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SELF-REGULATION OF STAR FORMATION IN LOW-METALLICITY CLOUDS

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ABSTRACT

We investigate the process of self-regulated star formation via the photodissociation of hydrogen molecules in low-metallicity clouds. We evaluate the scale of the influence region for a massive star in low-metallicity gas clouds whose temperatures are between 10^2 and 10^4 K. A single O star can photodissociate H_2 in the whole of the host cloud. If the metallicity is less than about $10^{-2.5}$ of the solar metallicity, the depletion of coolants in the host cloud is very serious, so that the cloud cannot cool in a free-fall time, and subsequent star formation is almost quenched. On the other hand, if the metallicity is greater than about $10^{-1.5}$ of the solar metallicity, star formation regulation via photodissociation is not efficient. The typical metallicity when this transition occurs is $\sim 10^{-2}$ of the solar metallicity. This indicates that stars do not form efficiently before the metallicity becomes larger than about 10^{-2} of the solar metallicity, and we consider that this value is the lower limit of the metallicity of luminous objects such as galaxies.

Subject headings: cosmology: theory — early universe — galaxies: formation — H II regions — ISM: clouds — stars: formation

1. INTRODUCTION

After the recombination era, little information is accessible until $z \sim 5$, after which we can observe objects such as galaxies and QSOs. On the other hand, the reionization of the intergalactic medium and the presence of heavy elements at high z suggest that there are other populations of luminous objects that precede normal galaxies. Thus, a theoretical approach to revealing the formation mechanism of such unseen luminous objects is very important.

The formation process of a luminous object is roughly divided into three steps: formation of cold clouds by H and/or H_2 line cooling, formation of the first-generation stars in the cold clouds, and then star formation throughout the clouds. However, the masses of the first-generation stars are estimated through detailed investigation to be fairly large (Nakamura & Umemura 1999; Omukai & Nishi 1998). Thus, the third step is disturbed by feedback from the massive stars formed in the clouds. The main feedback consists of two different processes, ultraviolet (UV) radiation from the stars and energy input by supernovae (SNe). Through ionization of H (Lin & Murray 1992) and dissociation of H_2 (Silk 1977; Omukai & Nishi 1999), UV radiation has a negative feedback on further star formation in the host clouds. In particular, H_2 is dissociated in such a large region that all the ordinary low-mass cosmological objects are influenced by one O5 type star (Omukai & Nishi 1999). For the case of a metal-free gas cloud, the influence region is much wider than the H II region, and the metal-free host cloud lacks coolants and cannot cool. Thus, next-generation stars are hardly formed before the first-generation stars die. However, the lifetimes of massive stars are much shorter than the cosmological timescale, and they die as SNe. By these SN explosions, the cloud's gas is often dispersed before a significant amount of the total gas is transformed into stars (e.g., Mac Low & Ferrara 1999; Ciardi et al. 2000; Nishi & Susa 1999). On the other hand, if the gas binding is not disrupted, next-generation stars are formed in a cloud that is slightly polluted by heavy elements. Even in the case in which the host cloud is disrupted by SN explosions, if the remnant gas does not escape from the host pregalactic object, next-generation clouds, which are slightly polluted, will be formed, and star formation will follow. In these polluted clouds, heavy elements will become important coolants, if their abundances increase to some degree. After the host cloud is sufficiently polluted that star formation regulation by UV radiation is not efficient, the effective star formation can start. Thus, the pregalactic object, which is a cloud complex, will evolve into a luminous object such as galaxy.

In this paper, we investigate the self -regulation of star formation via UV radiation and assess the critical metallicity that enables the formation of luminous objects.

2. INFLUENCE REGION OF A MASSIVE STAR IN A LOW-METALLICITY CLOUD

Around an OB star, hydrogen is photoionized, and an H II region is formed. Lin & Murray (1992) considered the star formation regulation via photoionization. However, the regulation can be efficient outside the H II region via photodissociation of H_2 in a low-metallicity cloud, where H_2 line emissions are the most important coolant.

Although ionizing photons hardly escape from the H II region, photons whose radiation energies are below the Lyman limit can get away. Such UV photons photodissociate H_2, and a photodissociation region (PDR) is formed around the H II region. In a PDR in a metal-free cloud, the H_2 dissociation effect is very efficient, so that a region that is larger than the whole of the cosmological low-mass cloud is influenced by only one O5-type star (Omukai & Nishi 1999). However, after a cloud is polluted by heavy elements, the situation becomes complicated, since other thermal processes may be important in a PDR in a cloud with heavy elements. In this region, CO molecules are also dissociated, since the threshold UV energy of H_2 and CO dissociation are close, and C, Si, and Fe in the gas phase are ionized, since the ionizing energies of C, Si, and Fe are lower than that of H. Thus, C^+, Si^+, and Fe^+ cooperate with H_2 as main coolants in a low-metallicity cloud. On the other hand, dust photoelectric heating becomes an important heating source in a polluted cloud. In this section, we study how much mass in a low-metallicity cloud is affected by UV photons from an OB star and as a result becomes unable to cool in a free-fall time.

To calculate the heating rate (Γ) and the cooling rate (Λ) per unit volume, we use Wolfire et al. (1995, and references therein), and the rates of Galli & Palla (1999; processes related to H_2). However, we do not include the effects of X-rays and cosmic rays. We assume the ionization degree, x_e, as

$$x_e = x_{H^+} + x_{C^+} + x_{Si^+} + x_{Fe^+} , \quad (1)$$

where x_i is the abundance of element i. We assume that abundances of heavy elements are determined from the cosmic abundances by scaling proportional to $Z/Z_\odot$.[1] The values adopted are $x_C = 10^{-3.52} Z/Z_\odot$, $x_O = 10^{-3.34} Z/Z_\odot$, $x_{Si} = 10^{-5.45} Z/Z_\odot$, and $x_{Fe} = 10^{-6.15} Z/Z_\odot$. We assume that all of these elements are ionized. Here we add the extra term x_{H^+} to evaluate the effect of relic ionization of cosmological recombination and/or previous SNe, etc. We investigate the cases of $x_{H^+} = 10^{-4}$ and $x_{H^+} = 0$. However, the overall tendency is not affected by the value of x_{H^+}. Thus, we hereafter mainly show the results for the case of $x_{H^+} = 10^{-4}$. In a PDR, H_2 molecules are dissociated mainly via the two-step photodissociation process by the Lyman and Werner (LW) band photons. For the H_2 number density, we use the equilibrium value with the initial ionization degree (Omukai & Nishi 1999). This treatment may result an overestimation of x_{H_2}, and hence in an overestimation of the cooling rate (see, e.g., Nishi & Susa 1999). However, in deriving the lower bound of the region affected by the photodissociating UV radiation from a massive star, we use the equilibrium value.

H_2 is formed mainly via the H^- process,

$$H + e^- \to H^- + \gamma , \quad (2)$$

$$H + H^- \to H_2 + e^- . \quad (3)$$

The rate-determining stage of the H^- process is the reaction given by equation (2), whose rate coefficient k_{H^-} is (de Jong 1972)

$$k_{H^-} = 1.0 \times 10^{-18} T \ s^{-1} \ cm^3 . \quad (4)$$

In a PDR, H_2 is dissociated mainly via the two-step photodissociation process,

$$H_2 + \gamma \to H_2^* \to 2H , \quad (5)$$

where the rate coefficient, k_{2step}, is given by (Kepner, Babul, & Spergel 1997; Draine & Betoldi 1996)

$$k_{2step} = 1.13 \times 10^8 F_{LW} \ s^{-1} . \quad (6)$$

Here F_{LW} (ergs s^{-1} cm^{-2} Hz^{-1}) is the averaged radiation flux in the LW bands. Thus, the equilibrium number density of H_2 under ionization degree x_e is

$$n_{H_2} = \frac{k_{H^-}}{k_{2step}} x_e n_H^2 \quad (7)$$

$$= 0.88 \times 10^{-26} x_e F_{LW}^{-1} T n_H^2 , \quad (8)$$

where n_H is the number density of the hydrogen nuclei. On the other hand, the averaged flux in the LW bands is approximately given by

$$F_{LW} = \frac{L_{LW}}{4\pi r^2} . \quad (9)$$

Then, if the self-shielding effect can be neglected, n_{H_2} is proportional to r^2, and x_{H_2} becomes abundant enough for efficient cooling only at the very outer region.

However, the self-shielding effect is important for ordinary clouds considered in this paper. If the column density of H_2 becomes larger than 10^{14} cm^{-2}, F_{LW} decreases because of self-shielding (Draine & Betoldi 1996). At the outer region, where LW band radiation is shielded, H_2 is dissociated via thermal collision and n_{H_2} assumes the thermal equilibrium value. In this case, H_2 is more abundant if the temperature is lower.

Figure 1 shows the change of cooling and heating rates per unit volume (Λ and Γ) with distance from the central massive star for the typical cloud ($n = 10$ cm^{-3}, $T = 3000$ K). Here we have assumed the existence of one O5 star, whose mass is $\sim 40\ M_\odot$, and that the luminosity of the LW bands is $\sim 10^{24}$ ergs s^{-1} Hz^{-1} at the center of the cloud,[2] and we have also assumed that n, T, and x_e are constant in

[1] The depletion of the gas-phase abundance of heavy elements is serious in the interstellar clouds. However, the main coolants apart from H_2 are C^+ and O, and the depletion of C and O is not so large. Moreover, the depletion of C may not be serious considering dust formation in SN ejecta (e.g., Kozasa, Hasegawa, & Nomoto 1989). However, it may be possible that the depletion of C and O is more significant. In this case, we should consider that the gas-phase abundances of C and O represent the heavy-element abundance, and hence the main results in this paper are almost the same.

[2] Note that although the total luminosity of a star depends strongly on the mass, the dependence of the luminosity in the LW bands is rather weak.

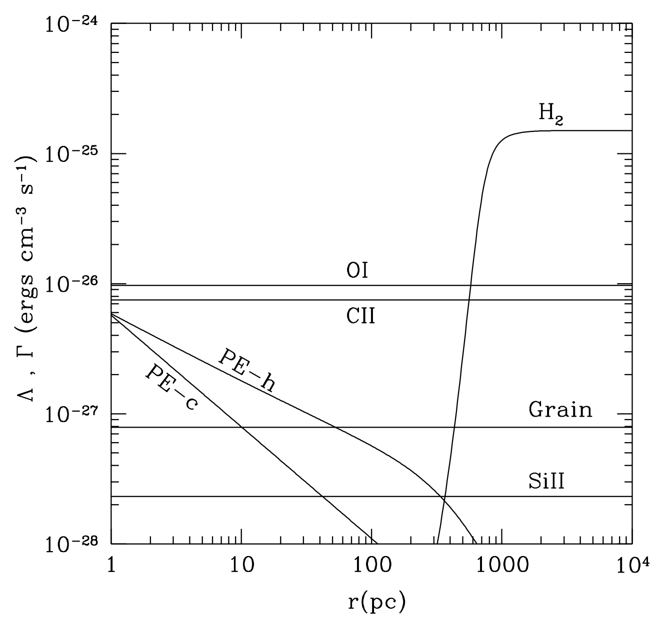

FIG. 1.—Change of the cooling and heating rates per unit volume (Λ and Γ) with the distance from the central massive star for the typical cloud ($n = 10$ cm^{-3}, $T = 3000$ K, and $Z = 10^{-2} Z_\odot$). The central star is assumed to be one O5 star. "PE-h" and "PE-c" in the figure denote the grain photoelectric heating and cooling rates, respectively. The others denote radiative cooling rates of H_2, O I, C II, Si II, and grains, respectively.

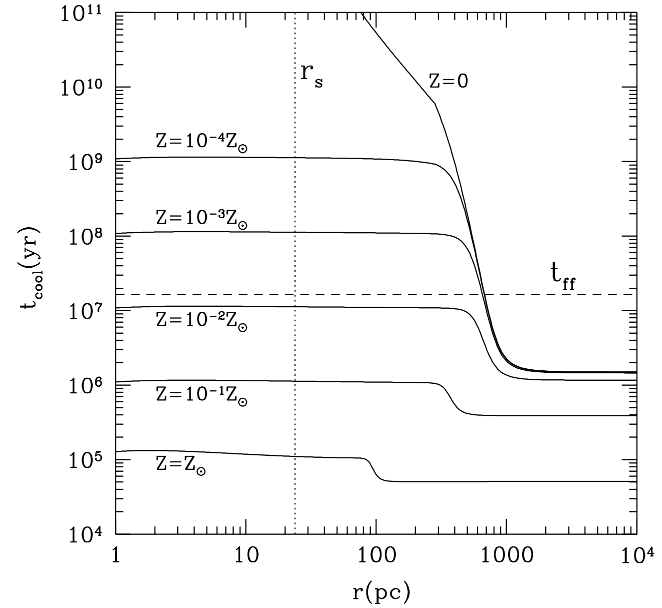

FIG. 2.—Cooling time, t_{cool}, at various distances from the center for a typical cloud ($n = 10$ cm^{-3} and $T = 3000$ K). The metallicity is $Z/Z_\odot = 0$, 10^{-4}, 10^{-3}, 10^{-2}, 10^{-1}, and 1. The dotted line denotes the Strömgren radius, r_S, for an O5 star. The dashed line denotes the free-fall time, t_{ff}.

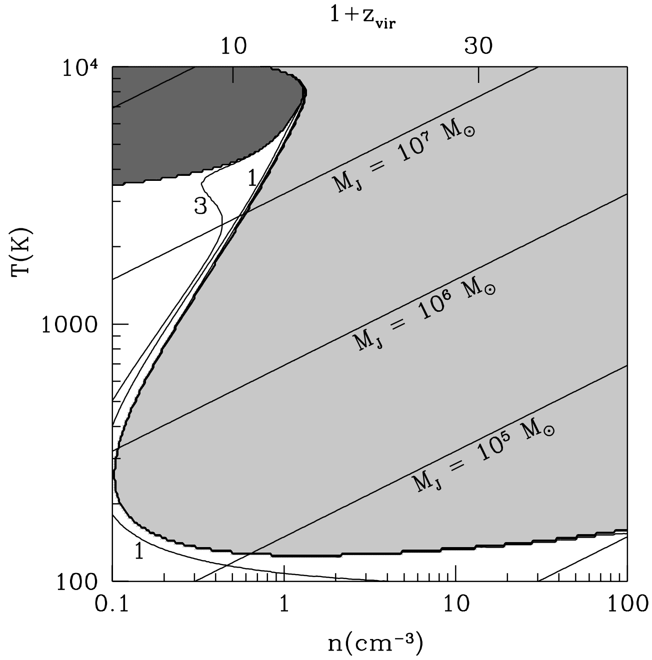

FIG. 4.—Same as Fig. 2, but for the case of $Z/Z_\odot = 10^{-1.5}$

space, for simplicity. At the inner region, where LW band radiation is not shielded, O I and C II line cooling is the dominant cooling process, and hence the cooling rate is approximately proportional to Z. On the other hand, at the outer region, where LW band radiation is shielded, H_2 line cooling becomes dominant for metal-poor clouds.

We calculate the cooling time,

$$t_{cool} = \frac{(3/2)nkT}{\Lambda_{eff}} , \quad (10)$$

where n and T are the number density and the temperature of the cloud, respectively, and Λ_{eff} ($\equiv \Lambda - \Gamma$) is the effective cooling rate. Figure 2 shows the change of the cooling time with the distance from the center.

If metallicity is slightly high ($Z/Z_\odot \gtrsim 10^{-2}$), the cooling time is shorter than the free-fall time, t_{ff} ($\equiv [3\pi/(32G\mu m_H n)]^{1/2} \simeq 1.5 \times 10^7$ yr $[n/10$ cm$^{-3}]^{-1/2}$) over the whole region. Here G is the gravitational constant, μ is the mean atomic weight, and m_H is the hydrogen mass. Considering the H I region, almost all hydrogen is atomic, and $\mu \simeq 1.4$. On the other hand, if the metallicity is very low, $t_{ff} < t_{cool}$ at the inner region and $t_{ff} > t_{cool}$ at the outer region. For the low-density ($n \sim 1$ cm^{-3}) and high-metallicity ($Z/Z_\odot \sim 1$) case, there exists no cool region where net cooling is negative (heating occurs) at the inner region.

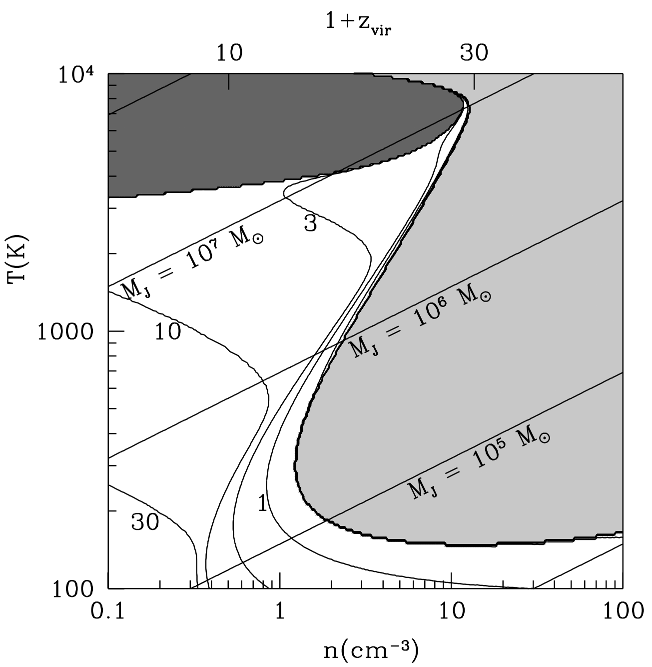

FIG. 3.—Ratio of the cooling radius to the Jeans length, r_{cool}/r_J, for the case of $Z/Z_\odot = 10^{-2}$. The numbers in the figure denote the value of r_{cool}/r_J of the contour. For dense cloud, in the right lightly shaded region, $t_{cool} < t_{ff}$ for the whole H I region. In this case, $r_{cool} = r_S \ll r_J$. For a low-density and high-temperature cloud, in the upper left shaded region, $t_{cool} > t_{ff}$ for the whole H I region. In this case, $r_{cool} \gg r_J$. The values of the Jeans mass are also shown. For reference, considering the cosmological objects, the related virialized redshift is also shown on the upper axis for the flat universe with $\Omega_b = 1$.

For all cases shown in Figure 2, $t_{cool} < t_{ff}$ is achieved at the region outside a certain distance from the center, and we call this transition radius the cooling radius (r_{cool}). However, in the cases of $Z/Z_\odot = 10^{-1}$ and 1, the Strömgren radius, r_S, is larger than r_{cool}. If $r < r_S$, the gas is fully ionized and the temperature becomes high, and star formation is strongly suppressed. Thus, we take as the actual r_{cool} the largest radius between r_{cool} and r_S. In a region $r \le r_{cool}$, next-generation stars are hardly formed before the death of the central star, since cooling is inefficient ($t_{cool} > t_{ff}$). Thus, we consider this region as an influence region. For the much lower metallicity case, $Z/Z_\odot \lesssim 10^{-2.5}$, r_{cool} depends on Z very weakly, since the main coolant becomes H_2 at the outer region. If we adopt $x_{H^+} = 0$, n_{H_2} depends a little on Z,

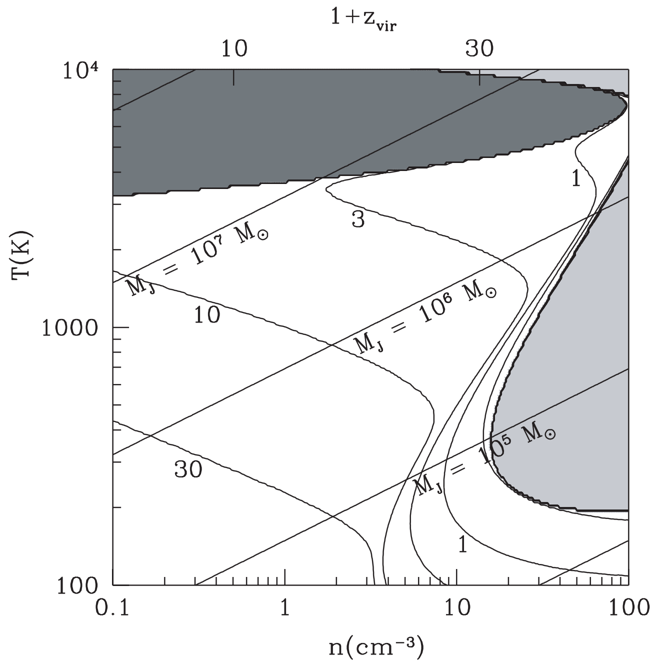

FIG. 5.—Same as Fig. 2, but for the case of $Z/Z_{\odot} = 10^{-2.5}$

so that the influence radius depends a little on Z. But the overall tendency does not change from the case of $x_{\mathrm{H}^+} = 10^{-4}$.

The Strömgren radius is

$$r_{\mathrm{S}} \simeq 24\ \mathrm{pc}\left(\frac{n_{\mathrm{H}}}{10\ \mathrm{cm}^{-3}}\right)^{-2/3}\left(\frac{Q_*}{5.1 \times 10^{49}\ \mathrm{s}^{-1}}\right)^{1/3}, \quad (11)$$

where Q_* is the flux of ionizing photons by an OB star and $Q_* \simeq 5.1 \times 10^{49}\ \mathrm{s}^{-1}$ for an O5 star. The mass within r_{S} is

$$M_{\mathrm{S}} \simeq 2 \times 10^4\ M_{\odot}\left(\frac{n_{\mathrm{H}}}{10\ \mathrm{cm}^{-3}}\right)^{-1}\left(\frac{Q_*}{5.1 \times 10^{49}\ \mathrm{s}^{-1}}\right). \quad (12)$$

Here M_{S} is somewhat smaller than the typical cloud Jeans mass (see Figs. 3–5). Note that r_{S} and Q_* depend strongly on the mass of the central star. Thus, if we consider a less massive central star ($M \simeq 10\ M_{\odot}$), r_{S} becomes much smaller, but other features of these figures hardly change.

For some cases, especially for the high-temperature and very low metallicity case, t_{cool} cannot be shorter than t_{ff} for any distance case, because of the insufficiency of x_{H_2}. In this case, r_{cool} is estimated to be ∞.

3. STAR FORMATION REGULATION IN LOW-METALLICITY CLOUDS

To evaluate the strength of the star formation regulation, we calculate the ratio of the cooling radius to the Jeans length, $r_{\mathrm{cool}}/r_{\mathrm{J}}$, in the n-T plane for the clouds with various metallicities ($0 \leq Z/Z_{\odot} \leq 1$). Here, $r_{\mathrm{J}} \equiv [\pi k T/(G \mu m_{\mathrm{H}} \rho)]^{1/2} \simeq 2.1 \times 10^2\ \mathrm{pc}(T/3000\ \mathrm{K})^{1/2}(n/10\ \mathrm{cm}^{-3})^{-1/2}$. As shown in Figure 3, for the case of $Z/Z_{\odot} = 10^{-2}$, the n-T plane is divided into two regions. In the region with higher density and/or lower temperature, $r_{\mathrm{cool}}/r_{\mathrm{J}}$ is smaller than unity. In other words, the influence radius of a massive star is smaller than the typical scale of the cloud (r_{J}). Thus, the regulation is considered to be ineffective. In the other region, $r_{\mathrm{cool}}/r_{\mathrm{J}} > 1$, and the regulation works well. For dense clouds (Fig. 3, *right light-shaded region*), $t_{\mathrm{cool}} < t_{\mathrm{ff}}$ for the whole H I region. In this case, $r_{\mathrm{cool}} = r_{\mathrm{S}} \ll r_{\mathrm{J}}$. For low-density and high-temperature clouds (Fig. 3, *top left shaded region*), $t_{\mathrm{cool}} > t_{\mathrm{ff}}$ for the whole H I region. In this case, $r_{\mathrm{cool}} \gg r_{\mathrm{J}}$.

Figure 4 shows $r_{\mathrm{cool}}/r_{\mathrm{J}}$ on the n-T plane for the case of $Z/Z_{\odot} = 10^{-1.5}$. In this case, $r_{\mathrm{cool}}/r_{\mathrm{J}} < 1$ for almost the whole region. Thus, star formation regulation by UV radiation is inefficient. In contrast, as shown in Figure 5, for the case of $Z/Z_{\odot} = 10^{-2.5}$, $r_{\mathrm{cool}}/r_{\mathrm{J}} > 1$ for almost the whole region, and the regulation works very well. The transition occurs when the metallicity is $10^{-2.5} \lesssim Z/Z_{\odot} \lesssim 10^{-1.5}$, and the typical metallicity when the transition occurs is estimated as $Z/Z_{\odot} \simeq 10^{-2}$.

For the extremely low metallicity case ($Z/Z_{\odot} \lesssim 10^{-4}$), the effect of heavy elements on the thermal process is almost negligible. Thus, considering the thermal process, it is difficult to distinguish a cloud with $Z/Z_{\odot} \lesssim 10^{-4}$ from a cloud with $Z/Z_{\odot} = 0$.

4. DISCUSSION

For the primordial gas clouds, if we consider the self-shielding effect, the mass within the region of influence is obtained as (Omukai & Nishi 1999)

$$M^{(\mathrm{inf})} = 8 \times 10^6\ M_{\odot}\left(\frac{x_{\mathrm{e}}}{10^{-4}}\right)^{-1} \times \left(\frac{L_{\mathrm{LW}}}{10^{24}\ \mathrm{ergs\ s^{-1}\ Hz^{-1}}}\right) \times \left(\frac{T}{3 \times 10^3\ \mathrm{K}}\right)^{-1}\left(\frac{n}{10\ \mathrm{cm}^{-3}}\right)^{-1}. \quad (13)$$

This mass is larger than the typical cloud mass (see Figs. 3–5). On the other hand, for the low-metallicity gas clouds, the strength of the self-regulation changes with the metallicity.

As shown in the previous section, in the case of very low metallicity clouds ($Z/Z_{\odot} \lesssim 10^{-2.5}$), star formation regulation by UV radiation is very effective. In this case, the star formation rate is very low, since only one massive star can stop the evolution of the whole host cloud. If SNe do not disrupt the gas binding, the host cloud is polluted by heavy elements little by little, and the following continuous star formation is possible. Even in the case in which the host cloud is disrupted by SN explosions, if the remnant gas does not escape from the host pregalactic object, next-generation clouds, which are slightly polluted, will be formed, and star formation will follow. After the metallicity becomes high enough ($Z/Z_{\odot} \gtrsim 10^{-2}$), effective star formation can start. Thus, the lower limit of the metallicity of luminous objects is roughly estimated at $Z/Z_{\odot} \sim 10^{-2}$. Note that a SN release several $M_{\odot}$ of heavy elements, and the typical cloud mass ($\sim \rho r_{\mathrm{J}}^3$) is $\sim 10^6\ M_{\odot}$. Thus, if we consider that the cloud mass is $10^6\ M_{\odot}$ and the mass of the heavy elements released by one SN is 4 $M_{\odot}$, the metallicity increase per SN is $\sim 4 \times 10^{-6}$, and the change in the metallicity is $\sim 2 \times 10^{-4}\ Z_{\odot}$; hence, before efficient star formation begins, about 50 cycles of star formation and SN explosion are required. This implies that at the effective star formation epoch, there should exist some quantity of heavy elements and dust, which are scattered well. Then it is expected that it will be difficult for Lyα photons to escape from the clouds. Moreover, the inefficiency of star formation in low-metallicity clouds may explain the G dwarf problem (e.g., Rocha-Pinto & Maciel 1996), and may also imply that the reionization epoch of the universe may come later than proposed in previous studies (e.g., Fukugita & Kawasaki 1994; Haiman & Loeb 1997; Gnedin & Ostriker 1997).

The lifetime of very massive stars is $\sim 3 \times 10^6$ yr. However, as noted above, the mass dependence of the luminosity in the LW bands is rather weak, and hence even if the mass of the central star is smaller ($\lesssim 10\ M_\odot$), in this case the lifetime of the star is longer, and star formation regulation can be efficient. Moreover, these less massive stars may not evolve into Type II SNe. Thus, after the death of the first OB star, star formation could occur somewhere in the cloud, and another OB star could form successively. Therefore, star formation regulation often becomes serious even for clouds whose $t_{\rm ff}$ is longer than $\sim 3 \times 10^6$ yr.

Lin & Murray (1992) considered regulation only via photoionization. In such a case, the region affected by an OB star becomes the region within a Strömgren sphere. For the very lower metallicity case, $r_{\rm S}$ is somewhat smaller than $r_{\rm cool}$. However, if the metallicity is fairly high ($Z/Z_\odot \gtrsim 10^{-1.5}$), $r_{\rm cool}$ becomes $r_{\rm S}$ for almost the whole region in the n-T plane (see Fig. 4, *lightly shaded region*). Thus, for the higher metallicity case ($Z/Z_\odot \gtrsim 10^{-1.5}$), it is a good approximation to consider that star formation regulation occurs only via photoionization, but in this case the influence region is much smaller than the host cloud, and hence the regulation is not effective.

Recently, it has been shown that the earlier phase of the chemical evolution of the Galaxy can be explained by models with the assumption that SN-induced star formation is the only star formation process (e.g., Tsujimoto, Shigeyama, & Yoshii 1999; Ishimaru & Wanajo 1999). As shown above, when the metallicity is lower ($Z/Z_\odot \lesssim 10^{-2}$), star formation can occur only after previous massive stars have died. If these massive stars die with SN explosions, we can consider that SN-induced star formation is the only star formation process, since stars can form only after SN explosions.

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