rate of change of the orbital eccentricity ($\dot{e}$) is a factor of about 7 smaller than $\dot{P}_{\rm orb}/P_{\rm orb}$. Thus, the theoretically expected value for $\dot{e}$ is a factor of ∼20 smaller than the current observational limit (Kaspi et al. 1996).

Prior to the supernova that produced the pulsar, it is likely that the rotational angular momentum of the stars and the orbital angular momentum were all aligned and synchronized; this is because the tidal interactions were much stronger when the progenitor star went through a giant phase, and there may have been episodes of mass transfer between the stars. Thus, the present retrograde rotation of the B star, as well as the misalignment of the spin and orbital angular momentum axes inferred by Lai et al. (1995) and Kaspi et al. (1996), suggest that the supernova must have had a dipole asymmetry that gave a net angular momentum kick to the system and reversed the sign of the orbital angular momentum. The large peculiar velocity of neutron stars is widely interpreted as evidence for asymmetric supernova (Lyne & Lorimer 1994); we believe that the dynamics of the PSR J0045−7319 binary provides more direct evidence for the asymmetry.

Kumar, Ao, & Quataert (1995) found that the amplitude of the light variation from the B star of the SMC binary caused by the equilibrium tide at periastron is about 5 millimag, and that the shape of the light curve depends sensitively on the inclination angle of the orbit. These predictions remain unaffected by the differential rotation of the star described in this paper; i.e., the amplitude and the shape of the light curve should be as is given in Figure 3 of Kumar et al. An observational detection of this light curve would thus provide an independent means of determining the orbital inclination angle and thus the masses of the stars. The frequencies and amplitudes of the dynamical tide (tidally excited g-modes) estimated by those authors will likely be modified by the differential rotation of the B star; the observational detection of these modes would provide interesting information regarding the star's internal structure.

We thank Jeremy Goodman, Dong Lai, and Vicky Kaspi for discussions and comments. We are grateful to an anonymous referee for a careful reading of the paper and for suggesting a number of improvements to our presentation.

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