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VELOCITY BIAS IN A Λ COLD DARK MATTER MODEL

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ABSTRACT

We use a high-resolution N-body simulation to study the velocity bias of dark matter halos, the difference in the velocity fields of dark matter and halos, in a flat low-density cold dark matter (ΛCDM) model. The high force, 2 h^{-1} kpc, and mass, 10^9 h^{-1} $M_\odot$, resolution allows dark matter halos to survive in very dense environments of groups and clusters, making it possible to use halos as galaxy tracers. We find that the velocity bias, $b_{v,12}$, measured as a ratio of pairwise velocities of the halos to that of the dark matter, evolves with time and depends on scale. At high redshifts ($z \sim 5$), halos generally move faster than the dark matter on almost all scales: $b_{v,12}(r) \approx 1.2$, $r > 0.5$ h^{-1} Mpc. At later moments, the bias decreases and gets below unity on scales less than $r \approx 5$ h^{-1} Mpc: $b_{v,12}(r) \approx (0.6\text{–}0.8)$ at $z = 0$. We find that the evolution of the pairwise velocity bias follows and probably is defined by the spatial antibias of the dark matter halos at small scales. The one-point velocity bias, b_v, defined as the ratio of the rms velocities of halos and dark matter, provides a more direct measure of the difference in velocities, because it is less sensitive to the spatial bias. We analyze b_v in clusters of galaxies and find that halos are "hotter" than the dark matter: $b_v = 1.2\text{–}1.3$ for $r = (0.2\text{–}0.8)r_{\text{vir}}$, where r_{vir} is the virial radius. At larger radii, b_v decreases and approaches unity at $r = (1\text{–}2)r_{\text{vir}}$. We argue that dynamical friction may be responsible for this small positive velocity bias ($b_v > 1$) found in the central parts of clusters. We do not find significant systematic difference in the velocity anisotropy of halos and the dark matter. The velocity anisotropy function, β, of dark matter particles can be approximated as $\beta(x) = 0.15 + 2x/(x^2 + 4)$, where the distance x is measured in units of the virial radius.

Subject headings: galaxies: clusters: general — large-scale structure of universe — methods: n-body simulations

1. INTRODUCTION

Peculiar velocities of galaxies arise as a result of the gravitational pull of surrounding overdense regions and therefore reflect the underlying density field. The statistical study of galaxy velocities is important in cosmology, since it can be used as a tool to constrain cosmological models. The connection between theoretical predictions and the observed statistics usually requires an additional quantity: the difference between galaxy and dark matter (DM) velocities, termed the velocity bias. The current state of predictions of the velocity bias is rather confusing. There is a wide range of estimates of the velocity bias. Values change from strong antibias, with galaxies moving twice as slow as the dark matter (Gelb & Bertschinger 1994; Klypin et al. 1993), to almost no bias (Klypin et al. 1999, hereafter KGKK; Ghigna et al. 1998), to slight positive bias (Diaferio et al. 1999; Okamoto & Habe 1999). Following Carlberg (1994) and Summers, Davis, & Evrard (1995), we distinguish two forms of the velocity bias. The one-point velocity bias, b_v, is defined as the ratio of the rms velocity of galaxies or galactic tracers to that of the dark matter:

$$b_v = \frac{\sigma_{\text{gal}}}{\sigma_{\text{DM}}}\,, \tag{1}$$

where the rms velocity σ is estimated on some scale. Traditionally, this measure of the velocity bias is used for clusters of galaxies. Two-particle or pairwise velocity bias, $b_{v,12}$, compares the relative velocity dispersion in pairs of objects separated by distance r:

$$b_{v,12} = \frac{\sigma_{\text{gal,gal}}(r)}{\sigma_{\text{DM,DM}}(r)}\,. \tag{2}$$

The pairwise velocity dispersion (PVD) was often used to complement the analysis of the two-point spatial correlation function. At small scales, the cosmic virial theorem (Peebles 1980) predicts that the PVD of galaxies should be proportional to the product of the mean density of the universe and the two-point correlation function. The PVD of galaxies has been estimated for the CfA catalog (Davis & Peebles 1983; Zurek et al. 1994; Somerville, Davis, & Primack 1997) and recently for the Las Campanas Redshift Survey by Landy, Szalay, & Broadhurst (1998) and Jing, Mo, & Börner (1998). The latter two studies gave 363 ± 44 km s^{-1} and 570 ± 80 km s^{-1}, respectively, for a 1 h^{-1} Mpc separation. Jing & Börner (1998) show that the discrepancy between these two studies is due to the difference in treatment of the infall velocities. The value of $\sigma_{\text{gal,gal}}$ as well as the infall velocities depend on which regions (clusters or field) are included in the surveyed sample.

The PVD of the dark matter, $\sigma_{\text{DM,DM}}$, has also been estimated for a variety of cosmological models (e.g., Davis et al. 1985; Carlberg & Couchman 1989; Carlberg, Couchman, & Thomas 1990; Klypin et al. 1993; Colín, Carlberg, & Couchman 1997; Jenkins et al. 1998). If galaxies were a random sample of the mass distribution, we would expect $\sigma_{\text{gal,gal}}$ to be approximately equal to $\sigma_{\text{DM,DM}}$. Davis et al. (1985) showed that an $\Omega_0 = 1$ model with $\sigma_8 = 1$ produces a PVD that is too large compared to observations. Here σ_8 is the rms of mass fluctuation estimated with a top-hat window of radius 8 h^{-1} Mpc. This is an example of a model that needs some kind of bias to be compatible with the observations.

The notion of the pairwise velocity bias, $b_{v,12}$, was introduced by Carlberg & Couchman (1989). They found that the dark matter had a PVD a factor of 2 higher than that of the simulated "galaxies." In a further analysis, Carlberg et al. (1990) suggested that an $\Omega_0 = 1$ model with $\sigma_8 = 1$ could be made consistent with the available data for $b_{v,12} \sim 0.5$ (velocity antibias) and almost no spatial bias. Estimates of the pairwise velocity bias are in the range of 0.5–0.8 (Couchman & Carlberg 1992; Cen & Ostriker 1992; Gelb & Bertschinger 1994; Evrard, Summers, & Davis 1994; Colín et al. 1997; Kauffmann et al. 1999a). Differences between the estimates (especially the early ones) can be attributed to some extent to numerical effects ("overmerging problem") and to different methods of identifying galaxy tracers. Only recently have N-body simulations achieved a high dynamic range in a relatively large region of the universe necessary for a large number of galaxy-size halos to survive in clusters and groups (e.g., KGKK, Ghigna et al. 1998; Colín et al. 1999). The estimates of the pairwise velocity bias start showing a tendency to converge. For example, the results of Kauffmann et al. (1999a, 1999b) for a low-density model with a cosmological constant and the results presented in this paper for the same cosmological model agree reasonably well in spite of the fact that we use very different methods. Results point systematically to an antibias of $b_{v,12} = 0.6$–0.7.

One-point velocity bias for clusters and groups of galaxies tells a different story. Values of b_v are typically larger than those for $b_{v,12}$ and range from 0.7 to 1.1 (Carlberg & Dubinski 1991; Katz & White 1993; Carlberg 1994; Ghigna et al. 1998; Frenk et al. 1996; Metzler & Evrard 1997; Okamoto & Habe 1999; Diaferio et al. 1999). Carlberg & Dubinski (1991) suggested that if the pairwise velocity antibias is significant, galaxies in clusters should have orbital velocities lower than the dark matter. However, this may not necessarily be true. In this paper (see also, e.g., Kauffmann et al. 1999a) we argue that galaxy tracers do not need to move more slowly in clusters to have the pairwise velocity bias $b_{v,12} < 1$. In particular, we find that while $b_{v,12} < 1$ for halos in our study, the halos in many clusters actually move somewhat *faster* than dark matter. Ghigna et al. (1998) also do not detect a significant difference between the orbits of DM particles and halos. They find that the cluster radial velocity dispersion of halos is within a few percent of that of the DM particles. Okamoto & Habe (1999) used hundreds of galaxy-size halos in their simulated cluster. They are able to compute the halo velocity dispersion profile. Their results suggest that in the range $0.3 \lesssim r \lesssim 0.6$ Mpc, halos have a velocity dispersion slightly larger than that of the DM particles. Diaferio et al. (1999), using a technique that combines N-body simulations and semianalytic hierarchical galaxy formation modeling, also find that galaxies in clusters have higher orbital velocities than the underlying dark matter field. They suggest that this effect is due to the infall velocities of blue galaxies. In this paper we find a similar effect: galaxy-size halos are "hotter" than the dark matter in clusters.

The paper is organized as follows. In § 2 brief descriptions of the model, simulation, and group-finding algorithm are given. In § 3 the DM and halo PVDs, as well as the corresponding velocity bias, are computed at four epochs. We take a sample of the most massive clusters in our simulation and compute an average halo and DM velocity dispersion profile. A cluster velocity bias is then defined and computed. A discussion of the main results is presented in § 4. The conclusions are given in § 5.

2. MODEL, SIMULATION, HALO-FINDING ALGORITHM

We use a flat low-density cold dark matter (ΛCDM) model with $\Omega_0 = 1 - \Omega_\Lambda = 0.3$ and $\sigma_8 = 1$. Cluster mass estimates (e.g., Carlberg et al. 1996), evolution of the abundance of galaxy clusters (e.g., Eke et al. 1998), baryon fraction in clusters (e.g., Evrard 1997), and the galaxy tracer two-point correlation function (e.g., Colín et al. 1999; Benson et al. 2000) favor a low-density universe with $\Omega_0 \sim 0.3$ (see also Roos & Harun-or-Rashid 1998). On the other hand, various observational determinations of h (the Hubble constant in units of 100 km s^{-1} Mpc^{-1}) converge to values between 0.6 and 0.7. Our model was set to $h = 0.7$, which gives an age for the universe of 13.4 Gyr, in close agreement with the oldest globular cluster age determinations (Chaboyer 1998). The approximation for the power spectrum is that given by Klypin & Holtzman (1997). The adopted normalization of the power spectrum is consistent with both the *COBE* observations and the observed abundance of galaxy clusters.

The Adaptive Refinement Tree code (ART; Kravtsov, Klypin, & Khokhlov 1997) was used to run the simulation, as described by Colín et al. (1999). The simulation followed the evolution of 256^3 dark matter particles in a 60 h^{-1} Mpc box, which gives a particle mass of $1.1 \times 10^9\ h^{-1}\ M_\odot$. The peak force resolution reached in the simulation is ~ 2 h^{-1} kpc. The mass resolution is sufficient for resolving and identifying galaxy-size halos with at least 30 particles. The force resolution allows halos to survive within regions of very high density (such as those found in groups and clusters of galaxies). In dense environment of clusters, the mass of halos is not well defined. Therefore, we use the maximum circular velocity

$$V_{\rm max} = \left[\frac{GM(<r)}{r}\right]^{1/2}_{\rm max}, \qquad (3)$$

where $M(<r)$ is the mass of the halo inside radius r, as a "proxy" for mass.

Halos begin to form at very early epochs. For example, at $z \sim 6$ we identify more than 3000 halos with maximum circular velocity, $V_{\rm max}$, greater than 90 km s^{-1}. The numbers of halos that we find at $z = 3$, 1, and 0 are 14102, 14513, and 10020, respectively. We use a limit of 90 km s^{-1} on the circular velocity, which is slightly lower than the completeness limit, $\sim$110–120 km s^{-1} (Colín et al. 1999), of our halo catalog. This $V_{\rm max}$ value increases the number of halos quite substantially (a factor of 2, as compared to the limit of 120 km s^{-1}), and thus reduces the statistical noise. We checked that our main results are only slightly affected by the partial incompleteness of the sample.

Our halo identification algorithm, the bound density maxima (BDM; see KGKK), is described in detail elsewhere (Klypin & Holtzman 1997). The main idea of the BDM algorithm is to find positions of local maxima in the density field smoothed at the scale of interest (20 h^{-1} kpc). BDM applies physically motivated criteria to test whether a

TABLE 1

PHYSICAL PARAMETERS OF CLUSTERS

M_{vir} (h^{-1} $M_\odot$) (1)	σ_{3D} (km s^{-1}) (2)	V_{max} (km s^{-1}) (3)	R_{vir} (h^{-1} Mpc) (4)	n_{halo} with $V_{max} > 90$ km s^{-1} (5)
6.5×10^{14}......	1645	1402	1.43	246
2.4×10^{14}......	1022	910	1.28	132
1.9×10^{14}......	992	831	1.17	98
1.6×10^{14}......	975	789	1.11	95
1.4×10^{14}......	887	747	1.05	58
1.3×10^{14}......	887	730	1.02	55
1.1×10^{14}......	831	695	0.98	45
1.0×10^{14}......	820	680	0.95	33
9.9×10^{13}......	789	673	0.94	74
9.7×10^{13}......	789	668	0.94	60
9.3×10^{13}......	753	659	0.92	67
8.3×10^{13}......	720	635	0.89	64

NOTE.—Col. (1): Virial mass of the cluster; col. (2): three-dimensional velocity dispersion of dark matter particles; col. (3): maximum circular velocity; col. (4): cluster radius; col. (5): number of galaxy-size halos with $V_{max} > 90$ km s^{-1}.

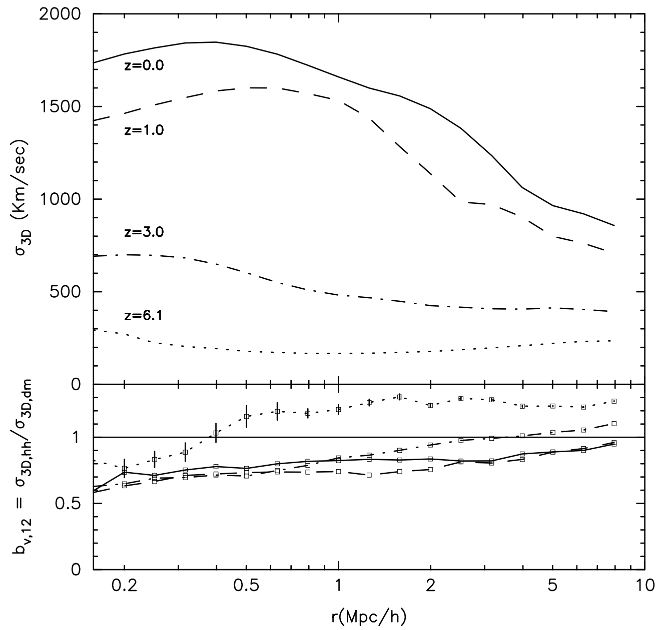

FIG. 1.—*Top*: Three-dimensional pairwise rms velocity of the dark matter at four different epochs, as indicated in the figure. *Bottom*: Pairwise velocity bias for galaxy-size halos with circular velocities $V_{max} > 90$ km s^{-1}. Curves are labeled in the same way as in the top panel. At very early epochs and on large scales, halos tend to move faster than the dark matter. At later moments, the pairwise velocity bias becomes smaller than unity.

group of DM particles is a gravitationally bound halo. The major virtue of the algorithm is that it is capable of finding both isolated halos and halos orbiting within larger dense systems. Cluster-size halos were also located by the BDM algorithm. The physical properties of a sample of the 12 most massive groups and clusters[1] are shown in Table 1. The total number of clusters chosen for the sample is a compromise between taking a relatively large number of clusters, so that we could talk about cluster average properties, and using clusters with a relatively high number of halos. This cluster sample is used to compute the average DM and halo velocity dispersion profiles, as well as the average DM and halo velocity anisotropy profiles.

3. RESULTS

3.1. *The Pairwise Velocity Bias*

The three-dimensional PVD is defined as

$$\sigma_{3D}^2(r) = \langle \boldsymbol{v}_{12}^2 \rangle - \langle \boldsymbol{v}_{12} \rangle^2 , \quad (4)$$

where $\boldsymbol{v}_{12}$ is the relative velocity vector of a pair of objects separated by a distance r, and angle brackets indicate averaging over all pairs with the separation r. Figure 1 shows the PVD for the dark matter, $\sigma_{3D,DM}$, at four epochs (*top*). At 1 h^{-1} Mpc, the radial PVD is about 1100 km s^{-1} at $z = 0$. For the same cosmological model, Jenkins et al. (1998) find a radial PVD of $\sim$910 km s^{-1}. Jenkins et al. used a slightly lower normalization for the model ($\sigma_8 = 0.9$) and a larger simulation box ($L_{box} = 141.3$ h^{-1} Mpc). When the differences in σ_8 are taken into account, the Jenkins et al. value increases to 1120 km s^{-1}. Thus, the two estimates roughly agree.

The ratio of the halo and the dark matter PVDs, the pairwise velocity bias $b_{v,12}$, is shown in the bottom panel of Figure 1. All halos with $V_{max} > 90$ km s^{-1} were included in the computation. At very early epochs and on large scales, halos tend to move faster than the dark matter. At later moments, the pairwise velocity bias becomes smaller than unity. It is interesting to compare the evolution of $b_{v,12}$ with the changes in the spatial bias for the same cosmological model (Colín et al. 1999). The spatial bias is defined as the square root of the ratio of correlation functions $[\xi_{hh}(r)/\xi_{DM}(r)]^{1/2}$, where ξ_{hh} and ξ_{DM} are the two-point correlation functions of halos and dark matter particles, respectively. In general, the biases evolve in the same way. At high redshifts, both biases are positive ($b > 1$), and they decline as the redshift decreases. At low redshifts, biases dive below unity (antibias) and stop evolving. In spite of similarities, there are some differences. The pairwise velocity bias becomes less than unity at $z = 3$ on scales below 3 h^{-1} Mpc. At the same redshift, the spatial bias is still positive on all scales.

Colín et al. (1999) and Kravtsov & Klypin (1999) interpret the evolution of the spatial bias as the result of several competing effects. Statistical bias (higher peaks are more clustered) tends to produce large positive bias and explains bias evolution at high redshifts. At later epochs, halos of a given mass or circular velocity become less rare and start merging inside forming groups of galaxies. Both effects lead to a decrease of bias. The merging becomes less important as clusters with large velocity dispersions form at $z < 1$. This results in a very slow evolution of the halo correlation function and bias. It is likely that the same processes define the evolution of the pairwise velocity bias. The differences can be explained by the known fact that the PVD is strongly dominated by the few largest objects (e.g., Zurek et al. 1994; Somerville et al. 1997); merging of halos inside forming groups at $z = 3$ results in fewer pairs with large relative velocities and in velocity antibias on $\approx$1 h^{-1} Mpc scales. If this interpretation is correct, the pairwise velocity bias mostly measures the spatial bias, not the differences in velocities.

[1] Cluster 8, in descending order in mass, was excluded from the sample because it has a group close to it that produces too much disturbance to the cluster.

3.2. *The Velocity Anisotropy* β

A sample of 12 groups and clusters (see Table 1) was used to compute various average cluster velocity statistics. In order to reduce the noise in the profiles caused by the small number of clusters in the sample, we double the sample by also using the same clusters at a slightly different time $z = 0.01$. For each cluster, the halo distances to the cluster center are divided by the corresponding cluster virial radius (normalized distances). The halo velocities (averaged in spherical bins) are divided by the corresponding cluster circular velocity at the virial radius (normalized velocities). In Figure 2 we show radial profiles, in normalized units, for halos and DM: the mean radial velocity (v_r), and the radial (σ_r) and tangential (σ_t) velocity dispersions. All halos are given equal weight. We have accounted for the Hubble flow when we compute σ_r and σ_t (so proper, not peculiar, velocities are used); no correction for the mean radial velocity was made. The trend in both the velocity dispersion and the anisotropy velocity is slightly affected if the mean radial velocity is subtracted at distances $\gtrsim 0.6$, and it is not affected at all at smaller distances.

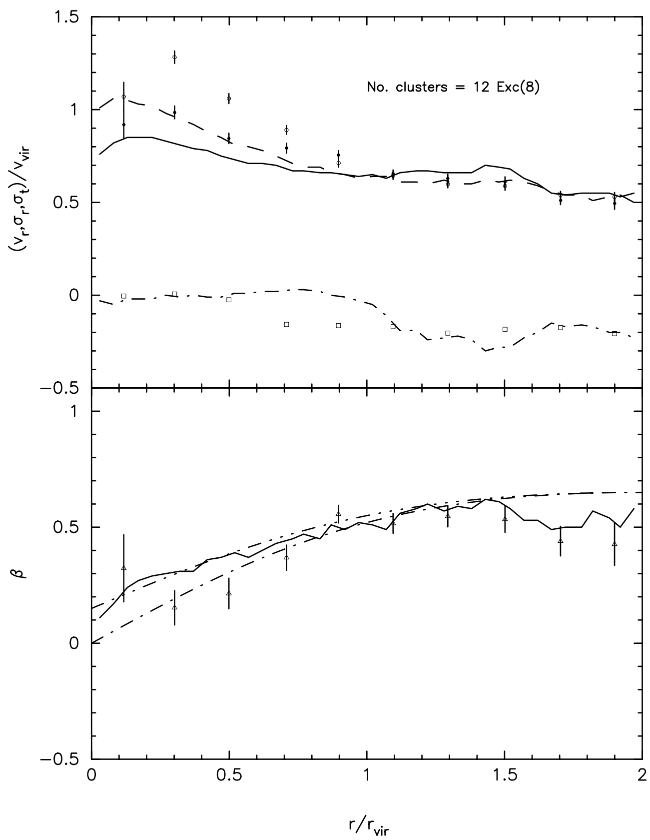

FIG. 2.—Velocity profiles averaged over all clusters in Table 1, excluding cluster 8. Curves show the dark matter, and different symbols show halos with circular velocity $V_{max} > 90$ km s^{-1}. Distances to the cluster centers are divided by the corresponding cluster virial radius, and halo velocities (averaged in spherical bins) are divided by the corresponding cluster circular velocity at the virial radius. Error bars show 1 σ errors of the mean. *Top*: Mean radial velocity (*dot-dashed curve*), radial velocity dispersion (*solid curve*), and tangential velocity dispersion (*dashed curve*) of the dark matter. Open squares, filled circles, and open circles show the radial velocity, radial velocity dispersion, and tangential velocity dispersion, respectively, for halos. *Bottom*: Velocity anisotropy for halos (*filled triangles*) and for the dark matter (*solid line*). The dot-dashed and triple-dot-dashed lines represent the fitting $\beta = 4r\beta_m/(r^2 + 4) + \beta_0$ for two pairs of (β_m, β_0): (0.65, 0) and (0.5, 0.15), respectively.

The velocity anisotropy function,

$$\beta = 1 - \frac{\sigma_t^2}{2\sigma_r^2}, \tag{5}$$

is presented in the bottom panel of Figure 2 for halos and for DM. For pure radial orbits $\beta = 1$, while an isotropic velocity dispersion implies $\beta = 0$. The two lines added to the panel show a fitting formula (Carlberg et al. 1997),

$$\beta = \beta_m \frac{4r}{r^2 + 4} + \beta_0, \tag{6}$$

for two pairs of parameters (β_m, β_0) at (0.65, 0) and (0.5, 0.15). The first set of parameters gives a better approximation for halos. It explicitly assumes that $\beta = 0$ at the center. The second set of parameters allows a small anisotropy at the center. It provides a better fit for the dark matter. Note that while the halos have a tendency for more isotropic velocities (with the possible exception of the center), the difference between halos and the dark matter is not statistically significant.

The variances of σ_r and σ_t are computed using standard expressions for errors; for example, for σ_r,

$$\mathrm{Var}\,(\sigma_r) = \frac{\mu_4 - \mu_2^2}{4n\mu_2}, \tag{7}$$

where $\mu_2 = \sum_i (v_{r,i} - \bar{v}_{r,i})^2$ and $\mu_4 = \sum_i (v_{r,i} - \bar{v}_{r,i})^4$, and n is the number of halos. The statistical error is thus given by the square root of Var (σ_r). The variance of β is given by

$$[\mathrm{Var}\,(\beta)]^2 = \left[\frac{\mathrm{Var}\,(\sigma_t^2)}{2\sigma_r^2}\right]^2 + \left[\frac{\mathrm{Var}\,(\sigma_r^2)}{2\sigma_r^4}\,\sigma_t^2\right]^2. \tag{8}$$

3.3. *The Cluster Velocity Bias*

The three-dimensional velocity dispersions for both halos and DM are shown in the top panel of Figure 3. The bottom panel shows the cluster velocity bias, defined here as $b_v = \sigma_{3D,\mathrm{halo}}/\sigma_{3D,\mathrm{DM}}$. It is surprising that halos in clusters appear to have larger, by about 20%, velocity dispersions

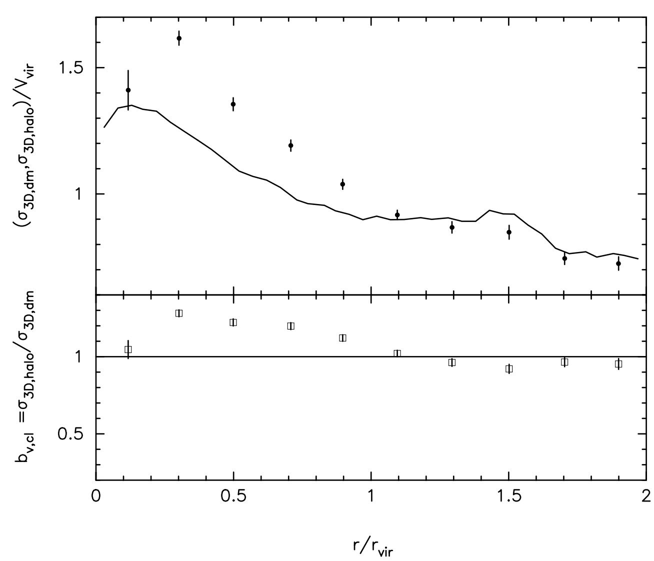

FIG. 3.—*Top*: Three-dimensional velocity dispersion profiles for halos (*filled circles*) and dark matter (*solid line*) in units of the mean virial velocity. *Bottom*: Cluster velocity bias profile. Errors correspond to 1 σ errors of the mean.

than the DM particles (positive bias). The trend is the same regardless of what *type* of velocity dispersion (three-dimensional, tangential, or radial) we use in the velocity bias definition. There is almost no bias in the very center of clusters. However, the b_v value of the innermost bin increases if we exclude the "cD" halos (defined as those halos that lie within the inner $\sim 100\ h^{-1}$ kpc radius and have maximum circular velocities greater than about 300 km s^{-1}). Their exclusion increases the positive velocity bias to 1.22, a value that is comparable to that found in the adjacent bin.

The cluster positive velocity bias is robust to changes in the limit of the circular velocity, V_{max}. Only the innermost bin experiences significant changes when this limit is increased. For example, when we increase V_{max} from 90 to 150 km s^{-1} (more massive halos are chosen), the value of b_v in the innermost bin reduces to 0.6. This favors a picture in which in the central regions of clusters, large galaxy-size halos feel more the slowing effect of the dynamical friction. All the other bins (within the virial radius) continue to show small positive velocity biases. The positive velocity bias is also robust to changes in the number of clusters of the sample. For instance, one might suspect that the most massive cluster weights so much that it alters the statistics.[2] This is true to some degree, but it *does not change* the "sign" of the bias. For example, when we exclude the most massive cluster and take $V_{max} = 150$ km s^{-1}, all bins continue to show positive bias (within the virial radius), except the innermost bin, where $b_v = 0.5$. The results for the innermost bin should be taken with caution, because the effects of overmerging may still be present in the central 100 h^{-1} kpc of the clusters.

The difference in velocity dispersions of halos and dark matter particles indicates that their velocity distribution functions (VDF) are different. We have examined both differential and cumulative VDFs for the analyzed clusters and found that the halo VDFs are generally skewed toward higher velocities as compared to the dark matter VDF, at $r/r_{vir} \lesssim 0.8$. The two VDFs are approximately the same for larger radii. The observed differences in the velocity distribution may be caused either by the differences in velocity fields of infalling halos and dark matter (if, for example, halos are accreted preferentially along filaments resulting in orbits of higher ellipticity) or by effects of dynamical friction operating on halos, but not on dark matter, in clusters. The dynamical friction may affect the slowest halos more efficiently because the dynamical friction time is proportional to the cube of the halo velocity. The slowest halos may therefore merge more efficiently, thereby skewing the velocity distribution of the surviving halos toward higher velocities.

One could ask whether or not this positive cluster velocity bias persists in the next set of twelve clusters or groups, in descending order of mass (with virial masses below those clusters shown in Table 1). Because this new sample of clusters have an average virial mass lower than the average mass of clusters of Table 1, dynamical friction is expected to operate more efficiently (e.g., West & Richstone 1988; Diaferio et al. 1999). The number of halos per cluster or group in this new sample is small; we therefore use the whole group velocity dispersion. We find integral b_v values that are in general lower than 1, and in some cases there are groups that exhibit a strong velocity antibias (ratios close to 0.6). This is contrary to what we find for the clusters of Table 1, where the majority of clusters have an integral positive velocity bias.

4. DISCUSSION

Literature on the velocity bias is very extensive and results are often contradictory. In this section we review some of the published results and compare them with our results. There are some reasons for the chaotic state of the field. One is the confusion of two different notions of the velocity bias, the single-point, b_v, and the pairwise, $b_{v,12}$, biases. The biases have different natures, and thus give different results. Another source of confusion is the way in which galaxies are identified or approximated in theoretical models. When we combine this uncertainty with the many physical processes that we believe can create and change velocity bias, the situation becomes rather complicated.

Velocity profiles.—These seems to be the easiest part of the picture. In this paper we present results that are less noisy and based on a more homogeneous set of clusters than in most previous publications. Our results on the average cluster profiles for the dark matter (v_r and σ_r) roughly agree with the results of Cole & Lacey (1996), Tormen, Bouchet, & White (1997), and Thomas et al. (1998). For example, Tormen et al. (1997) find a DM velocity anisotropy $\beta_{DM} \lesssim 0.2$ at $r/r_{vir} \lesssim 0.1$ and $\beta_{DM} \simeq 0.5$ at $r/r_{vir} \sim 1$, which is close to our results. The structure of galaxy clusters in various cosmologies is analyzed in detail by Thomas et al. (1998). From a total sample of 208 clusters, they choose a subsample that shows no significant substructure. They find a more isotropic averaged β profile ($\beta_{DM} \sim 0.3$ at $r/r_{180} = 1$) in their ΛCDM model. The differences between our result and theirs can be accounted for by the fact that their clusters were selected not to have significant substructure. More substructure in a cluster likely means a more anisotropic cluster. The β value at the cluster center (innermost bins) is around 0.1, which is close to our results.

Pairwise velocity bias.—This is very sensitive to the number of pairs found in rich clusters of galaxies. Removing a few pairs may substantially change the bias. Thus, it mostly measures the spatial bias (or antibias) and is less sensitive to real differences in velocities. The value of $b_{v,12}$ that we find at $z = 0$ is typically higher than previous estimates reported in the literature, computed for the $\Omega_0 = 1$ CDM model (Carlberg & Couchman 1989; Carlberg et al. 1990; Gelb & Bertschinger 1994; Summers et al. 1995). Some of the results are difficult to compare, because the pairwise velocity bias is expected to evolve with time and vary from model to model.

The first interesting result of this paper, which emerges from the evaluation of $b_{v,12}$ at very high redshift, is that the halo PVD *can be* greater than that of the DM. This positive velocity bias had not been detected before (but see below) partly because of the lack of simulations with very high resolution that could overcome the overmerging problem. This result is surprising, in part because halos are expected to be born dynamically cool.[3] In fact, this is one of the reasons given in the literature to explain the present-day pairwise velocity bias (e.g., Evrard et al. 1994). The other is

[2] In fact, the most massive cluster of our simulation has had a recent major merger, and halos may still have large ("overheated") velocities (e.g., Katz & White 1993)

[3] Halos tend to form near the peaks of the DM density distribution (e.g., Frenk et al. 1988).

the dynamical friction (e.g., Carlberg et al. 1990). We offer the following explanation for this positive velocity bias. Those halos that are formed at very high redshift come from very high density peaks. They are dynamically *cooler* than an average DM particle from the region in which they were born, but *hotter* than most of the matter. The pairwise velocity bias, $b_{v,12}$, rapidly becomes smaller than 1 at nonlinear scales. As time goes on, the mergers inside forming groups reduce the number of high-velocity halos, while velocities of DM particles increase. As a result, the average halo random relative velocities are reduced to below that of the DM.

Using a semianalytical method to track the formation of galaxies, Kauffmann et al. (1999a, 1999b) also find a pairwise velocity bias greater than 1 at high redshifts. They find that the galaxy PVD is greater than that of the DM at $z > 1.1$ (see their Fig. 11, τCDM model). A $b_{v,12} > 1$ is expected at higher redshift in their ΛCDM model as well.

Single-point velocity bias.—This appears to be the most difficult and controversial quantity. It is important because it is a more direct measure of the velocity differences. It still depends on the spatial bias, but to much lesser degree than the pairwise bias. An interesting result was found when we evaluated the average cluster halo velocity dispersion profile and compared it with that of the DM particles: within the virial radius *halos move faster than the dark matter*.

We believe that the explanation for this fact comes from a combination of two known physical mechanisms: the dynamical friction and the merging of halos. One might naively expect that the dynamical friction should always slow down halos, which must result in halos moving slower than the dark matter particles. This is not true. While on a short timescale the dynamical friction reduces the velocity of a halo, the halo may decrease or increase its velocity depending on the distribution of mass in the cluster and on the trajectory of the halo. For example, if a halo moves on a circular orbit inside a cluster with the Navarro-Frenk-White (NFW) profile, its velocity will first increase as it spirals from the virial radius to $2.2R_s$, where $R_s \approx 200$–300 kpc is the characteristic radius of the core of the cluster. The halo velocity will then decrease at smaller radii. When the halo comes close to the center of the cluster, it merges with the central cD halo, which will have a tendency to increase the average velocity of remaining halos. It appears that the Jeans equation provides a better tool for understanding the velocity bias.

We will use the Jeans equation as a guide through the jungle of contradictory results. It cannot be used more than a hint, because it assumes that a cluster is stationary and spherical, which is generally not the case. If a system is in a stationary state and is spherically symmetric, the mass $M(<r)$ inside radius r is related to the radial velocity dispersion σ_r,

$$M(<r) = \frac{r\sigma_r^2}{G} A, \tag{9}$$

$$A \equiv -\left(\frac{d\ln\sigma_r^2}{d\ln r} + \frac{d\ln\rho}{d\ln r} + 2\beta\right), \tag{10}$$

where ρ is the (number) density profile, and β is the velocity anisotropy function. The left-hand side of this equation (the total mass) is the same for both halos and dark matter. Thus, if the term A is the same for the dark matter and halos, then there should be no velocity bias: halos and dark matter must have the same σ_r. Numerical estimates of the term A are inevitably noisy, because we must differentiate noisy data. Nevertheless, we find that the value of the term A for halos is systematically smaller than for the dark matter. This gives a tendency for σ_r to be larger for halos. In turn, this produces a positive velocity bias. The main contribution comes typically from the logarithmic slope of the density: the halo density profile is shallower in the central part as compared to that of the dark matter. The halo profile is shallower, likely because of merging in the central part of the cluster, which gave rise to a central cD halo found in each of our clusters. We note that while the Jeans equation shows the correct tendency for the bias, it fails to reproduce the correct magnitude of the effect: variations of the term A are smaller than the measured velocity bias.

One can also use the Jeans equation in a different way—as an estimator of mass. We have computed $M(<r)$ for our average cluster using both DM and halos. At $\langle r/r_{\rm vir}\rangle = 0.25$, where b_v is close to its maximum, the halo mass determination is larger than that of the DM by a factor of 1.4. This is due to the larger halo velocity dispersion. Because the term A is actually higher for DM by about 10%, the overestimation is reduced from 1.56 to 1.4. As the distance to the cluster center approaches the virial radius, the mass overestimation disappears. At the virial radius both mass estimations agree, essentially because β, σ_r, and the sum of the logarithm derivatives are the same for both halos and DM, and are within 10%–15% of the true mass.

Using the Jeans equation for a spherically symmetric system and assuming an isotropic velocity field, Carlberg (1994) showed that a cool tracer population, $b_v < 1$, moving inside a cluster with a power-law density profile (the density profile for the tracer is also assumed to be a power law), produced a mass segregation. That is, the tracer population had a steeper density profile. We can invert this reasoning and say that a more centrally concentrated halo distribution produces a velocity antibias. We do not find this kind of mass segregation in our halo cluster distribution. In fact, we see the opposite—halos are less concentrated than DM. Dynamical friction, along with merging, produces a lack of halos in the center of the cluster. This very likely explains differences between our and Carlberg's results for the velocity bias.

Carlberg & Dubinski (1991) simulated a spherical region of 10 Mpc radius and 64^3 DM particles. They were unable to find galaxy-size halos inside the cluster at $z = 0$ because of insufficient resolution: their softening length was 15 kpc, instead of the $\sim 2\ h^{-1}$ kpc needed for the survival of halos (KGKK). Their identification of "galaxies" with those DM particles that were inside high-density groups found at high redshift may have produced a spurious cluster velocity antibias. Using different galaxy tracers, Carlberg (1994) also found an integral cluster velocity bias lower than 1. This result could still be affected by numerical resolution ($\epsilon = 9.7$ h^{-1} kpc). Evrard et al. (1994) ran a two-fluid simulation in a small box, $L_{\rm box} = 16$ Mpc, and stopped it at $z = 1$. Each DM particle had a mass of $9.7 \times 10^8\ M_\odot$ and an effective resolution of 13 kpc (at $z = 1$). The initial conditions were constrained to assure that a poor cluster could form in their simulation. Their "globs" (galaxy-like objects) exhibit a velocity bias lower than 1. This velocity bias appeared not to depend on epoch and mass. Their velocity antibias qualitatively agrees with our results for groups and poor clusters. At the same time, their value for the pairwise velocity bias also agrees with our results.

Metzler & Evrard (1997) used an ensemble of two-fluid simulations to compute the structure of clusters. Unfortunately, their simulations do not have the high mass resolution necessary to allow the gas in their simulations to cool and form "galaxies" (which could also allow for some feedback). Instead, they use a high-density peak recipe to convert groups of gas particles into galaxy particles. They find a one-point "galaxy" velocity bias that depends on cluster mass: the higher the cluster mass, the higher the b_v value. We find a similar result when we do the analysis of the velocity bias cluster by cluster.[4] Their ensemble-averaged bias parameter is 0.84. Their recipe for galaxy formation produces a galaxy number density profile that is steeper than that of the DM. This is likely the reason why they find a b_v value lower than 1 (Carlberg 1994; see above).

Frenk et al. (1996) simulated a Coma-like cluster with a P^3M + SPH code that includes the effects of radiative cooling. The mass per gas particle is $2.4 \times 10^9\ M_{\odot}$, with a softening parameter $\epsilon = 35$ kpc of the Plummer potential. Their galaxies have two extreme representations: one as pure gas clumps and the other as lumps of the stellar component. They find a mass segregation in both representations—galaxies are more clustered than DM toward the center of the cluster—which is not seen in our halo distribution.[5] Once again, according to the Carlberg (1994) analysis, this would result in a one-point velocity bias lower than 1 ($b_v \simeq 0.7$). Because of a strong cooling, their "galaxies" can acquire high density contrasts, which helps galaxies to survive inside cluster. At the same time, poor force resolution (35 kpc) could have affected their results.

There are two studies in which b_v values greater than 1 are obtained. Okamoto & Habe (1999) simulate a spherical region of 30 Mpc radius using a constrained random field method. They use a multimass initial condition to reach high resolution. Their high-resolution simulated region, in which the cluster ends up, has a softening length of $\epsilon = 5$ kpc and mass per particle of $m \sim 10^9\ M_{\odot}$. They find a cluster velocity bias lower than 1 *only* in the innermost part of the cluster, where dynamical friction is expected to be more efficient. A small positive bias ($b_v > 1$) is found in the range $0.3 < r < 0.6$ Mpc. Based on the previous work by Kauffman et al. (1998), Diaferio et al. (1999) study properties of galaxy groups and clusters. They also find that galaxies in clusters are "hotter" than the underlying dark matter field. They suggest that this effect is due to the infall velocities of blue galaxies. Infall could explain the positive velocity bias of the outermost bin (within the virial radius) of our Figure 3, but it definitely cannot account for the $b_v > 1$ value seen in the inner bins (the mean radial velocity is close to zero for both DM and halos in the three innermost bins).

There are several differences between our simulation and those mentioned above. First, some of the papers cited above simulate only a region that ends up as a cluster. Thus, they have structure for *only* one cluster. The single-cluster one-point velocity bias could not represent an *average* velocity bias, found using a sufficiently large sample of clusters. For example, if our small positive velocity bias is influenced by nonequilibrium cluster features, then when one selects a cluster that is in *good* dynamical equilibrium (this could be defined, for example, by the absence of substructure in the cluster) and computes the one-point velocity bias, it could be biased toward low values ($b_v < 1$) because dynamical friction has had more time to operate. Second, we simulate a relatively large random volume that gives us many clusters in which effects such as tidal torques, infall, and mergers are included naturally. A cluster simulated region or a random large region without sufficient resolution may not have a sufficiently large number of galaxy tracers, and thus might introduce high statistical errors. Our relatively large number of halos in clusters significantly reduces the statistical errors in the computation of b_v and makes them suitable for the determination of, e.g., the radial dependence of the velocity bias. Third, in view of the Okamoto & Habe (1999) and Ghigna et al. (1998) results, and our own results, it seems that numerical resolution not only plays an important role in determining the whole cluster velocity bias value (both spatial and velocity bias intervene to affect its value), but is also important in determining the radial dependence of b_v (almost pure velocity bias).

The velocity bias in clusters is difficult to measure because it is small. Figure 3 may be misleading, because it shows the average trend but does not give the level of fluctuations for a single cluster. Note that the errors in the plot correspond to the error of the mean obtained by averaging 12 clusters and two close moments of time. Fluctuations for a single cluster are much larger and come from two sources: poor statistics (small number of halos) and the noise produced by residual nonequilibrium effects (substructure). A comparable (but slightly smaller) value of b_v was recently found in simulations by Ghigna et al. (1999) for a cluster in the same mass range as in Figure 3. Unfortunately, it is difficult to make detailed comparison with their results because they use only one cluster for a different cosmological model. It is very likely that their results are dominated by the noise due to residual substructure. New results on cluster velocity bias by V. Springel (2000, private communication) agree with ours.

What could account for the small positive velocity bias that we see in our average cluster? We have examined both the differential and the cumulative radial velocity distribution functions. We use the radial velocity to highlight any contribution of infall velocities to the velocity bias. The cumulative radial velocity distribution function is shown in Figure 4 for four different radial bins. In the top-left panel (the innermost bin) we see a higher fraction of low-velocity halos at small v_r values. This is due to central cD halos, which move very slowly relative to clusters themselves. At large v_r values, we observe the contrary—a higher slope, which means that there are many fast-moving halos. If we do not include the cD halos, the velocity bias becomes larger than unity even in the central radial bin. However, as we noted earlier in § 3.3, a velocity antibias can appear in the central bin if the value of $V_{\max}$ is increased. It is clear that the deficiency of low- and moderate-v_r halos produces the positive velocity bias measured at $r = (0.2\text{–}0.8)r_{\rm vir}$ (see Fig. 4, *top left and bottom right*). We have used the Kolmogorov-Smirnov test to evaluate whether or not the halo and the DM velocity distribution functions are statistically different. We find that the probability that these functions were drawn from the same distribution is smaller

[4] On individual clusters, we take only integral velocity dispersions.

[5] The reader might want to compare the Figure 11 in Frenk et al. (1996) with Figure 2 in Colín et al. (1999).

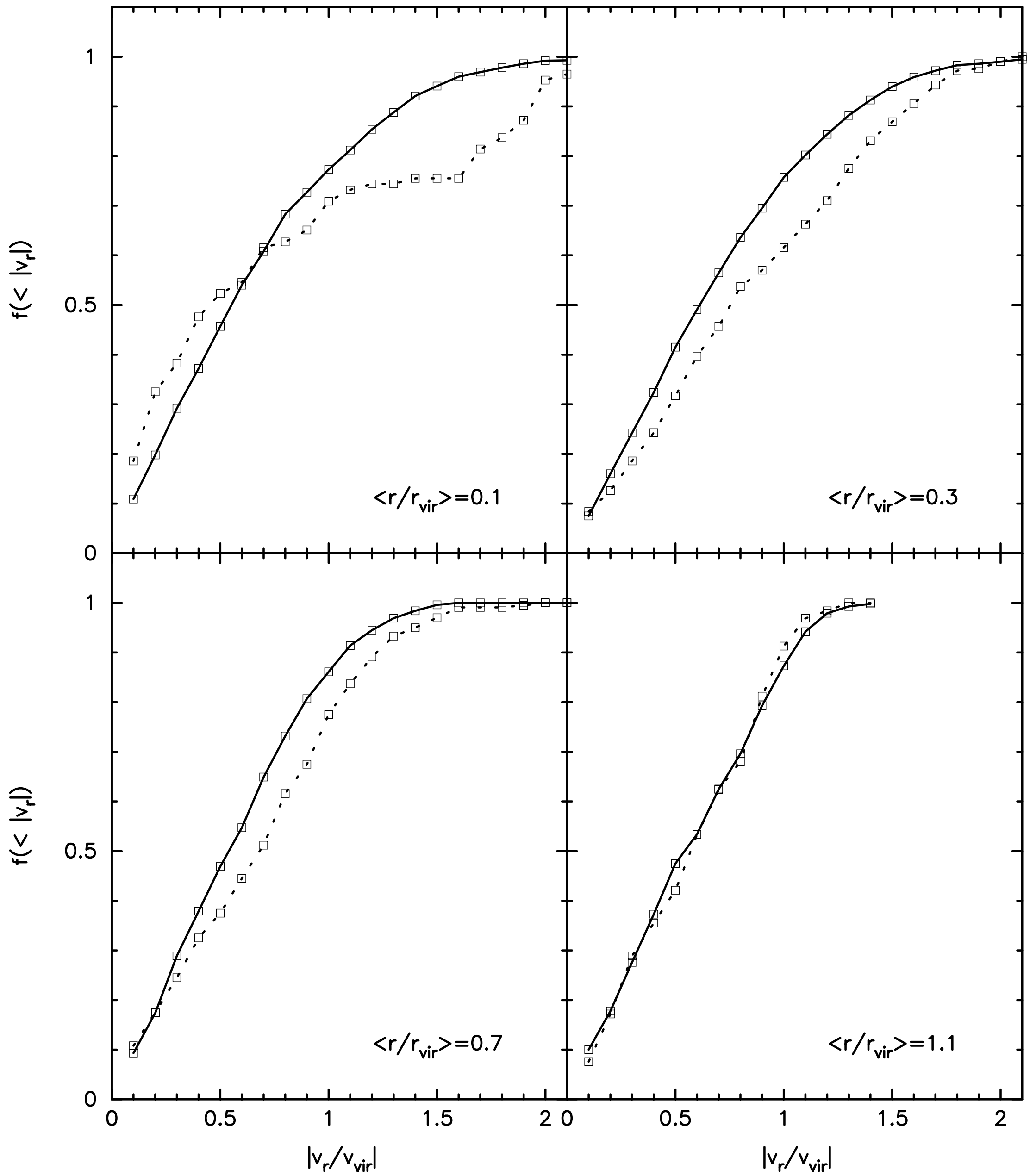

FIG. 4.—Cumulative radial velocity distribution functions in four radial bins for halos (*dotted line*) and dark matter (*solid line*)

than 0.01 in all radial bins that are within the virial radius. As mentioned in § 3.3, the dynamical friction may have affected the slow-moving halos more significantly, because the dynamical friction timescale is proportional to the cube of the halo velocity. It is thus expected that low-velocity halos merge sooner than their high-speed counterparts, thereby skewing the VDF toward high-velocity halos.[6] Infall could also be an important source of positive velocity bias for the outermost bins.

5. SUMMARY

1. We have found that galaxy-size halos have a time- and scale-dependent pairwise velocity bias. At high redshifts ($z \sim 5$) this bias is larger than unity (≈ 1.2). It declines with time and becomes ≈ 0.6–0.8 at $z = 0$. The evolution of the pairwise velocity bias follows and probably is defined by the spatial bias of the dark matter halos. These results are in qualitative agreement with those of Kauffmann et al. (1999b).

2. We have evaluated the velocity anisotropy function, $\beta(r)$, for both halos and DM particles. For both halos and DM, β is a function that increases with radius and reaches a value of $\simeq 0.5$ at the virial radius. The difference between this value and that found by Thomas et al. (1998) likely can be explained by the fact that Thomas et al. (1998) selected a sample of clusters that had little substructure. Our simulations indicate that the halo velocity anisotropy closely follows (but lies slightly below) that of the underlying dark matter.

3. Halos in our clusters move faster than DM particles:

[6] It should be also kept in mind that as halos move to orbits of smaller radii, they could acquire higher velocities because the DM velocity dispersion increases toward the cluster center.

$b_v = 1.2$–1.3 for $r = (0.2$–$0.8)r_{\rm vir}$. This result disagrees with many previous estimates of the cluster velocity bias. This difference appears to be due to differences in numerical resolution. More work needs to be done to settle the issue. Nevertheless, it is encouraging that Diaferio et al. (1999) and Okamoto & Habe (1999) found results similar to ours.

4. The usual argument that dynamical friction slows down galaxies and thus must produce velocity antibias is not correct. Galaxy tracers in clusters move through an environment that has a steep density gradient. A sinking halo may either decrease *or increase* its velocity, depending on the distribution of cluster mass and on the trajectory of the halo. A combination of dynamical friction and merging appears to be the most compelling hypothesis that could account for our small positive velocity bias.

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