

THE PHYSICS OF TYPE Ia SUPERNOVA LIGHT CURVES. II. OPACITY AND DIFFUSION

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ABSTRACT

We examine the nature of the opacity and radiation transport in Type Ia supernovae. The dominant opacity arises from line transitions. We discuss the nature of line opacities and diffusion in expanding media and the appropriateness of various mean and expansion opacities used in light-curve calculations. Fluorescence is shown to be the dominant physical process governing the rate at which energy escapes the supernova. We present a sample light curve that was obtained using a time-dependent solution of the radiative transport equation with a spectral resolution of 80 km s^{-1} and employing an LTE equation of state. The result compares favorably with light curves and spectra of typical supernovae and is used to illustrate the physics controlling the evolution of the light curve and especially the secondary maxima seen in infrared photometry.

Subject headings: distance scale — radiative transfer — supernovae: general

1. INTRODUCTION

In Paper I of this series (Pinto & Eastman 2000), we presented an analytic solution for the conversion of radioactive decay energy into the light curve of a Type Ia supernova (SN Ia). This allowed a simple and direct demonstration of the effects of varying such fundamental properties of the explosion as its energy, mass, density structure, ^{56}Ni yield, and the radial distribution of ^{56}Ni in the ejecta. Fundamental to that solution was the assumption that the opacity of the supernova ejecta was constant in time. No attempt was made either to justify this assumption or to determine what the appropriate choice of the opacity might be, other than achieving the correct rise time and peak width empirically.

It is obvious from that discussion that the opacity in the ejecta of SNe Ia is a crucial element in determining the behavior of the light curve. In this paper we examine the nature of this opacity and its effect upon the light curve. SNe Ia differ significantly from other astrophysical objects in that they combine a very large velocity gradient with a composition entirely constituted of heavy elements. This allows a number of interesting effects, present at a low level in many systems, to take on a dominant role. The most important of these follows from the extreme complexity of the atomic physics in iron group ions.

The monochromatic opacity in SNe Ia is very large in the ultraviolet and decreases strongly with increasing wavelength. We will show that it is overwhelmingly likely that radiation that is absorbed in the ultraviolet will suffer repeated fluorescences that result in the emission of a cascade of lower energy photons. This allows for an efficient transport of energy downward in frequency, which ends when the photons arrive at optically thin regions of the spectrum and escape. This downward transport in energy has a profound effect upon the radiative diffusion time. It also allows for a process, akin to thermalization but not

mediated by collisions, that can lead to a spectrum that appears thermal but that needs have little to do with the local gas temperature anywhere in the object.

After discussing the nature of this opacity, we present a sample light curve determined by solving the time-dependent, multifrequency transport equations without resorting to the use of any mean opacity. We then use this detailed solution to examine the behavior of the mean opacity by directly averaging monochromatic opacities. We find that the most natural a priori choice of a mean opacity, the Rosseland mean, underestimates the true flux mean by up to a factor of 4. The true flux mean opacity appears to be remarkably constant both in time and in radius through the peak in the light curve but declines significantly after peak.

Finally, we discuss how the secondary maximum that is often seen in infrared light curves of SNe Ia is a consequence of a sharp reduction in mean opacity that occurs when the photosphere recedes to near the center of the ejecta. With further study, this phenomenon may provide a useful diagnostic of the composition of the innermost regions of the explosion.

2. OPACITY AND PHOTON ESCAPE

2.1. The Monochromatic Opacity

If we are to discriminate between various models for SNe Ia based upon the behavior of their light curves, it is clear from the above that an accurate understanding and determination of the opacity is crucial. Harkness (1991), Wheeler, Swartz, & Harkness (1991), Höflich, Müller, & Khokhlov (1993), and Paper I have stressed that the bolometric rise time and peak luminosity depend as sensitively upon the opacity as on any of the other physical properties characterizing the explosion: mass, kinetic energy, or ^{56}Ni mass and distribution. A factor of 2 change in opacity has nearly the same effect on the peak of the light curve as a factor of 4 change in explosion energy or a 50% change in the mass of the ejecta.

In this section we examine the monochromatic opacity in SNe Ia and the mechanisms by which energy that has been deposited in the interior diffuses to the surface. The results

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of this section will then be applied to an examination of frequency-averaged mean opacities.

A simple application of the analytic model of Paper I shows that, for explosion models with $0.7 M_{\odot} < M < 1.4 M_{\odot}$ and $0.35 M_{\odot} < M(^{56}\text{Ni}) < 1.4 M_{\odot}$, the central temperature near maximum luminosity is $1.5 \times 10^4 \text{ K} \lesssim T \lesssim 2.5 \times 10^4 \text{ K}$ and the density is $10^{-14} \lesssim \rho \lesssim 10^{-12} \text{ g cm}^{-3}$. Under these conditions, the continuum opacity at optical wavelengths is dominated by electron scattering. For central temperatures $T_c > 1.5 \times 10^4 \text{ K}$, the peak of the Planck function is in the UV ($\lambda_{\text{BB}} \lesssim 1900 \text{ \AA}$) where the opacity is dominated by bound-bound transitions. The opacity from a thick forest of lines is greatly increased by velocity shear Doppler broadening (Karp et al. 1977).

Figure 1 displays the various sources of opacity for a mixture of ^{56}Ni (20%), ^{56}Co (70%), and ^{56}Fe (10%), at a density of $10^{-13} \text{ g cm}^{-3}$ and temperature of $2.5 \times 10^4 \text{ K}$, typical (perhaps) of maximum light in a Chandrasekhar-mass explosion. The excitation and ionization were computed from the Saha-Boltzmann equation. The opacity approximation of Eastman & Pinto (1993—also see below) was used for the line opacity, which greatly exceeds that from electron scattering. Bound-free and free-free transitions contribute negligibly to the overall opacity, but they are important contributors to the coupling between the radiation field and the thermal energy of the gas. The opacity is very strongly concentrated in the UV and falls off steeply toward optical wavelengths. As was noted by Montes & Wagoner (1995), the line opacity between 2000 and 4000 $\text{\AA}$ falls off roughly as $d \ln \kappa_{\lambda} / d \ln \lambda \sim -10$. We will show that the steepness of this decline toward the optical

has important implications for the effective opacity in SNe Ia and the way in which energy escapes.

2.2. Diffusion

Not only is the opacity from lines greater than that of electron scattering, it is also fundamentally different in character from a continuous opacity. In a medium where the opacity varies slowly with wavelength, photons have an exponential distribution of path lengths. Their progress through an optically thick medium is a random walk with a mean path length given by $(\rho\kappa)^{-1}$. In a supersonically expanding medium dominated by line opacity, however, there is a bimodal distribution of path lengths. The line opacity is concentrated in a finite number of isolated resonance regions. Within these regions, where a photon has Doppler shifted into resonance with a line transition, the mean free path is very small. Outside these regions, the path length is determined either by the much smaller continuous opacity or by the distance the photon must travel to have Doppler shifted into resonance with the next transition of longer wavelength. For the physical conditions of Figure 1, the mean free path of a UV photon goes from approximately $5 \times 10^{14} \text{ cm}$ in the continuum (because of electron scattering) to less than $\sim 10^6 \text{ cm}$ when in a line. The usual random walk description of continuum transport must be modified to take this bimodal distribution into account.

Within a line, a photon scatters on average $N \sim 1/p$ times, where p is the probability per scattering for escape, which is accomplished by Doppler shifting out of resonance. In spite of the possibly large number of scatterings needed for escape, a photon spends only a small fraction of

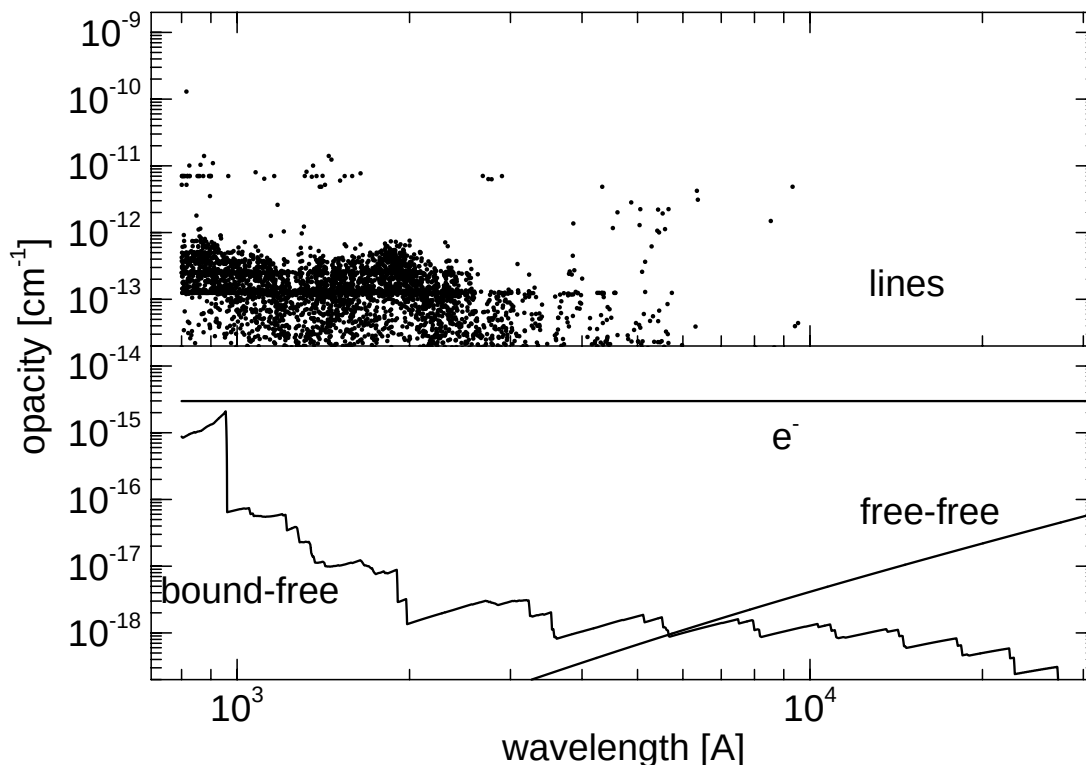


FIG. 1.—Monochromatic opacity sources at maximum light for a Chandrasekhar-mass model of a Type Ia supernova from a time-dependent, multi-frequency, LTE calculation. The physical conditions are $\rho = 10^{-13} \text{ g cm}^{-3}$, $T = 2.5 \times 10^4 \text{ K}$ and $t = 14 \text{ days}$. The line opacity shown here is the frequency bin-averaged *expansion* opacity, as given by equation (9) (see text) and not the much larger monochromatic Sobolev opacity. The line opacity has not been plotted below $2 \times 10^{-14} \text{ cm}^{-1}$, so as not to obscure the contributions from continuum opacity sources.

its flight time in resonance with lines. The effect is quite different from that of a similar observer-frame optical depth arising from a continuous opacity.

Because of the very supersonic expansion of the supernova's ejecta, we can make use of Sobolev theory to describe the path of a photon, following the discussion of Eastman & Pinto (1993). The Sobolev optical depth of a line transition with Einstein coefficient B_{lu} is (ignoring stimulated emission)

$$\tau_s = \frac{h}{4\pi} \frac{n_l B_{lu}}{|\partial\beta/\partial l|}, \quad (1)$$

where n_l is the lower level number density and $\partial\beta/\partial l \sim 1/ct$ is the velocity gradient over the speed of light. The probability of escape from the line transition is

$$p = \frac{1 - e^{-\tau_s}}{\tau_s} \quad (2)$$

(Castor 1970). Consider a photon that is emitted in a resonance at frequency displacement x and subsequently reabsorbed at displacement $x' < x$, having traveled a distance $(x - x')\Delta v_D/(\partial\beta/\partial l)$, where Δv_D is the thermal Doppler width. Assume for the moment that the line has a negligible photon destruction probability (we will address thermal destruction below). The optical depth between emission and absorption is

$$\tau(x, x') = \tau_s \int_{x'}^x \phi(t) dt, \quad (3)$$

where $\phi(x)$ is the normalized line absorption profile. The probability that a photon emitted at x will travel a distance $(x - x')ct$ to be reabsorbed at x' is

$$\tau_s \phi(x) \phi(x') \exp[-\tau(x, x')]. \quad (4)$$

Assuming complete redistribution, the average value of $x - x'$ is then

$$\langle x - x' \rangle = \left\{ \tau_s \int_{-\infty}^{\infty} \phi(x) \int_{-\infty}^x (x - x') \phi(x') \times \exp[-\tau(x, x')] dx' dx \right\} [1 - (1 - e^{-\tau_s})/\tau_s]^{-1}. \quad (5)$$

Here, the denominator is just the total probability of reabsorption, $1 - p$. For a Doppler line profile $\phi(x)$, this can be approximated as $\langle x - x' \rangle \sim 0.8/(1 + \tau_s/5)$ (Eastman & Pinto 1993). The typical distance traveled between scatterings in the transition is then

$$\delta r = \langle x - x' \rangle \Delta v_D ct, \quad (6)$$

where in homologous expansion, $\partial\beta/\partial l = 1/ct$. On average, the photon will scatter $N \sim 1/p$ times, and the total distance covered while trapped in the line resonance will be

$$\delta r_L = \left(1 - \frac{1}{p}\right) \delta r. \quad (7)$$

In a very optically thick line, the photon will thus travel a distance equal to $4v_{th}/v_{exp}$ times the radius of the ejecta while trapped within a line. Since $v_{th}/v_{exp} < 10^{-2}$ at all times, the photon travels a negligible distance, and hence spends a negligible time, while in resonance with any one line.

This means that each time an optically thick line absorbs a photon, the effect is as though the photon is instantane-

ously reemitted in a random direction. From the point of view of the diffusion of radiation through the ejecta, each interaction with an optically thick line acts like a single scattering event, independent of the optical depth of the transition. The distribution of mean free paths will thus be determined by the continuum opacity and by the distribution (and density) of lines in energy space. In a medium with little continuous opacity, the mean free path of a photon is the average distance a photon travels between resonances, and the effective mean free path thus has little to do with any conventionally defined monochromatic opacity.

For the purpose of estimating the diffusion time, the effective total "optical depth" of the supernova for a photon emitted from the center of the remnant at frequency ν is the number of line interactions a photon undergoes in Doppler shifting from ν to $(1 - v_{exp}/c)\nu$. This optical depth may be written as

$$\tau(\nu) = \sum_{\{k: \nu \geq \nu_k \geq \nu(1 - v_{exp}/c)\}} [1 - \exp(-\tau_k)]. \quad (8)$$

The sum is over all lines with Sobolev optical depths τ_k and transition frequencies ν_k lying in the interval $\nu \geq \nu_k \geq \nu(1 - v_{exp}/c)$.

The evolution of this optical depth with time depends on whether the lines are optically thick or thin. In the limit that all lines are optically thick, $\tau_k \gg 1$, $\tau(\nu)$ is just the number of lines in the range $[\nu(1 - v_{exp}/c), \nu]$ and, barring significant changes in excitation conditions, is constant with time. In the other extreme, where all lines are optically thin, $\tau_k \ll 1$ and $\tau(\nu) = \sum_l \tau_l$. Since $\tau_l \propto t^{-2}$ (again, barring changes in excitation conditions), $\tau(\nu) \propto t^{-2}$ and behaves like a continuum optical depth that is proportional to the ejecta column density.

2.3. Expansion Opacities

We can derive an effective monochromatic opacity coefficient by setting $\rho\kappa(\nu)R_{max} = \tau(\nu)$. This would correspond to a global average over the frequency range $[\nu(1 - v_{exp}/c), \nu]$. A more local quantity can be obtained by reducing this range to $(\nu, \nu + \Delta\nu)$, where $\Delta\nu \sim \nu \Delta r \partial\beta/\partial r$, giving

$$\kappa(\nu) = \frac{\nu}{\rho \Delta\nu} \frac{\partial\beta}{\partial r} \sum_{\{k: \nu \leq \nu_k \leq \nu + \Delta\nu\}} [1 - \exp(-\tau_k)]. \quad (9)$$

This is the expansion opacity introduced by Eastman & Pinto (1993). It has the advantage of being a purely local quantity. The expansion opacity formulation of Karp et al. (1977) and its descendents, on the other hand, always average over a mean free path. This distance can easily become larger than the distance over which the material properties change or even than the supernova itself. We mention as a technical note another problem with the formulation of Karp et al. (1977) that was pointed out by Blinnikov (1996). The Karp et al. opacity is expressed as an average over a mean free path in the *downwind* frequency direction; the probability that a photon travels a distance $d\tau$ depends upon the presence of lines it has not yet encountered. As Blinnikov (1996) showed in a derivation from the Boltzmann equation, the correct formulation of such an effective opacity should average in the *upwind* direction so that the probability of having penetrated to $d\tau$ depends upon lines *already* encountered. Since the effective opacity described by equation (9) makes reference only to lines actually encountered in $d\tau$, such an ambiguity does not present itself. In practice, for the conditions encountered in

SNe Ia, the upwind and downwind averages give numerically similar results. We note further that equation (9) is just a generalization of the effective opacity developed by Castor, Abbott, & Klein (1975) in the context of line-driven stellar winds.

In Figure 2 we show the density of lines per 10^4 km s^{-1} [which we denote $D(\nu)$], for the same conditions as were used for Figure 1, at 18 days past explosion, with lines taken from the Kurucz line list (Kurucz 1991, 1992). The uppermost curve is the spectral density of all lines included in the calculation. Below that is all lines with $\tau_s > \frac{2}{3}$, and further down are all lines with $\tau_s > 6.7$ and 67. If we exclude lines for which $\tau_s \lesssim 1$, then $D(\nu) \approx \tau(\nu)$. Superposed upon these curves for reference are three blackbody distributions (the vertical scale is arbitrary). For temperatures above 10^4 K , most photons “see” a very large value of $\tau(\nu)$ and remain trapped with an ever-increasing diffusion time.

In terms of the effective total optical depth, $\tau(\nu)$, the mean free path is $l \approx R(t)/\tau(\nu)$, and the diffusion time can be written $t_d \approx \tau(\nu)^2 l/c \approx \tau(\nu)R(t)/c$. The light curve peaks when the diffusion time roughly equals the elapsed time. Setting $t_d = t$ and substituting $R(t) = v_{\text{exp}} t$ and $v_{\text{exp}} \sim 10^4 \text{ km s}^{-1}$, one finds that $\tau(\nu) \sim c/v_{\text{exp}} \sim 30$; the flux mean optical depth must decline below ~ 30 at peak. In the UV, $\tau(\nu) \gg 30$, and since the Sobolev optical depth of many of these lines is $\gg 1$, $\tau(\nu)$ will remain approximately constant with time. Individual line Sobolev optical depths decline as $1/t^2$, and it will not be until 50 days that the $2 \times 10^4 \text{ K}$ Planck mean of $\tau(\nu)$ falls below ~ 30 . What, then, accounts for the fact that the light curve peaks near 18 days and not 50 days?

One possibility may be, at least in part, that as photons random walk their way out, they are Doppler shifted to longer wavelengths, where the effective optical depth is much smaller. If photons must scatter on order $\tau(\nu)^2$ times to escape and the mean free path is $R/\tau(\nu)$, we can ask what the accumulated Doppler shift might be, following this path. Since, for homologous expansion $dv/dr = v_{\text{max}}/R = 1/t$, the total Doppler shift is $\tau(\nu) v_{\text{max}}/c \sim \nu$, i.e., of order of the entire energy of the photon! Given the rapid decline of $\tau(\nu)$ with decreasing frequency, this implies that, in the absence of photon destruction mechanisms (electron collisional de-excitation, fluorescence), a photon emitted in the interior will scatter off lines until it has accumulated sufficient redshift to put it at a frequency where $\tau(\nu)$ is small enough to permit escape.

2.4. Photon Collisional Destruction and the Thermalization Length

Our discussion of line opacity has so far assumed that photons interact with “pure scattering” lines, with no account taken of photon “destruction.” By this we mean any alternative channel for depopulating the upper state of the line transition other than emission and escape of a photon in the original line. The two most important mechanisms for this are collisional depopulation by thermal electron collisions and emission into another line (fluorescence).

We first examine the efficiency for collisional destruction: that upon being excited to the upper level of the transition, the absorbing atom is collisionally depopulated and the photon’s energy is added to (or subtracted from) the thermal kinetic energy of the gas. The collisional destruc-

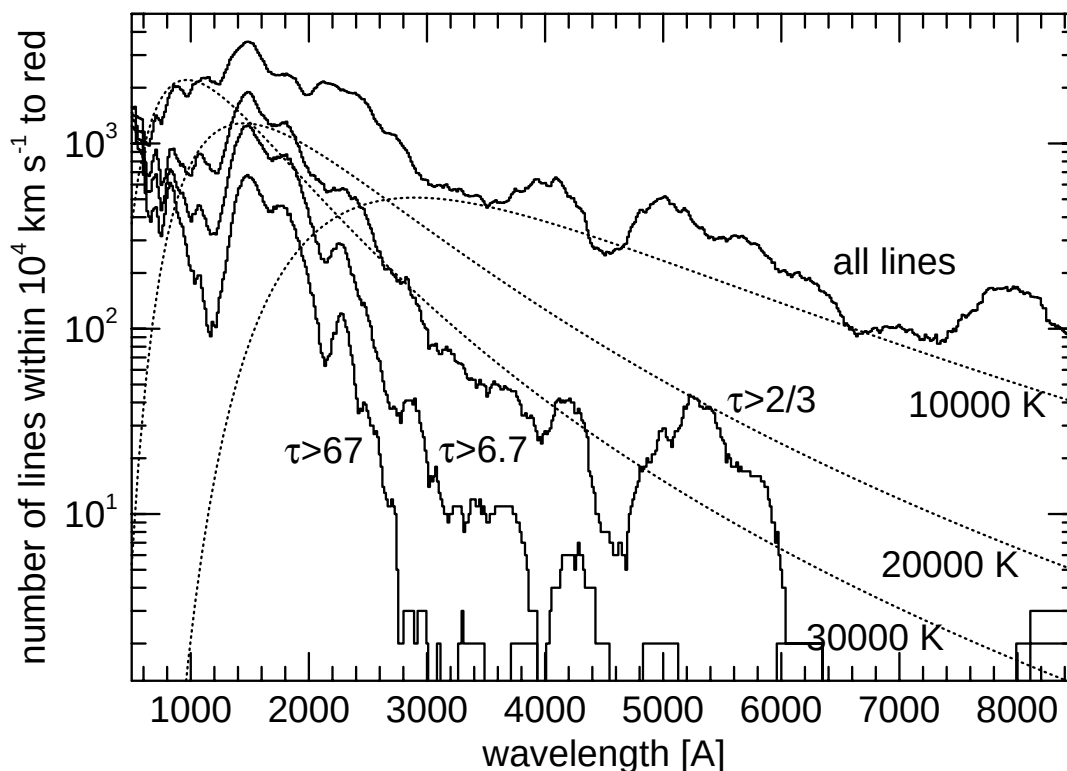


FIG. 2.—Density of lines in energy space at maximum light (18 days). The histograms are the number of lines in 10^4 km s^{-1} to the red of a given wavelength, binned in energy. The heavy line includes all lines with an average (over volume) Sobolev optical depth greater than $\frac{2}{3}$. The two lower histograms show the same quantity, but include lines with average Sobolev optical depth greater than 6.7 and 67. Roughly speaking, the optical depth $\frac{2}{3}$ curve will decline to resemble the optical depth 6.7 curve at 31 days. Data are taken from the same calculation as for Fig. 1. The dotted lines are schematic flux distributions for blackbody radiation fields of 1, 2, and $3 \times 10^4 \text{ K}$, shown for comparison.

tion probability per line interaction to upper level u (not per single scattering) can be written

$$\epsilon_u = \frac{n_e \sum_l C_{ul}}{n_e \sum_l C_{ul} + \sum_l p_{ul} A_{ul}}, \quad (10)$$

where, for $E_u > E_l$,

$$n_e C_{ul} = \frac{8.629 \times 10^{-6} \Omega_{ul}(T) n_e}{g_u \sqrt{T}} \quad (11)$$

(cf. Osterbrock 1989) is the rate per atom of collisions from state u , with statistical weight g_u , and collision strength Ω_{ul} . The quantity $p_{ul} A_{ul}$ is the effective radiative de-excitation rate. To investigate the effect of electron collisions on the effective line opacity one can use equation (9), with each line k weighted by the probability for thermalization:

$$\kappa_{\text{thm}}(\nu) = \frac{\nu}{\rho} \frac{\partial \beta}{\partial \nu} \sum_{\{k: \nu \leq \nu_k \leq \nu + \Delta \nu\}} [1 - \exp(-\tau_k)] \epsilon_k, \quad (12)$$

where ϵ_k is the thermalization probability for line k (eq. [10]) and $\Delta \nu$ is the width of a frequency bin. The top panel of Figure 3 shows κ_{thm} for the same line list and conditions as in Figure 1. Van Regemorter's formula (Van Regemorter 1962) was used to calculate the collision strengths Ω_{ul} in the absence of detailed data, but in no case was the resulting collision strength allowed to be less than unity for permitted transitions. For this particular example, κ_{thm} peaks at a value of $\sim 10^{-4} \text{ cm}^2 \text{ g}^{-1}$ near 1500 Å. To put this in context we must consider the question of what value of κ_{thm} is sufficient to bring the gas and radiation field into thermal equilibrium. This will occur when $\tau_{\text{thm}} \equiv \rho R_{\text{max}} (\kappa \kappa_{\text{thm}})^{1/2} \gtrsim 1$, where κ is the total opacity. For a $1.4 M_{\odot}$ uniform-density sphere expanding at 10^9 km s^{-1} , the column density at 18

days is $\rho R_{\text{max}} \sim 276 \text{ g cm}^{-2}$. At 1500 Å, $\kappa_{\text{thm}} \sim 10^{-4} \text{ cm}^2 \text{ g}^{-1}$, while the total opacity is $\kappa \sim 1 \text{ cm}^2 \text{ g}^{-1}$, so $\tau_{\text{thm}} \sim 2.8$ —barely adequate to thermalize the radiation field to the local gas temperature. Longward of 1500 Å, the situation is somewhat different. At 3000 Å, for instance, $\kappa \sim 10^{-3.5}$ and $\kappa_{\text{thm}} \sim 10^{-6}$, so $\tau_{\text{thm}} \sim 5 \times 10^{-3}$ —much less than 1 and entirely insufficient for thermalization.

While these numbers should be taken as no more than order-of-magnitude estimates, they are accurate enough for us to conclude that near maximum light, the electron density in Type Ia supernova ejecta is too low for collisional destruction to bring about thermalization between the gas and the radiation field, at least at wavelengths $\lambda \gtrsim 2000 \text{ Å}$. We expect, therefore, that the radiation field at longer wavelengths may be significantly different from a Planck function at the local gas temperature. This is a very important point. It means that there is no depth in the supernova to apply the usual equilibrium radiative diffusion inner boundary condition, where it is assumed that $J(\nu) = B_{\nu}(T_{\text{gas}})$. We show below that line scattering results in a pseudocontinuous spectrum; this must not be confused, however, with a continuum that arises from thermalization mediated by electron collisions. Even at the peak of the light curve, the *entire* supernova must be included in models of the spectrum.

2.5. Fluorescence and Enhanced Escape

Another possible fate for a photon trapped in a line resonance is that it will decay from the upper level, not in the transition it was absorbed in, but to some other state of lower energy. (While photoexcitation out of the level is also a way of “destroying” a photon, for the UV transitions that contribute most to the opacity in SNe Ia, this is a much less likely outcome.) The probability that a photon trapped in a

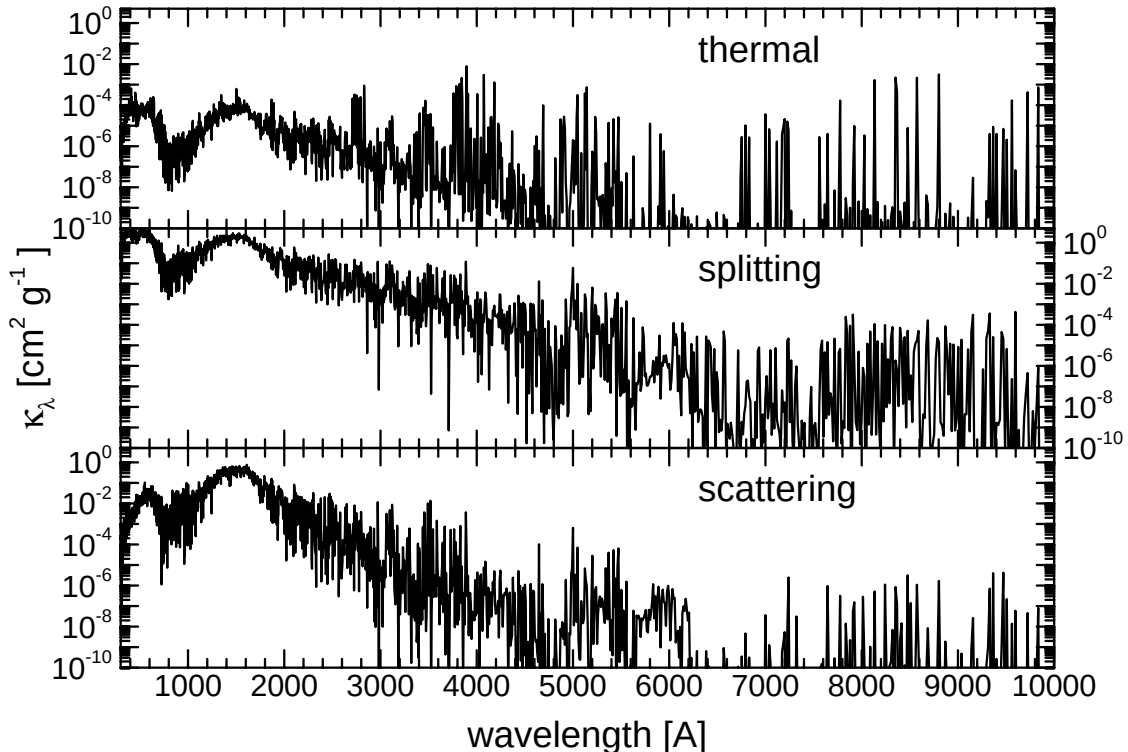


FIG. 3.—Opacity of a gas at the same conditions as in Fig. 1, decomposed into thermal destruction (*top*: eq. [12]), fluorescence (*middle*: eq. [14]), and coherent scattering (*bottom*: eq. [15]). LTE level populations were employed.

resonance with upper state u will escape the resonance region via downward transition l , is given by the branching ratio

$$b_{ul} = \frac{p_l A_l}{n_e \sum_j C_j + \sum_k p_k A_k}, \quad (13)$$

where the sum k is over all lines with the same upper level u , including line l . While UV lines tend to have the highest Einstein A -values (for electric dipole permitted transitions $A_l \propto \lambda^{-2}$), they also tend to arise from levels nearest the ground level to have higher optical depths and, therefore, smaller escape probabilities. This may lead to a larger probability of decay into another, longer wavelength transition and further cascade into yet other transitions. The effect of this fluorescence is to split a photon's energy up into a series of longer wavelength photons. This process is well observed in nebulae, for example, where each photon of higher energy transitions in the Lyman series of hydrogen is "degraded" to emerge from the lowest energy transitions in lower energy series; the energy in $\text{Ly}\gamma$ photons is "split" into $\text{Ly}\alpha$, $\text{H}\alpha$, and $\text{Br}\alpha$ photons. While the inverse process—absorption from a high-lying state and decay into a higher energy transition—can and does occur, it must do so less frequently by obvious thermodynamic considerations (Rosseland's theorem of cycles—cf. Mihalas 1978). Pinto

(1988) noted the importance of fluorescence to spectrum formation in SNe Ia in the nebular phase. Li & McCray (1996) showed that it is an important effect in SN 1987A at late times as well.

In Figure 4 we show the relative probability that a photon's energy, absorbed into a transition at a given wavelength, is reemitted at one or more other wavelengths, weighted by the probability of absorption into the initial transition. It is much like the more familiar recombination cascade matrix, but instead of a collisional process pumping the high-energy states, in this case it is line absorption. There is a considerable tendency for energy absorbed in the UV to come out in the optical and infrared. This tendency is enhanced by the radiative transfer through the ejecta, as a photon emitted in the UV is overwhelmingly likely to be reabsorbed in another thick transition nearby and given another opportunity for fluorescence, while those photons emitted at longer wavelengths are more likely to escape. Note that while there is some probability below the diagonal of the plot corresponding to photoexcitation followed by emission into a shorter wavelength line, this outcome is relatively less likely. In addition, the rate of electron collisions coupling states of similar energies is much larger than those that remove a substantial fraction of the photon's energy to the gas. These collisions between states of similar energies have the effect of opening up an even larger

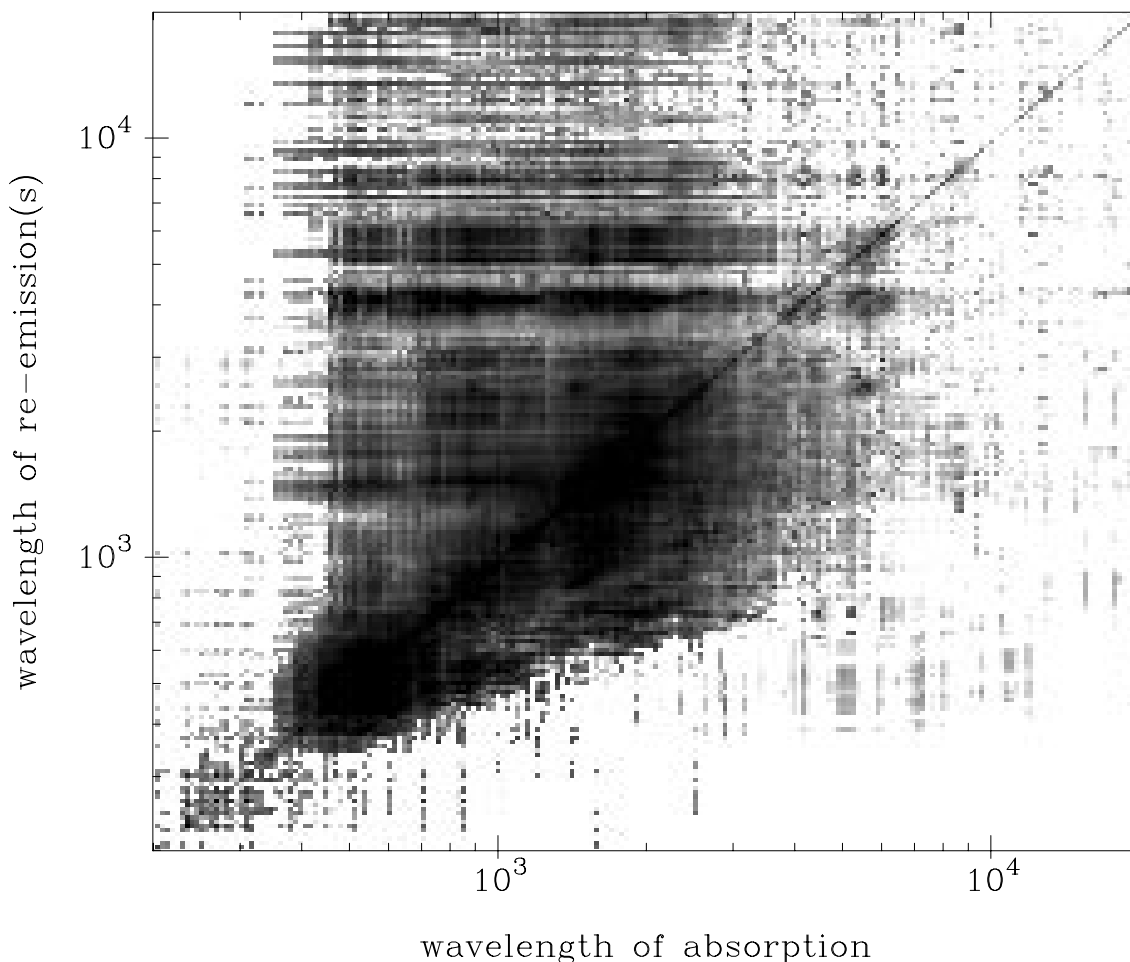


FIG. 4.—Cascade matrix for a photon absorbed into a transition with wavelength given by the abscissa and emitted into (possibly many) wavelengths given by the ordinate. The intensity represents the probability of branching multiplied by the probability of being trapped in the absorbing transition. Thus, absorptions at low optical depth, which may nonetheless lead to multiple splittings, are suppressed. The line list and physical conditions are again the same as for Fig. 1.

number of subordinate transitions, enhancing the rate of this photon “splitting” beyond that of radiative fluorescence alone.

As was done for electron collisional destruction, we can define an effective line opacity for fluorescence as

$$\kappa_{\text{spl}}(v) = \frac{v}{\rho \Delta v} \frac{\partial \beta}{\partial r} \sum_{\{k: v \leq v_k \leq v + \Delta v\}} [1 - \exp(-\tau_k)] \times \sum_{l \neq k} b_{u(k)l}, \quad (14)$$

where $b_{u(k)l}$ is as given by equation (13) and $u(k)$ is the upper level of transition k . The sum is over all downward transitions except l (and thus ignores the effects of small ΔE electron collisions). We can similarly define an effective scattering opacity as the line opacity multiplied by the probability that a photon absorbed into a transition ultimately escapes as a single photon at the same energy:

$$\kappa_{\text{scat}}(v) = \frac{v}{\rho \Delta v} \frac{\partial \beta}{\partial r} \sum_{\{k: v \leq v_k \leq v + \Delta v\}} [1 - \exp(-\tau_k)] b_{u(k)k}. \quad (15)$$

Examples of $\kappa_{\text{spl}}(v)$ and $\kappa_{\text{scat}}(v)$ are shown in the middle and bottom panels of Figure 3, respectively.

With the exception of a few optical wavelengths where κ_{thm} dominates, the bulk of the line opacity may be described as primarily “fluorescent” and “scattering.” This then provides another path for energy to escape from the supernova. Even deep within the ejecta where the UV optical depth is quite great, there is a significant “leak” of energy *downward in frequency* to energies where the optical depth is much lower. In the iron-rich composition of SNe Ia, fluorescence is a much more efficient mechanism for downgrading photons in energy than the Doppler shift accumulated through repeated scatterings described in the previous section.

2.6. Thermalization from Fluorescence

It is instructive to examine the results of a few simple, schematic models. In these, the supernova is taken to be a constant-density sphere expanding homologously with an outer velocity of 10^4 km s^{-1} . Photons are emitted uniformly throughout the volume of this sphere with a blackbody energy distribution. While emitting the photons at the center of the sphere (and hence at a larger average optical depth) would have provided more extreme demonstrations of the scattering physics, uniform emission is closer to the case in a real supernova. The fate of a large number of emitted photons is followed with a simple Monte Carlo procedure.

The first calculation illustrates the progressive redshift of trapped radiation that results from multiple scatterings. The opacity is due only to scattering that is coherent in the comoving frame. The total optical depth of the “supernova” is 10. This corresponds to the electron opacity of a Chandrasekhar mass with an outer velocity of 10^4 km s^{-1} at 15 days past explosion, ionized on average to Co v. In this calculation a typical photon loses 8% of its energy before escaping. (See Fig. 5.)

In the next calculations, we have used a picket-fence opacity with a line density $D(\lambda)$, which is a rough approximation to that shown in Figure 2:

$$D(\lambda) = \begin{cases} 400, & \lambda < 1500, \\ 10^3(\lambda/1500)^{-4.5}, & \lambda > 1500. \end{cases} \quad (16)$$

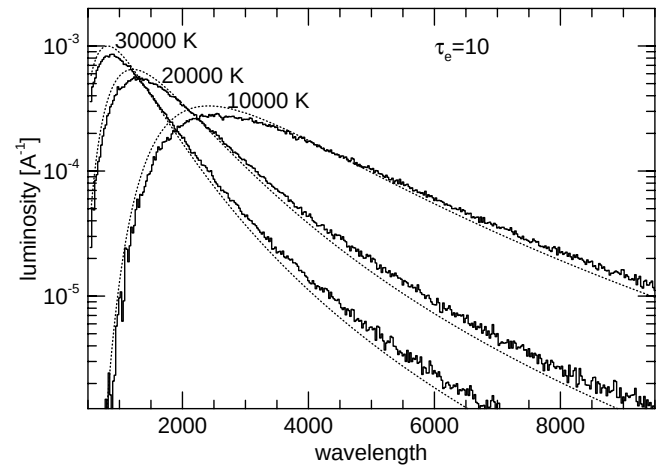


FIG. 5.—Photons with a Planck distribution in energy and at three different temperatures were emitted uniformly throughout the volume of a constant-density sphere and followed by a Monte Carlo simulation until emergence. The model has an outer velocity of 10^4 km s^{-1} and total, pure-scattering optical depth of $\tau_e = 10$.

All of the lines have a Sobolev optical depth of 10 [consequently, $\tau(\lambda) = D(\lambda)$], although as expected from the discussion above we have verified that the result is unchanged with any value of the line optical depth greater than unity. Upon absorbing a photon, a line has a given probability σ of fluorescing into a pair of photons with longer wavelengths. Energy is conserved by requiring that the combined photon energies equal that of the absorbed photon. This is a far more random splitting than that imposed by a real set of atomic data where each line has a fixed set of branching ratios into a finite (though typically large) number of wavelengths. It nonetheless captures the spirit of fluorescence and is computationally expedient. In these calculations we have ignored electron collisional destruction.

In Figure 6 we have taken the fluorescence probability to be $\sigma = 0.25$ and computed three spectra corresponding to emission temperatures of 1, 2, and $3 \times 10^4 \text{ K}$, but in each case with the same total (energy) emission rate. The result of this calculation is remarkably different in character from

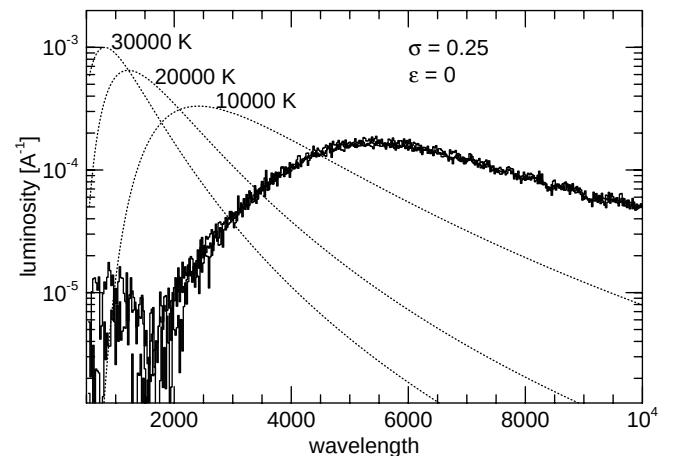


FIG. 6.—Initially Planckian radiation field at three different temperatures (dotted lines) was transported through the line opacity described in the text. The total injected energy is the same in each case, and the emergent spectra (histograms) are virtually identical.

the coherent scattering case. One can see from the figure that as long as the peak wavelength of the blackbody emission spectrum, λ_{Wien} , lies at a wavelength characterized by $\tau(\lambda_{\text{Wien}}) \gg 1$, the *shape* of the emitted flux is identical in all cases; any memory of the shape of the thermal emission spectrum has been lost.

Figure 7 shows the effect on the emergent spectrum of varying the fluorescence probability. We have chosen three values—0.25, 0.1, and 0.025—and a single emission spectrum at 2×10^4 K. In all three cases the emergent spectrum is completely insensitive to the temperature of the emission spectrum (as established by calculations similar to those in Fig. 6). However, it is quite sensitive to the splitting probability σ . A remarkable feature of these calculations is that the emergent spectrum comes very close in shape to a Planck spectrum. The solid lines in the figure, which follow the emergent spectra, are blackbodies at temperatures of 4500, 5500, and 7500 K. In detail, the emergent flux has a Rayleigh-Jeans tail and a Wien cutoff and is just slightly more peaked than an actual blackbody; they are characterized by a small, but nonzero, chemical potential. The peak wavelength, λ_{peak} , is determined by the condition $\sigma\tau^2(\lambda_{\text{peak}}) \sim 1$. Any departure in the spectral shape from an exact Planck function would most likely be impossible to discern in spectra of real supernovae, as the pseudocontinuum produced by multiple fluorescences has superposed upon it a number of strong lines as well as structure from deviations in $D(\lambda)$ from the smooth power law employed for these examples.

The fact that one can use the basic physical picture these examples outline to produce a spectrum that is nearly a Planck function in shape but that has nothing to do with the “thermal emission” leads one to suspect that observed color temperatures in SNe Ia may have less to do with close thermal coupling between the gas and the radiation field and more to do with the distribution of lines and branching ratios in the complex atomic physics of the iron group.

The physical process underlying the production of these pseudoblackbodies is simple: the energy in short-wavelength photons is continuously redistributed to longer wavelengths until an equilibrium is reached between the injection of energy and its loss from the system. The simi-

larity to equilibrium thermodynamics suggests an interesting possibility: if the redistribution in energy from multiple fluorescences is sufficiently large and random, perhaps a Planck distribution is established, and a calculation that assumes this thermodynamic limit would not be too much in error. The occupation numbers of the atomic states in the scattering system might not be too far from their LTE values. In this case, using LTE level populations might not be a bad approximation. The approach to this thermodynamic limit is fundamentally different, however. In a gas, the distribution of particles in phase space approaches a Maxwellian distribution because of the essentially infinite number of ways collisions can redistribute momentum. In the usual LTE radiation transport picture, either a large bound-free continuum optical depth or the dominance of collisional rates over radiative ones ensures that the thermodynamic equilibrium established by gas particles is strongly coupled to the radiation field, giving it an equilibrium distribution as well. This is in spite of the relatively restricted possibilities for energy redistribution in the photon gas through radiative processes. In the present case, the extreme complexity of the radiative processes (in a gas composed entirely of complex ions) allows the photon gas to reach something of a thermodynamic equilibrium on its own. This equilibrium will then drive the level populations to LTE values through radiative processes. We may expect, then, that the electron gas will be driven toward a similar equilibrium state through its (albeit weak) coupling to the photon gas. In a less schematic calculation, of course, the emergent spectrum would reflect the gaps and bumps of Figure 2, and the result would be a line spectrum similar in character to that emitted by a supernova. The important point is that the shape of the emergent spectrum reflects the variation in D and not the underlying radiation temperature.

L. B. Lucy (1998, private communication) has recently demonstrated this effect in a more dramatic way, using an improved version of the NLTE (non-LTE) Monte Carlo transport code of Mazzali & Lucy (1993). This code assumes a lower boundary condition with a fixed luminosity. For SNe Ia maximum-light conditions, he computed spectra with both a blackbody inner boundary spectrum and an inner boundary spectrum that emitted a constant flux only between 1500 and 2000 Å but with the same total luminosity. The resulting spectra are virtually identical throughout the optical region and differ only at wavelengths long enough that the ejecta are nearly transparent down to the inner boundary, where the blackbody input shines through in one case and there is no emission in the other.

There is another indication that such a scattering-mediated “thermalization” may be taking place in SNe Ia. Baron, Hauschildt, & Mezzacappa (1996) note that the agreement of their calculated maximum-light spectra with observations is significantly improved by the inclusion of additional thermalization. They have attributed this to a lack of accurate collision cross sections. Under this assumption, they empirically fit an enhancement to the collision rates they employ, using the agreement of their simulations with observed spectra as a criterion. This may well be an appropriate exercise. On the other hand, the approximations they employ to determine collision rates (the same as we have employed above) work well, on average, in other astrophysical and terrestrial applications; in particular,

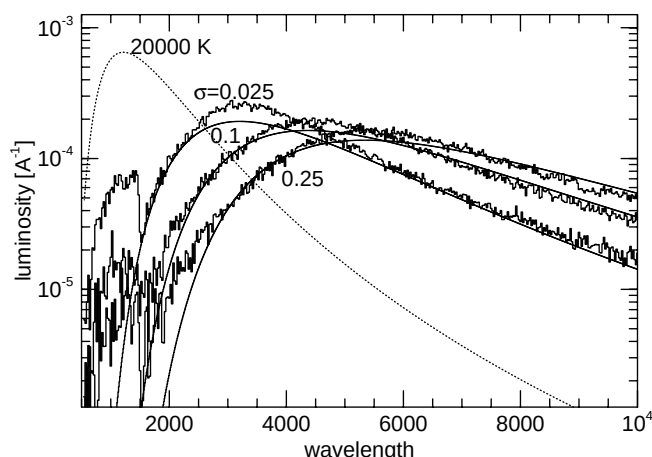


FIG. 7.—Photons from the blackbody shown by the dotted line were scattered through the line opacity described in the text. While the shape of the emergent spectrum (*histograms*) is sensitive to the fluorescence probability σ , it is independent of the input blackbody's temperature. The solid lines following the histograms are blackbodies at 4500, 5500, and 7500 K.

there is no reason to suspect that these approximations are *systematically* incorrect (other than the disagreement of computed supernova spectra with observations).

The additional thermalization resulting from fluorescence in a sufficiently large number of lines is another, less artificial, way to achieve this same effect. Höflich (1995) has expressed the source function for lines in the equivalent two-level atom (ETLA) formulation and has made an analytic fit to the coefficients from detailed rates from NLTE calculations. This procedure thus includes the effect of fluorescence in addition to the pure thermalization employed by Baron et al. (1996). While the fidelity of these fits to the actual variation of the parameters in detail may be questioned (see below), the average magnitude of the effect is clearly what is necessary, and the spectra that result show a similar agreement with reality as those of Baron et al. (1996) without any artificial adjustment of parameters.

3. LIGHT-CURVE CALCULATIONS

We have undertaken a series of light-curve calculations with a more modern version of the code EDDINGTON, described by Eastman & Pinto (1993). Here we present calculations from just one explosion model and discuss only the evolution of its radiation field and opacity. Paper III (P. A. Pinto et al. 2000, in preparation) will contain a more systematic examination of the light curves predicted for a variety of SNe Ia models and compare them with observations.

The explosion we have employed is model DD4 of Woosley & Weaver (1994). This is a typical delayed-

detonation model, which produces $0.63 M_{\odot}$ of ^{56}Ni in the 1.1×10^{51} ergs explosion of a Chandrasekhar-mass C/O white dwarf. Its abundance profiles are shown in Figure 8. For the present discussion, the details of this model are unimportant. We note only that for the phases of the light curve discussed here, energy from radioactive decay is deposited virtually in situ, so there is little deposition outside the ^{56}Ni region interior to $1 M_{\odot}$. We note also that the central region burned at very high density and was thus subject to extensive electron capture, leading to a composition dominated by stable iron group elements. The fact that the very center of the supernova is not heated by radioactive decay leads to an *inward* flux from surrounding heated regions, which we shall comment on below in discussion of the infrared secondary maximum.

We describe here briefly the physical ingredients of this calculation, leaving to a future paper a detailed discussion of our method and the many sources of atomic data. A detailed treatment is given to He, C, O, Si, S, Fe, Co, and Ni. These elements have been included in their first 10 ionization stages. In all, nearly 8000 energy levels and continua and more than 175,000 line transitions are included. An LTE equation of state has been assumed in order to reduce the required memory and time to a manageable size (650 MB of core and 3 days on a single 175 MHz MIPS R10000 processor). As discussed above, within the flux mean photosphere, where the luminosity is determined, Saha-Boltzmann level populations are probably not a bad approximation. In layers above this photosphere (the entire object after about 35 days!), the assumption of LTE will tend to underestimate both the temperature and the ioniza-

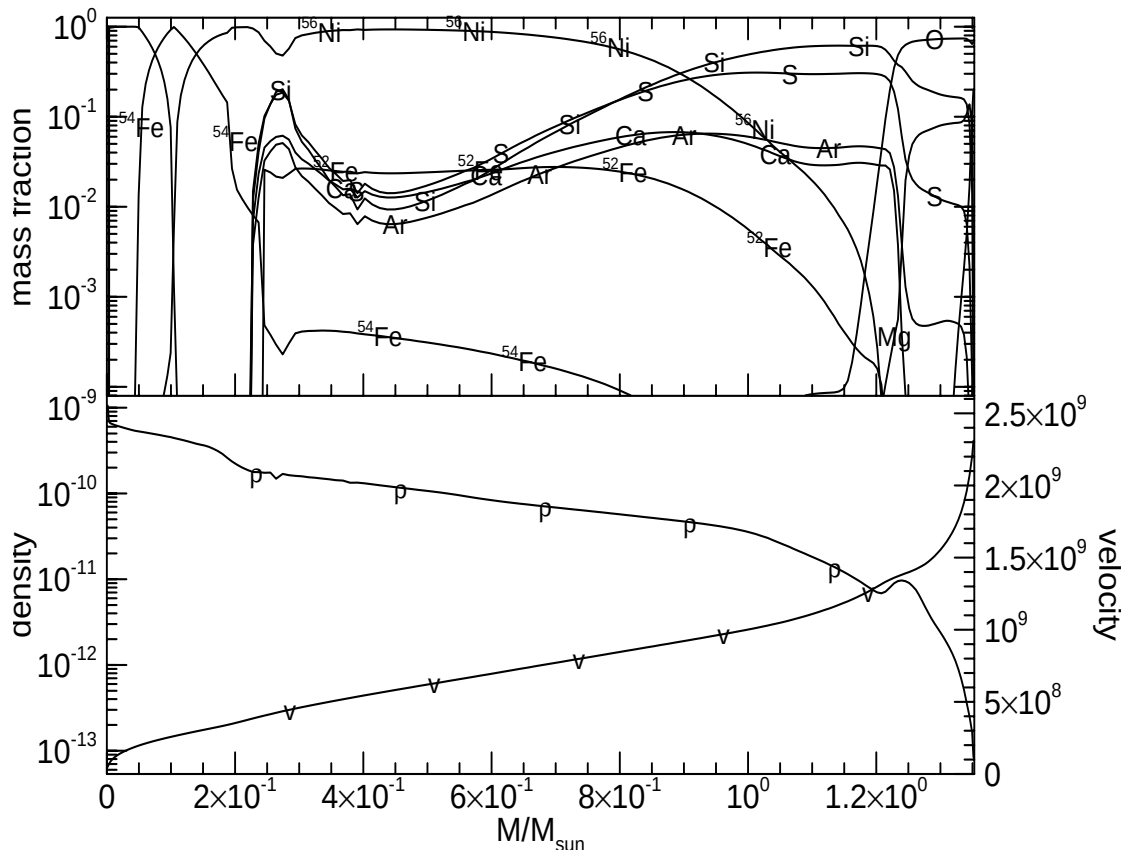


FIG. 8.—Delayed-detonation explosion model DD4 of Woosley & Weaver (1994)

tion of the supernova. Thus, for example, strong line features formed significantly above the photosphere will not be accurately determined. In addition to the lines from the atomic models, the opacity from an additional 425,000 lines is included as blanketing (we discuss this addition below).

The radiation transport equations are solved to $O(v/c)$ in the comoving frame in a fully time-dependent manner using Feautrier's method and variable Eddington factors (cf. Eastman & Pinto 1993). The frequency grid employs an average bin width of $c \Delta v/v = 80 \text{ km s}^{-1}$ from $1000 \text{ \AA}$ to $5 \text{ \mu m}$, with additional points more coarsely distributed from $100 \text{ \AA}$ to $50 \text{ \mu m}$. The lines from the atomic models are explicitly treated in the Sobolev limit and have source functions given by the ETLA formalism. The ETLA parameters for each line are calculated from the radiative rates and the LTE level populations using the radiation field and thermodynamic conditions at each time step. The source functions thus correctly and self-consistently reflect the effects of fluorescence, coherent scattering, and thermalization described above, to the extent that the populations are described by LTE, individually for each line. Because the level populations and rates are not available for the additional lines, in each frequency bin these lines are represented by the expansion opacity of expression (9) and assumed to be pure absorbers.

In EDDINGTON, the gas temperature is determined by solving the time-dependent first law of thermodynamics. Heating is from radiative absorption, fast particles from Compton scattering of γ -rays, and decay positrons. Cooling is due to expansion and radiative emission. Near peak, the energy density is overwhelmingly dominated by the radiation; the same gas temperature would be obtained simply by balancing heating and cooling, ignoring the gas pressure

contribution to PdV losses. The PdV work done by the radiation, implicitly taken into account by the transport solution, must still be included, however. The local energy deposition from radioactive decay is determined with reasonable accuracy, especially before 50 days, by doing a separate deterministic transport calculation for each γ -ray line emitted in the decay cascade from ^{56}Ni through ^{56}Fe , as described in Woosley et al. (1994). Energy from ^{56}Co positrons (irrelevant to the light curve near peak) was deposited in situ.

The velocity field employed was determined by evolving the model to homologous expansion in the explosion calculation and was then held fixed for the light-curve calculation; while decay energy is expended in doing work on the gas, the velocity of the gas is not increased accordingly. The error this introduces is not large (see Paper III), occurs primarily in the first few days, and is not relevant to our present purpose of investigating the nature of the opacity in SNe Ia. Blinnikov et al. (1998) obtain excellent agreement in results on test problems run with EDDINGTON and with the multigroup implicit radiation-hydrodynamics code STELLA (Blinnikov & Bartunov 1993; Bartunov et al. 1994) when the same approximations are used in both codes for the line opacity.

3.1. Results

The resulting *UBVRI* and bolometric light curves are shown along with data for SN 1991T in Figure 9 and data for SN 1992A (Hamuy et al. 1996) in Figure 10. The light curves are in general too narrow to match SN 1991T and too broad to match SN1992A and so fit comfortably within the range of observed SNe Ia light-curve widths. No attempt was made to fit a particular supernova with a par-

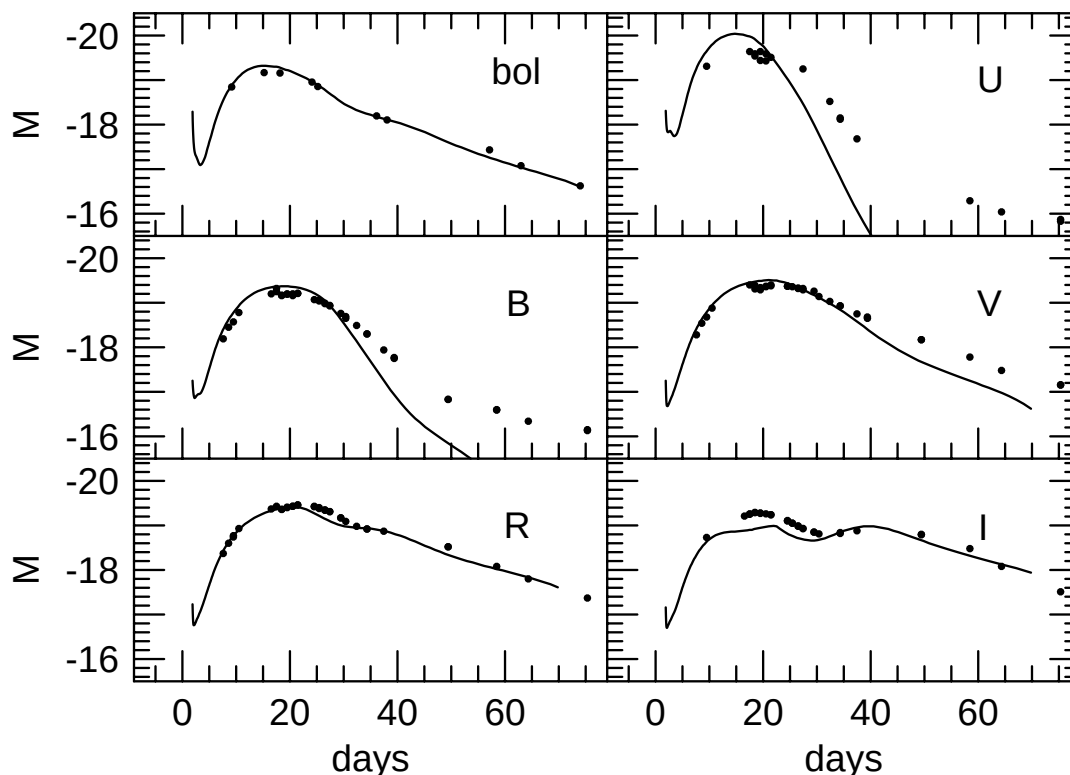


FIG. 9.—Light curves calculated from the delayed detonation model DD4 of Woosley & Weaver (1994) as described in the text compared with data for SN 1991T (Hamuy et al. 1996) and bolometric data from Suntzeff (1995).

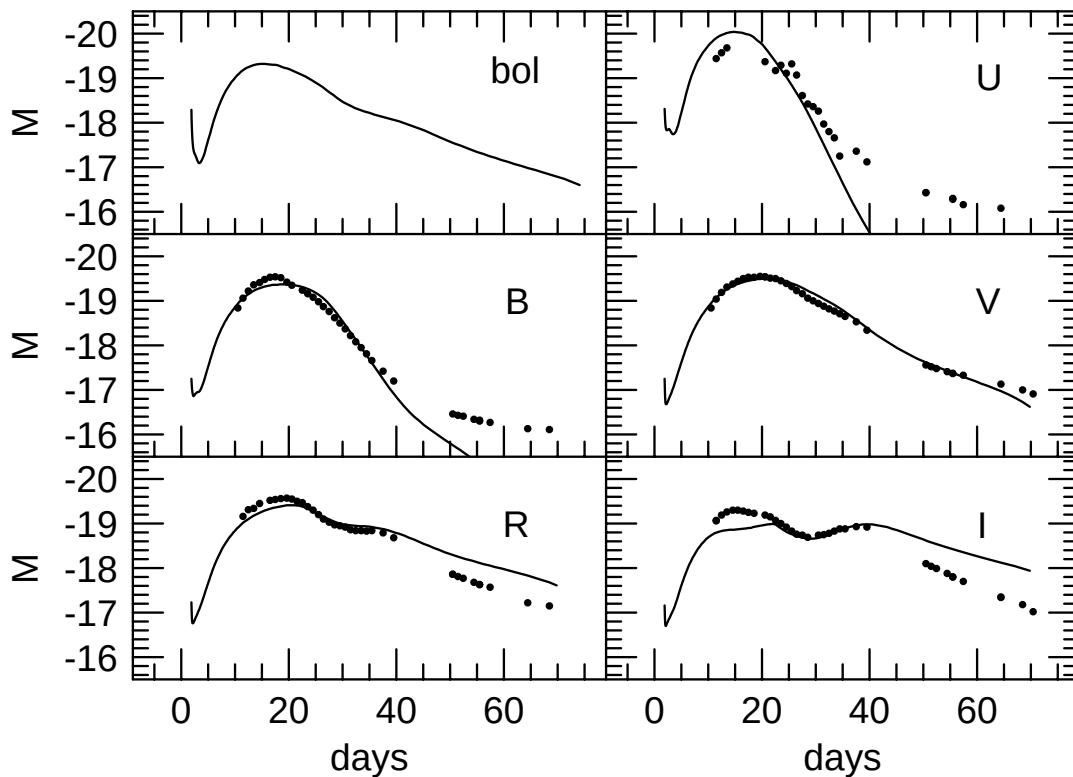


FIG. 10.—Same light curves as in the previous figure, compared with data for SN 1992A from Hamuy et al. (1996)

ticular explosion model; the lack of a better fit is of little concern in the present context.

The light curves exhibit many of the general features of a typical SN Ia. The *B* light curve rises to maximum at 20 days, preceded by *U* (but possibly to too great an extent in this model) and followed by maxima in *V*, *R*, and *I*. The *R* and *I* light curves exhibit secondary maxima of typical size and timing, while the *I* light curve rises too slowly before maximum and is strangely flat near peak. The colors from the model are also generally too red after the peak phase is over, even for 1992A. After 40 or so days, SNe Ia are observed to make a transition to a quasi-nebular, emission-line spectrum. An LTE calculation would, thus, not be expected to be a reasonable representation of the dominant physics, and we suspect that whatever agreement there is between models and observations at such a late stages is to some degree fortuitous.

At each time step a formal solution of the time-dependent transport equation is obtained in the gas frame, and the emergent intensity is Lorentz transformed into the observer frame and used to compute the observer frame spectrum. One such spectrum from the light-curve calculation, at a time 5 days past *B* maximum, is shown in Figure 11 and compared with that of SN 1992A at the same time from *Hubble Space Telescope* (*HST*) and Cerro Tololo Inter-American Observatory (CTIO) observations (Kirshner et al. 1993). Overall, the agreement with observed data is quite good shortward of 6500 Å, with virtually all features in the observed spectrum reproduced in the model. To the red, the flux in the model falls off somewhat more rapidly than in 1992A. The excellent agreement in the UV out to 2000 Å gives us some confidence that the bolometric luminosity is being determined with reasonable accuracy.

It must be remembered that this is the direct result of a time-dependent calculation and not the postprocessing of a light-curve model with a more detailed treatment of the radiation transport and a non-LTE equation of state. For example, the emission part of the Si II $\lambda\lambda 5958, 5980$ doublet is too strong while not extending to a sufficiently high velocity; this is probably due at least in part to the LTE equation of state above the photosphere. Because the LTE approximation leads to an underestimate of temperature and ionization in optically thin layers, this calculation will tend to

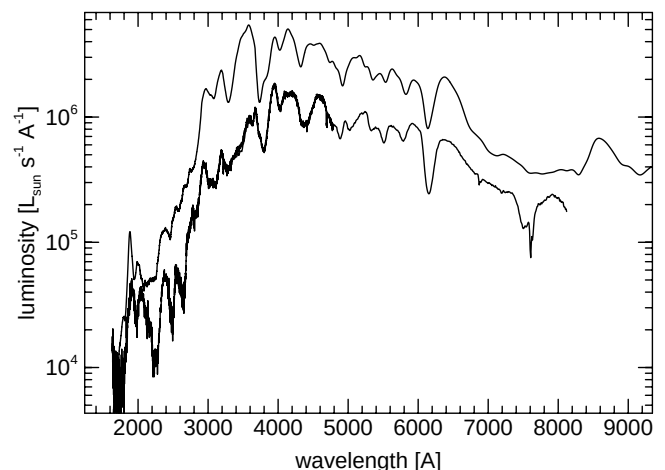


FIG. 11.—Emergent spectrum from the light-curve calculation (top curve) at 5 days past *B* maximum, compared with spectra of SN 1992A taken by the *Hubble Space Telescope* and at CTIO (Kirshner et al. 1993) at the same time. The top curve has been multiplied by a factor 3 from the normalization used in the light-curve plots above, for clarity.

overestimate the abundance of Si II near the photosphere and underestimate it at higher velocities.

3.2. Uncertainties

Several uncertainties concerning the line-blanketing opacity arise from approximations in its numerical representation. The first is the use of expansion opacities. In both the opacity of equation (9) and the variants of Karp et al. (1977) opacity, the mean free path of a photon is just the distance between lines in the line list. While we have shown that this is approximately the correct physical picture, it is not exact, and in principle some error may result in calculated diffusion times. To date, a comparison of exact results with those from expansion opacities has not been made.

For light curves in which all lines are treated by the expansion opacity of equation (9), we have determined that the calculations are not sensitive to the number of frequency bins used to represent the opacity, as long as the resolution of the bins is below $\sim 500 \text{ km s}^{-1}$. In addition, we have calculated light curves using 50 km s^{-1} wavelength bins and the opacity given by equation (9) and compared them with calculations in which the 10^5 most important lines from the line list are treated explicitly in the Sobolev limit, but with the same pure-absorption source function assumed for the expansion opacity lines. The results are virtually identical. Thus, though more work remains to be done to firmly establish the adequacy of the expansion opacity approximation, these experiments give us some confidence that at least in the formulation presented here it yields reasonably correct results.

For the majority of lines ($\sim 90\%$), the computed ETLA parameters give line source functions not very different from those computed under the assumption of pure absorp-

tion (e.g., scattering fractions $\lesssim 0.2$). This is not surprising given the dominance of the photon destruction processes (mostly splitting) discussed above. In Figure 12 we show two light curves from calculations that are identical but for the source functions employed. The solid curves are from an ETLA calculation, and the dashed curves, from one employing thermal source functions. In both cases, only lines from the detailed atomic models were employed. While the shapes of the light curves are different in detail, they are in general quite similar, as one would expect from the similarity of the source functions. The ETLA parameters tend to be more scattering-dominated in the UV, but in detail they vary strongly and nearly at random with both wavelength and lower level energy, defeating any attempt to fit these parameters and avoid a detailed rate computation. We are at present computing non-LTE models to assess the appropriateness of the LTE approximation employed here, and will report on these in a subsequent paper.

Another source of uncertainty concerns the atomic data themselves. The line lists most commonly employed in supernova modeling are those of Kurucz (1991, 1992). Because the number of lines that contribute to the total opacity is so large, numbering in the hundreds of thousands or more, there is no way at present to know either how accurate or how complete the list is, especially with respect to the weaker lines. The calculation is far more sensitive to the line list employed than to the difference between thermal and ETLA source functions discussed above. Figure 13 shows the effect of adding an additional $\sim 450,000$ lines not included in the detailed atomic models. The effect is strongest in the *U* and *B* bands, where flux is forced to emerge at longer wavelengths in the blanketed models. In the absence of even more elaborate line lists, it is

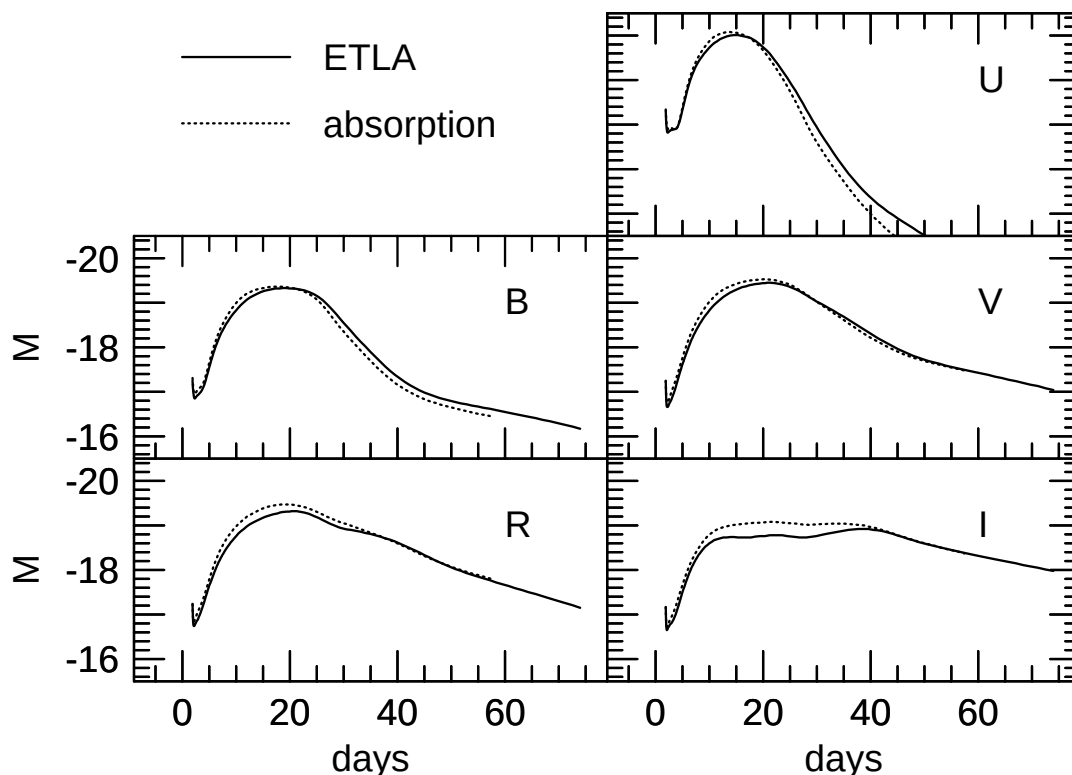


FIG. 12.—Light curves calculated with ETLA and pure absorption (thermal) source functions. Both calculations employed only lines from the detailed atomic models, and all other aspects of the calculation are identical.

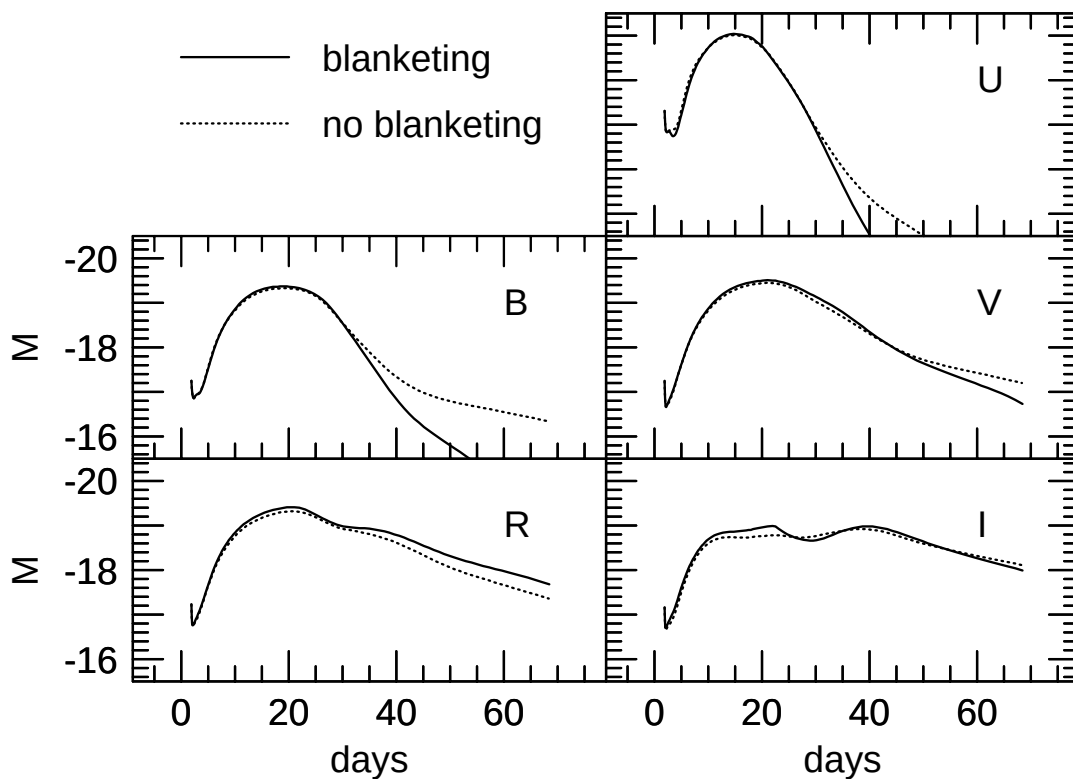


FIG. 13.—Light curves calculated employing only the $\sim 175,000$ lines from the detailed atomic models and with the addition of $\sim 450,000$ lines treated as pure absorbers.

difficult to assess the effect of the lists' completeness on our results.

Yet another approximation is the use of the Sobolev theory itself. It has been noted by several authors in the context of supernova calculations (e.g., Höflich 1995; Baron et al. 1996) that the Sobolev theory, developed for isolated lines, is in error when lines populate wavelength space so densely that there is significant overlap of the intrinsic line profiles in the comoving frame. Since a model that accurately resolved all of the line profiles in the list we employ would require more than a million frequency points, limitations of computer size and memory prevent us from attempting a direct calculation for comparison. Eastman & Pinto (1993) compared supernova spectra calculated with and without the Sobolev approximation (with a much smaller line list, yet still with many overlapping lines) and found no significant differences that could be attributed to this source of error. Thus, while the objection is correct in principle, it does not appear to be numerically relevant in practice for models of supernovae.

3.3. The Radiation Field

In Figures 14 and 15 we show the radiation field in the supernova at 20 and 40 days past explosion, respectively. In the bottom panels, the log of the mean intensity in the comoving frame is shown as a function both of velocity (*vertical axis*) and of comoving wavelength (*horizontal axis*). In the middle panels we show the comoving flux, and in the top panels the emergent flux is shown (again in the comoving frame). It is obvious from these figures that at maximum light the bulk of the radiation energy in the supernova is stored at UV wavelengths where the opacity is very large and there is essentially no flux. Significant flux

arises only at optical and IR wavelengths and occurs from significant depths even at early times. By 40 days, the peak of the *I*-band secondary maximum, much of this reservoir of trapped energy has been depleted, and the photosphere has receded nearly to the center.

In these plots, features, such as the blue edge of strong absorption lines, that are constant in comoving wavelength appear as vertical lines. Freely streaming radiation, however, moves longward in comoving wavelength as it traverses the velocity gradient. Many features can be seen that arise at depths where J is large and follow the slanted photon characteristics outward. It is, thus, clear from these figures that the rate at which energy can be emitted at these longer wavelengths, at all depths, governs the luminosity of the supernova more than the rate at which the typical photon in the UV can diffuse outward.

The dark band in the flux at low velocities is due to the lack of radioactivity in this region of the model. At 40 days, the lack of heating leads to a lower flux. At earlier times, with no energy deposition heating them, these layers are somewhat cooler than the surrounding gas, and energy flows inward.

The energy source that powers the secondary maximum in the infrared light curve is also apparent from these figures. Near 40 days, the ionization in regions of trapped radiation falls to include significant amounts of once-ionized species (Ca II, Fe II, and Co II) that can emit strongly in the near infrared. Note especially the rise of the Ca II $\lambda\lambda 8498, 8542, 8662$ infrared triplet in the emergent spectrum. This emission occurs at more optically thin wavelengths and enhances the “leak” of trapped radiation, thus reducing the flux mean opacity and draining the remaining trapped radiation at an enhanced rate. This leads in turn to

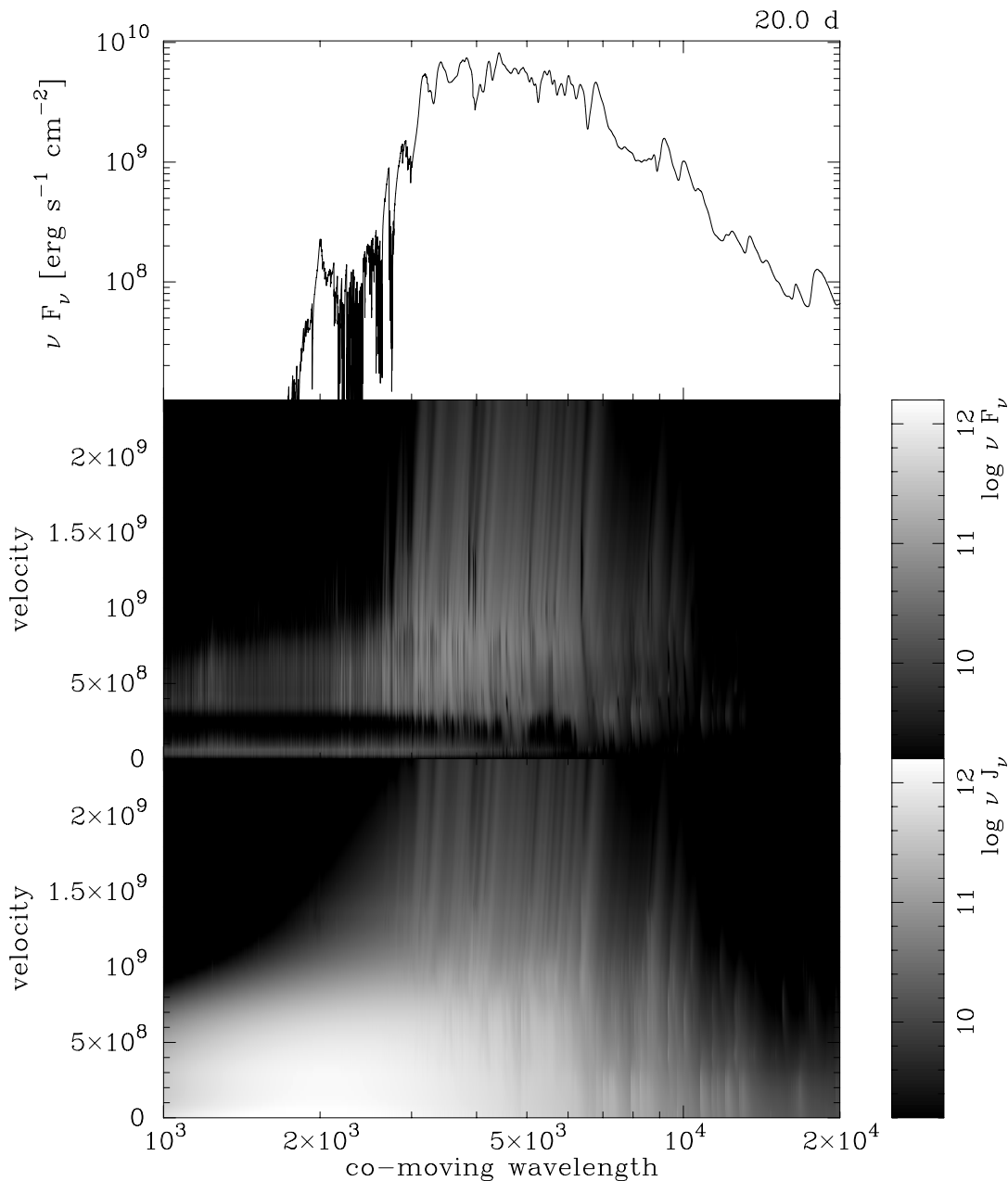


FIG. 14.—Radiation field at *B* maximum, 20 days past explosion. The emergent spectrum is shown in the top panel, and in the next two panels, the flux and the mean intensity are shown as functions of velocity, all as functions of wavelength in the comoving frame. Note the storage of energy in the UV and its transport at longer wavelengths.

a temporary rise in luminosity until the excess radiation is exhausted. When this occurs, the supernova arrives at the radioactive “tail” of the light curve and the bolometric luminosity becomes equal to the instantaneous deposition thereafter. It is important to note that this decrease in flux mean opacity is not due to a drop, at any wavelength, in the monochromatic opacity. Rather, it is due to an increase in emission at wavelengths where the monochromatic opacity is already small. A more detailed discussion of the infrared secondary maximum is the subject of Paper III.

4. FREQUENCY-INTEGRATED MEAN OPACITIES

While the bolometric flux of the supernova is, thus, determined more by the wavelengths at which radiation is

emitted than where it is absorbed, it is useful to be able to discuss the progress of the bolometric light curve in terms of frequency-integrated mean opacities, so that we can make simple analyses such as those of Paper I. In addition, nearly all radiation hydrodynamic calculations rely upon the existence of a well-defined mean opacity. A natural choice for this mean opacity is the Rosseland mean, which has a long history of use in astrophysical contexts. The formally correct choice is, of course, the flux mean, but the whole point of using a mean opacity is to avoid the multigroup calculation of the flux, which is essential to the computation of the flux mean in the first place! In this section we use the results of multigroup calculations to compare the Rosseland and flux mean opacities and to examine their evolution and effect upon the light curve.

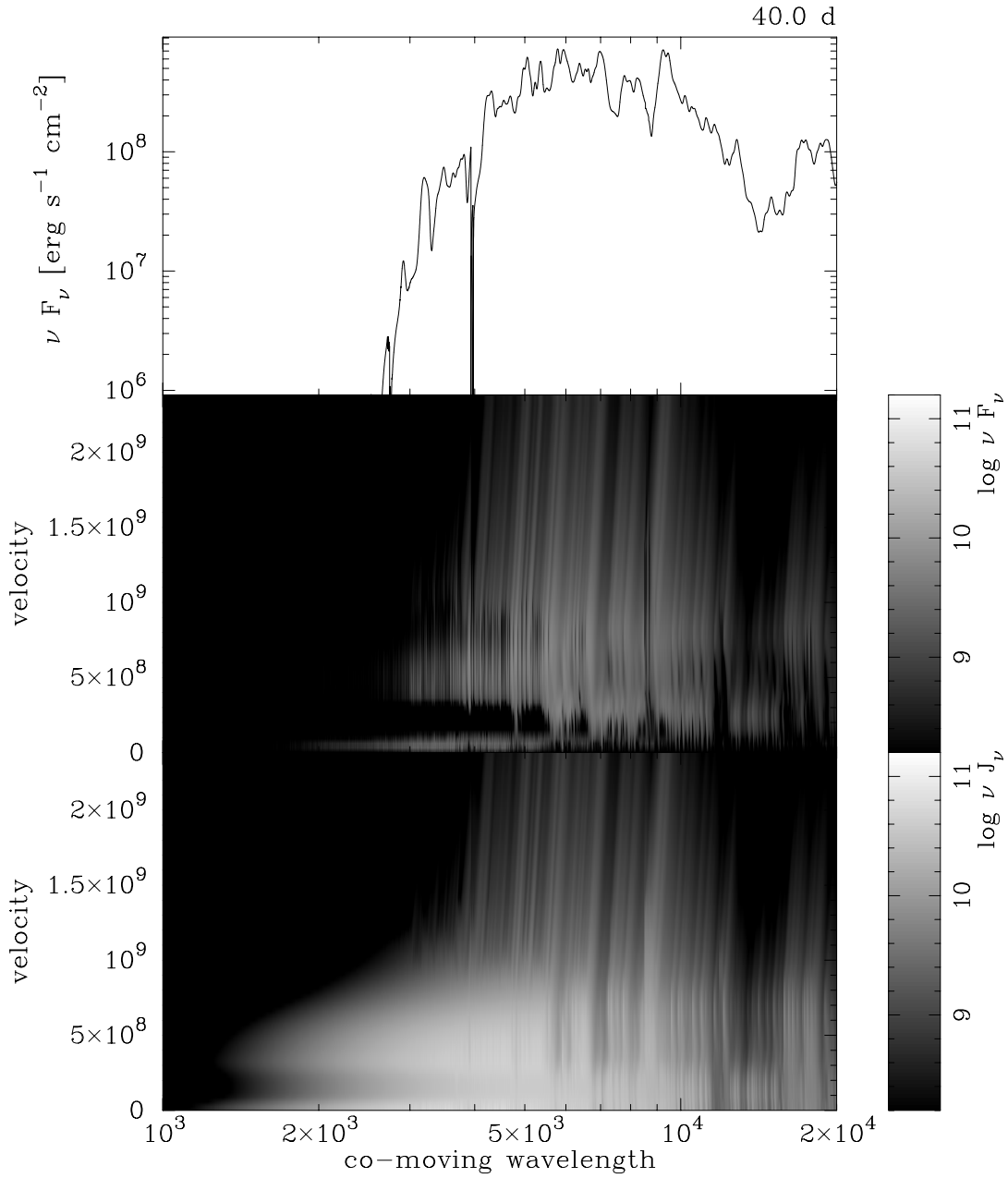


FIG. 15.—Radiation field, as in Fig. 14, at the peak of the *I*-band secondary maximum, 40 days past explosion. At this time, most of the luminosity originates in regions of small diffusion time.

The second moment of the monochromatic radiation transport equation may be written as

$$\begin{aligned} \frac{1}{c^2} \frac{DF_v}{Dt} + \frac{\partial P_v}{\partial r} + \frac{3P_v - E_v}{r} + \frac{2}{c^2} \frac{v}{r} (F_v + 4\pi N_v) \\ + \frac{1}{c^2} \frac{\partial v}{\partial r} (2F_v - 4\pi N_v) - \frac{\partial}{\partial \ln v} \\ \times \left[\frac{v}{r} F_v + \left(\frac{\partial v}{\partial r} - \frac{v}{r} \right) 4\pi N_v \right] = -\frac{1}{c} \rho \kappa_v F_v, \quad (17) \end{aligned}$$

where N_v is the third angular moment of the specific intensity. If this is integrated over frequency, one obtains

$$\begin{aligned} \frac{1}{c^2} \frac{DF}{Dt} + \frac{\partial P}{\partial r} + \frac{3P - E}{r} + \frac{2}{c^2} \left(\frac{\partial v}{\partial r} + \frac{v}{r} \right) F \\ = -\frac{1}{c} \int_0^\infty \chi_v F_v dv. \quad (18) \end{aligned}$$

The right-hand side of equation (18) can be written as $-\rho \kappa_F F/c$, where the flux mean opacity, κ_F , is defined as

$$\kappa_F \equiv F^{-1} \int_0^\infty \kappa_v F_v dv. \quad (19)$$

This is clearly the correct quantity to use for the mean opacity in this context. The problem, as mentioned above, is that the calculation of the flux mean requires prior knowledge of F_v and thus requires solution of the frequency-

dependent problem. On the other hand, having calculated F_ν by some much more complex calculation for a variety of models, we can try to discover regularities in the behavior of κ_F that may be useful in more approximate calculations.

At large enough optical depth ($\tau_{\text{thm}} \gg 1$), the gas and radiation field will be thermalized and $E_\nu \approx 4\pi B_\nu(T)$. The radiation field will be isotropic, so that $P_\nu \approx E_\nu/3$. The expansion is homologous, so $v/r = \partial v/\partial r = 1/t$. Finally, since $\rho\kappa_\nu ct \gg 1$, the time-derivative term in equation (17), $c^{-2}DF_\nu/Dt$, may be set to zero. Substituting these relations into equation (17) gives

$$\frac{\partial F_\nu}{\partial \ln \nu} - (3 + \rho\kappa_\nu ct)F_\nu = \frac{4\pi ct}{3} \frac{\partial T}{\partial r} \frac{\partial B_\nu(T)}{\partial T}, \quad (20)$$

where we have written $[\partial B_\nu(T)/\partial T](\partial T/\partial r)$. This equation is straightforward to solve for the flux:

$$F_\nu = -\frac{4\pi ct}{3} \frac{\partial T}{\partial r} \int_\nu^\infty \frac{\partial B_{\nu'}(T)}{\partial T} \left(\frac{\nu}{\nu'}\right)^3 \times \exp\left(-ct \int_\nu^{\nu'} \rho\kappa_{\nu''} \frac{d\nu''}{\nu''}\right) \frac{d\nu'}{\nu'}. \quad (21)$$

The Rosseland mean opacity can now be determined from the definition

$$F = \frac{-4\pi}{3\rho\kappa_R} \frac{\partial T}{\partial r} \int_0^\infty \frac{\partial B_\nu(T)}{\partial T} d\nu. \quad (22)$$

Integrating equation (21) over ν and combining with equation (22) gives

$$\kappa_R = \frac{\rho\pi t}{aT^3} \int_0^\infty d\nu \int_\nu^\infty \frac{\partial B_{\nu'}(T)}{\partial T} \left(\frac{\nu}{\nu'}\right)^3 \times \exp\left(-ct \int_\nu^{\nu'} \rho\kappa_{\nu''} \frac{d\nu''}{\nu''}\right) \frac{d\nu'}{\nu'}. \quad (23)$$

Note that we have used several approximations to derive this expression: a large optical depth, a thermalized radiation field, and a weak time dependence of the radiation field. Since the luminosity is set in the diffusion region at depth inside the supernova, the assumption of large optical depth is appropriate. As discussed in Paper I, the time derivative may safely be neglected in the radiation momentum equation (though not in the radiation energy equation).

The most important assumption is that the radiation field be thermalized. From the discussion above, we should not expect this to be the case. At depth near maximum light, the radiation energy density is that of a blackbody with a temperature near 20,000 K. While the precise distribution of J_ν is not a Planck function, the typical photon energy is not too far from the blackbody peak. Thus, energy is “stored” in the supernova near 1000–2000 Å where the opacity is very great. At the same time, the supernova is much less optically thick in the optical, and the radiation can more freely escape at these longer wavelengths. Moreover, where the flux is greatest, the color temperature of the radiation has little to do with the gas temperature, which determines the opacity. Thus the assumption of diffusion used in the derivation of the Rosseland mean fails badly on two counts.

We note in passing that in the first moment equation, the correct mean opacity is weighted by the mean intensity. This is far more closely represented by a blackbody at the local gas temperature, and thus the Planck mean opacity will depart much less severely from the J mean.

Figures 16, 17, and 18 display the run of κ_F , κ_R , κ_J , and κ_B at 10, 20, and 40 days past explosion. At the earliest time, the supernova is optically thick at nearly all wavelengths, and the leak of radiation is small compared with the rate at which energy is being added to the radiation field by deposition. Equilibrium radiative diffusion is thus a good approximation to the energy flow, and the diffusion means (κ_R and κ_B) are thus quite close to their exact counterparts.

The downward spike of the flux mean opacity near 0.15 $M_\odot$ is a consequence of the lack of radioactive ^{56}Ni in this region. Since there is less deposition in this material while it is still being cooled just as strongly by expansion, the temperature gradient becomes small and even negative in this region. This leads to a negative flux; where the flux passes through zero, the flux mean becomes ill-defined, and large gyrations in the plotted quantity result.

By the time of maximum light, however, the transport has departed significantly from the equilibrium diffusion paradigm, and we see that the flux mean has become significantly larger than the Rosseland mean. We can understand this from the fact that the monochromatic opacity is such a steeply decreasing function of wavelength. The Rosseland mean weights the monochromatic opacity by the Rayleigh-Jeans tail of a Planck function. The flux, however, falls off more rapidly than ν^{-3} . Thus, the effective wavelength of the Rosseland mean is longer than that of the flux mean, and a lower mean opacity results.

After 35 days, significant IR emission begins to occur, however, and the effective wavelength of the flux mean opacity shifts rapidly to wavelengths longer than that of the Rosseland mean. The flux mean thus drops below the Rosseland mean, leading to the increase in luminosity that finally exhausts the supply of energy accumulated at earlier times.

Even at maximum light, the departure from radiative equilibrium is significant. In the bottom panel of the previous three figures is shown the ratio of the frequency integrated mean intensity to $acT^4/4\pi$ (“ J/B ”). At maximum light, the criterion that the radiation field be thermalized, required in the derivation of the Rosseland mean opacity, is violated. Where $J/B < 1$, the color of the radiation field is hotter than a blackbody at the same energy density. Again, the Rosseland mean, which is weighted by $\partial B_\nu/\partial T$ at the local gas temperature, samples the opacity at longer wavelengths than does the flux mean. Because the opacity falls off so strongly with increasing wavelength, the resulting Rosseland mean is lower than the flux mean.

Also shown is the flux mean thermalization optical depth, τ_{thm} . As discussed above, the mean thermalization depth has increased to the order of the radius of the ejecta by maximum light.

4.1. Infrared Secondary Maxima

The infrared light curves of many SNe Ia exhibit a secondary maximum similar to that in the calculations presented, and it is instructive to inquire as to the cause of this secondary peak. At least in those few SNe Ia for which bolometric light curves have been assembled (N. B. Suntzeff 1998, private communication), the increase in the IR bands is sufficient to cause an inflection in the bolometric light curve for a period of 10–20 days. Unless one postulates an additional source of energy that presents itself several tens of days past explosion, an increase in bolometric luminosity can be produced only by a decrease in the flux mean

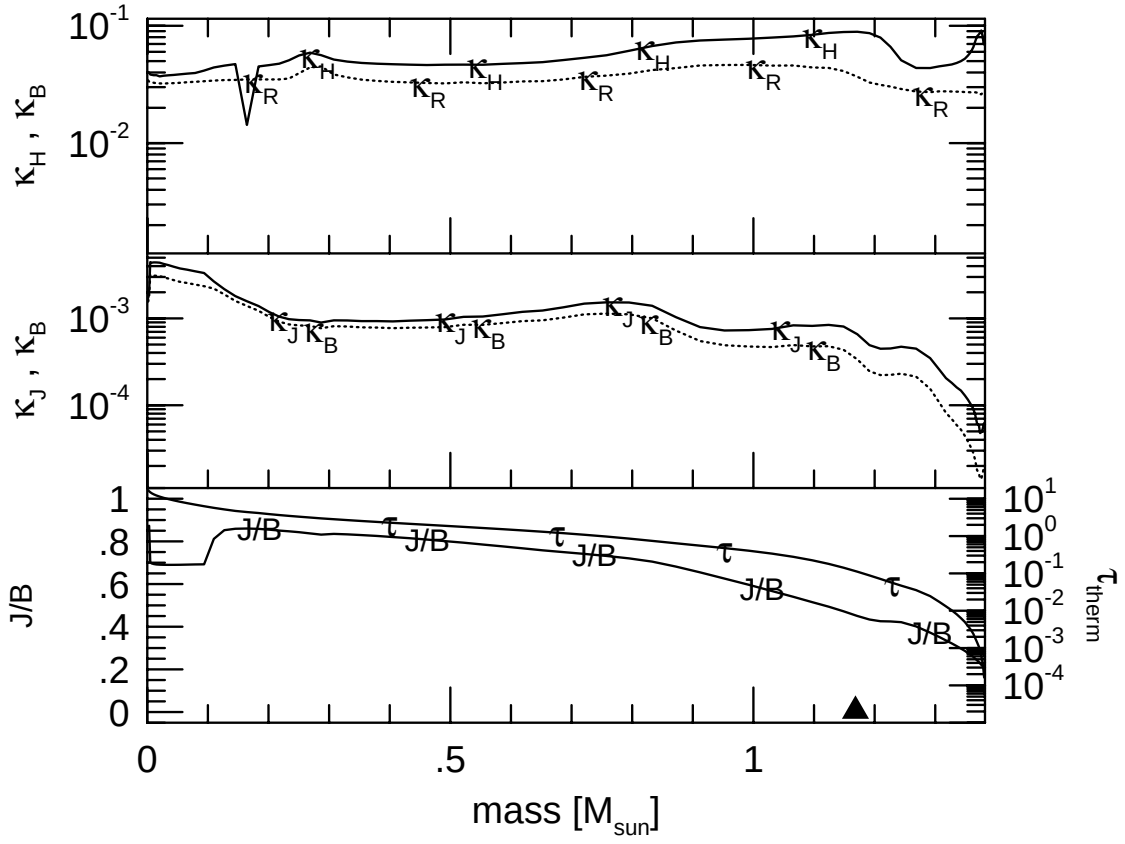


FIG. 16.—Comparison of flux mean (κ_F), Rosseland mean (κ_R), intensity mean (κ_J), and Planck mean (κ_B) mass-opacity coefficients in model DD4 vs. Lagrangian mass coordinate at 10 days. Also shown is the ratio of the frequency-integrated radiation field mean intensity to the Planck energy density at the local gas temperature (J/B). Where $J/B \neq 1$, the gas and radiation field are not in equilibrium. The flux mean photosphere is indicated by a triangle on the mass axis.

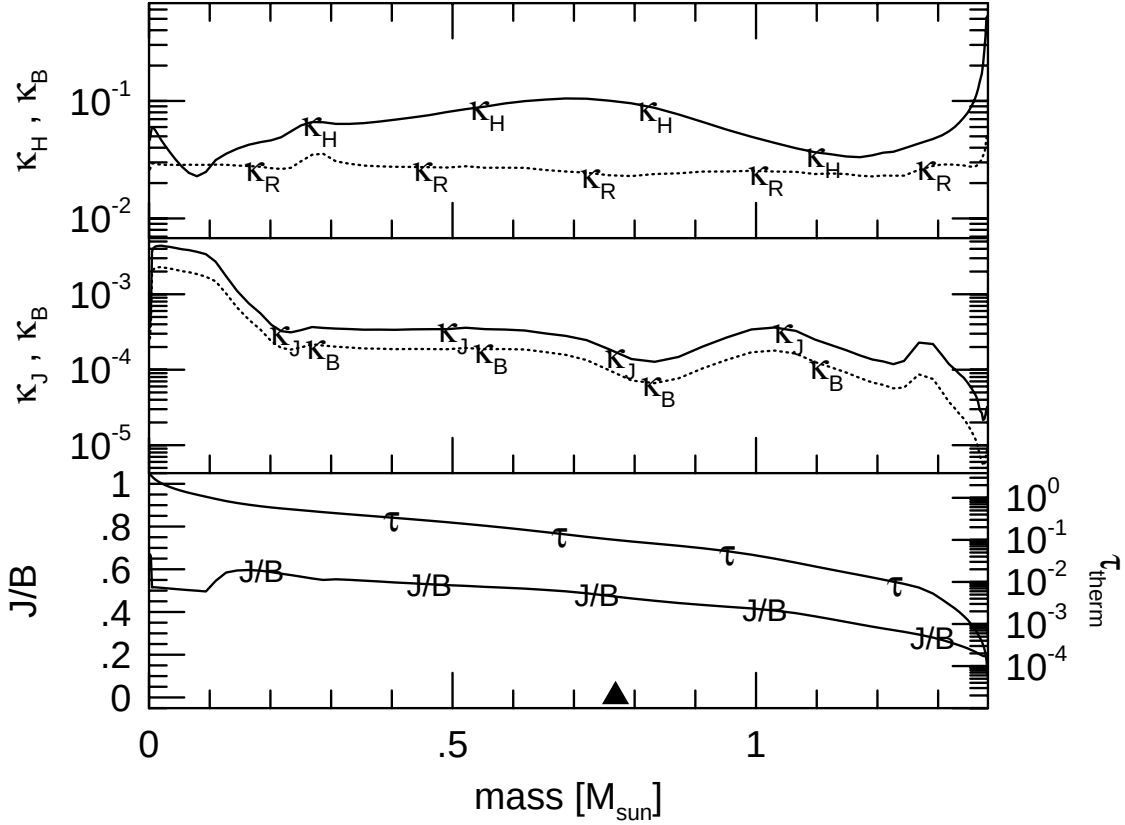


FIG. 17.—Mean opacities as in Fig. 16 at 20 days

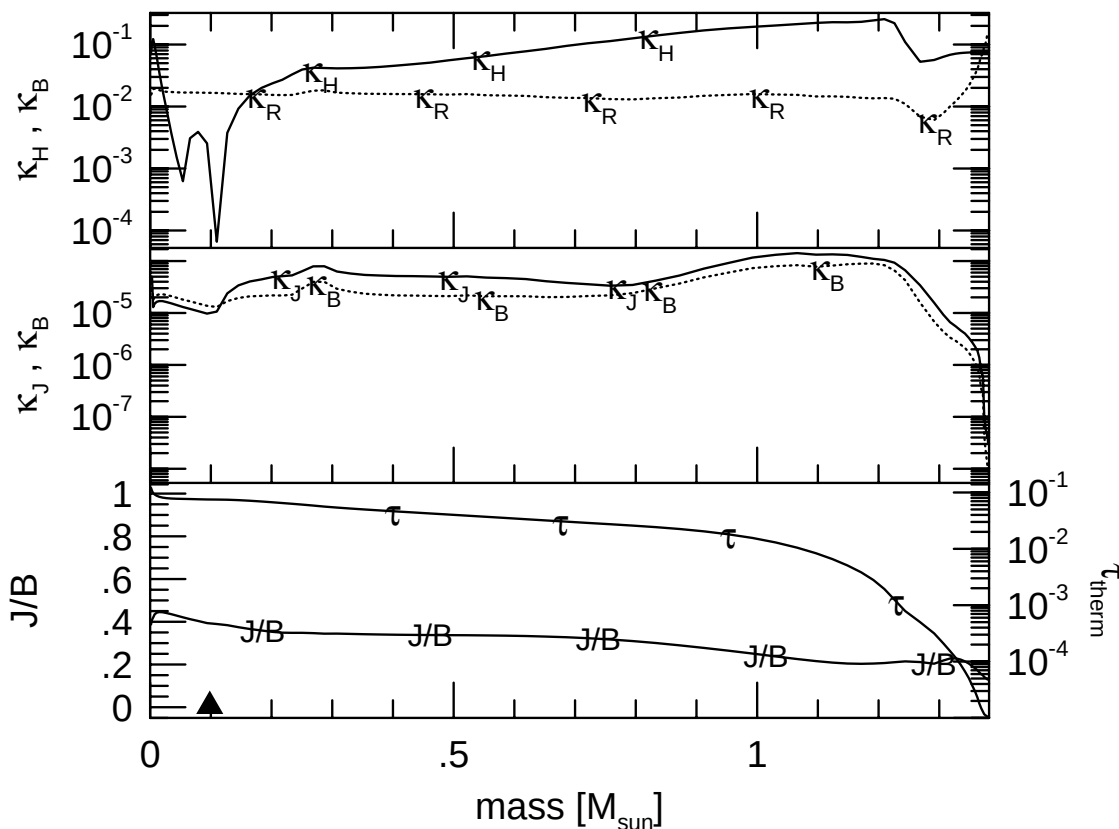


FIG. 18.—Mean opacities as in Fig. 16 at 40 days

opacity. A decrease in opacity reduces the diffusion time, allowing the reservoir of trapped radiation to escape more rapidly, leading in turn to an increase in the luminosity.

The effects of trapping and diffusion can affect the light curve only in regions within the photosphere. An opacity change in outer layers where the radiation is already streaming will have no effect on the bolometric light curve (although it may affect the *shape* of the emitted spectrum). As the photosphere recedes, an increasing fraction of the luminosity comes from energy deposition outside the photosphere, where the diffusion time is small. The light curve approaches the instantaneous balance of deposition and emission that characterizes the radioactive “tail.”

Figure 19 compares the bolometric light curve with the instantaneous energy deposition rate from radioactive decay. Evidence of the secondary maximum in the infrared is clearly seen. It is an *increasing* departure from the deposition curve, and the extra luminosity is due to an increasing rate of release of energy stored within the photosphere. As an approximation to the overall behavior of the opacity inside the photosphere, we show the opacity at the photosphere as a function of time in Figure 20. After an initial dip, throughout the observable phase of rise to maximum, the opacity is seen to be constant to within $\sim 25\%$. This constancy leads to the near coincidence of the deposition and luminosity at maximum, as discussed in Paper I. During this phase the photosphere approximately follows the transition between the second and third ionization states of the iron group elements and remains at a nearly constant temperature of $\sim 13,000$ K. This in turn leads to a nearly constant opacity.

After peak, the temperature of the photosphere and the ionization state of the gas slowly declines. The combination of lower temperatures and lower ionization states, in which there is more emissivity at longer wavelengths, leads to a reduction in opacity. By the time the photosphere has receded to $\sim 0.2 M_\odot$, the temperature has dropped to

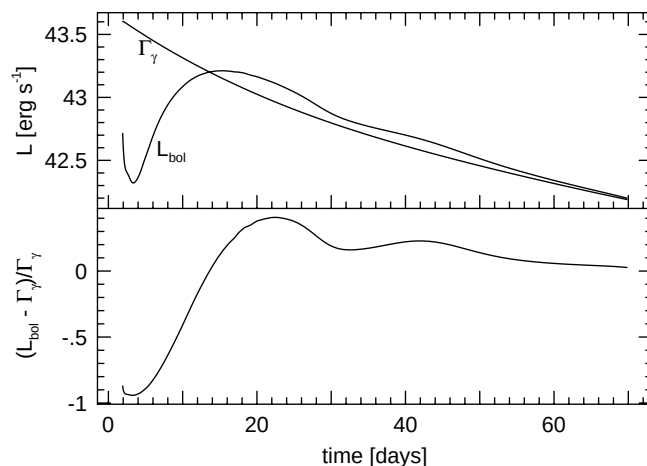


FIG. 19.—Bolometric luminosity (L_{bol}) and the instantaneous energy deposition rate (Γ_γ) as functions of time. The time at which $L_{\text{bol}} = \Gamma_\gamma$ occurs 1 day before the bolometric maximum. In the lower panel is shown the relative fraction of the deposition rate, which emerges as thermalized radiation. Evidence of the IR secondary maximum is clearly seen and should be compared with the evolution of the photospheric opacity in Fig. 20.

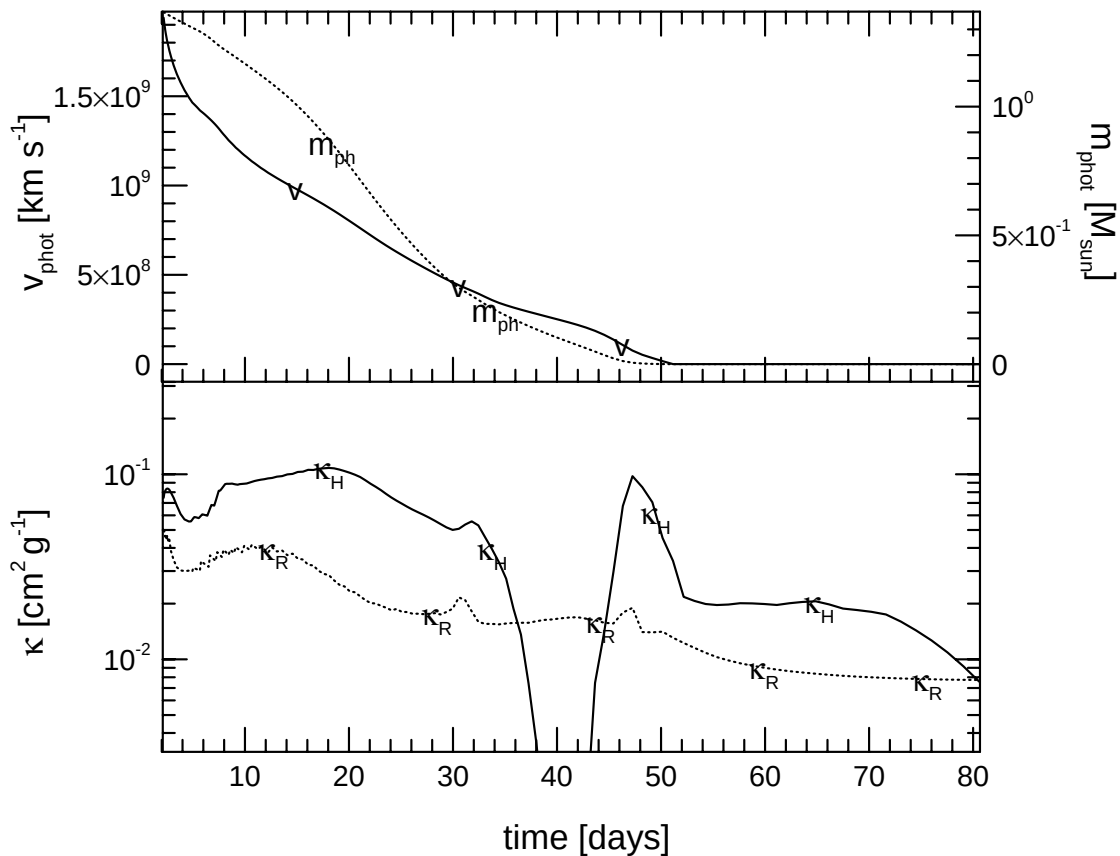


FIG. 20.—Opacity at the flux mean photosphere as a function of time, with the position of the photosphere in mass and velocity indicated in the top panel

~ 8000 K and the dominant ionization states are Fe II and Fe III. As Fe II becomes more abundant, the IR emissivity greatly increases, at wavelengths where the monochromatic opacity is very low. This leads to a sharp reduction in the flux mean opacity. The diffusion time is thus reduced, and the residual stored energy is released, leading to the infrared peak.

This effect is magnified by the fact that most of the inner $\sim 0.1 M_{\odot}$ is not radioactive in this model; by the time the photosphere has receded to $0.1 M_{\odot}$, the temperature gradient in the core is negative, and the decreasing temperature leads to a further reduction in mean opacity. The sharp rise in opacity just before 50 days occurs when the photosphere encounters radioactive material in the very center of the model. Because by this time very little mass is involved, the rise has no discernible effect on the light curve.

Understanding the secondary maximum as a phenomenon that occurs at the center of the supernova is particularly interesting in the context of explosion models that ignite near center. Might the secondary maximum be telling us something about the ignition conditions of the explosion? The neutron-rich isotopes that make up the non-radioactive core in this model constitute a serious problem for its nucleosynthesis. Nonetheless, it is interesting to note that there is a potentially observable consequence, the duration and magnitude of the IR secondary maximum, of such a “cold core.” We will defer further discussion of this phenomenon to Paper III.

5. CONCLUSIONS

The rate at which radiation can be emitted at long wavelengths where the monochromatic opacity is small domi-

nates the energy flow in the supernova. This rate is greatly enhanced by the complexity of the heavy ions that dominate the composition. Energy can be emitted at these optically thin wavelengths as a result of both fluorescence and collisional excitation. The exact balance between these two processes is clouded by our assumption of an LTE equation of state. By employing an ETLA approximation for the line source functions, the greatly enhanced photon destruction rates that result from fluorescence have been taken into account. There is, however, probably too much emission of collisionally excited radiation at longer wavelengths, as LTE overestimates the excited-state populations that form the lower levels of these longer wavelength transitions. We have shown that even at the peak of the light curve, the collisional thermalization depth of the supernova is small. On the other hand, our Monte Carlo experiments suggest that repeated fluorescences may provide an alternate mechanism for thermalizing the level populations, but at this point we can only conjecture that the effect is sufficient to lead all the way to the thermodynamic limit.

Given the prohibitive computational effort a time-dependent non-LTE calculation would entail, this question cannot be settled at present. Time-independent calculations can give at best an approximate indication. The problem is that the timescale for achieving this thermodynamic limit must be comparable to the time it takes for originally emitted photons to have their energy randomly distributed into many lines. This in turn must be comparable to the flux mean diffusion time if such a redistribution is the primary route to energy loss. Since the light-curve peak occurs when this diffusion time is comparable to the elapsed time, the physics is poorly represented by a steady state.

Subject to these uncertainties, we find that the flux mean opacity is constant on the rise to peak, but declines significantly thereafter. If this constancy is a general feature of these explosions, the simple analytic model presented in Paper I can be used to analyze observed distributions of rise times to determine fundamental properties of the explosions. Though difficult to achieve, an accurate measurement of the rise times for a variety of supernovae would thus be very useful.

The rate of the opacity decline after peak will affect the postmaximum decline rate of the light curve, which is central to the use of SNe Ia as cosmological probes (Phillips 1993; Riess, Press, & Kirshner 1995). Understanding how this decline varies from explosion to explosion will be essential to understanding the peak magnitude–decline rate relation.

We have shown that a decrease in mean opacity as the photosphere recedes through the innermost layers of the ejecta contributes to the infrared secondary maximum. The infrared light curve can exhibit a secondary maximum without this decrease in opacity within the photosphere. In Paper III we demonstrate that the rise in the *I* band, for example, is caused in large part by a rise in the strength of the Ca II infrared triplet. This can occur simply as a result of a shift in the ionization balance of Ca in regions of significant energy deposition. However, if the infrared maxima are strong enough to reduce the *bolometric* decline rate, we

have shown that the only additional energy source must be residual trapped energy within the photosphere released more abruptly by a decline in opacity. If the core of the supernova is cooler because of the presence of nonradioactive isotopes (not necessarily the result of electron capture, as in the current model), this release may be more abrupt. This is a potentially significant diagnostic of the innermost regions of the explosion. We therefore stress the need for infrared photometry to enable the construction of bolometric light curves in the postmaximum phase.

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