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VERY FAST OPENING OF A THREE-DIMENSIONAL TWISTED MAGNETIC FLUX TUBE

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ABSTRACT

This Letter is devoted to the still open problem of the evolution of a three-dimensional coronal flux tube embedded in a low-beta ideal plasma and having its footpoints twisted by slow photospheric motions. Such a process has been simulated with a recently developed magnetohydrodynamic code. In the particular calculation reported here, the system occupies a large cubic box. The field is initially potential, being generated by an underlying horizontal dipole, and it is twisted by two vortices located on the lower face $\{z = 0\}$ of the box, on both sides of the neutral line. In a first phase, the field roughly evolves quasi-statically through a sequence of force-free configurations. Thus, it enters a dynamical phase during which it suffers a very fast expansion, closely approaching after some finite time a semiopen configuration. The energy increases monotonically during all the evolution, and it tends to a limit, which is equal to about 80% of the energy of the totally open field associated with B_z.

Subject headings: MHD — stars: coronae — stars: flare — stars: magnetic fields

1. INTRODUCTION

Coronal mass ejections and large two-ribbon flares are very interesting phenomena that involve at some stage a fast opening of a part of the magnetic field of the solar corona. From the observational viewpoint, it is now well established that they are produced in regions in which the magnetic shear is large along the neutral line (Heyvaerts & Hagyard 1991 and references therein). Consequently, most of the theoretical work aimed at their understanding has concentrated on the following basic question of magnetohydrodynamics (MHD): In which conditions does a coronal magnetic field embedded in a low-beta highly conducting plasma and evolving as a result of slow photospheric motions imposed on its footpoints lose equilibrium, thus entering a phase of rapid evolution leading to an opening of the magnetic lines?

During the past two decades, this problem has been mainly addressed in the framework of a simple model in which the field occupies the half-space $\{z > 0\}$ and is x-invariant. In this case, the behavior of the field is now quite well understood from both the analytical (see, e.g., Aly 1990) and the numerical (see, e.g., Amari et al. 1996, hereafter ALAT) points of view. For instance, dynamical simulations reported in ALAT (and effected with our recently developed MHD code METEOSOL, described in detail in Amari & Luciani 1995) have shown that the system follows an almost quasi-static evolution as long as resistivity is neglected, the field expanding gently while electric currents concentrate into a sheet. In the presence of resistivity, the evolution is not changed as long as the shear is not very large. But when the latter exceeds a critical value, reconnection actually develops, and a plasmoid is ejected by the system at roughly the Alfvén speed.

Very recently, several authors have considered the shearing problem in cylindrical geometry, with the field occupying the exterior of a sphere—the main difference with the previous model arising from the fact that the field now decreases to zero at infinity in the three spatial directions (and not in only two of them). This simple fact is at the origin of an important qualitative change in the behavior of the system: at some stage, the field starts expanding at a rate much larger than the driving velocity on the boundary, and eventually, the quasi-static approximation is no longer able to describe quantitatively the evolution of the field (Mikic & Linker 1994; Roumeliotis, Sturrock, & Antiochos 1994; Lynden-Bell & Boily 1994; Aly 1995; Wolfson 1995).

As for the fully three-dimensional problem, it has not been dealt with very much so far. Klimchuk & Sturrock (1992) presented examples of evolving equilibria, with their calculations showing no evidence for the development of nonequilibrium phenomena. But they were able to impose shears of only a moderate value. Dahlburg, Antiochos, & Zang (1991) also computed the evolution of some structures, which turned out to have a gentle behavior. But they made an assumption of incompressibility of the plasma, which is not realistic and certainly prevents a large expansion of the system.

The aim of this Letter is to report the results of a detailed study of the evolution of a three-dimensional flux tube occupying a half-space and having its footpoints submitted to twisting motions. It has been effected with our numerical code quoted above, which is able to follow an evolution for quite a long time. Most recently, related calculations have been done independently by Mikic, Linken, & Schnack (1996). However, their work differs from ours in several respects. In particular, they consider a quadrupolar region rather than a dipolar one, and they find a slow opening of the lines associated with weak fields, while we exhibit in our simulations a very fast opening of the lines associated with strong fields. The Letter is organized as follows. In § 2, we present the model. In § 3, we report and analyze our results, while our conclusions are presented in § 4.

2. DESCRIPTION OF THE MODEL AND NUMERICAL METHOD

We consider a model in which the corona is represented by the half-space $D = \{z > 0\}$, which is filled up with a nonresistive, slightly viscous plasma. The latter obeys the simplified

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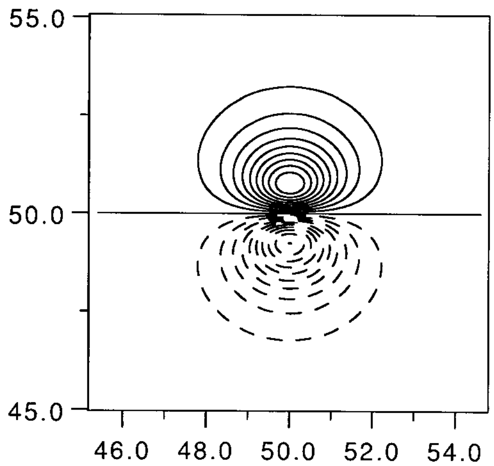

FIG. 1*a*

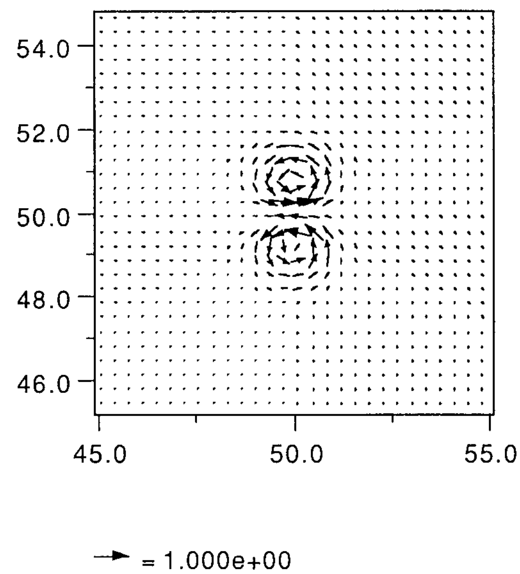

FIG. 1*b*

FIG. 1.—(*a*) Photospheric distribution of the normal component of the initial magnetic configuration. The contours correspond to a dipole located below the plane $\{z = 0\}$ at a depth $z = -1.5$ and having its axis parallel to $\hat{\boldsymbol{y}}$. (*b*) Photospheric velocity field ($v_x^{\rm ph}$, $v_y^{\rm ph}$) applied to the footpoints of the initial dipolar magnetic configuration, used as a Dirichlet boundary condition for $\boldsymbol{v}$.

MHD equations (written in nondimensionalized form; see ALAT)

$$\rho \frac{\partial \boldsymbol{v}}{\partial t} = -\rho(\boldsymbol{v} \cdot \nabla \boldsymbol{v}) + (\nabla \times \boldsymbol{B}) \times \boldsymbol{B} + \nu \rho \Delta \boldsymbol{v}, \tag{1}$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B}), \tag{2}$$

$$\nabla \cdot \boldsymbol{B} = 0, \tag{3}$$

where $\boldsymbol{B}$ is the magnetic field, and $\boldsymbol{v}$, ρ, and ν are, respectively, the velocity, mass density, and kinematic viscosity of the plasma. The pressure term is neglected, and ρ is taken to be a prescribed function of the position $\boldsymbol{r}$. We choose here $\rho = B^2$, which makes the Alfvén velocity $v_{\rm A}$ uniform and equal to 1.

For the numerical computations, D is approximated by the finite box $\{0 < x < L_x, 0 < y < L_y, 0 < z < L_z\}$, the size of which is very large compared to the characteristic spatial scale of the system (which is our reference length). Here, we take $L_x = L_y = L_z = 300$ (the cases $L_x = L_y = L_z = 100$ and $L_x = L_y = L_z = 600$ have also been considered, with similar results being obtained). Inside that box, the MHD equations are discretized on a nonuniform mesh. An important fraction of the $101 \times 103 \times 99$ nodes are concentrated near the central plane $\{y = 0\}$—i.e., in the region where it is expected on theoretical grounds (Aly 1990) that electric currents are going to concentrate—and near the bottom of the box, where the field will be stronger. The horizontal size $\Delta x = \Delta y$ of the cells varies between 0.03 and 12, while their vertical size Δz is ranges between 0.04 and 12.

At time $t = 0$, we take $\boldsymbol{B}$ to be the potential field created by a dipole of moment $\boldsymbol{m} = 4\hat{\boldsymbol{y}}$ located at (0, 0, −1.5) (Fig. 1*a*), i.e.,

$$\boldsymbol{B}_0 = \frac{4}{[x^2 + y^2 + (z + 1.5)^2]^{5/2}} \times [3xy, 2y^2 - x^2 - (z + 1.5)^2, 3y(z + 1.5)]. \tag{4}$$

For $t \geq 0$, we impose the footpoints on the bottom of the box to suffer a twisting velocity field

$$\boldsymbol{v}^{\rm ph}(x, y, t) = \nabla^{\perp}\{\phi[B_z(x, y, 0), t]\}, \tag{5}$$

which has the property of preserving the normal component $B_z(x, y, 0)$ of the field on the boundary (Aly 1991). In equation (5), $\nabla^{\perp} = (\partial_y, -\partial_x)$ and

$$\phi(B_z, t) = v_0 f(t) B_z^2 \exp\left(\frac{B_z^2 - B_{z\,\max}^2}{\delta B^2}\right), \tag{6}$$

where $B_{z\,\max}$ is the maximum value of $B_z(x, y, 0)$, $\delta B = 10$, and $v_0 = 10^{-2}$ (then v_0 is small compared to the Alfvén speed $v_{\rm A} = 1$), and the ramp function f (which is used to switch on or switch off the velocity field smoothly) is first linear and thus stays constant. The velocity field is shown in Figure 1*b*. It corresponds to two parallel vortices rotating in the same direction.

On the other faces of the box, we take the homogeneous Dirichlet condition $\boldsymbol{v} = 0$, which is natural for a viscous plasma in contact with a nonmoving wall.

The calculation of the evolution of the field is done with the code METEOSOL, which uses a semi-implicit numerical scheme (for details, see Amari & Luciani 1995). A typical run is made as follows:

1. In a first step, we impose a zero velocity on the boundary and a large viscosity (1–10), and we let the initial field freely evolve up to $t = 200$ (the unit of time is the characteristic Alfvén time $\tau_{\rm A}$ of the structure). We thus obtain a numerically satisfying initial equilibrium, in which the residual current and Lorentz force densities (generated by the discretization errors) are smaller than 10^{-6}. The divergenceless condition for the magnetic field is well satisfied.

2. Thus we start twisting that initial state, using a linear ramp of $20\tau_{\rm A}$ to reach a maximum velocity 10^{-2} at $t = 20$ (time having been reinitialized). The photospheric twisting

motions are applied up to about $t = 500$. The time advance is performed with a time step between 0.05 and 0.1. Thanks to our numerical scheme, this allows us to effect the simulation in a reasonable computational time. The viscosity is fixed at 10^{-2}.

3. RESULTS

Qualitatively, the evolution of the field can be roughly divided into two phases.

1. In the first phase, the evolution of the field is approximately quasi-static. Near the neutral line, where quite a lot of shear is introduced, the magnetic structure evolves quite similarly to the x-invariant sheared arcade considered in ALAT. When we move toward the spot centers, the corresponding field lines become sensitive to twist, and they assume a characteristic S shape, which is reminiscent of the shape of some coronal loops recently observed by *Yohkoh* (the flare of 1994 October 25, reported by Van Driel-Gesztelyi 1995). Correlatively, we observe a relatively slow (velocity smaller than 10^{-2}) and progressive inflation of the field lines in all three spatial directions. However, the outermost field lines (which correspond to regions of weak magnetic field and shear), rather than expanding, lean sideward to let through the twisted inner field lines associated with stronger magnetic field regions (Fig. 2 [Pl. L5]; actually, even the field lines whose footpoints are fixed, coinciding with the knots of the photospheric velocity field, emerge through the outer weaker field lines). This feature is characteristic of a fully three-dimensional field. In an x- or ϕ-invariant field, indeed, the outer lines are forced to expand by the inner ones, merely because of the strong symmetry of the system.

2. The second phase starts at about $t = 395$–405 and is mainly characterized by a tremendous expansion of the field, which occurs at a high rate, the velocities reaching values of the order of 0.4–0.7. The electric current density increases in the middle plan $\{y = L_y/2\}$. As can be seen in Figure 3 (Plate L6), this eventually leads to what we characterize as a (possibly partial) opening of the magnetic field. (Of course, a true opening is forbidden because of our boundary conditions on the outer faces of the box, which confine the system. When we say that the field opens, we mean here that it becomes quite similar to an open field in the inner part of the box, where the effects of the outer boundary conditions are very small for the durations of our runs.)

More quantitative information on the evolution has been obtained by monitoring two important quantities: the flux tube volume F and the ratio $w = W/W_0$ of the magnetic energy W to the energy W_0 of the potential field associated with $B_z(x, y, 0)$.

1. F is defined by the relation

$$F(x, y, t) = \frac{\delta V}{\delta \Phi} = \int_{\mathscr{C}} \frac{dl}{\|\boldsymbol{B}\|}, \quad (7)$$

where $\mathscr{C}$ is the field line originating from the point $(x, y, 0)$ of the lower boundary, and δV and $\delta\Phi$ are, respectively, the volume and the magnetic flux of an infinitely thin flux tube surrounding $\mathscr{C}$. The evolution of the maximum value of F is shown in Figure 4 (all the individual lines originating from the strong field region exhibit similar F-profiles). The transition observed on the velocity is confirmed by the evolution in time of the flux tube volume, which increases at least exponentially

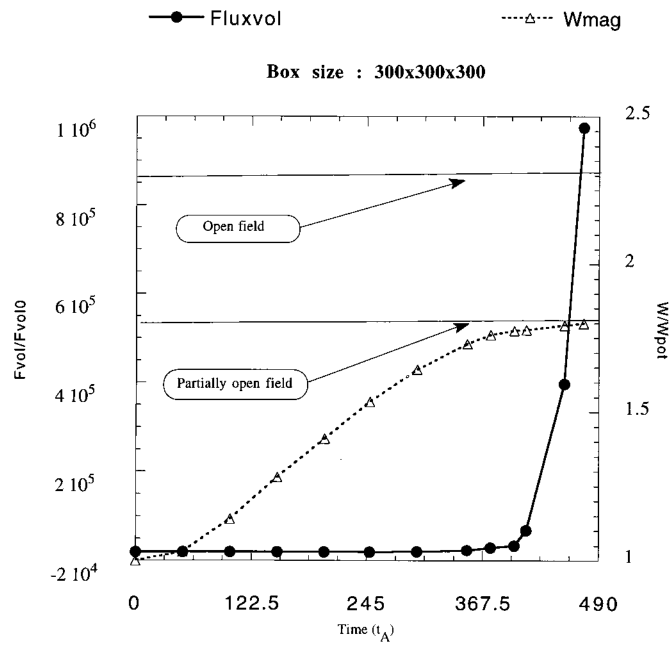

FIG. 4.—Flux tube volume F in the strong magnetic field middle region and relative magnetic energy w, as functions of time t. The field tends toward a semiopen field, the energy of which is lower than that of the totally open field. The transition to opening occurs at $t \approx 417$, when the energy stored in the field is close to that of the semiopen field.

fast after the critical time. The lines are allowed to reach large values of the flux tube volume before feeling the effect of the finite size of the box shown by the last point position (a tube hitting the outer boundary gets compressed afterward by the underlying ones). It should be noted that the authors quoted in § 1 have taken the maximum height of a given field line rather than F as the criterion for the opening of the field. Simple estimates show, however, that both criteria are roughly equivalent—a point that will be discussed in detail in a forthcoming paper.

2. The evolution of the "relative" magnetic energy w stored in the system is also shown in Figure 4. We can make a few striking points about this curve: (a) w is a monotonically increasing function of time. (b) w does not exceed the energy of the open field associated with the value of $B_z(x, y, 0)$ (the latter is found to be $w_{op} = 2.3$)—a result that is in accordance with theoretical arguments developed in the framework of the quasi-static theory (Aly 1984, 1991; Sturrock 1991). (c) In fact, w appears to approach a lower limit $w_l = 1.8$, the transition to an open field occurring when w is of the order of 99% of that limit. We take these indications as the strongest evidence that the field opens only partially—the lines emerging in the region of strong field becoming open, and those originating in the weak field region staying closed. It is worth noticing that a complete opening could result eventually in a very long ideal run. However, it is clearly the partial opening that we have shown here that is important from a physical point of view, as it will certainly lead to immediate reconnection when resistivity will be introduced into the code.

4. CONCLUSIONS

In this Letter, we have reported new results on the ideal MHD behavior of a magnetic flux tube driven into evolution by twisting motions imposed on its footpoints on a boundary.

Our results have been obtained by using the three-dimensional version of our dynamical METEOSOL code that was used before to treat the simplest two-and-one-half–dimensional (ALAT). For a field initially potential (being created by a single horizontal subphotospheric dipolar moment) and twisted by two vortices located on either side of the neutral line, we have obtained the following results:

1. During a first phase, the field evolves approximately through a sequence of force-free configurations. Several interesting features were noticed: (*a*) The magnetic lines assume a characteristic S shape in the inner part of the system, close to the neutral line. (*b*) They do suffer a global slow inflation, mainly localized in the region of stronger photospheric magnetic field. (*c*) The exterior field lines, rooted in the region of weaker magnetic field where the imposed twist is small, lean sideward in response to the volumic force-free currents.

2. In a second phase, the field evolves dynamically. This phase is characterized by the following features: (*a*) The fluid velocity reaches values that are a nonnegligible fraction of the Alfvén speed. (*b*) The rate of increase of the flux tube volume shows a sharp transition, rising in a few Alfvén times from a moderate value to a tremendous one. The field can be considered as being partially open after some finite time. This result appears to be the first extension to the fully three-dimensional case of results previously discussed by several authors for axisymmetric fields (Lynden-Bell & Boily 1994; Mikic & Linker 1994; Roumeliotis et al. 1994; Aly 1995; Wolfson 1995).

3. The relative magnetic energy $w = W/W_0$ increases monotonically in time, and it tends eventually to a limit (1.8), which is smaller than the energy (2.3) of the totally open field associated with the value of $B_z(x, y, 0)$, this latter quantity being the least upper bound to the energy which can be stored quasi-statically in a force-free field (Aly 1984, 1991; Sturrock 1991).

These conclusions appear to be relevant for the modeling of solar (and stellar) flares and coronal mass ejections. They show in particular that deformation of the field of a coronal region by photospheric motions may lead eventually to the fast opening that seems to be implied by the solar observations (Pneumann & Kopp 1971; Heyvaerts & Hagyard 1991). In particular, they could give an explanation to the recent measures of the evolution of the height of a prominence during a coronal ejection that was found to exhibit at some stage a sharp transition to a high rate, with a rapid ejection of the prominence (Mouradian, Soru-Escaut, & Pojoga 1995).

The ideal evolution of three-dimensional configurations with various geometries, as well as their resistive evolution, is currently under investigation, and the results will be reported in a forthcoming paper.

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REFERENCES

Aly, J. J. 1984, ApJ, 283, 349
———. 1990, Comput. Phys. Commun., 59, 13
———. 1991, ApJ, 375, L61
———. 1995, ApJ, 439, L63
Amari, T., & Luciani, J. F. 1995, J. Comput. Phys., submitted
Amari, T., Luciani, J. F., Aly, J. J., & Tagger, M. 1996, A&A, 306, 913 (ALAT)
Dahlburg, R. B., Antiochos, S. K., & Zang, T. A. 1991, ApJ, 383, 420
Heyvaerts, J., & Hagyard, M. J. 1991, in Proc of the Flares 22 Workshop, Dynamics of Solar Flares, ed. B. Schmieder & E. R. Priest (Paris: Observatoire de Paris, DASOP), 1
Klimchuk, J. A., & Sturrock, P. 1992, ApJ, 353, 385
Lynden-Bell, D., & Boily, C. 1994, MNRAS, 267, 46
Mikic, Z., & Linker, J. A. 1994, ApJ, 430, 898
Mikic, Z., Linker, J. A., & Schnack, D. D. 1996, in Proc. Sac Peak Workshop, Summer 1996, in press
Mouradian, Z., Soru-Escaut, I., & Pojoga, S. 1995, Sol. Phys., 158, 269
Pneuman, G. W., & Kopp, R. A. 1971, Sol. Phys., 18, 258
Roumeliotis, G., Sturrock, P. A., & Antiochos, S. K. 1994, ApJ, 423, 847
Sturrock, P. 1991, ApJ, 380, 655
Van Driel-Gesztelyi, L. 1995, private communication
Wolfson, R. 1995, ApJ, 443, 810

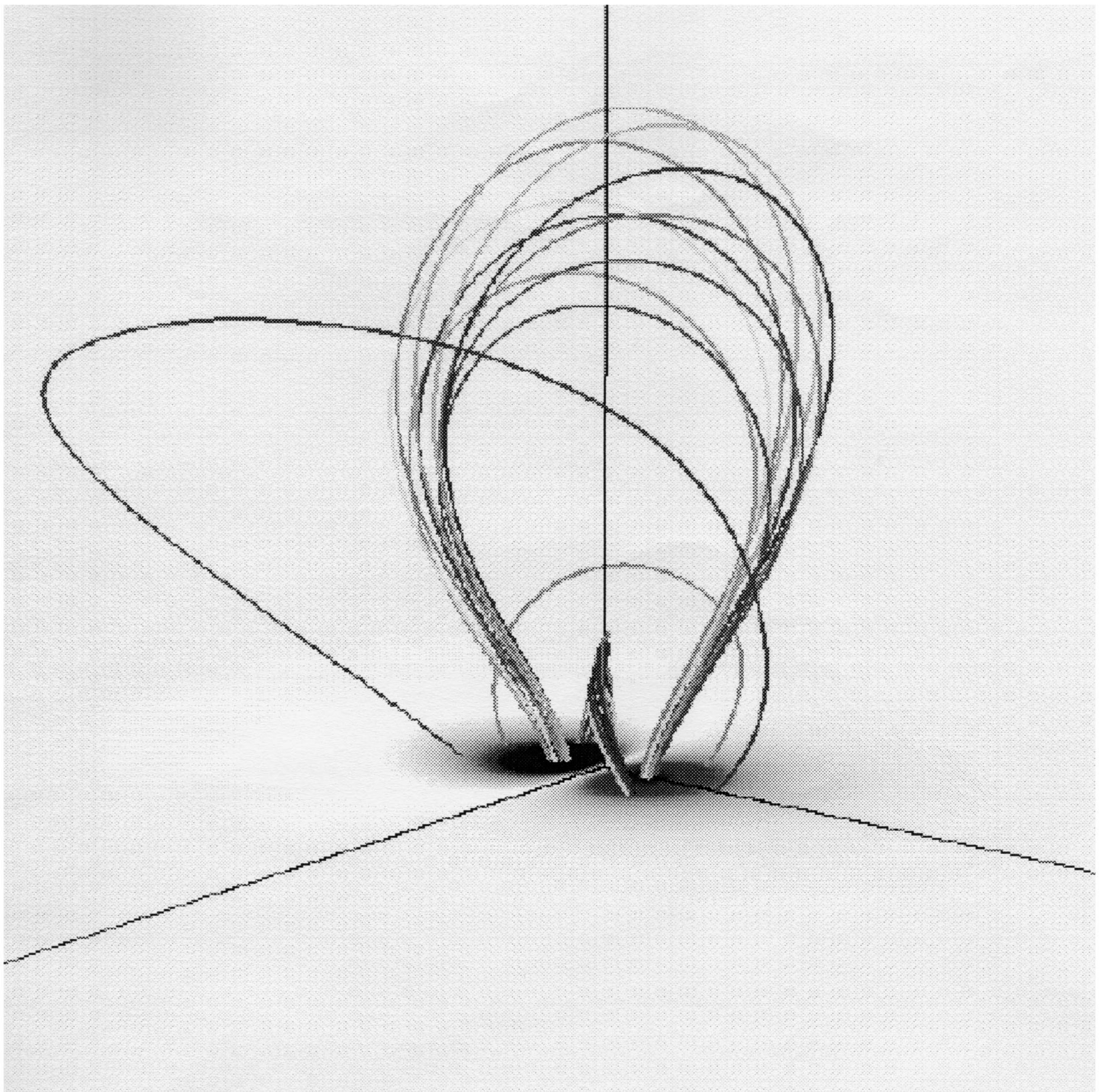

FIG. 2.—Selected set of field lines at t = 330. Note the following feature, characteristic of three-dimensional evolution: slightly twisted outer field lines coming from weaker magnetic field regions can lean sideward to allow field lines coming from a stronger field region to emerge (the flux tube comes from the center of the spots, and its center field line is untwisted).

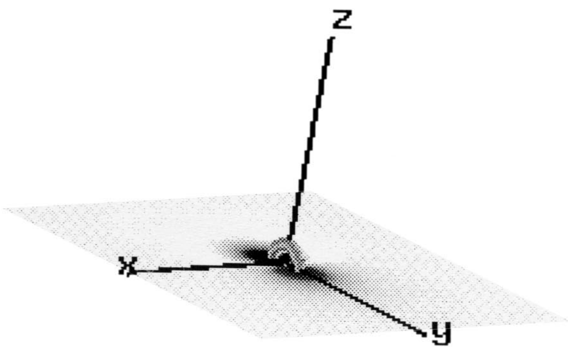

FIG. 3*a*

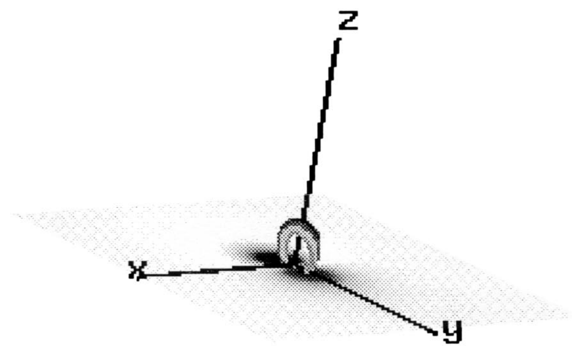

FIG. 3*b*

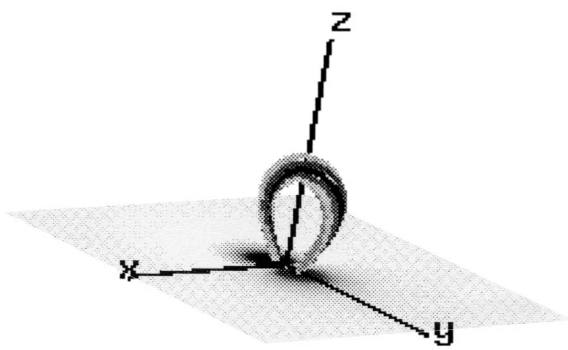

FIG. 3*c*

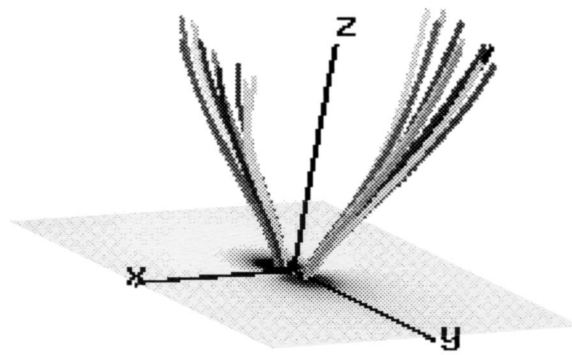

FIG. 3*d*

FIG. 3.—Ideal MHD evolution of the magnetic structure, which is found to stay quasi-static in a first phase and then becomes dynamic in a second one. This evolution leads to the opening of the magnetic configuration. The figures show the evolution of a set of selected field lines forming a flux tube at four particular stages corresponding to (*a*) $t = 0$, (*b*) $t = 200$, (*c*) $t = 330$, and (*d*) $t = 420$.