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ISOCHRONES AND LUMINOSITY FUNCTIONS FOR OLD WHITE DWARFS

HARVEY B. RICHER,[1,2] BRAD HANSEN,[3] MARCO LIMONGI,[1] ALESSANDRO CHIEFFI,[4]
OSCAR STRANIERO,[5] AND GREGORY G. FAHLMAN[2]
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ABSTRACT

Using a new grid of models of cooling white dwarfs, we calculate isochrones and luminosity functions in the Johnson-Kron/Cousins and *Hubble Space Telescope* (*HST*) filter sets for systems containing old white dwarfs. These new models incorporate a nongray atmosphere that is necessary to properly describe the effects of molecular opacity at the cool temperatures of old white dwarfs. The various functions calculated and extensively tabulated and plotted are meant to be as utilitarian as possible for observers, so all results are listed in quantities that observers will obtain. The tables and plots developed should eventually prove critical in interpreting the results of *HST*'s Advanced Camera observations of the oldest white dwarfs in nearby globular clusters, in understanding the results of searches for old white dwarfs in the Galactic halo, and in determining ages for star clusters using white dwarfs. As a practical application we demonstrate the use of these results by deriving the white dwarf cooling age of the old Galactic cluster M67.

Subject headings: galaxy: globular clusters: general — stars: evolution — stars: interiors — stars: luminosity function, mass function — stars: white dwarfs

1. INTRODUCTION

The search for and use of old white dwarfs in determining the ages and star formation histories in stellar systems was given an important lift recently with the publication of new sets of models of cooling white dwarfs (Hansen 1998, 1999; Saumon & Jacobson 1999). These models included nongray atmospheres that are critical in understanding the luminosity and emergent spectrum of white dwarfs whose temperatures fall below 4000 K. It is the atmosphere that regulates changes in the white dwarf's largely isothermal core and, hence, its cooling time. Also, the behavior of the atmosphere is strongly dependent on its composition, particularly the amount of hydrogen and helium (helium does not form molecules, whereas hydrogen does at cool temperatures). Hydrogen molecules thus provide a dramatic opacity source that must be included in the modeling in order to understand the emergent flux from the star. Therefore, careful treatment of the physics is essential to properly interpret the luminosities and colors of old white dwarfs.

As an added incentive, the recent microlensing results in the direction of the Magellanic Clouds (Alcock et al. 1997a, 1997b; Renault et al. 1997) suggest that a sizeable fraction (perhaps half or even larger) of the dark matter in the Galactic halo could be tied up in stellar objects with masses near 0.5 $M_{\odot}$. This suggests old white dwarfs as the likely candidate, although other possibilities remain (neutron stars, primordial black holes, or other exotica). Although all of these candidates for the Galactic dark matter have their problems, numerous searches are now under way to attempt to locate these objects, and there already exists a few possible old white dwarf candidates in the Hubble Deep Field (Ibata et al. 1999) and general field of the Galaxy (Harris et al. 1999). All the searches for old white dwarfs must be guided by appropriate cooling models, isochrones, and luminosity functions. In this paper we present all of the above as an aid in directing these endeavors. The compilations here are much more extensive than those in Chabrier (1999) and use a different set of models, notably those of Hansen. The tables and plots are all in the observers plane in the Johnson-Kron/Cousins *VRI* color system or the *Hubble Space Telescope* (*HST*) system, and attempts have been made to make the data as utilitarian for observers as possible. For this reason we have presented not just the cooling models, but have developed white dwarf isochrones and luminosity functions both for clusters and for the Galactic halo. These are the quantities that will actually be observed when the Advanced Camera for Surveys on *HST* eventually penetrates the faint end of the white dwarf cooling sequence in a globular cluster or when a wide-area, ground-based survey detects a sizeable sample of old halo white dwarfs.

2. THE WHITE DWARF COOLING MODELS

The white dwarf models presented here are based on the code of Hansen & Phinney (1998). The addition of new atmospheric models (Hansen 1998, 1999) has led to a revision of the cooling ages and observational appearance of old white dwarfs and it is these that we will use. The only models we present are those for C-O cores (without separation energy) and with hydrogen-rich atmospheres. Chemical separation may lengthen the ages slightly (Salaris et al. 1997; Hansen 1999). As such, the omission of this contribution represents the most conservative assumption, that is the fastest cooling. It is only for hydrogen-rich atmospheres that the white dwarfs remain bright (brighter than $M_V = 18$) for times comparable to the Hubble time. This is due to the strong opacity of molecular hydrogen. The helium-rich models, which do not possess this opacity source, cool much more rapidly and become fainter than $M_V = 18$ on a

[1] Osservatorio Astronomico di Roma Via Frascati 33, Monteporzio Catone 00040, Italy.
[2] Department of Physics and Astronomy University of British Columbia, Vancouver, BC V6T 1Z1, Canada.
[3] Canadian Institute for Theoretical Astrophysics University of Toronto, 60 St. George Street, Toronto, ON M5S 3H8, Canada.
[4] Istituto di Astrofisica Spaziale CNR C.P. 67, I-00044, Frascati, Italy.
[5] Osservatorio Astronomico di Collurania, 64100, Teramo, Italy.

timescale less than 6 Gyr. Because of the strongly non-blackbody colors of cool hydrogen-rich white dwarfs caused by the hydrogen molecules redistributing the emergent flux, the stars actually become bluer in $V-I$ when $T_{\rm eff}$ drops below about 3500 K. At this point there is little change in M_V as the stars evolve but the VRI colors become very different from those of black bodies of any temperature and these colors may be a key to the discovery of old white dwarfs.

In Figure 1 we illustrate the effect that the added H_2 opacity has on the observed cooling track of a 0.5 $M_\odot$ C-O core, H-rich (DA) white dwarf. Here we compare the cooling sequence for such a model by Hansen with an extrapolated model using Wood's (1992), (1995) interiors and Bergeron, Wesemael, & Beauchamp's (1995) atmospheres. Down to an M_V of about 16 (age slightly older than 7 Gyr for both models), both sets of models agree quite well, but as the star gets cooler and more molecules are able to form, the effect of the molecular opacity increases and the two models differ enormously. In $V-I$, old white dwarfs are blue, not red.

Table 1 contains these new cooling models in Johnson/Kron-Cousins VRI and Table 2 contains these in the HST filters, while Figure 2 plots the M_V, $V-I$ cooling sequences from Table 1. The colors can be quite different in these two different filter sets since the strong H_2 opacity produces sharp flux peaks in the emergent white dwarf spectrum. The colors measured by the observer, then, depend critically on the positions of the transmission peaks of the filters. The HST colors are calculated using the Holtzmann et al. (1995) bandpasses and the transformations they use to express fluxes in V, R, and I.

The mass range in the models varies from 0.5–0.9 $M_\odot$ and the sequences all begin at about an age of 0.2 Gyr since

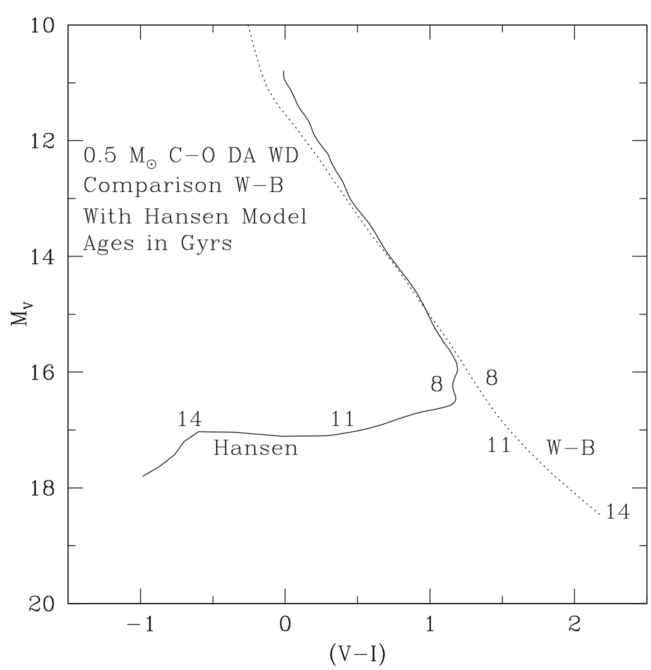

FIG. 1.—New 0.5 $M_\odot$ white dwarf cooling model of Hansen (1998, 1999) compared with a similar mass model constructed from the interiors of Wood (1992) and Bergeron et al. (1995) atmospheres (W-B). The main differences appeared around 8 Gyr, where the effects of atmospheric H_2 opacity become important.

TABLE 1

WHITE DWARF COOLING MODELS: C-O CORES, HYDROGEN ATMOSPHERES, JOHNSON-KRON/COUSINS FILTERS

Age (Gyr)	$T_{\rm eff}$	M_V	$V-R$	$V-I$
		0.5 $M_\odot$		
0.123......	15095.03	10.790	0.044	−0.009
0.135......	14654.43	10.860	0.024	−0.010
0.149......	14222.96	10.932	0.011	−0.005
0.164......	13780.73	11.013	0.012	0.009
0.180......	13337.02	11.102	0.027	0.032
0.198......	12906.18	11.194	0.042	0.053
0.218......	12488.92	11.288	0.052	0.068
0.240......	12085.52	11.383	0.058	0.085
0.264......	11691.84	11.479	0.065	0.110
0.291......	11305.23	11.576	0.076	0.139
0.320......	10926.06	11.674	0.094	0.163
0.352......	10555.34	11.775	0.117	0.180
0.387......	10193.22	11.883	0.135	0.197
0.426......	9840.420	12.001	0.139	0.223
0.469......	9498.634	12.121	0.140	0.260
0.516......	9167.637	12.235	0.158	0.296
0.567......	8846.488	12.361	0.166	0.315
0.624......	8537.167	12.519	0.168	0.349
0.686......	8237.403	12.691	0.180	0.394
0.755......	7948.685	12.859	0.193	0.423
0.831......	7671.349	13.000	0.214	0.448
0.914......	7404.970	13.139	0.240	0.484
1.005......	7149.014	13.279	0.267	0.532
1.106......	6902.436	13.421	0.284	0.577
1.215......	6667.564	13.562	0.292	0.614
1.325......	6463.478	13.693	0.302	0.645
1.435......	6285.780	13.813	0.320	0.674
1.544......	6132.283	13.923	0.340	0.702
1.654......	6000.452	14.024	0.359	0.729
1.764......	5887.130	14.115	0.376	0.757
1.873......	5787.598	14.198	0.391	0.783
1.983......	5698.339	14.275	0.404	0.807
2.093......	5616.067	14.347	0.415	0.829
2.202......	5539.106	14.415	0.425	0.849
2.312......	5466.304	14.479	0.433	0.868
2.421......	5397.410	14.541	0.441	0.885
2.531......	5333.680	14.598	0.447	0.900
2.641......	5272.595	14.653	0.453	0.913
2.750......	5216.421	14.705	0.459	0.926
2.860......	5153.561	14.763	0.464	0.939
2.970......	5093.876	14.819	0.470	0.951
3.079......	5045.101	14.866	0.475	0.961
3.189......	5006.260	14.903	0.479	0.968
3.299......	4970.547	14.938	0.483	0.975
3.408......	4938.016	14.970	0.486	0.981
3.518......	4913.123	14.995	0.489	0.986
3.628......	4893.929	15.015	0.491	0.990
3.737......	4876.708	15.032	0.493	0.993
3.847......	4860.749	15.048	0.495	0.996
3.957......	4841.101	15.068	0.497	1.001
4.066......	4820.696	15.089	0.499	1.004
4.176......	4799.458	15.110	0.502	1.008
4.286......	4775.066	15.135	0.505	1.013
4.395......	4746.954	15.164	0.508	1.018
4.505......	4715.784	15.196	0.512	1.025
4.614......	4680.278	15.232	0.516	1.033
4.724......	4648.286	15.266	0.520	1.040
4.834......	4619.301	15.297	0.524	1.047
4.943......	4589.228	15.329	0.528	1.055
5.053......	4555.379	15.366	0.532	1.064
5.163......	4522.656	15.401	0.537	1.073
5.272......	4489.496	15.438	0.541	1.083

TABLE 1—*Continued*

Age (Gyr)	$T_{\rm eff}$	M_V	$V-R$	$V-I$
5.382	4458.713	15.472	0.545	1.092
5.492	4430.572	15.503	0.549	1.101
5.601	4404.640	15.533	0.552	1.110
5.711	4381.141	15.559	0.555	1.117
5.821	4357.899	15.586	0.558	1.125
5.930	4333.108	15.614	0.562	1.133
6.040	4305.524	15.645	0.565	1.141
6.150	4277.585	15.676	0.569	1.149
6.259	4246.783	15.711	0.574	1.158
6.369	4215.137	15.746	0.578	1.166
6.478	4181.689	15.783	0.583	1.174
6.588	4150.566	15.817	0.587	1.180
6.698	4120.335	15.849	0.592	1.185
6.807	4088.863	15.882	0.596	1.188
6.917	4056.221	15.916	0.601	1.190
7.027	4023.599	15.949	0.606	1.191
7.136	3990.353	15.982	0.611	1.189
7.246	3954.138	16.017	0.617	1.185
7.356	3920.099	16.049	0.622	1.179
7.465	3887.626	16.080	0.627	1.174
7.575	3853.812	16.113	0.632	1.168
7.685	3819.187	16.148	0.636	1.163
7.794	3784.345	16.185	0.641	1.159
7.904	3749.789	16.224	0.645	1.157
8.014	3714.956	16.266	0.648	1.158
8.123	3675.609	16.317	0.651	1.163
8.233	3641.318	16.364	0.654	1.170
8.343	3610.248	16.407	0.655	1.176
8.452	3576.533	16.453	0.657	1.179
8.562	3540.475	16.500	0.658	1.175
8.671	3504.480	16.541	0.659	1.162
8.781	3471.429	16.573	0.660	1.139
8.891	3438.031	16.601	0.661	1.107
9.000	3404.297	16.625	0.662	1.067
9.110	3370.412	16.648	0.662	1.023
9.220	3335.544	16.671	0.661	0.976
9.329	3301.808	16.695	0.660	0.932
9.439	3269.609	16.720	0.657	0.892
9.549	3236.274	16.749	0.653	0.853
9.658	3203.143	16.779	0.648	0.816
9.768	3170.244	16.811	0.642	0.780
9.878	3141.138	16.840	0.636	0.747
9.987	3110.630	16.872	0.629	0.710
10.097	3080.182	16.904	0.622	0.671
10.207	3050.325	16.935	0.614	0.630
10.316	3022.236	16.965	0.606	0.587
10.426	2998.492	16.989	0.599	0.547
10.536	2973.438	17.015	0.591	0.501
10.636	2940.729	17.046	0.581	0.436
10.800	2907.066	17.075	0.571	0.363
10.965	2873.790	17.097	0.560	0.286
11.513	2755.530	17.109	0.529	−0.019
12.061	2631.510	17.038	0.506	−0.345
12.609	2500.087	17.029	0.490	−0.599
13.158	2371.128	17.198	0.474	−0.698
13.706	2243.651	17.428	0.443	−0.763
14.254	2118.142	17.631	0.394	−0.869
14.802	2003.111	17.804	0.341	−0.986
		0.6 $M_\odot$		
0.177	14843.15	11.101	0.032	−0.010
0.195	14366.82	11.174	0.014	−0.008
0.215	13898.19	11.253	0.010	0.003
0.236	13438.67	11.341	0.023	0.027
0.260	12990.45	11.435	0.040	0.050
0.286	12551.13	11.532	0.051	0.066

TABLE 1—*Continued*

Age (Gyr)	$T_{\rm eff}$	M_V	$V-R$	$V-I$
0.315	12122.97	11.631	0.058	0.083
0.346	11708.42	11.730	0.065	0.109
0.381	11307.59	11.829	0.076	0.139
0.419	10920.09	11.928	0.094	0.164
0.461	10547.16	12.028	0.118	0.180
0.507	10186.72	12.135	0.136	0.197
0.558	9838.626	12.250	0.139	0.224
0.614	9503.302	12.368	0.139	0.259
0.675	9180.363	12.478	0.158	0.294
0.743	8870.609	12.596	0.166	0.314
0.817	8573.980	12.745	0.167	0.344
0.899	8289.914	12.905	0.178	0.386
0.989	8016.144	13.068	0.189	0.418
1.087	7751.372	13.203	0.207	0.440
1.196	7497.785	13.333	0.230	0.470
1.316	7263.138	13.458	0.256	0.510
1.448	7054.518	13.575	0.276	0.549
1.592	6865.231	13.684	0.286	0.583
1.750	6688.593	13.791	0.291	0.611
1.908	6536.831	13.886	0.297	0.634
2.066	6400.791	13.970	0.308	0.655
2.224	6276.921	14.059	0.321	0.675
2.382	6158.933	14.143	0.336	0.696
2.540	6047.026	14.227	0.353	0.719
2.698	5945.456	14.307	0.368	0.742
2.856	5848.571	14.386	0.382	0.767
3.013	5754.589	14.465	0.396	0.792
3.171	5661.967	14.545	0.409	0.817
3.329	5572.308	14.623	0.421	0.841
3.487	5495.300	14.699	0.430	0.861
3.645	5412.929	14.765	0.439	0.881
3.803	5328.751	14.840	0.448	0.901
3.961	5256.578	14.905	0.455	0.917
4.119	5198.637	14.958	0.460	0.930
4.277	5134.595	15.017	0.466	0.943
4.434	5076.699	15.072	0.472	0.955
4.592	5039.023	15.108	0.476	0.962
4.750	5005.241	15.140	0.479	0.968
4.908	4968.070	15.176	0.483	0.976
5.066	4928.367	15.215	0.487	0.983
5.224	4894.569	15.248	0.491	0.989
5.382	4866.263	15.276	0.494	0.995
5.540	4840.229	15.302	0.497	1.000
5.698	4815.657	15.326	0.500	1.005
5.856	4787.219	15.355	0.503	1.010
6.013	4756.879	15.385	0.507	1.016
6.171	4725.292	15.417	0.511	1.023
6.329	4691.195	15.452	0.515	1.030
6.487	4656.167	15.489	0.519	1.038
6.645	4622.911	15.524	0.524	1.046
6.803	4590.039	15.559	0.528	1.055
6.961	4556.122	15.595	0.532	1.064
7.119	4517.374	15.637	0.537	1.074
7.277	4476.308	15.682	0.543	1.087
7.435	4430.533	15.733	0.549	1.101
7.592	4381.882	15.787	0.555	1.117
7.750	4338.176	15.836	0.561	1.131
7.908	4293.258	15.887	0.567	1.145
8.066	4243.015	15.943	0.574	1.159
8.224	4192.182	15.999	0.581	1.172
8.382	4138.105	16.057	0.589	1.182
8.540	4087.839	16.111	0.596	1.188
8.698	4034.327	16.165	0.604	1.191
8.856	3975.993	16.222	0.613	1.187
9.013	3920.523	16.275	0.622	1.180
9.171	3869.634	16.324	0.629	1.171

TABLE 1—*Continued*

Age (Gyr)	$T_{\rm eff}$	M_V	$V-R$	$V-I$
9.329	3818.512	16.375	0.636	1.163
9.487	3763.022	16.435	0.643	1.157
9.645	3707.316	16.501	0.649	1.159
9.803	3647.716	16.581	0.653	1.169
9.961	3592.017	16.658	0.656	1.178
10.119	3541.803	16.723	0.658	1.175
10.277	3487.057	16.784	0.660	1.151
10.435	3434.827	16.828	0.662	1.103
10.592	3385.314	16.863	0.662	1.043
10.750	3336.989	16.895	0.661	0.978
10.908	3284.393	16.933	0.658	0.910
11.066	3232.627	16.977	0.652	0.849
11.224	3182.985	17.024	0.644	0.794
11.382	3131.988	17.075	0.634	0.736
11.540	3084.716	17.124	0.623	0.677
11.698	3031.762	17.180	0.609	0.602
11.856	2981.371	17.232	0.594	0.516
12.013	2938.354	17.273	0.581	0.431
12.157	2926.146	17.283	0.577	0.405
12.631	2776.846	17.341	0.534	0.038
13.026	2651.867	17.276	0.509	−0.295
13.421	2516.529	17.246	0.491	−0.576
13.815	2375.756	17.416	0.474	−0.696
14.052	2293.782	17.566	0.458	−0.733
14.289	2208.270	17.713	0.431	−0.789
14.526	2124.721	17.847	0.397	−0.863
14.763	2041.982	17.972	0.359	−0.946
15.000	1961.336	18.092	0.321	−1.030
		0.7 $M_\odot$		
0.242	14913.46	11.342	0.035	−0.010
0.266	14418.44	11.416	0.016	−0.008
0.292	13931.65	11.496	0.010	0.002
0.322	13450.78	11.587	0.023	0.026
0.354	12976.53	11.685	0.040	0.051
0.389	12507.30	11.788	0.051	0.068
0.428	12044.80	11.895	0.059	0.087
0.471	11613.16	11.998	0.067	0.116
0.519	11211.43	12.097	0.079	0.146
0.570	10830.39	12.195	0.100	0.168
0.628	10466.15	12.294	0.123	0.183
0.690	10116.31	12.399	0.138	0.201
0.759	9780.568	12.512	0.138	0.229
0.835	9462.165	12.623	0.141	0.264
0.919	9166.861	12.723	0.159	0.296
1.011	8894.094	12.826	0.167	0.313
1.112	8643.276	12.948	0.166	0.335
1.223	8414.625	13.073	0.173	0.368
1.346	8197.749	13.198	0.181	0.399
1.480	7980.196	13.326	0.191	0.421
1.628	7769.401	13.432	0.206	0.438
1.791	7559.454	13.539	0.224	0.461
1.970	7338.874	13.655	0.247	0.496
2.168	7121.523	13.774	0.270	0.537
2.382	6908.543	13.895	0.284	0.575
2.597	6708.771	14.014	0.290	0.608
2.812	6526.905	14.128	0.298	0.635
3.027	6344.902	14.248	0.313	0.664
3.242	6148.930	14.385	0.338	0.698
3.457	5954.893	14.533	0.366	0.740
3.672	5794.888	14.665	0.390	0.781
3.887	5661.806	14.779	0.409	0.817
4.102	5540.478	14.885	0.425	0.849
4.317	5440.402	14.973	0.436	0.874
4.532	5347.771	15.056	0.446	0.896
4.747	5263.535	15.132	0.454	0.915

TABLE 1—*Continued*

Age (Gyr)	$T_{\rm eff}$	M_V	$V-R$	$V-I$
4.961	5208.143	15.183	0.459	0.927
5.176	5163.271	15.224	0.464	0.937
5.391	5123.507	15.261	0.467	0.945
5.606	5075.434	15.306	0.472	0.955
5.821	5041.870	15.338	0.475	0.961
6.036	5007.492	15.371	0.479	0.968
6.251	4971.232	15.405	0.483	0.975
6.466	4929.016	15.446	0.487	0.983
6.681	4882.581	15.491	0.492	0.992
6.896	4834.008	15.539	0.498	1.001
7.111	4785.358	15.587	0.503	1.011
7.326	4729.533	15.643	0.510	1.022
7.540	4671.635	15.702	0.517	1.035
7.755	4611.470	15.764	0.525	1.049
7.970	4549.051	15.830	0.533	1.065
8.185	4480.140	15.905	0.542	1.086
8.400	4409.324	15.983	0.551	1.108
8.615	4337.092	16.064	0.561	1.131
8.830	4258.954	16.151	0.572	1.155
9.045	4168.789	16.250	0.585	1.176
9.260	4088.764	16.335	0.596	1.188
9.475	4001.292	16.424	0.609	1.190
9.690	3906.217	16.514	0.624	1.177
9.904	3812.124	16.607	0.637	1.162
10.198	3716.670	16.715	0.648	1.158
10.334	3627.638	16.835	0.654	1.173
10.549	3531.468	16.962	0.658	1.173
10.764	3446.799	17.045	0.661	1.116
10.979	3355.300	17.109	0.662	1.003
11.194	3265.593	17.175	0.656	0.887
11.409	3174.044	17.258	0.643	0.784
11.624	3076.448	17.359	0.621	0.666
11.839	2987.764	17.451	0.596	0.528
12.054	2909.311	17.523	0.571	0.368
12.269	2829.705	17.566	0.548	0.177
12.483	2748.437	17.557	0.527	−0.038
12.698	2660.447	17.506	0.511	−0.273
12.913	2571.178	17.466	0.498	−0.480
13.128	2485.013	17.490	0.488	−0.618
13.343	2406.382	17.589	0.479	−0.681
13.558	2323.072	17.737	0.465	−0.719
13.773	2244.531	17.877	0.444	−0.763
13.988	2168.899	18.002	0.416	−0.822
14.203	2101.429	18.107	0.387	−0.885
14.418	2031.461	18.213	0.354	−0.957
14.633	1964.061	18.313	0.322	−1.027
14.847	1849.694	18.488	0.271	−1.140
15.062	1695.594	18.748	0.218	−1.252
		0.8 $M_\odot$		
0.347	14827.20	11.607	0.032	−0.010
0.382	14318.27	11.684	0.013	−0.007
0.420	13825.06	11.767	0.012	0.007
0.463	13339.78	11.860	0.027	0.032
0.509	12852.63	11.962	0.044	0.056
0.560	12350.56	12.073	0.054	0.073
0.616	11862.71	12.187	0.062	0.098
0.677	11417.10	12.294	0.072	0.131
0.745	11005.48	12.397	0.089	0.159
0.820	10629.32	12.496	0.112	0.177
0.902	10304.04	12.588	0.131	0.191
0.992	10025.96	12.675	0.139	0.208
1.091	9771.160	12.762	0.138	0.230
1.200	9515.225	12.851	0.139	0.258
1.321	9257.279	12.938	0.152	0.287
1.453	8984.493	13.034	0.169	0.310

TABLE 1—*Continued*

Age (Gyr)	$T_{\rm eff}$	M_V	$V-R$	$V-I$
1.598	8644.942	13.191	0.166	0.334
1.758	8304.034	13.380	0.177	0.384
1.934	8015.659	13.551	0.189	0.418
2.127	7761.395	13.679	0.207	0.439
2.340	7494.761	13.815	0.230	0.470
2.574	7179.554	13.983	0.264	0.526
2.831	6841.066	14.175	0.287	0.587
3.112	6563.108	14.346	0.296	0.630
3.393	6299.760	14.519	0.318	0.671
3.673	6051.851	14.698	0.352	0.718
3.954	5847.120	14.861	0.382	0.767
4.235	5678.406	15.004	0.407	0.812
4.515	5530.119	15.133	0.426	0.852
4.796	5441.573	15.212	0.436	0.874
5.077	5337.787	15.304	0.447	0.899
5.358	5273.102	15.363	0.453	0.913
5.638	5218.652	15.412	0.458	0.925
5.919	5162.725	15.464	0.464	0.937
6.200	5112.481	15.511	0.468	0.947
6.480	5060.477	15.559	0.473	0.958
6.761	5003.271	15.613	0.479	0.969
7.042	4933.282	15.679	0.487	0.982
7.323	4859.581	15.750	0.495	0.996
7.603	4780.419	15.828	0.504	1.012
7.884	4693.962	15.915	0.515	1.030
8.165	4595.114	16.017	0.527	1.053
8.445	4486.675	16.133	0.541	1.084
8.726	4363.449	16.270	0.558	1.123
9.007	4234.122	16.414	0.575	1.161
9.287	4099.712	16.560	0.595	1.187
9.568	3946.773	16.711	0.618	1.184
9.849	3783.074	16.873	0.641	1.159
10.130......	3615.818	17.086	0.655	1.175
10.410......	3446.500	17.280	0.661	1.116
10.691......	3292.242	17.388	0.659	0.920
10.972......	3116.982	17.551	0.631	0.718
11.252......	2936.514	17.735	0.580	0.427
11.533......	2772.573	17.799	0.533	0.026
11.814......	2589.899	17.704	0.500	−0.441
12.095......	2432.296	17.784	0.482	−0.665
12.375......	2310.931	17.993	0.462	−0.724
12.656......	2233.273	18.130	0.440	−0.771
12.937......	2072.729	18.385	0.374	−0.914
		0.9 $M_\odot$		
0.484	14710.72	11.894	0.026	−0.010
0.532	14165.67	11.977	0.010	−0.004
0.586	13649.91	12.068	0.016	0.015
0.644	13163.69	12.164	0.034	0.042
0.709	12697.61	12.264	0.048	0.061
0.779	12290.85	12.355	0.055	0.076
0.857	11933.71	12.437	0.061	0.093
0.943	11587.54	12.519	0.067	0.118
1.038	11255.32	12.601	0.078	0.143
1.141	10931.75	12.683	0.093	0.163
1.256	10560.71	12.781	0.117	0.179
1.381	10125.64	12.909	0.137	0.201
1.519	9716.503	13.046	0.138	0.236
1.671	9381.225	13.161	0.145	0.273
1.839	9062.250	13.270	0.166	0.305
2.023	8757.424	13.398	0.165	0.322
2.225	8434.758	13.569	0.172	0.365
2.447	8057.051	13.788	0.187	0.414
2.692	7663.667	13.990	0.214	0.449
2.961	7291.167	14.184	0.252	0.505
3.258	6939.941	14.380	0.283	0.570

TABLE 1—*Continued*

Age (Gyr)	$T_{\rm eff}$	M_V	$V-R$	$V-I$
3.583	6595.978	14.586	0.294	0.625
3.939	6267.617	14.802	0.322	0.677
4.294	5977.769	15.016	0.363	0.735
4.649	5737.974	15.213	0.398	0.796
5.005	5560.277	15.367	0.422	0.844
5.360	5475.945	15.442	0.432	0.866
5.715	5342.835	15.560	0.446	0.898
6.070	5254.616	15.639	0.455	0.917
6.426	5174.179	15.712	0.463	0.935
6.781	5091.327	15.788	0.470	0.952
7.136	4992.645	15.880	0.480	0.971
7.491	4857.250	16.010	0.495	0.997
7.847	4712.883	16.153	0.512	1.026
8.202	4532.780	16.340	0.535	1.070
8.557	4326.192	16.569	0.563	1.135
8.912	4081.319	16.835	0.597	1.189
9.268	3789.772	17.122	0.640	1.159
9.623	3471.486	17.515	0.660	1.139
9.978	3155.130	17.767	0.639	0.763
10.334......	2889.188	18.027	0.565	0.322
10.689......	2700.601	18.020	0.518	−0.168
11.044......	2571.432	17.954	0.498	−0.479
11.399......	2374.063	18.132	0.474	−0.697
11.755......	1855.517	18.968	0.273	−1.135

it takes this long for the white dwarfs to lose the imprint of their initial conditions. The models terminate when $T_{\rm eff}$ drops below about 2000 K, which is the limit of the opacity tables used.

Figure 3 plots the color-color diagram in the Johnson/Kron-Cousins photometric system for a white dwarf of mass 0.7 $M_\odot$ with ages indicated along the sequence. The

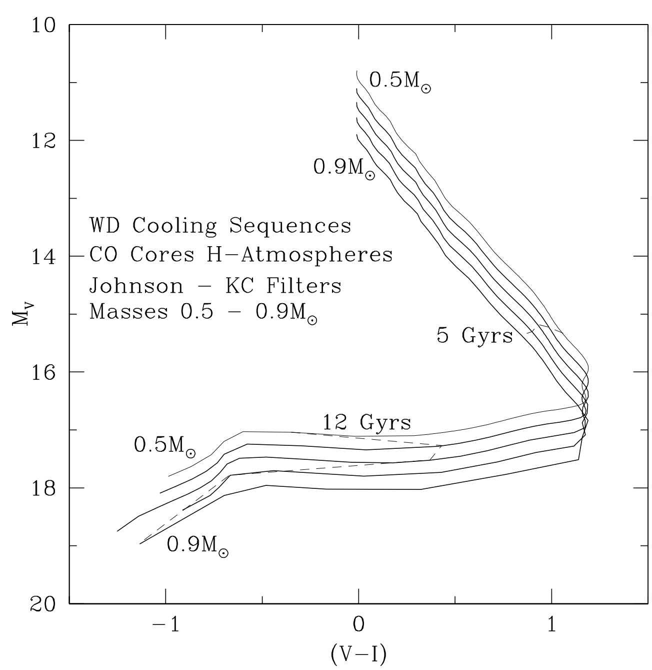

FIG. 2.—Cooling sequences for C-O core, hydrogen-rich white dwarfs of varying mass. The lines shown are for Johnson-Kron/Cousins filters. Constant ages of 5 and 12 Gyr are indicated on the diagram.

TABLE 2

THE WHITE DWARF COOLING MODELS: C-O CORES, HYDROGEN ATMOSPHERES, *HST* FILTERS

Age (Gyr)	$T_{\rm eff}$	M_V	$V-R$	$V-I$
		0.5 $M_\odot$		
0.123......	15095.03	10.774	−0.034	−0.110
0.135......	14654.43	10.873	−0.012	−0.060
0.149......	14222.96	10.971	0.009	−0.012
0.164......	13780.73	11.071	0.029	0.031
0.180......	13337.02	11.167	0.044	0.065
0.198......	12906.18	11.250	0.050	0.081
0.218......	12488.92	11.324	0.046	0.083
0.240......	12085.52	11.404	0.048	0.087
0.264......	11691.84	11.504	0.063	0.108
0.291......	11305.23	11.616	0.085	0.141
0.320......	10926.06	11.725	0.102	0.176
0.352......	10555.34	11.825	0.109	0.205
0.387......	10193.22	11.930	0.119	0.233
0.426......	9840.420	12.056	0.142	0.265
0.469......	9498.634	12.191	0.170	0.300
0.516......	9167.637	12.312	0.184	0.335
0.567......	8846.488	12.440	0.210	0.375
0.624......	8537.167	12.604	0.234	0.420
0.686......	8237.403	12.775	0.233	0.452
0.755......	7948.685	12.938	0.243	0.486
0.831......	7671.349	13.076	0.262	0.528
0.914......	7404.970	13.223	0.292	0.580
1.005......	7149.014	13.380	0.330	0.643
1.106......	6902.436	13.535	0.359	0.701
1.215......	6667.564	13.681	0.377	0.749
1.325......	6463.478	13.813	0.393	0.786
1.435......	6285.780	13.936	0.411	0.820
1.544......	6132.283	14.049	0.430	0.854
1.654......	6000.452	14.154	0.449	0.889
1.764......	5887.130	14.249	0.468	0.926
1.873......	5787.598	14.336	0.486	0.961
1.983......	5698.339	14.417	0.502	0.993
2.093......	5616.067	14.492	0.518	1.022
2.202......	5539.106	14.563	0.532	1.047
2.312......	5466.304	14.631	0.546	1.069
2.421......	5397.410	14.695	0.558	1.088
2.531......	5333.680	14.754	0.570	1.104
2.641......	5272.595	14.812	0.581	1.119
2.750......	5216.421	14.865	0.591	1.131
2.860......	5153.561	14.926	0.602	1.145
2.970......	5093.876	14.984	0.613	1.158
3.079......	5045.101	15.033	0.622	1.169
3.189......	5006.260	15.073	0.628	1.178
3.299......	4970.547	15.110	0.635	1.186
3.408......	4938.016	15.144	0.640	1.194
3.518......	4913.123	15.170	0.645	1.201
3.628......	4893.929	15.191	0.648	1.205
3.737......	4876.708	15.210	0.651	1.210
3.847......	4860.749	15.227	0.654	1.214
3.957......	4841.101	15.248	0.658	1.219
4.066......	4820.696	15.270	0.661	1.225
4.176......	4799.458	15.293	0.665	1.230
4.286......	4775.066	15.319	0.669	1.237
4.395......	4746.954	15.350	0.674	1.245
4.505......	4715.784	15.384	0.680	1.254
4.614......	4680.278	15.423	0.686	1.264
4.724......	4648.286	15.458	0.692	1.274
4.834......	4619.301	15.491	0.697	1.282
4.943......	4589.228	15.525	0.703	1.291
5.053......	4555.379	15.563	0.709	1.302
5.163......	4522.656	15.600	0.715	1.312
5.272......	4489.496	15.638	0.721	1.323
5.382......	4458.713	15.674	0.727	1.333

TABLE 2—*Continued*

Age (Gyr)	$T_{\rm eff}$	M_V	$V-R$	$V-I$
5.492	4430.572	15.706	0.733	1.342
5.601	4404.640	15.736	0.738	1.350
5.711	4381.141	15.764	0.742	1.358
5.821	4357.899	15.791	0.747	1.365
5.930	4333.108	15.819	0.752	1.373
6.040	4305.524	15.851	0.757	1.381
6.150	4277.585	15.884	0.763	1.389
6.259	4246.783	15.919	0.770	1.398
6.369	4215.137	15.955	0.776	1.406
6.478	4181.689	15.993	0.784	1.414
6.588	4150.566	16.028	0.791	1.421
6.698	4120.335	16.062	0.798	1.426
6.807	4088.863	16.097	0.806	1.431
6.917	4056.221	16.133	0.814	1.435
7.027	4023.599	16.168	0.823	1.438
7.136	3990.353	16.203	0.832	1.440
7.246	3954.138	16.240	0.842	1.440
7.356	3920.099	16.275	0.852	1.440
7.465	3887.626	16.309	0.861	1.439
7.575	3853.812	16.345	0.870	1.438
7.685	3819.187	16.382	0.879	1.438
7.794	3784.345	16.422	0.886	1.439
7.904	3749.789	16.462	0.893	1.442
8.014	3714.956	16.505	0.898	1.447
8.123	3675.609	16.557	0.902	1.455
8.233	3641.318	16.604	0.903	1.464
8.343	3610.248	16.647	0.904	1.470
8.452	3576.533	16.692	0.905	1.474
8.562	3540.475	16.738	0.906	1.473
8.671	3504.480	16.781	0.909	1.462
8.781	3471.429	16.815	0.913	1.442
8.891	3438.031	16.845	0.918	1.413
9.000	3404.297	16.873	0.924	1.378
9.110	3370.412	16.900	0.930	1.340
9.220	3335.544	16.927	0.935	1.299
9.329	3301.808	16.955	0.939	1.261
9.439	3269.609	16.983	0.942	1.229
9.549	3236.274	17.015	0.942	1.198
9.658	3203.143	17.048	0.941	1.169
9.768	3170.244	17.082	0.937	1.140
9.878	3141.138	17.113	0.933	1.114
9.987	3110.630	17.146	0.927	1.086
10.097......	3080.182	17.180	0.919	1.054
10.207......	3050.325	17.213	0.909	1.019
10.316......	3022.236	17.244	0.899	0.982
10.426......	2998.492	17.269	0.889	0.947
10.536......	2973.438	17.296	0.877	0.906
10.636......	2940.729	17.329	0.861	0.847
10.800......	2907.066	17.359	0.843	0.779
10.960......	2873.790	17.383	0.825	0.706
11.513......	2755.530	17.401	0.772	0.415
12.061......	2631.510	17.336	0.738	0.103
12.609......	2500.087	17.329	0.710	−0.139
13.158......	2371.128	17.497	0.674	−0.232
13.706......	2243.651	17.731	0.628	−0.289
14.254......	2118.142	17.946	0.574	−0.383
14.802......	2003.111	18.134	0.521	−0.487
		0.6 $M_\odot$		
0.177	14843.15	11.102	−0.021	−0.081
0.195	14366.82	11.205	0.002	−0.028
0.215	13898.19	11.307	0.024	0.020
0.236	13438.67	11.406	0.042	0.058
0.260	12990.45	11.494	0.050	0.080
0.286	12551.13	11.571	0.047	0.083
0.315	12122.97	11.653	0.047	0.086
0.346	11708.42	11.754	0.062	0.106

TABLE 2—*Continued*

Age (Gyr)	$T_{\rm eff}$	M_V	$V-R$	$V-I$
0.381......	11307.59	11.869	0.085	0.141
0.419......	10920.09	11.979	0.102	0.176
0.461......	10547.16	12.078	0.109	0.205
0.507......	10186.72	12.182	0.119	0.233
0.558......	9838.626	12.306	0.143	0.265
0.614......	9503.302	12.438	0.169	0.299
0.675......	9180.363	12.555	0.184	0.334
0.743......	8870.609	12.675	0.208	0.372
0.817......	8573.980	12.829	0.232	0.415
0.899......	8289.914	12.989	0.233	0.447
0.989......	8016.144	13.148	0.239	0.477
1.087......	7751.372	13.279	0.255	0.515
1.196......	7497.785	13.413	0.280	0.560
1.316......	7263.138	13.552	0.313	0.615
1.448......	7054.518	13.682	0.343	0.667
1.592......	6865.231	13.799	0.363	0.709
1.750......	6688.593	13.909	0.376	0.745
1.908......	6536.831	14.006	0.387	0.773
2.066......	6400.791	14.096	0.398	0.798
2.224......	6276.921	14.182	0.412	0.822
2.382......	6158.933	14.269	0.426	0.847
2.540......	6047.026	14.355	0.442	0.876
2.698......	5945.456	14.438	0.458	0.906
2.856......	5848.571	14.521	0.475	0.939
3.013......	5754.589	14.604	0.492	0.973
3.171......	5661.967	14.688	0.509	1.006
3.329......	5572.308	14.770	0.526	1.037
3.487......	5495.300	14.842	0.540	1.061
3.645......	5412.929	14.918	0.556	1.084
3.803......	5328.751	14.996	0.571	1.105
3.961......	5256.578	15.064	0.584	1.122
4.119......	5198.637	15.119	0.594	1.135
4.277......	5134.595	15.181	0.606	1.149
4.434......	5076.699	15.238	0.616	1.162
4.592......	5039.023	15.276	0.623	1.170
4.750......	5005.241	15.310	0.629	1.178
4.908......	4968.070	15.348	0.635	1.187
5.066......	4928.367	15.389	0.642	1.197
5.224......	4894.569	15.424	0.648	1.205
5.382......	4866.263	15.454	0.653	1.213
5.540......	4840.229	15.481	0.658	1.219
5.698......	4815.657	15.508	0.662	1.226
5.856......	4787.219	15.538	0.667	1.234
6.013......	4756.879	15.571	0.673	1.242
6.171......	4725.292	15.605	0.678	1.251
6.329......	4691.195	15.642	0.684	1.261
6.487......	4656.167	15.680	0.691	1.271
6.645......	4622.911	15.717	0.697	1.281
6.803......	4590.039	15.754	0.703	1.291
6.961......	4556.122	15.792	0.709	1.302
7.119......	4517.374	15.836	0.716	1.314
7.277......	4476.308	15.883	0.724	1.327
7.435......	4430.533	15.935	0.733	1.342
7.592......	4381.882	15.991	0.742	1.358
7.750......	4338.176	16.042	0.751	1.371
7.908......	4293.258	16.094	0.760	1.385
8.066......	4243.015	16.151	0.770	1.399
8.224......	4192.182	16.209	0.782	1.412
8.382......	4138.105	16.270	0.794	1.423
8.540......	4087.839	16.325	0.806	1.431
8.698......	4034.327	16.383	0.820	1.437
8.856......	3975.993	16.444	0.836	1.440
9.013......	3920.523	16.501	0.852	1.440
9.171......	3869.634	16.555	0.866	1.439
9.329......	3818.512	16.610	0.879	1.438
9.487......	3763.022	16.673	0.891	1.441
9.645......	3707.316	16.741	0.899	1.448

TABLE 2—*Continued*

Age (Gyr)	$T_{\rm eff}$	M_V	$V-R$	$V-I$
9.803	3647.716	16.821	0.903	1.462
9.961	3592.017	16.897	0.905	1.473
10.119......	3541.780	16.962	0.906	1.473
10.277......	3487.057	17.025	0.911	1.452
10.435......	3434.827	17.073	0.919	1.410
10.592......	3385.314	17.113	0.927	1.357
10.750......	3336.989	17.151	0.935	1.301
10.908......	3284.393	17.195	0.941	1.243
11.066......	3232.627	17.243	0.942	1.195
11.224......	3182.985	17.294	0.939	1.151
11.382......	3131.988	17.348	0.931	1.106
11.540......	3084.716	17.400	0.920	1.059
11.698......	3031.762	17.458	0.902	0.995
11.856......	2981.371	17.513	0.881	0.920
12.013......	2938.354	17.556	0.860	0.842
12.157......	2926.146	17.566	0.853	0.818
12.631......	2776.846	17.633	0.780	0.471
13.026......	2651.867	17.573	0.743	0.151
13.421......	2516.529	17.546	0.713	−0.117
13.815......	2375.756	17.715	0.675	−0.230
14.052......	2293.782	17.866	0.647	−0.262
14.289......	2208.270	18.019	0.613	−0.312
14.526......	2124.721	18.161	0.577	−0.377
14.763......	2041.982	18.296	0.539	−0.451
15.000......	1961.336	18.427	0.501	−0.525
		0.7 $M_\odot$		
0.242	14913.46	11.338	−0.025	−0.090
0.266	14418.44	11.444	−0.000	−0.034
0.292	13931.65	11.549	0.023	0.016
0.322	13450.78	11.652	0.041	0.057
0.354	12976.53	11.744	0.050	0.080
0.389	12507.30	11.825	0.047	0.083
0.428	12044.80	11.915	0.048	0.088
0.471	11613.16	12.026	0.067	0.114
0.519	11211.43	12.141	0.090	0.150
0.570	10830.39	12.246	0.104	0.184
0.628	10466.15	12.343	0.111	0.211
0.690	10116.31	12.447	0.122	0.239
0.759	9780.568	12.570	0.148	0.271
0.835	9462.165	12.694	0.171	0.303
0.919	9166.861	12.800	0.185	0.335
1.011	8894.094	12.905	0.206	0.369
1.112	8643.276	13.031	0.229	0.405
1.223	8414.625	13.158	0.235	0.434
1.346	8197.749	13.282	0.233	0.456
1.480	7980.196	13.406	0.241	0.482
1.628	7769.401	13.508	0.254	0.512
1.791	7559.454	13.617	0.273	0.548
1.970	7338.874	13.743	0.302	0.596
2.168	7121.523	13.877	0.334	0.650
2.382	6908.543	14.009	0.359	0.700
2.597	6708.771	14.132	0.374	0.741
2.812	6526.905	14.247	0.387	0.775
3.0277......	6344.902	14.369	0.404	0.808
3.242	6148.930	14.511	0.428	0.850
3.457	5954.893	14.665	0.457	0.903
3.672	5794.888	14.802	0.485	0.958
3.887	5661.806	14.922	0.509	1.006
4.102	5540.478	15.033	0.532	1.047
4.317	5440.402	15.126	0.551	1.077
4.532	5347.771	15.211	0.567	1.101
4.747	5263.535	15.290	0.583	1.121
4.961	5208.143	15.343	0.593	1.133
5.176	5163.271	15.386	0.601	1.143
5.391	5123.507	15.425	0.608	1.152
5.606	5075.434	15.472	0.616	1.162

TABLE 2—*Continued*

Age (Gyr)	T_{eff}	M_V	$V-R$	$V-I$
5.821	5041.870	15.506	0.622	1.170
6.036	5007.492	15.540	0.628	1.178
6.251	4971.232	15.577	0.635	1.186
6.466	4929.016	15.620	0.642	1.197
6.681	4882.581	15.668	0.650	1.208
6.896	4834.008	15.719	0.659	1.221
7.111	4785.358	15.770	0.668	1.234
7.326	4729.533	15.830	0.678	1.250
7.540	4671.635	15.892	0.688	1.267
7.755	4611.470	15.958	0.699	1.285
7.970	4549.051	16.028	0.710	1.304
8.185	4480.140	16.106	0.723	1.326
8.400	4409.324	16.186	0.737	1.349
8.615	4337.092	16.269	0.751	1.372
8.830	4258.954	16.359	0.767	1.395
9.045	4168.789	16.461	0.787	1.417
9.260	4088.764	16.550	0.806	1.431
9.475	4001.292	16.644	0.829	1.439
9.690	3906.217	16.742	0.856	1.440
9.904	3812.124	16.842	0.880	1.438
10.119......	3716.670	16.955	0.898	1.446
10.334......	3627.638	17.074	0.904	1.467
10.549......	3531.468	17.201	0.907	1.471
10.764......	3446.799	17.289	0.917	1.421
10.979......	3355.300	17.363	0.932	1.322
11.194......	3265.593	17.438	0.942	1.225
11.409......	3174.044	17.529	0.938	1.144
11.624......	3076.448	17.635	0.918	1.050
11.839......	2987.764	17.732	0.884	0.930
12.054......	2909.311	17.808	0.844	0.784
12.269......	2829.705	17.854	0.803	0.603
12.483......	2748.437	17.849	0.770	0.397
12.698......	2660.447	17.803	0.745	0.172
12.913......	2571.178	17.765	0.725	−0.025
13.128......	2485.013	17.790	0.706	−0.157
13.343......	2406.382	17.888	0.685	−0.216
13.558......	2323.072	18.036	0.658	−0.250
13.773......	2244.531	18.180	0.628	−0.289
13.988......	2168.899	18.311	0.597	−0.341
14.203......	2101.429	18.424	0.566	−0.397
14.418......	2031.461	18.539	0.534	−0.460
14.633......	1964.061	18.648	0.503	−0.523
14.847......	1849.694	18.837	0.451	−0.623
15.062......	1695.594	19.108	0.388	−0.721
		0.8 $M_\odot$		
0.347	14827.20	11.609	−0.021	−0.080
0.382	14318.27	11.718	0.004	−0.023
0.420	13825.06	11.824	0.027	0.026
0.463	13339.78	11.925	0.044	0.065
0.509	12852.63	12.016	0.049	0.082
0.560	12350.56	12.103	0.046	0.083
0.616	11862.71	12.207	0.054	0.096
0.677	11417.10	12.330	0.078	0.131
0.745	11005.48	12.447	0.099	0.169
0.820	10629.32	12.546	0.108	0.199
0.902	10304.04	12.636	0.115	0.224
0.992	10025.96	12.724	0.128	0.247
1.091	9771.160	12.820	0.149	0.272
1.200	9515.225	12.920	0.169	0.298
1.321	9257.279	13.014	0.180	0.324
1.453	8984.493	13.114	0.200	0.360
1.598	8644.942	13.274	0.229	0.405
1.758	8304.034	13.465	0.233	0.445
1.934	8015.659	13.631	0.239	0.478
2.127	7761.395	13.755	0.255	0.513
2.340	7494.761	13.895	0.280	0.561

TABLE 2—*Continued*

Age (Gyr)	T_{eff}	M_V	$V-R$	$V-I$
2.574	7179.554	14.082	0.326	0.636
2.831	6841.066	14.291	0.365	0.714
3.112	6563.108	14.465	0.385	0.768
3.393	6299.760	14.642	0.409	0.817
3.673	6051.851	14.826	0.442	0.875
3.954	5847.120	14.997	0.475	0.940
4.235	5678.406	15.147	0.506	1.000
4.515	5530.119	15.282	0.534	1.050
4.796	5441.573	15.364	0.550	1.076
5.077	5337.787	15.460	0.569	1.103
5.358	5273.102	15.521	0.581	1.118
5.638	5218.652	15.573	0.591	1.131
5.919	5162.725	15.626	0.601	1.143
6.200	5112.481	15.675	0.610	1.154
6.480	5060.477	15.726	0.619	1.166
6.761	5003.271	15.783	0.629	1.179
7.042	4933.282	15.853	0.641	1.196
7.323	4859.581	15.929	0.654	1.214
7.603	4780.419	16.012	0.668	1.236
7.884	4693.962	16.104	0.684	1.260
8.165	4595.114	16.212	0.702	1.290
8.445	4486.675	16.334	0.722	1.324
8.726	4363.449	16.475	0.746	1.363
9.007	4234.122	16.623	0.772	1.401
9.287	4099.712	16.773	0.803	1.430
9.568	3946.773	16.936	0.845	1.440
9.849	3783.074	17.110	0.887	1.439
10.130......	3615.818	17.325	0.904	1.469
10.410......	3446.500	17.524	0.917	1.421
10.691......	3292.242	17.649	0.940	1.251
10.972......	3116.982	17.825	0.928	1.092
11.252......	2936.514	18.018	0.859	0.838
11.533......	2772.573	18.090	0.778	0.460
11.814......	2589.899	18.003	0.729	0.011
12.095......	2432.296	18.084	0.692	−0.202
12.375......	2310.931	18.293	0.653	−0.255
12.656......	2233.273	18.434	0.624	−0.296
12.937......	2072.729	18.705	0.553	−0.423
13.217......	2218.510	18.456	0.618	−0.305
		0.9 $M_\odot$		
0.484	14710.72	11.903	−0.015	−0.066
0.532	14165.67	12.019	0.012	−0.006
0.586	13649.91	12.130	0.034	0.042
0.644	13163.69	12.228	0.048	0.074
0.709	12697.61	12.310	0.048	0.083
0.779	12290.85	12.381	0.046	0.083
0.857	11933.71	12.458	0.052	0.092
0.943	11587.54	12.548	0.068	0.116
1.038	11255.32	12.644	0.088	0.146
1.141	10931.75	12.734	0.102	0.175
1.256	10560.71	12.830	0.109	0.204
1.381	10125.64	12.956	0.122	0.238
1.519	9716.503	13.107	0.153	0.277
1.671	9381.225	13.234	0.175	0.311
1.839	9062.250	13.348	0.193	0.350
2.023	8757.424	13.478	0.219	0.388
2.225	8434.758	13.654	0.235	0.432
2.447	8057.051	13.869	0.237	0.472
2.692	7663.667	14.067	0.263	0.529
2.961	7291.167	14.276	0.309	0.608
3.258	6939.941	14.492	0.356	0.693
3.583	6595.978	14.705	0.382	0.762
3.939	6267.617	14.925	0.413	0.824
4.294	5977.769	15.146	0.453	0.896
4.649	5737.974	15.353	0.495	0.979
5.005	5560.277	15.515	0.528	1.041

TABLE 2—*Continued*

Age (Gyr)	T_{eff}	M_V	$V-R$	$V-I$
5.360	5475.945	15.593	0.544	1.067
5.715	5342.835	15.715	0.568	1.102
6.070	5254.616	15.798	0.584	1.123
6.426	5174.179	15.874	0.599	1.141
6.781	5091.327	15.954	0.613	1.159
7.136	4992.645	16.051	0.631	1.181
7.491	4857.250	16.188	0.655	1.215
7.847	4712.883	16.341	0.681	1.255
8.202	4532.780	16.539	0.713	1.309
8.557	4326.192	16.774	0.753	1.375
8.912	4081.319	17.050	0.808	1.432
9.268	3789.772	17.359	0.885	1.439
9.623	3471.486	17.757	0.913	1.442
9.978	3155.130	18.039	0.935	1.127
10.334......	2889.188	18.312	0.834	0.741
10.689......	2700.601	18.315	0.755	0.273
11.044......	2571.432	18.253	0.725	−0.025
11.399......	2374.063	18.431	0.675	−0.231
11.755......	1855.517	19.317	0.453	−0.618

wild deviations from blackbody colors are evident in this diagram as the oldest and coolest white dwarfs get dramatically bluer in the $V-I$ color and somewhat bluer in $V-R$.

3. WHITE DWARF ISOCHRONES

When white dwarfs are observed in an open or globular cluster, it is not strictly correct to compare their location in the cluster color-magnitude diagram with a theoretical cooling sequence of some mass as has generally been done in the past (e.g., Richer et al. 1995, 1997, 1998; Cool, Piotto, & King 1996; Renzini et al.1996). The reason for this is that the oldest white dwarfs in these clusters have evolved from the most massive stars originally in the cluster (up to the maximum mass that produces white dwarfs), and because more massive progenitors produce more massive remnants, the older white dwarfs should be more massive. This has generally not been a problem with the clusters observed thus far, since the range in white dwarf masses has been relatively small, and since the initial mass function of clusters is expected to yield fewer massive stars and, hence, few massive white dwarfs. However, when large ground-based telescopes or *HST* eventually penetrate the termination point of the white dwarf cooling sequence in a globular cluster, and thus cover a wide range in white dwarf masses, it will be extremely important to have white dwarf isochrones ready to interpret the data as opposed to just cooling sequences for some mass.

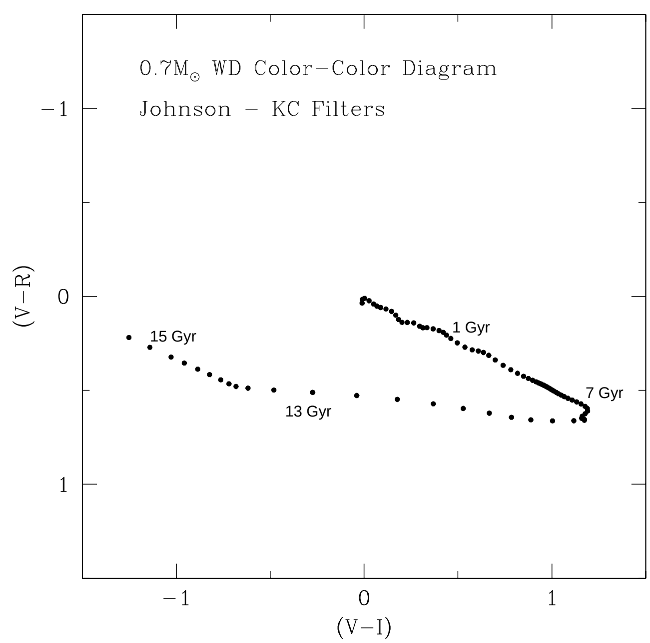

FIG. 3.—$V-R$, $V-I$ color-color diagram for 0.7 $M_\odot$ hydrogen-rich white dwarfs in Johnson-Kron/Cousins filters. When H_2 opacity becomes important for ages older than about 8 Gyr, the colors strongly deviate from those of blackbodies.

For these reasons we have calculated isochrones for white dwarfs in star clusters with a wide range in age. All the isochrones shown are derived from solar metallicity models. The isochrones were constructed by (1) beginning with a white dwarf mass of 0.9 $M_\odot$, the maximum mass model that we had available; (2) using an initial-final mass relation constructed from Herwig's (1995) data at the high-mass end mated to the results from Gibson et al. (1999) for M67 and M4 at the low-mass end to determine the mass of the main-sequence progenitor. (In using these data we are mixing metal-rich and metal-poor relations, however, and that is all that can be done for the moment if we wish to stick with empirical results.) (3) The stellar evolutionary models of Dominguez et al. (1999) were then employed to determine the lifetime of the main-sequence star (A_{ms}) up to the end of the asymptotic giant branch (AGB). (4) The age of the white dwarf was then simply $T_{iso} - A_{ms}$, where $T_{iso}$was the age of the isochrone that we calculated. (5) The absolute magnitude and color of the white dwarf of interest was then obtained from the cooling model for a white dwarf of mass 0.9 $M_\odot$ and age $T_{iso} - A_{ms}$. (6) We then decremented the mass, interpolating within the models, and repeated the process until a white dwarf mass of 0.5 $M_\odot$ was reached (the minimum white dwarf mass model available), at which point the calculations were halted.

Figure 4 illustrates these isochrones for a range of ages likely to be of interest in any application. The hook to the blue in the isochrones for ages less than about 7 Gyr is not due to the effects of H_2 opacity (these stars are too hot for H_2 to form), but is caused by the fact that the white dwarfs at the bottoms of these curves come from massive main-sequence stars. These produce heavier white dwarfs that follow cooling sequences that lie below those of lighter degenerates (more massive white dwarfs have smaller radii, thus are less luminous at a given temperature). The effect of the H_2 opacity is seen only in those older than 8 Gyr.

Figure 5 illustrates some detail for the 11 Gyr isochrone, showing the mass of the white dwarf itself and that of its main-sequence precursor. Tables 3 and 4 list selected isochrones in both the Johnson-Kron/Cousins system and in the *HST* filters. The columns in these tables are the white dwarf mass ($M_\odot$), the mass of the progenitor ($M_\odot$), and T_{eff}, M_V, $V-R$, and $V-I$ of the white dwarf. Details of the initial-final mass relation that we used can be obtained from these tables.

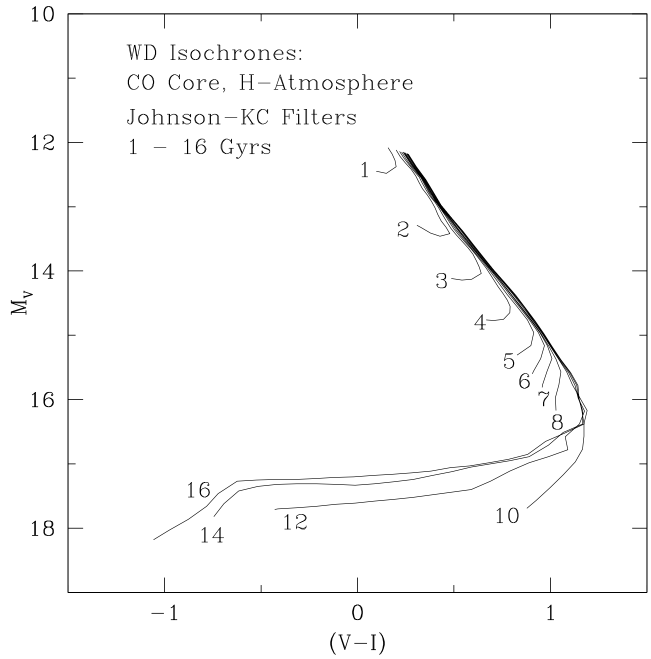

FIG. 4.—White dwarf isochrones in Johnson-Kron/Cousins filters. Isochrone ages are indicated.

4. CLUSTER WHITE DWARF LUMINOSITY FUNCTIONS

The white dwarf luminosity function in a star cluster, in the absence of dynamical evolution, contains information on the initial mass function (IMF) and age of the cluster. In fact, it will eventually be possible to use the white dwarf luminosity function in an old star cluster (e.g., a globular cluster) to extend the observed main-sequence mass function up to massive stars that many billions of years ago evolved into white dwarfs (Richer et al. 1997).

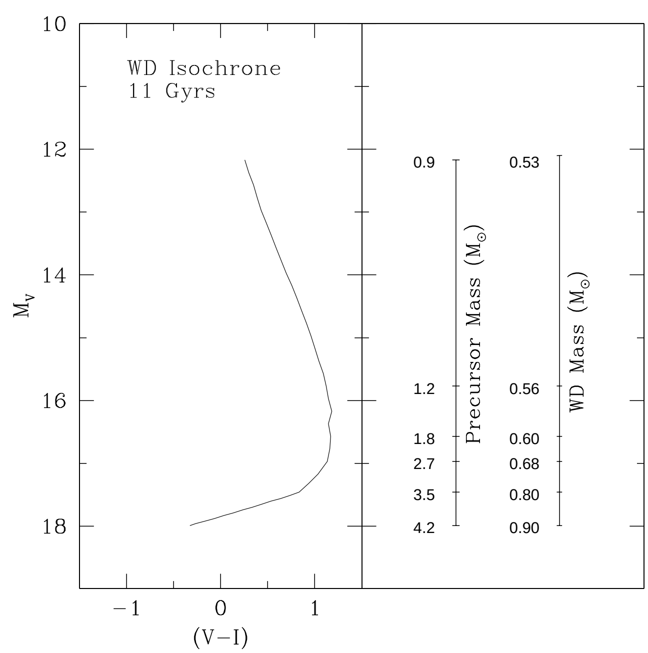

FIG. 5.—Details of the 11 Gyr white dwarf isochrone indicating white dwarf and precursor masses.

TABLE 3

WHITE DWARF ISOCHRONES: C-O CORES, HYDROGEN ATMOSPHERES, JOHNSON-KRON/COUSINS FILTERS

WD Mass ($M_\odot$)	MS Mass ($M_\odot$)	WD $T_{\rm eff}$ (K)	M_V	$V-R$	$V-I$
		1 Gyr			
0.6590077......	2.4217548	10911	12.08	0.0988	0.1583
0.6674698......	2.5204812	10623	12.18	0.1075	0.1788
0.6795970......	2.6579666	10367	12.28	0.1206	0.1940
0.6980980......	2.8661022	10163	12.38	0.1358	0.2002
0.8300148......	3.7059693	11038	12.48	0.0981	0.1500
0.8961619......	4.1469267	11840	12.45	0.0642	0.0999
0.9000000......	4.1818182	11886	12.45	0.0620	0.0975
		2 Gyr			
0.5930743......	1.6525180	10178	12.12	0.1346	0.2004
0.5944701......	1.6688181	9884	12.22	0.1399	0.2214
0.5960071......	1.6867498	9608	12.32	0.1408	0.2480
0.5978213......	1.7079153	9334	12.42	0.1488	0.2781
0.5999605......	1.7328720	9065	12.52	0.1612	0.3023
0.6021966......	1.7589603	8834	12.62	0.1662	0.3181
0.6043859......	1.7845019	8647	12.72	0.1666	0.3371
0.6067719......	1.8123393	8479	12.82	0.1718	0.3618
0.6095369......	1.8445967	8313	12.92	0.1787	0.3847
0.6128131......	1.8828200	8152	13.02	0.1857	0.4020
0.6176407......	1.9391419	7973	13.12	0.1975	0.4154
0.6241702......	2.0153186	7802	13.22	0.2107	0.4337
0.6337733......	2.1273555	7656	13.32	0.2250	0.4589
0.6510722......	2.3291762	7560	13.42	0.2312	0.4782
0.7356074......	3.1506320	7870	13.46	0.2041	0.4281
0.8192829......	3.6428404	8373	13.41	0.1783	0.3781
0.8772036......	3.9835504	8833	13.32	0.1702	0.3280
0.9000000......	4.1818182	9011	13.29	0.1661	0.3084
		4 Gyr			
0.5597911......	1.2642281	9857	12.15	0.1410	0.2283
0.5601979......	1.2689759	9569	12.25	0.1481	0.2502
0.5606154......	1.2738463	9308	12.35	0.1505	0.2736
0.5610509......	1.2789266	9065	12.45	0.1545	0.3003
0.5615260......	1.2844702	8829	12.55	0.1661	0.3290
0.5620330......	1.2903853	8612	12.65	0.1735	0.3464
0.5625564......	1.2964909	8411	12.75	0.1793	0.3646
0.5631193......	1.3030588	8216	12.85	0.1858	0.3847
0.5637120......	1.3099730	8038	12.95	0.1959	0.4109
0.5643516......	1.3174357	7866	13.05	0.2064	0.4345
0.5650667......	1.3257784	7692	13.15	0.2181	0.4576
0.5659418......	1.3359878	7508	13.25	0.2309	0.4784
0.5669364......	1.3475916	7327	13.35	0.2439	0.5017
0.5680758......	1.3608843	7150	13.45	0.2580	0.5278
0.5694031......	1.3763689	6979	13.55	0.2741	0.5566
0.5710011......	1.3950127	6816	13.65	0.2905	0.5865
0.5729676......	1.4179555	6662	13.75	0.3025	0.6145
0.5754015......	1.4463511	6515	13.85	0.3108	0.6404
0.5785170......	1.4826978	6375	13.95	0.3184	0.6647
0.5825599......	1.5298655	6242	14.05	0.3291	0.6872
0.5877787......	1.5907517	6118	14.15	0.3430	0.7084
0.5946285......	1.6706655	6002	14.25	0.3590	0.7298
0.6041758......	1.7820514	5907	14.35	0.3746	0.7538
0.6174150......	1.9365081	5827	14.45	0.3861	0.7761
0.6373649......	2.1692568	5763	14.55	0.3939	0.7910
0.6794081......	2.6558415	5757	14.65	0.3946	0.7903
0.7685893......	3.3446428	5896	14.75	0.3753	0.7563
0.8477993......	3.8105842	6136	14.77	0.3438	0.7061
0.9000000......	4.1818182	6331	14.76	0.3172	0.6675
		6 Gyr			
0.5460852......	1.1043257	9725	12.16	0.1436	0.2410

TABLE 3—*Continued*

WD Mass ($M_\odot$)	MS Mass ($M_\odot$)	WD $T_{\rm eff}$ (K)	M_V	$V-R$	$V-I$
0.5465165......	1.1093589	9186	12.36	0.1543	0.2845
0.5469649......	1.1145911	8735	12.56	0.1675	0.3380
0.5474689......	1.1204708	8328	12.76	0.1822	0.3754
0.5480539......	1.1272956	7948	12.96	0.2017	0.4193
0.5487206......	1.1350742	7595	13.16	0.2270	0.4701
0.5495923......	1.1452432	7232	13.36	0.2528	0.5188
0.5506661......	1.1577708	6884	13.56	0.2780	0.5716
0.5520665......	1.1741095	6569	13.76	0.3085	0.6276
0.5539508......	1.1960931	6282	13.96	0.3338	0.6825
0.5565609......	1.2265437	6018	14.16	0.3595	0.7354
0.5600036......	1.2667085	5771	14.36	0.3896	0.7859
0.5644768......	1.3188961	5547	14.56	0.4189	0.8384
0.5709858......	1.3948344	5342	14.76	0.4457	0.8915
0.5814442......	1.5168488	5150	14.96	0.4658	0.9380
0.6056783......	1.7995797	4998	15.16	0.4802	0.9698
0.7308424......	3.1226025	5098	15.36	0.4703	0.9500
0.8765881......	3.9799298	5277	15.56	0.4532	0.9125
0.9000000......	4.1818182	5304	15.60	0.4506	0.9069
		10 Gyr			
0.5323001......	0.9434999	9592	12.17	0.1463	0.2535
0.5325008......	0.9458429	9063	12.37	0.1581	0.2958
0.5326989......	0.9481537	8639	12.57	0.1691	0.3471
0.5329189......	0.9507210	8246	12.77	0.1844	0.3863
0.5331854......	0.9538293	7863	12.97	0.2064	0.4278
0.5334888......	0.9573690	7502	13.17	0.2355	0.4814
0.5338629......	0.9617344	7142	13.37	0.2618	0.5348
0.5343096......	0.9669459	6801	13.57	0.2823	0.5867
0.5348562......	0.9733223	6487	13.77	0.3123	0.6404
0.5355448......	0.9813563	6202	13.97	0.3429	0.6958
0.5364376......	0.9917722	5943	14.17	0.3714	0.7528
0.5375502......	1.0047529	5697	14.37	0.4006	0.8071
0.5389025......	1.0205297	5466	14.57	0.4280	0.8590
0.5405271......	1.0394832	5250	14.77	0.4536	0.9094
0.5431643......	1.0702500	5044	14.97	0.4767	0.9573
0.5466219......	1.1105885	4848	15.17	0.4977	0.9986
0.5512312......	1.1643639	4663	15.37	0.5198	1.0416
0.5585138......	1.2493272	4490	15.57	0.5420	1.0850
0.5678283......	1.3579974	4319	15.77	0.5654	1.1143
0.5797717......	1.4973369	4164	15.97	0.5858	1.1520
0.6005202......	1.7394026	4029	16.17	0.6055	1.1905
0.6382178......	2.1792073	3910	16.37	0.6232	1.1735
0.7150974......	3.0299846	3887	16.57	0.6270	1.1746
0.7751725......	3.3833678	3830	16.77	0.6348	1.1660
0.8138217......	3.6107156	3731	16.97	0.6402	1.1285
0.8370207......	3.7471805	3604	17.17	0.6431	1.0667
0.8608766......	3.8875091	3473	17.37	0.6450	0.9990
0.8852748......	4.0479524	3336	17.57	0.6460	0.9257
0.9000000......	4.1818182	3252	17.69	0.6462	0.8773
		12 Gyr			
0.5280815......	0.8942830	9551	12.17	0.1471	0.2574
0.5282357......	0.8960833	9026	12.37	0.1592	0.2993
0.5283858......	0.8978338	8609	12.57	0.1697	0.3500
0.5285520......	0.8997735	8221	12.77	0.1851	0.3897
0.5287552......	0.9021440	7837	12.97	0.2077	0.4304
0.5289877......	0.9048563	7475	13.17	0.2379	0.4847
0.5292703......	0.9081535	7114	13.37	0.2646	0.5397
0.5296050......	0.9120583	6775	13.57	0.2838	0.5914
0.5300088......	0.9167697	6462	13.77	0.3133	0.6443
0.5305109......	0.9226270	6179	13.97	0.3451	0.6995
0.5311526......	0.9301137	5921	14.17	0.3747	0.7574
0.5319464......	0.9393746	5678	14.37	0.4037	0.8128
0.5329101......	0.9506175	5446	14.57	0.4306	0.8648
0.5340257......	0.9636335	5228	14.78	0.4557	0.9144
0.5358703......	0.9851539	5022	14.98	0.4789	0.9617

TABLE 3—*Continued*

WD Mass ($M_\odot$)	MS Mass ($M_\odot$)	WD $T_{\rm eff}$ (K)	M_V	$V-R$	$V-I$
0.5382041......	1.0123816	4824	15.18	0.5003	1.0032
0.5409172......	1.0440337	4637	15.38	0.5231	1.0475
0.5447784......	1.0890818	4459	15.58	0.5461	1.0953
0.5489003......	1.1371699	4278	15.78	0.5711	1.1281
0.5534137......	1.1898269	4095	15.98	0.5967	1.1435
0.5580600......	1.2440329	3935	16.18	0.6150	1.1764
0.5644956......	1.3191151	3756	16.38	0.6343	1.1502
0.5742476......	1.4328886	3572	16.58	0.6494	1.0773
0.5846065......	1.5537420	3447	16.78	0.6527	1.0902
0.6009637......	1.7445763	3345	16.89	0.6620	0.9902
0.6140871......	1.8976825	3265	16.98	0.6561	0.8901
0.6358945......	2.1521029	3178	17.11	0.6432	0.7900
0.6699392......	2.5492901	3095	17.27	0.6256	0.6899
0.7033720......	2.9254347	3034	17.40	0.6098	0.5895
0.7124349......	3.0143231	2990	17.44	0.5982	0.4893
0.7220160......	3.0706821	2947	17.48	0.5873	0.3893
0.7315430......	3.1267236	2904	17.52	0.5765	0.2893
0.7410831......	3.1828418	2861	17.55	0.5658	0.1888
0.7506197......	3.2389394	2818	17.58	0.5552	−0.0882
0.7601411......	3.2949479	2775	17.61	0.5447	−0.0123
0.7696420......	3.3508353	2732	17.63	0.5343	−0.1124
0.7791937......	3.4070216	2688	17.66	0.5239	−0.2129
0.7887459......	3.4632114	2645	17.68	0.5136	−0.3130
0.7983199......	3.5195287	2601	17.70	0.5033	−0.4131
0.7999000......	3.5288235	2594	17.71	0.5016	−0.4295
		14 Gyr			
0.5247511......	0.8554292	9519	12.18	0.1477	0.2605
0.5248749......	0.8568733	8996	12.38	0.1601	0.3021
0.5249939......	0.8582622	8585	12.58	0.1701	0.3522
0.5251257......	0.8597995	8202	12.78	0.1856	0.3924
0.5252877......	0.8616899	7818	12.98	0.2086	0.4324
0.5254741......	0.8638640	7453	13.18	0.2397	0.4872
0.5256982......	0.8664793	7093	13.38	0.2668	0.5434
0.5259620......	0.8695561	6756	13.58	0.2850	0.5950
0.5262772......	0.8732345	6443	13.78	0.3140	0.6473
0.5266652......	0.8777604	6162	13.98	0.3466	0.7023
0.5271570......	0.8834984	5905	14.18	0.3771	0.7608
0.5277630......	0.8905681	5663	14.38	0.4062	0.8171
0.5284941......	0.8990983	5431	14.58	0.4326	0.8691
0.5293290......	0.9088379	5213	14.78	0.4571	0.9181
0.5307185......	0.9250493	5007	14.98	0.4804	0.9648
0.5324652......	0.9454268	4808	15.18	0.5021	1.0064
0.5343252......	0.9671271	4620	15.38	0.5252	1.0513
0.5368506......	0.9965900	4440	15.58	0.5485	1.1012
0.5393552......	1.0258109	4257	15.78	0.5738	1.1369
0.5420318......	1.0570376	4067	15.98	0.6011	1.1457
0.5445807......	1.0867754	3898	16.18	0.6207	1.1687
0.5474384......	1.1201149	3731	16.38	0.6358	1.1670
0.5504041......	1.1547140	3580	16.51	0.6475	1.0669
0.5544903......	1.2023866	3405	16.71	0.6468	0.9900
0.5591135......	1.2563238	3257	16.89	0.6364	0.8898
0.5616498......	1.2859142	3192	16.95	0.6300	0.7897
0.5645764......	1.3200579	3128	17.00	0.6230	0.6896
0.5680684......	1.3607979	3060	17.05	0.6121	0.5895
0.5723963......	1.4112904	2989	17.12	0.5966	0.4894
0.5764596......	1.4586957	2929	17.18	0.5810	0.3892
0.5829595......	1.5345271	2876	17.24	0.5658	0.2892
0.5882234......	1.5959402	2835	17.28	0.5540	0.1890
0.5934374......	1.6567700	2795	17.31	0.5417	0.0886
0.5993974......	1.7263031	2758	17.33	0.5306	−0.0114
0.6037684......	1.7772976	2721	17.32	0.5234	−0.1121
0.6084082......	1.8314286	2682	17.31	0.5161	−0.2130
0.6139288......	1.8958362	2641	17.31	0.5087	−0.3160
0.6208082......	1.9760957	2593	17.32	0.5021	−0.4163
0.6303409......	2.0873110	2536	17.35	0.4948	−0.5166

TABLE 3—*Continued*

WD Mass ($M_{\odot}$)	MS Mass ($M_{\odot}$)	WD $T_{\rm eff}$ (K)	M_V	$V-R$	$V-I$
0.6443202......	2.2504019	2466	17.42	0.4858	−0.6167
0.6703429......	2.5538571	2363	17.62	0.4708	−0.6929
0.6999000......	2.8863750	2277	17.82	0.4529	−0.7450
		16 Gyr			
0.5220250......	0.8236246	9493	12.18	0.1483	0.2631
0.5221275......	0.8248206	8972	12.38	0.1609	0.3044
0.5222252......	0.8259609	8565	12.58	0.1705	0.3541
0.5223332......	0.8272210	8186	12.78	0.1860	0.3946
0.5224668......	0.8287794	7801	12.98	0.2093	0.4341
0.5226211......	0.8305793	7436	13.18	0.2411	0.4893
0.5228051......	0.8327258	7076	13.38	0.2686	0.5465
0.5230206......	0.8352406	6740	13.58	0.2860	0.5980
0.5232763......	0.8382233	6428	13.78	0.3145	0.6498
0.5235888......	0.8418688	6148	13.98	0.3478	0.7046
0.5239825......	0.8464631	5892	14.18	0.3789	0.7635
0.5244664......	0.8521081	5651	14.38	0.4081	0.8204
0.5250479......	0.8588926	5420	14.58	0.4343	0.8726
0.5257050......	0.8665578	5201	14.78	0.4583	0.9212
0.5268077......	0.8794236	4995	14.98	0.4816	0.9673
0.5281924......	0.8955785	4796	15.18	0.5035	1.0088
0.5295801......	0.9117678	4607	15.38	0.5268	1.0541
0.5314036......	0.9330417	4427	15.58	0.5502	1.1053
0.5331566......	0.9534935	4244	15.78	0.5756	1.1430
0.5350151......	0.9751758	4050	15.98	0.6037	1.1488
0.5367307......	0.9951920	3876	16.18	0.6245	1.1641
0.5384677......	1.0154570	3715	16.38	0.6378	1.1747
0.5402541......	1.0362977	3563	16.52	0.6490	1.0747
0.5420950......	1.0577747	3423	16.65	0.6494	0.9746
0.5445959......	1.0869519	3259	16.85	0.6350	0.8830
0.5461626......	1.1052299	3177	16.93	0.6252	0.7829
0.5473633......	1.1192380	3117	16.98	0.6161	0.6826
0.5487182......	1.1350458	3058	17.03	0.6065	0.5824
0.5501318......	1.1515376	3000	17.06	0.5955	0.4820
0.5516765......	1.1695592	2941	17.11	0.5818	0.3819
0.5531557......	1.1868170	2886	17.14	0.5690	0.2818
0.5545879......	1.2035250	2837	17.16	0.5567	0.1817
0.5566888......	1.2280360	2789	17.18	0.5456	0.0810
0.5581895......	1.2455444	2746	17.20	0.5356	−0.0198
0.5597393......	1.2636253	2702	17.21	0.5253	−0.1199
0.5614798......	1.2839314	2656	17.23	0.5149	−0.2207
0.5633927......	1.3062481	2609	17.24	0.5067	−0.3218
0.5654198......	1.3298975	2563	17.24	0.4985	−0.4225
0.5679091......	1.3589398	2508	17.25	0.4906	−0.5231
0.5707399......	1.3919654	2450	17.27	0.4816	−0.6232
0.5778270......	1.4746480	2324	17.46	0.4621	−0.7232
0.5851052......	1.5595602	2221	17.66	0.4346	−0.7830
0.5948751......	1.6735429	2111	17.86	0.3916	−0.8767
0.6050298......	1.7920148	2013	18.03	0.3464	−0.9767
0.6136000......	1.8920000	1931	18.18	0.3100	−1.0573

White dwarf luminosity functions for different ages were constructed in the following manner: (1) The isochrones and the initial-final mass relation were used to set the maximum and minimum main-sequence masses for a cluster of a particular age. (2) Based on the IMF used, a random extraction of a main-sequence mass in this range was made, yielding a white dwarf mass from the initial-final mass relation. (3) From the isochrones, the M_V of this white dwarf was then obtained. (4) This was repeated 1000 times and eventually renormalized to 100 white dwarfs for each cluster.

TABLE 4

WHITE DWARF ISOCHRONES: C-O CORES, HYDROGEN ATMOSPHERES, *HST* FILTERS

WD Mass ($M_{\odot}$)	MS Mass ($M_{\odot}$)	WD $T_{\rm eff}$ (K)	M_V	$V-R$	$V-I$
		1 Gyr			
0.6590077......	2.4217045	10911	12.12	0.0963	0.1729
0.6666745......	2.5112022	10646	12.22	0.1112	0.1971
0.6779121......	2.6390110	10396	12.32	0.1207	0.2191
0.6964207......	2.8472329	10176	12.42	0.1220	0.2356
0.7538974......	3.2582202	10369	12.52	0.1245	0.2211
0.8259034......	3.6817845	10988	12.52	0.0936	0.1711
0.8746027......	3.9682513	11584	12.49	0.0668	0.1210
0.9000000......	4.1818182	11886	12.47	0.0544	0.0963
		2 Gyr			
0.5930743......	1.6525297	10178	12.17	0.1223	0.2354
0.5943966......	1.6679598	9898	12.27	0.1389	0.2607
0.5957853......	1.6841618	9645	12.37	0.1579	0.2858
0.5973714......	1.7026667	9396	12.47	0.1751	0.3116
0.5993549......	1.7258075	9133	12.57	0.1877	0.3400
0.6016215......	1.7522503	8886	12.67	0.2072	0.3708
0.6036453......	1.7758617	8709	12.77	0.2223	0.3971
0.6059239......	1.8024451	8535	12.87	0.2297	0.4189
0.6085466......	1.8330437	8366	12.97	0.2295	0.4369
0.6117063......	1.8699067	8204	13.07	0.2327	0.4548
0.6160999......	1.9211651	8026	13.17	0.2426	0.4783
0.6219882......	1.9898622	7852	13.27	0.2585	0.5064
0.6298521......	2.0816081	7704	13.37	0.2750	0.5355
0.6422131......	2.2258190	7592	13.47	0.2874	0.5595
0.7268909......	3.0993580	7824	13.54	0.2578	0.5094
0.7947399......	3.4984698	8193	13.52	0.2374	0.4592
0.8449665......	3.7939204	8574	13.45	0.2168	0.4092
0.8973798......	4.1579986	8990	13.38	0.1983	0.3592
0.9000000......	4.1818182	9011	13.37	0.1975	0.3568
		4 Gyr			
0.5597911......	1.2642288	9857	12.21	0.1429	0.2653
0.5601795......	1.2687610	9581	12.31	0.1598	0.2928
0.5605712......	1.2733303	9333	12.41	0.1786	0.3199
0.5609692......	1.2779742	9106	12.51	0.1957	0.3464
0.5614226......	1.2832633	8879	12.61	0.2022	0.3709
0.5619195......	1.2890612	8660	12.71	0.2135	0.3971
0.5624375......	1.2951036	8454	12.81	0.2270	0.4240
0.5629850......	1.3014917	8259	12.91	0.2425	0.4523
0.5635564......	1.3081583	8084	13.01	0.2491	0.4764
0.5641749......	1.3153739	7911	13.11	0.2575	0.5011
0.5648408......	1.3231422	7744	13.21	0.2678	0.5267
0.5656700......	1.3328165	7564	13.31	0.2826	0.5575
0.5666039......	1.3437116	7385	13.41	0.2983	0.5896
0.5676505......	1.3559223	7210	13.51	0.3146	0.6226
0.5688071......	1.3694165	7050	13.61	0.3341	0.6578
0.5701672......	1.3852844	6898	13.71	0.3538	0.6930
0.5718050......	1.4043916	6751	13.81	0.3722	0.7273
0.5738363......	1.4280905	6606	13.91	0.3879	0.7605
0.5764118......	1.4581377	6466	14.01	0.4010	0.7922
0.5797481......	1.4970612	6331	14.11	0.4133	0.8211
0.5840709......	1.5474936	6203	14.21	0.4263	0.8466
0.5896278......	1.6123244	6082	14.31	0.4401	0.8704
0.5969144......	1.6973352	5975	14.41	0.4545	0.8978
0.6068493......	1.8132419	5887	14.51	0.4706	0.9300
0.6206434......	1.9741725	5813	14.61	0.4849	0.9575
0.6424870......	2.2290149	5755	14.71	0.4944	0.9724
0.6972379......	2.8564263	5781	14.81	0.4875	0.9629
0.7898159......	3.4695056	5935	14.90	0.4614	0.9128
0.8467203......	3.8042371	6132	14.90	0.4350	0.8625
0.9000000......	4.1818182	6331	14.88	0.4072	0.8124

TABLE 4—*Continued*

WD Mass ($M_\odot$)	MS Mass ($M_\odot$)	WD T_{eff} (K)	M_V	$V-R$	$V-I$
		6 Gyr			
0.5460852......	1.1043264	9725	12.22	0.1514	0.2782
0.5464968......	1.1091299	9207	12.42	0.1870	0.3343
0.5469215......	1.1140845	8776	12.62	0.2091	0.3844
0.5474209......	1.1199103	8361	12.82	0.2286	0.4339
0.5479915......	1.1265670	7985	13.02	0.2539	0.4894
0.5486267......	1.1339779	7642	13.22	0.2786	0.5458
0.5494340......	1.1433972	7290	13.42	0.3115	0.6116
0.5504228......	1.1549328	6957	13.62	0.3433	0.6780
0.5516499......	1.1692483	6651	13.82	0.3799	0.7449
0.5533000......	1.1885000	6368	14.02	0.4128	0.8118
0.5555781......	1.2150778	6106	14.22	0.4444	0.8787
0.5586196......	1.2505614	5860	14.42	0.4786	0.9381
0.5625558......	1.2964846	5636	14.62	0.5147	0.9983
0.5676213......	1.3555822	5432	14.82	0.5514	1.0645
0.5755913......	1.4485656	5242	15.02	0.5866	1.1235
0.5887972......	1.6026343	5067	15.22	0.6180	1.1648
0.6319979......	2.1066416	4970	15.42	0.6352	1.1872
0.8109065......	3.5935675	5198	15.62	0.5949	1.1356
0.9000000......	4.1818182	5304	15.75	0.5758	1.1116
		10 Gyr			
0.5323001......	0.9435001	9592	12.24	0.1599	0.2914
0.5324932......	0.9457538	9081	12.44	0.1955	0.3490
0.5326829......	0.9479671	8671	12.64	0.2163	0.3985
0.5329036......	0.9505425	8272	12.84	0.2312	0.4451
0.5331642......	0.9535824	7890	13.04	0.2575	0.5015
0.5334506......	0.9569235	7545	13.24	0.2876	0.5634
0.5337941......	0.9609317	7201	13.44	0.3232	0.6321
0.5342103......	0.9657863	6870	13.64	0.3529	0.6981
0.5347051......	0.9715596	6562	13.84	0.3862	0.7616
0.5353225......	0.9787621	6283	14.04	0.4209	0.8273
0.5361152......	0.9880106	6027	14.24	0.4557	0.8995
0.5371152......	0.9996777	5787	14.44	0.4923	0.9664
0.5383239......	1.0137788	5558	14.64	0.5299	1.0251
0.5397144......	1.0300012	5344	14.84	0.5674	1.0834
0.5417004......	1.0531711	5145	15.04	0.6037	1.1428
0.5447329......	1.0885509	4953	15.24	0.6383	1.1921
0.5483986......	1.1313166	4771	15.44	0.6709	1.2393
0.5538420......	1.1948239	4600	15.64	0.7024	1.2888
0.5611549......	1.2801409	4434	15.84	0.7352	1.3336
0.5707668......	1.3922793	4273	16.04	0.7702	1.3695
0.5834725......	1.5405119	4129	16.24	0.7969	1.4130
0.6061341......	1.8048982	3995	16.44	0.8319	1.4377
0.6493436......	2.3090092	3899	16.64	0.8563	1.4393
0.7254460......	3.0908587	3877	16.84	0.8637	1.4399
0.7850599......	3.4415288	3821	17.04	0.8775	1.4399
0.8167890......	3.6281707	3715	17.24	0.8900	1.4094
0.8393441......	3.7608475	3591	17.44	0.9013	1.3642
0.8625105......	3.8971204	3464	17.64	0.9123	1.3139
0.8861326......	4.0557510	3331	17.84	0.9227	1.2587
0.9000000......	4.1818182	3252	17.95	0.9289	1.2229
		12 Gyr			
0.5280815......	0.8942831	9551	12.24	0.1625	0.2955
0.5282301......	0.8960178	9043	12.44	0.1980	0.3535
0.5283741......	0.8976983	8639	12.64	0.2185	0.4028
0.5285411......	0.8996465	8245	12.84	0.2321	0.4485
0.5287406......	0.9019731	7862	13.04	0.2583	0.5050
0.5289592......	0.9045246	7516	13.24	0.2901	0.5684
0.5292177......	0.9075395	7175	13.44	0.3265	0.6382
0.5295297......	0.9111793	6845	13.64	0.3559	0.7042
0.5298984......	0.9154814	6537	13.84	0.3881	0.7667
0.5303506......	0.9207566	6259	14.04	0.4229	0.8320
0.5309223......	0.9274263	6004	14.24	0.4585	0.9048

TABLE 4—*Continued*

WD Mass ($M_\odot$)	MS Mass ($M_\odot$)	WD T_{eff} (K)	M_V	$V-R$	$V-I$
0.5316378......	0.9357743	5766	14.44	0.4958	0.9738
0.5324952......	0.9457772	5538	14.64	0.5337	1.0329
0.5334681......	0.9571282	5324	14.84	0.5713	1.0895
0.5348260......	0.9729697	5123	15.04	0.6076	1.1475
0.5369654......	0.9979294	4931	15.24	0.6422	1.1974
0.5392862......	1.0250054	4747	15.44	0.6751	1.2459
0.5422623......	1.0597272	4572	15.64	0.7074	1.2974
0.5459798......	1.1030972	4400	15.84	0.7415	1.3453
0.5501212......	1.1514135	4226	16.04	0.7812	1.3792
0.5545215......	1.2027512	4056	16.24	0.8188	1.4080
0.5591840......	1.2571463	3901	16.44	0.8422	1.4448
0.5659783......	1.3364132	3722	16.64	0.8822	1.4115
0.5750427......	1.4421647	3559	16.84	0.9106	1.3848
0.5861803......	1.5721035	3432	17.04	0.9100	1.3983
0.6026827......	1.7646315	3333	17.16	0.9358	1.2982
0.6209553......	1.9778114	3232	17.29	0.9409	1.1981
0.6559784......	2.3864147	3124	17.49	0.9286	1.0981
0.7017486......	2.9071719	3042	17.67	0.9050	0.9981
0.7114238......	3.0083756	2994	17.72	0.8858	0.8981
0.7217185......	3.0689324	2948	17.76	0.8674	0.7977
0.7319287......	3.1289926	2902	17.80	0.8493	0.6972
0.7420417......	3.1884808	2857	17.84	0.8314	0.5972
0.7521698......	3.2480577	2811	17.87	0.8136	0.4967
0.7622353......	3.3072663	2765	17.90	0.7960	0.3967
0.7723390......	3.3667000	2720	17.93	0.7784	0.2962
0.7824188......	3.4259927	2674	17.96	0.7609	0.1961
0.7925459......	3.4855642	2628	17.99	0.7434	0.0958
0.7999000......	3.5288235	2594	18.01	0.7307	0.0231
		14 Gyr			
0.5247511......	0.8554292	9519	12.24	0.1646	0.2988
0.5248706......	0.8568232	9012	12.44	0.2000	0.3572
0.5249852......	0.8581606	8613	12.64	0.2203	0.4063
0.5251176......	0.8597051	8224	12.84	0.2328	0.4513
0.5252773......	0.8615680	7840	13.04	0.2590	0.5077
0.5254522......	0.8636095	7493	13.24	0.2920	0.5725
0.5256567......	0.8659953	7154	13.44	0.3292	0.6430
0.5259030......	0.8688688	6824	13.64	0.3583	0.7091
0.5261927......	0.8722485	6518	13.84	0.3897	0.7709
0.5265440......	0.8763461	6240	14.05	0.4245	0.8357
0.5269828......	0.8814661	5987	14.25	0.4606	0.9089
0.5275293......	0.8878421	5750	14.45	0.4984	0.9795
0.5281819......	0.8954559	5523	14.65	0.5365	1.0389
0.5289220......	0.9040901	5308	14.85	0.5741	1.0944
0.5299217......	0.9157529	5107	15.05	0.6104	1.1509
0.5315627......	0.9348982	4915	15.25	0.6450	1.2014
0.5332201......	0.9542348	4730	15.45	0.6781	1.2506
0.5351798......	0.9770971	4554	15.65	0.7106	1.3031
0.5376172......	1.0055335	4381	15.85	0.7450	1.3523
0.5401187......	1.0347182	4203	16.05	0.7856	1.3871
0.5426530......	1.0642852	4025	16.25	0.8278	1.4095
0.5451153......	1.0930116	3866	16.45	0.8520	1.4445
0.5480984......	1.1278144	3694	16.65	0.8800	1.4288
0.5522686......	1.1764673	3497	16.85	0.9174	1.3316
0.5562356......	1.2227481	3344	17.05	0.9122	1.2998
0.5601245......	1.2681189	3232	17.17	0.9060	1.1996
0.5630193......	1.3018916	3161	17.24	0.9024	1.0996
0.5667049......	1.3448906	3085	17.31	0.8971	0.9995
0.5716530......	1.4026186	3001	17.39	0.8810	0.8994
0.5764075......	1.4580871	2929	17.46	0.8588	0.7992
0.5841944......	1.5489347	2872	17.53	0.8330	0.6991
0.5889515......	1.6044343	2829	17.57	0.8117	0.5987
0.5946709......	1.6711601	2787	17.61	0.7888	0.4983
0.6007104......	1.7416216	2748	17.62	0.7719	0.3968
0.6052376......	1.7944390	2708	17.61	0.7603	0.2955
0.6105900......	1.8568830	2666	17.61	0.7481	0.1896

TABLE 4—*Continued*

WD Mass ($M_\odot$)	MS Mass ($M_\odot$)	WD $T_{\rm eff}$ (K)	M_V	$V-R$	$V-I$
0.6167991......	1.9293230	2621	17.61	0.7363	0.0889
0.6251188......	2.0263864	2566	17.63	0.7241	−0.0113
0.6369736......	2.1646925	2501	17.68	0.7089	−0.1114
0.6631969......	2.4706302	2388	17.87	0.6775	−0.2115
0.6915188......	2.7920861	2299	18.07	0.6487	−0.2617
0.6999000......	2.8863750	2277	18.12	0.6410	−0.2732
		16 Gyr			
0.5220250......	0.8236249	9493	12.25	0.1663	0.3015
0.5221241......	0.8247811	8987	12.45	0.2017	0.3602
0.5222183......	0.8258805	8592	12.65	0.2218	0.4092
0.5223267......	0.8271454	8207	12.85	0.2334	0.4536
0.5224588......	0.8286865	7822	13.05	0.2594	0.5099
0.5226035......	0.8303738	7475	13.25	0.2935	0.5757
0.5227710......	0.8323285	7138	13.45	0.3313	0.6470
0.5229725......	0.8346789	6808	13.65	0.3603	0.7131
0.5232087......	0.8374353	6502	13.85	0.3909	0.7743
0.5234925......	0.8407453	6225	14.05	0.4258	0.8387
0.5238442......	0.8448491	5972	14.25	0.4623	0.9121
0.5242804......	0.8499382	5738	14.45	0.5003	0.9837
0.5248006......	0.8560072	5511	14.65	0.5388	1.0439
0.5253913......	0.8628982	5296	14.85	0.5764	1.0984
0.5261685......	0.8719655	5095	15.05	0.6126	1.1538
0.5274889......	0.8873703	4903	15.25	0.6472	1.2044
0.5287587......	0.9021851	4718	15.45	0.6803	1.2540
0.5301863......	0.9188405	4541	15.65	0.7129	1.3070
0.5319590......	0.9395221	4367	15.85	0.7474	1.3572
0.5336880......	0.9596934	4189	16.05	0.7881	1.3927
0.5354291......	0.9800066	4008	16.25	0.8329	1.4117
0.5370542......	0.9989652	3845	16.45	0.8587	1.4437
0.5388653......	1.0200956	3680	16.65	0.8803	1.4416
0.5408353......	1.0430785	3518	16.80	0.9142	1.3416
0.5432124......	1.0708113	3345	17.00	0.9205	1.2741
0.5456577......	1.0993393	3202	17.17	0.9031	1.1741
0.5470217......	1.1152526	3132	17.24	0.8921	1.0738
0.5485402......	1.1329685	3066	17.30	0.8825	0.9737
0.5501230......	1.1514348	3000	17.34	0.8702	0.8735
0.5518948......	1.1721059	2933	17.39	0.8513	0.7732
0.5535000......	1.1908335	2873	17.43	0.8333	0.6731
0.5550897......	1.2093803	2821	17.46	0.8135	0.5728
0.5572854......	1.2349968	2772	17.48	0.7955	0.4726
0.5588558......	1.2533171	2726	17.50	0.7772	0.3722
0.5605487......	1.2730683	2680	17.52	0.7583	0.2719
0.5624857......	1.2956667	2631	17.53	0.7419	0.1718
0.5646125......	1.3204797	2581	17.53	0.7271	0.0679
0.5670211......	1.3485799	2527	17.54	0.7134	−0.0328
0.5698071......	1.3810831	2469	17.56	0.6990	−0.1332
0.5758562......	1.4516554	2356	17.70	0.6674	−0.2333
0.5829396......	1.5342955	2250	17.90	0.6302	−0.2893
0.5914128......	1.6331493	2146	18.10	0.5871	−0.3605
0.6022097......	1.7591135	2040	18.30	0.5390	−0.4526
0.6130919......	1.8860727	1936	18.50	0.4914	−0.5456
0.6136000......	1.8920000	1931	18.51	0.4893	−0.5496

Figure 6 illustrates such luminosity functions for a Salpeter IMF [$n(m) \propto m^{-\alpha}$, where $\alpha = 2.35$; *solid line*] and a much flatter IMF ($\alpha = 1.3$; *dashed line*), which is more in line with the steepest IMFs being found at the low-mass end in globular clusters (Piotto and Zoccali 1999). The main feature to note in this diagram is the manner in which the peak of the cluster white dwarf luminosity function marches toward lower luminosity as the cluster age increases. This is then a potentially powerful technique for determining cluster ages that is largely independent of isochrone fitting to the turnoff region of a cluster. As can be seen, the effect of even a radical change in the IMF slope has a rather small influence on the morphology of the white dwarf luminosity function, and it appears that this is unlikely to be a sensitive method of investigating cluster IMFs. For this reason we only tabulate functions for Salpeter IMFs.

These luminosity functions are listed in Table 5 for those in Johnson-Kron/Cousins filters and in Table 6 for those calculated in the *HST* filter set. In these tables the number of white dwarfs is normalized to 100 and the columns are, respectively, the absolute V magnitude of the middle of the bin, the number of white dwarfs in that bin, the cumulative number of white dwarfs, and the mean mass of the white dwarfs and of the progenitors.

As a last point regarding white dwarf luminosity functions in clusters, we inquire whether information about the age of a cluster can be obtained if the turnover in the white dwarf luminosity function is *not* observed, but only if a bright portion (e.g., to $M_V = 15$) is seen. This, of course, has potential practical applications since the termination points of white dwarf sequences will only be possible to observe in the nearest globular clusters even with *HST* and the Advanced Camera for Surveys. To investigate this, in Figure 7 we superimpose synthetic white dwarf luminosity functions for 10, 12, and 14 Gyr old clusters. The numbers of white dwarfs indicated are those expected from a single WFPC2 field at six core radii from the center of the globular cluster M4. If the functions in Figure 7 are compared down to $M_V = 15$, it becomes clear that virtually no useful information is obtained regarding the age of the cluster. The turnover in the luminosity function must be observed in order to constrain the cluster age.

5. THE WHITE DWARF COOLING AGE OF M67

In an earlier paper Richer et al. (1998) presented and discussed the observed white dwarf luminosity function in the open cluster M67, which has a turnoff age of about 4 Gyr (Montgomery, Marschall, & Janes 1993). Here we compare this function with synthetic luminosity functions in order to derive the white dwarf cooling age of the cluster. In the previous paper we did not have access to such synthetic functions, so the current derivation of the cluster cooling age will supercede the results in the earlier paper.

Figure 8 displays the observed cumulative white dwarf luminosity function in M67 from Richer et al. (1998; *heavy solid line*) compared with synthetic luminosity functions for clusters with ages of 3, 4, and 5 Gyr. The synthetic functions have been shifted to represent a cluster with an apparent V distance modulus of 9.59 (Montgomery et al. 1993), and they have been normalized to contain the same number of white dwarfs that are observed in the cluster (58). All of the synthetic functions have Salpeter IMFs, but the actual choice of the IMF, within rather broad limits, has little effect on the final results as could be deduced from Figure 6. From the faint end of the luminosity function seen in M67, it is clear that the cooling age of the cluster is larger than 3 Gyr but less than 4 Gyr. This is somewhat younger than the cluster turnoff whose best estimate is currently near 4 Gyr (Montgomery et al. 1993).

Nevertheless, this result indicates that a properly constructed synthetic white dwarf luminosity function compared with data should be a robust and reliable age indicator, and that it will be an important tool in establishing ages for old clusters in the Galaxy.

We note in passing that the observed white dwarf lumi-

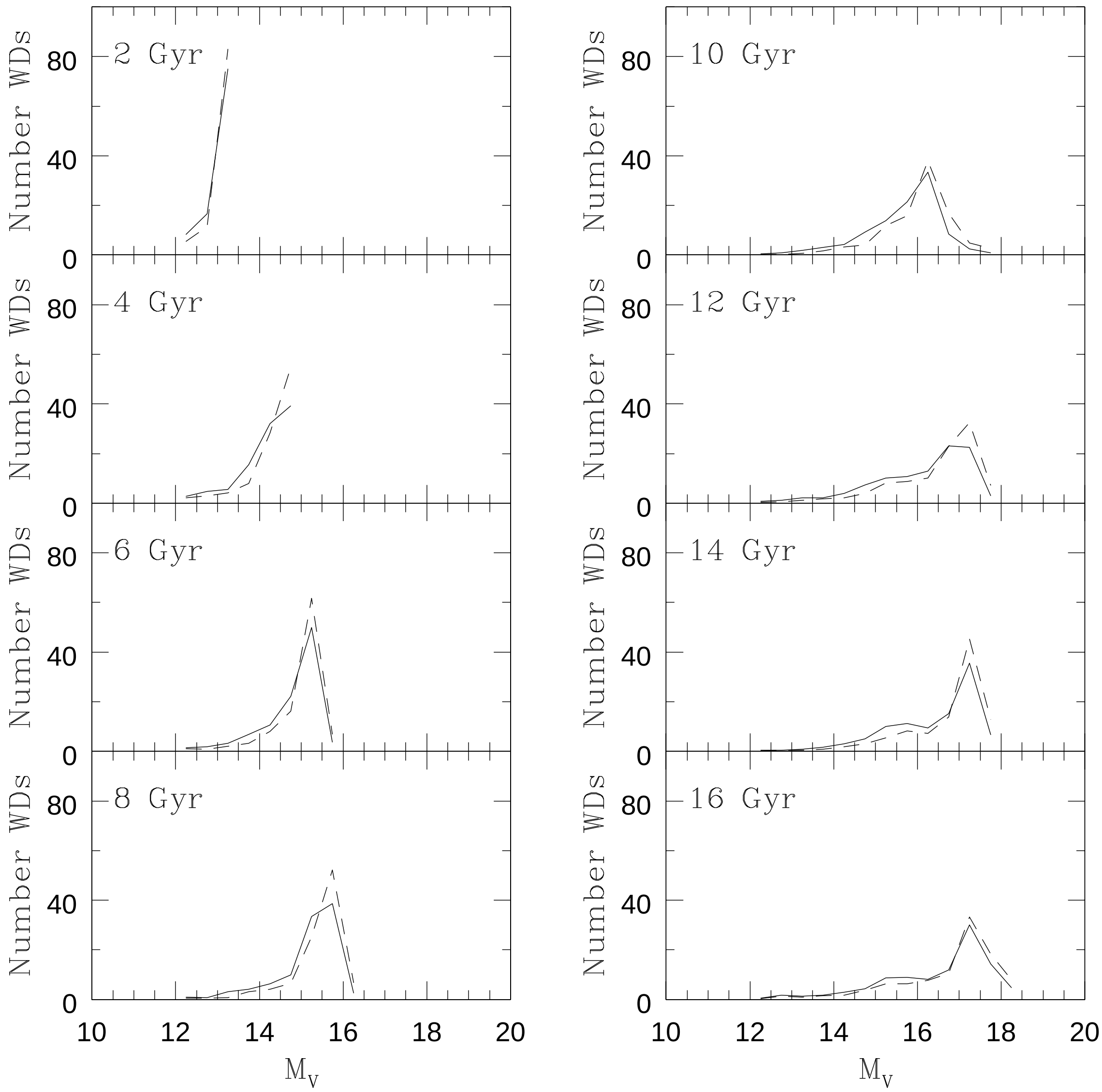

FIG. 6.—Differential luminosity functions for white dwarfs in clusters of varying ages. The solid lines are for Salpeter IMFs ($\alpha = 2.35$), while the dashed lines are for significantly flatter IMFs ($\alpha = 1.3$).

nosity function in M67 has a well-populated tail of stars to high luminosity, many more stars than are predicted by the models. Varying the IMF, even by a rather large amount, could not make the fit of the synthetic function to the observations significantly better since the precursor mass range among the bright M67 white dwarfs is quite small. The origin of this tail is not currently understood, but might be related to the high binary fraction in the cluster (see Richer et al. 1998 for further discussion) or to some deficiency in the cooling models, which *overestimates* the true rate of cooling of young white dwarfs. If the binary scenario is correct, the excess number of bright white dwarfs could be produced by making a relatively large number of helium-core white dwarfs via truncated stellar evolution. Such objects fade less rapidly than C-O white dwarfs since they have a greater heat capacity per unit mass.

6. THE WHITE DWARF LUMINOSITY FUNCTIONS IN THE GALACTIC HALO

The microlensing experiments in the direction of the LMC seem to be indicating that 60% $\pm$ 20% of the dark matter in the Galactic halo is tied up in $0.5^{+0.3}_{-0.2}\,M_{\odot}$ objects (Alcock et al. 1997a, 1997b; Renault et al. 1997). This naturally suggests old white dwarfs, although other possibilities exist (e.g., neutron stars, primordial black holes). The possibility that white dwarfs are important contributors to the Galactic dark matter has been considered for some time now (Larson 1986; Silk 1991; Carr 1994), but with the microlensing results this scenario has taken on increased viability.

Chabrier (1999), Chabrier, Segretain, & Méra (1996), Gibson & Mould (1997), and Adams & Laughlin (1996) have all pointed out that if, indeed, this scenario is correct, the IMF of the white dwarf precursors could not have had a Salpeter form, but might have been more Gaussian in shape and peaked near 2.7 $M_{\odot}$. For this reason we have calculated halo luminosity functions for both Salpeter IMFs and those of the form $\Phi(m) = \exp^{-(\overline{m}/m)^{\beta_1}}\, m^{-\beta_2}$, with $\overline{m} = 2.7$, $\beta_1 = 2.2$, and $\beta_2 = 5.75$ (Chabrier et al. 1996).

Under this scenario, old white dwarfs will be plentiful in the Galactic halo, but difficult to detect because of theirintrinsic faintness. The local number of such objects can be

TABLE 5

White Dwarf Cluster Luminosity Functions: Salpeter IMF, Johnson-Kron/Cousins Filters

M_V	Number of WDs	Cumulative Number	$\overline{M}_{\rm MS}$	$\overline{M}_{\rm WD}$
		2 Gyr		
12.25......	8.30E+00	8.30E+00	1.68918	0.59622
12.75......	1.66E+01	2.49E+01	1.80524	0.60616
13.25......	7.51E+01	1.00E+02	2.75499	0.71432
		4 Gyr		
12.25......	2.80E+00	2.80E+00	1.27107	0.56038
12.75......	4.70E+00	7.50E+00	1.29520	0.56245
13.25......	5.60E+00	1.31E+01	1.33988	0.56628
13.75......	1.56E+01	2.87E+01	1.44236	0.57506
14.25......	3.21E+01	6.08E+01	1.76255	0.60250
14.75......	3.92E+01	1.00E+02	2.81084	0.71192
		6 Gyr		
12.25......	1.50E+00	1.50E+00	1.10844	0.54644
12.75......	1.90E+00	3.40E+00	1.12036	0.54746
13.25......	3.30E+00	6.70E+00	1.14067	0.54920
13.75......	6.90E+00	1.36E+01	1.17995	0.55257
14.25......	1.06E+01	2.42E+01	1.25152	0.55870
14.75......	2.22E+01	4.64E+01	1.41787	0.57296
15.25......	4.99E+01	9.63E+01	2.27274	0.65292
15.75......	3.70E+00	1.00E+02	3.95566	0.86951
		8 Gyr		
12.25......	9.00E−01	9.00E−01	1.01184	0.53816
12.75......	8.00E−01	1.70E+00	1.01993	0.53885
13.25......	3.10E+00	4.80E+00	1.03195	0.53988
13.75......	4.10E+00	8.90E+00	1.05335	0.54172
14.25......	6.40E+00	1.53E+01	1.08678	0.54458
14.75......	9.90E+00	2.52E+01	1.16717	0.55147
15.25......	3.35E+01	5.87E+01	1.41758	0.57294
15.75......	3.87E+01	9.74E+01	2.43038	0.66703
16.25......	2.60E+00	1.00E+02	3.96584	0.86806
		10 Gyr		
12.25......	5.00E−01	5.00E−01	0.94536	0.53246
12.75......	8.00E−01	1.30E+00	0.95139	0.53298
13.25......	1.80E+00	3.10E+00	0.95895	0.53362
13.75......	3.10E+00	6.20E+00	0.97363	0.53488
14.25......	4.30E+00	1.05E+01	1.00014	0.53715
14.75......	9.20E+00	1.97E+01	1.04563	0.54105
15.25......	1.38E+01	3.35E+01	1.14265	0.54937
15.75......	2.15E+01	5.50E+01	1.35909	0.56792
16.25......	3.33E+01	8.83E+01	1.99388	0.62266
16.75......	8.50E+00	9.68E+01	3.11420	0.73475
17.25......	2.40E+00	9.92E+01	3.83451	0.85149
17.75......	8.00E−01	1.00E+02	4.06340	0.88630
		12 Gyr		
12.25......	7.00E−01	7.00E−01	0.89574	0.52821
12.75......	1.20E+00	1.90E+00	0.90027	0.52859
13.25......	2.10E+00	4.00E+00	0.90554	0.52905
13.75......	2.20E+00	6.20E+00	0.91663	0.53000
14.25......	4.00E+00	1.02E+01	0.93442	0.53152
14.75......	7.40E+00	1.76E+01	0.96880	0.53447
15.25......	1.02E+01	2.78E+01	1.02203	0.53903
15.75......	1.08E+01	3.86E+01	1.12981	0.54827
16.25......	1.29E+01	5.15E+01	1.28046	0.56118
16.75......	2.31E+01	7.46E+01	1.61569	0.58992
17.25......	2.25E+01	9.71E+01	2.42684	0.65994
17.75......	2.90E+00	1.00E+02	3.28954	0.75922
		14 Gyr		
12.25......	5.00E−01	5.00E−01	0.85643	0.52484
12.75......	5.00E−01	1.00E+00	0.85980	0.52513
13.25......	8.00E−01	1.80E+00	0.86610	0.52567
13.75......	1.70E+00	3.50E+00	0.87292	0.52625
14.25......	3.00E+00	6.50E+00	0.88707	0.52746
14.75......	5.10E+00	1.16E+01	0.91355	0.52973
15.25......	1.01E+01	2.17E+01	0.95535	0.53332
15.75......	1.13E+01	3.30E+01	1.02764	0.53951
16.25......	9.50E+00	4.25E+01	1.10176	0.54586
16.75......	1.52E+01	5.77E+01	1.23283	0.55710
17.25......	3.56E+01	9.33E+01	1.73916	0.60050
17.75......	6.70E+00	1.00E+02	2.60490	0.67500
		16 Gyr		
12.25......	5.00E−01	5.00E−01	0.82458	0.52211
12.75......	1.80E+00	2.30E+00	0.82673	0.52229
13.25......	1.40E+00	3.70E+00	0.83176	0.52272
13.75......	1.80E+00	5.50E+00	0.83864	0.52331
14.25......	3.00E+00	8.50E+00	0.85037	0.52432
14.75......	4.30E+00	1.28E+01	0.86894	0.52591
15.25......	8.80E+00	2.16E+01	0.90262	0.52880
15.75......	9.00E+00	3.06E+01	0.95273	0.53309
16.25......	8.20E+00	3.88E+01	1.00550	0.53761
16.75......	1.20E+01	5.08E+01	1.07851	0.54387
17.25......	3.01E+01	8.09E+01	1.28297	0.56140
17.75......	1.43E+01	9.52E+01	1.63110	0.59124
18.25......	4.80E+00	1.00E+02	1.82482	0.60784

determined simply from the local dark matter density (0.0079 $M_\odot$ pc^{-3}) (Alcock et al. 1997a; Chabrier & Méra 1997; Gould, Flynn, & Bahcall 1996) and the mean white dwarf mass ($\langle M_{\rm WD} \rangle$) through

$$\text{local number WDs pc}^{-3} = \frac{0.0079}{\langle M_{\rm wd} \rangle} .$$

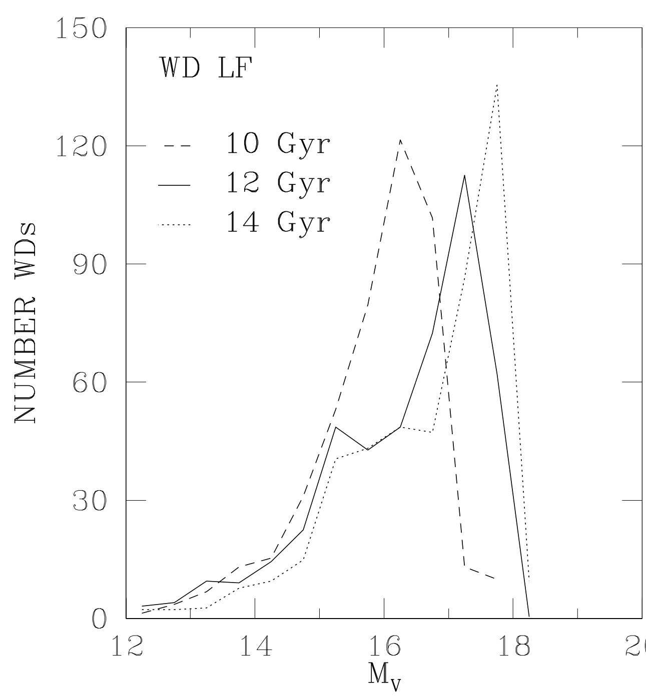

Fig. 7.—Synthetic white dwarf luminosity functions in the *HST* filters for clusters with ages of 10, 12, and 14 Gyr. The numbers of white dwarfs are those expected in the Galactic globular cluster M4 in a single WFPC2 field at 6 core radii from the cluster center (see Richer et al. 1997).

TABLE 6

WHITE DWARF CLUSTER LUMINOSITY FUNCTIONS: SALPETER IMF, *HST* FILTERS

M_V	Number of WDs	Cumulative Number	$\overline{M}_{\rm MS}$	$\overline{M}_{\rm WD}$
		2 Gyr		
12.25......	6.60E+00	6.60E+00	1.68163	0.59557
12.75......	1.43E+01	2.09E+01	1.77834	0.60386
13.25......	7.91E+01	1.00E+02	2.70997	0.70790
		4 Gyr		
12.25......	2.60E+00	2.60E+00	1.27030	0.56031
12.75......	4.30E+00	6.90E+00	1.29223	0.56219
13.25......	5.00E+00	1.19E+01	1.33101	0.56552
13.75......	1.01E+01	2.20E+01	1.40760	0.57208
14.25......	2.60E+01	4.80E+01	1.60390	0.58891
14.75......	5.20E+01	1.00E+02	2.59579	0.68561
		6 Gyr		
12.25......	1.10E+00	1.10E+00	1.10713	0.54633
12.75......	2.10E+00	3.20E+00	1.11809	0.54726
13.25......	2.80E+00	6.00E+00	1.13740	0.54892
13.75......	5.20E+00	1.12E+01	1.16906	0.55163
14.25......	9.50E+00	2.07E+01	1.22398	0.55634
14.75......	1.71E+01	3.78E+01	1.35109	0.56724
15.25......	4.74E+01	8.52E+01	1.92879	0.61968
15.75......	1.48E+01	1.00E+02	3.34459	0.78385
		8 Gyr		
12.25......	7.00E−01	7.00E−01	1.01130	0.53811
12.75......	8.00E−01	1.50E+00	1.01752	0.53864
13.25......	2.70E+00	4.20E+00	1.02987	0.53970
13.75......	3.40E+00	7.60E+00	1.04710	0.54118
14.25......	6.60E+00	1.42E+01	1.07852	0.54387
14.75......	7.00E+00	2.12E+01	1.14127	0.54925
15.25......	2.07E+01	4.19E+01	1.28268	0.56137
15.75......	4.90E+01	9.09E+01	1.98810	0.62571
16.25......	9.10E+00	1.00E+02	3.59036	0.81067
		10 Gyr		
12.25......	3.00E−01	3.00E−01	0.94439	0.53238
12.75......	8.00E−01	1.10E+00	0.94968	0.53283
13.25......	1.50E+00	2.60E+00	0.95673	0.53343
13.75......	2.90E+00	5.50E+00	0.97046	0.53461
14.25......	3.40E+00	8.90E+00	0.99072	0.53635
14.75......	6.90E+00	1.58E+01	1.02812	0.53955
15.25......	1.18E+01	2.76E+01	1.09306	0.54512
15.75......	1.77E+01	4.53E+01	1.25496	0.55900
16.25......	2.70E+01	7.23E+01	1.62699	0.59089
16.75......	2.26E+01	9.49E+01	2.49231	0.66880
17.25......	2.90E+00	9.78E+01	3.57085	0.80704
17.75......	2.20E+00	1.00E+02	3.96886	0.87262
		12 Gyr		
12.25......	7.00E−01	7.00E−01	0.89574	0.52821
12.75......	9.00E−01	1.60E+00	0.89969	0.52855
13.25......	2.10E+00	3.70E+00	0.90449	0.52896
13.75......	2.00E+00	5.70E+00	0.91433	0.52980
14.25......	3.20E+00	8.90E+00	0.92902	0.53106
14.75......	5.00E+00	1.39E+01	0.95437	0.53323
15.25......	1.08E+01	2.47E+01	0.99901	0.53706
15.75......	9.50E+00	3.42E+01	1.08715	0.54461
16.25......	1.08E+01	4.50E+01	1.20676	0.55487
16.75......	1.61E+01	6.11E+01	1.40608	0.57195
17.25......	2.50E+01	8.61E+01	1.92714	0.61661
17.75......	1.38E+01	9.99E+01	2.81984	0.69915
18.25......	1.00E−01	1.00E+02	3.52773	0.79971

TABLE 6—*Continued*

M_V	Number of WDs	Cumulative Number	$\overline{M}_{\rm MS}$	$\overline{M}_{\rm WD}$
		14 Gyr		
12.25......	5.00E−01	5.00E−01	0.85643	0.52484
12.75......	5.00E−01	1.00E+00	0.85980	0.52513
13.25......	6.00E−01	1.60E+00	0.86560	0.52562
13.75......	1.70E+00	3.30E+00	0.87179	0.52615
14.25......	2.10E+00	5.40E+00	0.88315	0.52713
14.75......	3.30E+00	8.70E+00	0.90085	0.52864
15.25......	9.00E+00	1.77E+01	0.93688	0.53173
15.75......	9.60E+00	2.73E+01	0.99254	0.53650
16.25......	1.08E+01	3.81E+01	1.06377	0.54261
16.75......	1.05E+01	4.86E+01	1.15726	0.55062
17.25......	1.92E+01	6.78E+01	1.34489	0.56670
17.75......	3.01E+01	9.79E+01	1.96724	0.62007
18.25......	2.10E+00	1.00E+02	2.79347	0.69165
		16 Gyr		
12.25......	4.00E−01	4.00E−01	0.82438	0.52209
12.75......	1.70E+00	2.10E+00	0.82641	0.52226
13.25......	1.20E+00	3.30E+00	0.83068	0.52263
13.75......	1.40E+00	4.70E+00	0.83576	0.52307
14.25......	2.30E+00	7.00E+00	0.84529	0.52388
14.75......	3.60E+00	1.06E+01	0.85856	0.52502
15.25......	7.40E+00	1.80E+01	0.88746	0.52750
15.75......	8.20E+00	2.62E+01	0.93028	0.53117
16.25......	8.10E+00	3.43E+01	0.97729	0.53520
16.75......	9.50E+00	4.38E+01	1.03547	0.54018
17.25......	2.12E+01	6.50E+01	1.15537	0.55046
17.75......	2.05E+01	8.55E+01	1.40733	0.57206
18.25......	1.44E+01	9.99E+01	1.72568	0.59934
18.75......	1.00E−01	1.00E+02	1.88973	0.61341

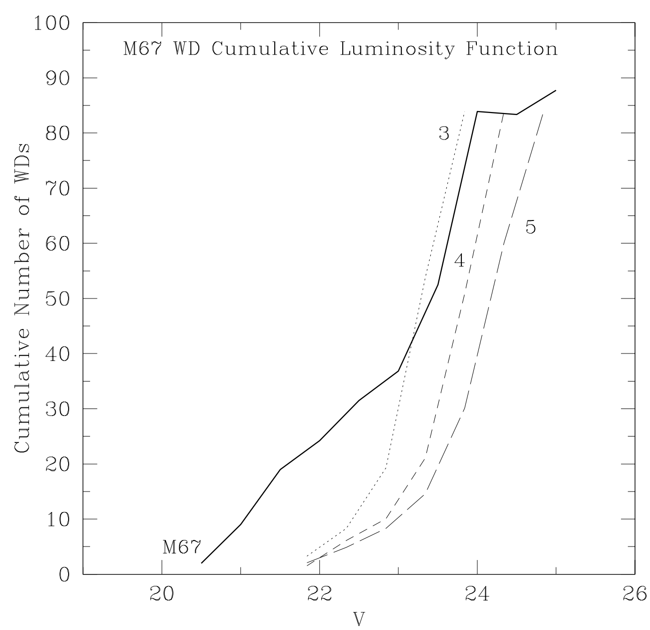

FIG. 8.—Cumulative M67 white dwarf luminosity function (*heavy solid line*) compared with synthetic functions of ages 3, 4, and 5 Gyr, all with Salpeter IMFs. The location of the break in the M67 luminosity function and the general fit to the synthetic functions suggest a white dwarf cooling age for M67 somewhat older than 3 Gyr, but less than 4 Gyr.

TABLE 7

Halo White Dwarf Luminosity Functions: Chabrier IMF, Johnson-Kron/Cousins Filters

V	WD deg^{-2}	Cumulative Number
10 Gyr	$\overline{M}_{\rm WD} = 0.64\ M_\odot$	
17.75......	1.75E−02	1.75E−02
18.25......	0.00E+00	1.75E−02
18.75......	1.75E−02	3.50E−02
19.25......	6.99E−02	1.05E−01
19.75......	1.57E−01	2.62E−01
20.25......	2.62E−01	5.24E−01
20.75......	4.72E−01	9.96E−01
21.25......	9.79E−01	1.97E+00
21.75......	2.38E+00	4.35E+00
22.25......	4.25E+00	8.60E+00
22.75......	8.55E+00	1.71E+01
23.25......	1.63E+01	3.34E+01
23.75......	3.19E+01	6.54E+01
24.25......	6.21E+01	1.27E+02
24.75......	1.28E+02	2.56E+02
25.25......	2.59E+02	5.15E+02
25.75......	5.14E+02	1.03E+03
26.25......	1.01E+03	2.04E+03
26.75......	1.99E+03	4.03E+03
27.25......	3.94E+03	7.97E+03
27.75......	7.76E+03	1.57E+04
14 Gyr	$\overline{M}_{\rm WD} = 0.62\ M_\odot$	
19.75......	1.75E−02	1.75E−02
20.25......	6.99E−02	8.74E−02
20.75......	8.74E−02	1.75E−01
21.25......	1.22E−01	2.97E−01
21.75......	5.59E−01	8.56E−01
22.25......	8.39E−01	1.70E+00
22.75......	1.96E+00	3.65E+00
23.25......	3.57E+00	7.22E+00
23.75......	7.52E+00	1.47E+01
24.25......	1.45E+01	2.92E+01
24.75......	2.99E+01	5.91E+01
25.25......	5.82E+01	1.17E+02
25.75......	1.18E+02	2.35E+02
26.25......	2.34E+02	4.69E+02
26.75......	4.60E+02	9.28E+02
27.25......	9.09E+02	1.84E+03
27.75......	1.79E+03	3.63E+03
16 Gyr	$\overline{M}_{\rm WD} = 0.59\ M_\odot$	
19.75......	1.75E−02	1.75E−02
20.25......	5.24E−02	6.99E−02
20.75......	0.00E+00	6.99E−02
21.25......	1.22E−01	1.92E−01
21.75......	3.15E−01	5.07E−01
22.25......	7.69E−01	1.28E+00
22.75......	1.35E+00	2.62E+00
23.25......	2.59E+00	5.21E+00
23.75......	5.47E+00	1.07E+01
24.25......	9.86E+00	2.05E+01
24.75......	2.16E+01	4.21E+01
25.25......	4.04E+01	8.25E+01
25.75......	8.49E+01	1.67E+02
26.25......	1.66E+02	3.34E+02
26.75......	3.31E+02	6.65E+02
27.25......	6.45E+02	1.31E+03
27.75......	1.26E+03	2.57E+03

TABLE 8

Halo White Dwarf Luminosity Functions: Chabrier IMF, *HST* Filters

V	No. WD deg^{-2}	Cumulative Number
10 Gyr	$\overline{M}_{\rm WD} = 0.64\ M_\odot$	
17.75......	1.75E−02	1.75E−02
18.25......	0.00E+00	1.75E−02
18.75......	0.00E+00	1.75E−02
19.25......	6.99E−02	8.74E−02
19.75......	1.05E−01	1.92E−01
20.25......	1.75E−01	3.67E−01
20.75......	4.19E−01	7.86E−01
21.25......	8.74E−01	1.66E+00
21.75......	1.22E+00	2.88E+00
22.25......	3.18E+00	6.06E+00
22.75......	5.92E+00	1.20E+01
23.25......	1.20E+01	2.40E+01
23.75......	2.35E+01	4.74E+01
24.25......	4.72E+01	9.46E+01
24.75......	9.84E+01	1.93E+02
25.25......	1.91E+02	3.84E+02
25.75......	3.75E+02	7.59E+02
26.25......	7.48E+02	1.51E+03
26.75......	1.46E+03	2.96E+03
27.25......	2.88E+03	5.84E+03
27.75......	5.68E+03	1.15E+04
14 Gyr	$\overline{M}_{\rm WD} = 0.62\ M_\odot$	
19.75......	1.75E−02	1.75E−02
20.25......	3.50E−02	5.24E−02
20.75......	3.50E−02	8.74E−02
21.25......	1.05E−01	1.92E−01
21.75......	3.32E−01	5.24E−01
22.25......	7.17E−01	1.24E+00
22.75......	1.33E+00	2.57E+00
23.25......	2.39E+00	4.96E+00
23.75......	5.35E+00	1.03E+01
24.25......	9.70E+00	2.00E+01
24.75......	2.02E+01	4.02E+01
25.25......	3.87E+01	7.89E+01
25.75......	8.05E+01	1.59E+02
26.25......	1.58E+02	3.18E+02
26.75......	3.10E+02	6.27E+02
27.25......	6.08E+02	1.24E+03
27.75......	1.19E+03	2.42E+03
16 Gyr	$\overline{M}_{\rm WD} = 0.61\ M_\odot$	
20.25......	1.75E−02	1.75E−02
20.75......	5.24E−02	6.99E−02
21.25......	8.74E−02	1.57E−01
21.75......	1.75E−01	3.32E−01
22.25......	4.89E−01	8.21E−01
22.75......	8.91E−01	1.71E+00
23.25......	1.85E+00	3.57E+00
23.75......	3.64E+00	7.20E+00
24.25......	7.01E+00	1.42E+01
24.75......	1.37E+01	2.79E+01
25.25......	2.83E+01	5.63E+01
25.75......	5.50E+01	1.11E+02
26.25......	1.10E+02	2.22E+02
26.75......	2.21E+02	4.42E+02
27.25......	4.32E+02	8.74E+02
27.75......	8.40E+02	1.71E+03

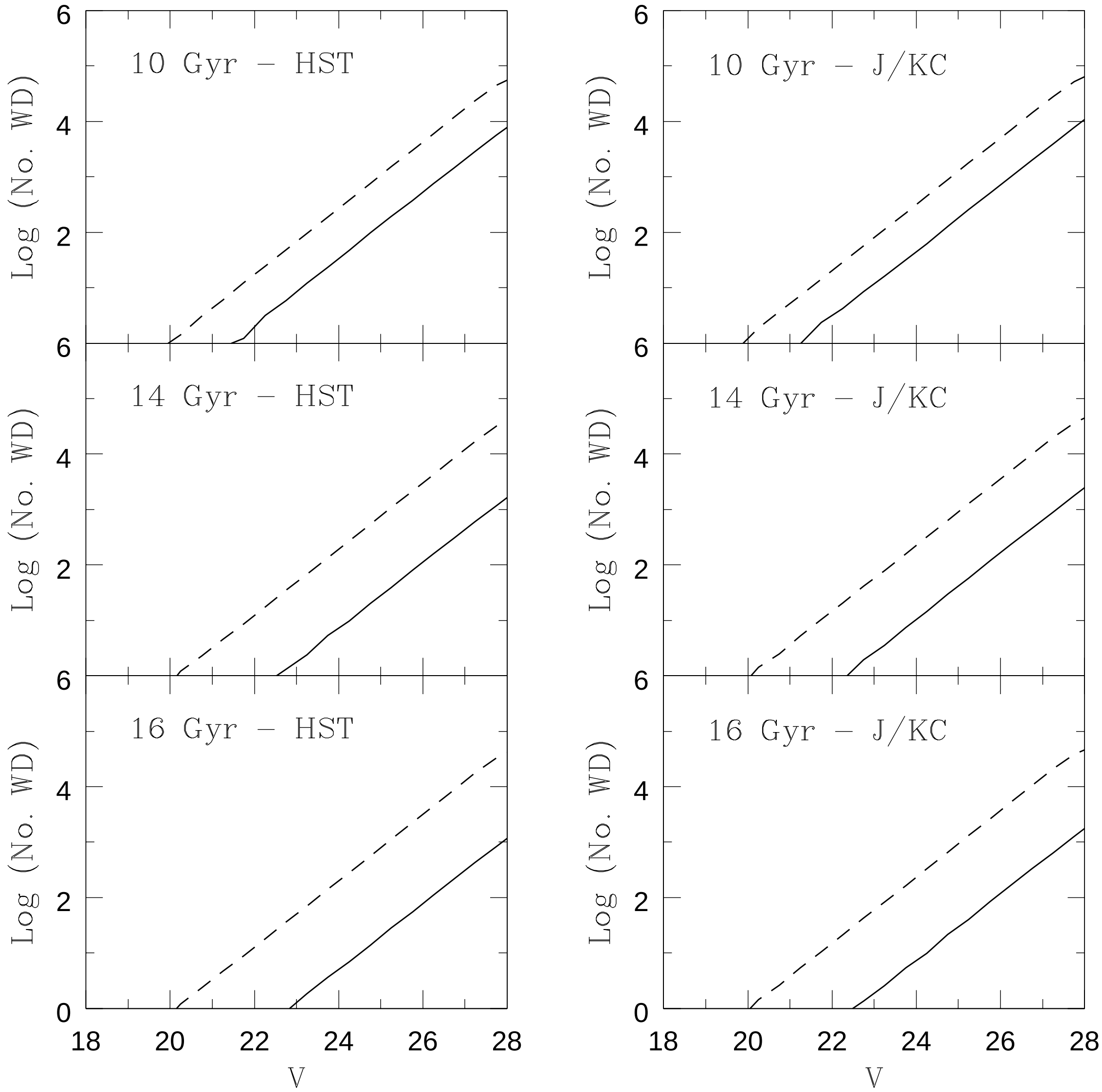

FIG. 9.—Cumulative white dwarf luminosity functions in the Galactic halo. The *y*-axis is the logarithm of the number of stars per square degree under the assumption that white dwarfs make up 100% of the local dark matter density and that they all have hydrogen atmospheres. Solid lines are for a Chabrier (1999)–type IMF, while the dashed line is for a Salpeter IMF.

A synthetic halo white dwarf luminosity function for a given age ($T_{\rm field}$) and limiting magnitude ($V_{\rm lim}$) was calculated as follows. First, from the cluster luminosity function for an age $T_{\rm field}$, we obtained the maximum and minimum main-sequence masses as before as well as the brightest white dwarf ($M_V^{\rm min}$) in the cluster. We then set the distance $R_{\rm max}$ out to which we would fill a volume with white dwarfs as

$$R_{\rm max} = 10^{(V_{\rm lim} - M_V{}^{\rm min} + 5)/5} .$$

From the IMF we then extracted a main-sequence star of a given mass and determined the associated white dwarf mass $M_{\rm wd}$ through the initial-final mass relation. This white dwarf was then placed randomly at a distance of $R_{\rm wd}$ inside the volume to keep the density constant. From the isochrone we then obtained the M_V of this object and obtained its apparent V magnitude from

$$V = M_V - 5 + 5 \log R_{\rm wd} .$$

The average mass of the white dwarfs in the volume came simply from

$$\langle M_{\rm wd} \rangle = \sum \frac{M_{\rm wd}}{N} ,$$

and the total number of white dwarfs observed in the volume would then be

$$N_{\rm tot} = \frac{0.0079}{\langle M_{\rm wd} \rangle} \times \frac{4}{3} \pi R_{\rm max}^3 .$$

This was done N times until $N \geq N_{\rm tot}$, at which point the calculations were terminated.

In this way we constructed synthetic halo luminosity functions including all the stars in the mass range allowed by the models for ages of 10, 14, and 16 Gyr. The volume size was chosen so that all the stars brighter than $V = 28$ would be counted. This limiting magnitude constitutes a reasonably faint limit, but not so faint that the halo density variation would be important for luminosity functions constructed with Chabrier et al. (1996) IMFs. With $V_{\rm lim} = 28$ and $M_V^{\rm min} =$ 14.5, 15.5, and 16.0 for ages of 10, 14, and 16 Gyr with a Chabrier IMF, $R_{\rm max}$ was 5.0, 3.2, and 2.5 kpc, respectively. However, when we used a Salpeter IMF it was 12.5 kpc for all ages. For the Chabrier IMFs the number of stars inserted in to the volume was $\sim 1.6 \times 10^5$ deg^{-2} of field for the 10 Gyr halo ($\sim 2.0 \times 10^4$ for 16 Gyr), while it was $\sim 2.5 \times 10^6$ for the Salpeter IMF at all three ages.

We can only reasonably calculate halo luminosity functions for stars that are very local (to keep the density constant and not to have the functions depend on the direction of viewing), so this is a reasonable assumption for Chabrier-type IMFs (about a 6% error is made by not including the R^{-2} halo density variation for a 16 Gyr halo for viewing toward the Galactic poles), but not very good for Salpeter functions (looking in the same direction, an overestimate of about a factor of 4 in the counts from the most distant region of the volume results from ignoring the density falloff). This is not a critical point, because—if the MACHOs are indeed old white dwarfs—their precursors are unlikely to have been formed with a Salpeter IMF. In any case, the Salpeter IMF counts exceed those with Chabrier IMFs by a much larger amount than that caused by density variations. For example, for a 14 Gyr halo and a 1 deg^2 field to $V = 27.5$, the cumulative counts with *HST* filters for a Salpeter IMF are 34,000 (assuming constant density) compared with 1210 for those with a Chabrier IMF. The average white dwarf mass for a halo of this age with a Chabrier IMF was 0.62 $M_\odot$, while it was 0.57 $M_\odot$ for a Salpeter function.

Tables 7 and 8 contain halo white dwarf luminosity functions for Johnson-Kron/Cousins and *HST* filter systems, respectively, for ages of 10, 14, and 16 Gyr and all with Chabrier IMFs. For reasons outlined above, the functions for Salpeter IMFs are not very accurate, so they should be considered as illustrative only. Even so, there are a number of interesting conclusions that can be derived from the plots (Fig. 9) and Tables 7 and 8. First, the white dwarf luminosity functions based on a Salpeter IMF always predict many more stars than do those for a Chabrier function. For example, the Salpeter-based function for 14 Gyr predicts that five old white dwarfs should be found in the Hubble Deep Field (HDF) to $V = 26.0$, whereas at most there is one possible candidate (Ibata et al. 1999; Hansen 1999). This, by itself, seems capable of excluding old white dwarfs formed with a Salpeter IMF as the entire source of the Galactic dark matter. By contrast, using the luminosity function generated with a Chabrier IMF suggests that only 0.2 should be found. This model is clearly not as yet excluded by the data.

For a 10 Gyr old halo and a Chabrier IMF, to $V = 28.0$, we expect to find 14 old white dwarfs in the HDF (if the dark halo is 100% hydrogen-rich white dwarfs), but only two if it is as old as 16 Gyr. This, coupled with the colors of the stars, is a potentially powerful method of establishing the time of formation of the Galactic halo, as well as possibly shedding some light on the nature of dark matter in the Galaxy. With the advent of the Advanced Camera for Surveys on *HST*, experiments of this sort covering larger areas than the HDF will become eminently feasible.

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