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AN UPPER LIMIT ON Ω_m USING LENSED ARCS

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ABSTRACT

We use current observations on the number statistics of gravitationally lensed optical arcs toward galaxy clusters to derive an upper limit on the cosmological mass density of the universe. The gravitational lensing statistics from foreground clusters combine properties of both cluster evolution, which is sensitive to the matter density, and volume change, which is sensitive to the cosmological constant. The uncertainties associated with the predicted number of lensing events, however, currently do not allow one to distinguish between flat and open cosmological models with and without a cosmological constant. Still, after accounting for known errors, and assuming that clusters in general have dark matter core radii of the order $\sim 35\ h^{-1}$ kpc, we find that the cosmological mass density, Ω_m, is less than 0.56 at the 95% confidence. Such a dark matter core radius is consistent with cluster potentials determined recently by detailed numerical inversions of strong and weak lensing imaging data. If no core radius is present, the upper limit on Ω_m increases to 0.62 (95% confidence level). The estimated upper limit on Ω_m is consistent with various cosmological probes that suggest a low matter density for the universe.

Subject headings: cosmology: observations — cosmology: theory — gravitational lensing

1. INTRODUCTION

A large number of cosmological probes now suggest that the universe is spatially flat with a low mass density (e.g., Perlmutter et al. 1998; Riess et al. 1998; Lineweaver 1998; Guerra & Daly 1998; Bahcall & Fan 1998). In addition to the mass density, gravitational lensing statistics have allowed limits to be placed on the cosmological constant. However, current limits on the cosmological constant from gravitational lensing arguments are based only on lensing statistics from foreground galaxies (e.g., Kochanek 1996b; Falco, Kochanek, & Muñoz 1998; Cheng & Krauss 1999; Cooray, Quashnock, & Miller 1999; Cooray 1999a; Quast & Helbig 1999).[1] An alternative approach is to consider lensing statistics from foreground galaxy clusters (e.g., Wu & Hammer 1993; Bartelmann et al. 1998; Cooray 1999b). It is well known that galaxy cluster evolution is strongly sensitive to the cosmological mass density of the universe (e.g., Bahcall & Fan 1998; Viana & Liddle 1998). Since lensing statistics are sensitive to the cosmological constant, it is likely that the number of lensed arcs due to galaxy clusters can provide strong constraints on both the mass density and the cosmological constant.

Since the first suggestion that lensed optical arcs can be used as a cosmological probe (Wu & Hammer 1993), several studies have addressed specific issues related to the statistical calculation. These include the effect of a cosmological constant (Wu & Mao 1996) and background source evolution (Hamana & Futamase 1997). The numerical works by Bartelmann et al. (1998), using simulated clusters in three cosmological models, suggested that current observational statistics on lensed arcs are consistent with predictions in an open universe ($\Omega_\Lambda = 0$) with $\Omega_m \sim 0.3$. In Cooray (1999b, hereafter C99), we extended the predictions to general cosmologies and also predicted the existence of lensed radio and submillimeter sources toward foreground clusters. Here, we extend the calculation in C99 by including various uncertainties in the predicted number of lensed optical sources in order to study the possibility of obtaining limits on cosmological parameters based on the observed number.

In § 2, we describe our calculation and inputs for the prediction. In § 3, we compare the predicted number of lensed arcs to the observed number and use a reliable lower limit on the observed number to derive an upper limit on the cosmological mass density of the universe. We follow the conventions that the Hubble constant, H_0, is 100 h km s^{-1} Mpc^{-1}, the present matter energy density in units of the closure density is Ω_M, and the normalized cosmological constant is Ω_Λ. Unless otherwise noted, quoted errors are 1 σ statistical errors.

2. GRAVITATIONAL LENSING STATISTICS

In this section, we briefly describe our calculation and, especially, foreground lensing clusters (§ 2.1) and background sources (§ 2.2). We also introduce a nonsingular isothermal sphere model to describe the galaxy cluster dark matter profile, which is primarily motivated by recent determinations of the cluster potentials using high-performance numerical inversions of combined strong and weak lensing data from a sample of galaxy clusters.

2.1. *Foreground Lenses*

The differential probability that a beam moving from a background source will encounter a foreground lens with a path length of dz_L is

$$d\tau = n(z_L)a_{\rm lens}\frac{c\,dt}{dz_L}\,dz_L\,, \tag{1}$$

where $n(z_L)$ is the number density of foreground lenses at redshift z_l, while $a_{\rm lens}$ is the lensing cross section (e.g., Fukugita et al. 1992).

Using the Press-Schechter mass function (Press & Schechter 1974, hereafter PS), we can write the comoving number density of galaxy clusters, $dn(M, z)$, at redshift z and

[1] We note that other techniques, such as the luminosity distance to Type Ia supernovae at high redshifts (e.g., Perlmutter et al. 1998; Riess et al. 1998), also allow constraints to be placed on the cosmological constant.

mass $(M, M + dM)$, as

$$\frac{dn(M, z)}{dM} = -\sqrt{\frac{2}{\pi}}\,\frac{\bar{\rho}}{M}\,\frac{d\sigma(M, z)}{dM}\,\frac{\delta_c}{\sigma^2(M, z)}\exp\left[\frac{-\delta_c^2}{2\sigma^2(M, z)}\right], \tag{2}$$

where $\bar{\rho}$ is the comoving background matter density, $\sigma^2(M, z)$ is the variance of the fluctuation spectrum averaged over a mass scale M, and δ_c is the linear overdensity of a perturbation that has collapsed and virialized. Taking an approach similar to the one presented in Viana & Liddle (1998), we write $\sigma(M, z)$ as a function of the comoving radius, R, which contains mass M at the current epoch:

$$\sigma(R, z) = \sigma_8(z)\left(\frac{R}{8\ h^{-1}\ \mathrm{Mpc}}\right)^{-\gamma(R)}, \tag{3}$$

where

$$\gamma(R) = (0.3\Gamma + 0.2)\left[2.92 + \log_{10}\left(\frac{R}{8\ h^{-1}\ \mathrm{Mpc}}\right)\right]. \tag{4}$$

Here, $\Gamma = 0.23 \pm 0.05$ (Peacock & Dodds 1994) is the cold dark matter (CDM) shape parameter; our results are insensitive to its specific value (e.g., Viana & Liddle 1998). In order to calculate growth evolution as a function of redshift in various cosmologies, we write $\sigma_8(z)$ as

$$\sigma_8(z) = \frac{\sigma_8(0)}{1 + z}\,\frac{g[\Omega_m(z)]}{g[\Omega_m(0)]}, \tag{5}$$

where, following Carroll, Press, & Turner (1992), the growth suppression factor is

$$g(\Omega_m) = \frac{5}{2}\,\Omega_m\left[\Omega_m^{4/7} - \Omega_\Lambda + \left(1 + \frac{\Omega_m}{2}\right)\left(1 + \frac{\Omega_\Lambda}{70}\right)\right]^{-1}. \tag{6}$$

The normalization for σ_8 comes from the local temperature function (Pen 1998):

$$\sigma_8(0) = \begin{cases}(0.53 \pm 0.05)\Omega_m^{-0.46}, & \Omega_\Lambda = 0,\\ (0.53 \pm 0.05)\Omega_m^{-0.53}, & \Omega_m + \Omega_\Lambda = 1.\end{cases} \tag{7}$$

In order to model the cluster-lensing potential, we use the nonsingular singular isothermal sphere model with the observed velocity dispersion and an a priori determined value for the core radius of the dark matter potential of the cluster. The evidence for a core radius in the dark matter profile of galaxy clusters primarily comes from the existence of gravitationally lensed arcs in the radial direction from the cluster center. For simple models of the cluster potential involving singular isothermal models, such arcs are located at a distance equivalent to the core radius of the cluster potential profile. Also, recent numerical inversions of galaxy cluster–lensing potentials using *Hubble Space Telescope* and other ground-based high-quality images clearly suggest the presence of a small core radius (Tyson, Kochanski, & Dell'Antonio 1998; I. Dell'Antonio 1999, private communication). Thus, it is necessary that we consider a lensing model that allows for the possible presence of a core radius. Following Hinshaw & Krauss (1987), we consider a isothermal sphere model with a core radius and write the density profile as

$$\rho = \frac{\sigma_{\mathrm{vel}}^2}{2\pi G(r^2 + r_c^2)}, \tag{8}$$

where σ_{vel} is the dark matter velocity dispersion and r_c is the core radius of the dark matter profile of the cluster. The conventional singular isothermal sphere (SIS) is recovered when r_c is zero. The lensing cross section for the nonsingular isothermal model is given by

$$a_{\mathrm{lens}} = 16\pi^3\left(\frac{\sigma_{\mathrm{vel}}}{c}\right)^4\left(\frac{D_{\mathrm{OL}}D_{\mathrm{LS}}}{D_{\mathrm{OS}}}\right)^2 f(\beta), \tag{9}$$

where D_{OL}, D_{OS}, and D_{LS} are observer-to-lens, observer-to-source, and lens-to-source distances. These distances are calculated under the filled-beam approximation. In equation (9), $f(\beta)$ is a correction factor that takes into account the nonsingular behavior of the density profile (see Hinshaw & Krauss 1987):

$$f(\beta) = 1 + 5\beta - \frac{\beta^2}{2} - \frac{\sqrt{\beta}(4 + \beta)^{3/2}}{2}, \tag{10}$$

where β is the ratio of core radius to critical radius of the lensing potential, with the latter measured at the redshift of the cluster:

$$\beta = \frac{r_c cH_0(1 + z_L)}{4\pi\sigma_{\mathrm{vel}}^2}\left(\frac{D_{\mathrm{OS}}}{D_{\mathrm{LS}}D_{\mathrm{OS}}}\right). \tag{11}$$

When the SIS model is considered, $\beta = 0$ and $f(\beta) = 1$. For small core radii, especially for the present case involving galaxy clusters, one can usually ignore higher order β terms associated with $f(\beta)$; we consider, however, the full formula in deriving cosmological parameters. Finally, the differential optical depth for the nonsingular isothermal model is

$$d\tau = 16\pi^3\left\{\int_{M_{\min}}^{\infty}\left[\frac{\sigma_{\mathrm{vel}}(M')}{c}\right]^4\frac{dn(M', z_L)}{dM'}f(\beta)dM'\right\} \times (1 + z_L)^3\left(\frac{D_{\mathrm{OL}}D_{\mathrm{LS}}}{D_{\mathrm{OS}}}\right)^2\frac{c\,dt}{dz_L}\,dz_L. \tag{12}$$

The total optical depth to a given background redshift, z_s, is given by

$$\tau(z_s) = \int_0^{z_s}\frac{d\tau}{dz_L}\,dz_L. \tag{13}$$

To calculate the lensing optical depth, we take a two-step approach in order to relate a cluster's velocity dispersion to its mass. We relate velocity dispersion to cluster temperature, using the recently updated σ-T relation (Wu, Fang, & Xu 1998),

$$\sigma_{\mathrm{vel}}(T) = 10^{2.57 \pm 0.03}\left(\frac{T}{\mathrm{keV}}\right)^{0.56 \pm 0.09}\ \mathrm{km\ s^{-1}}, \tag{14}$$

and then to mass, using a partly theoretical M-T relation (e.g., Barbosa et al. 1996),

$$T(M, z) = (6.8 \pm 0.5)\ h^{2/3}\ \mathrm{keV}\left[\frac{\Omega_m\Delta_c(\Omega_m, z)}{178}\right]^{1/3} \times\left(\frac{M}{10^{15}\ h^{-1}\ M_\odot}\right)^{2/3}(1 + z). \tag{15}$$

We have allowed for an extra uncertainty in the M-T relation by comparing various normalizations that have been suggested in the literature. Also, we note that β is dependent on cluster mass through velocity dispersion (eq. [11]). It is likely that cluster core radii are also dependent on individual cluster masses. Even though such variations have been observationally determined for galaxies (e.g., Lauer 1985), there is still no observational evidence for a dependence of galaxy cluster core radii with other physical properties, such as the X-ray luminosity or temperature. For the purpose of this calculation, we take a constant value for the core radius, based on the mean value of cluster core radii from numerical inversions ($\sim 35\ h^{-1}$ kpc; I. Dell'Antonio 1999, private communication). When deriving cosmological parameters, we vary the exact value of the core radius to investigate the parameter dependences on it; as we find later, our limit on Ω_m is weakly dependent on the core radius.

In Figure 1, as an illustration, we show the lensing optical depth due to foreground clusters with total masses greater than $7.5 \times 10^{14}\ h^{-1}\ M_\odot$ for a background source at a redshift of 1, as a function of Ω_m for open and flat cosmologies and considering a lensing potential in which $f(\beta) = 1$ (SIS model). Figure 1 shows the 95% upper and lower confidences that were produced in each case when we considered all the possible errors we have so far discussed. The uncertainty in the optical depth is primarily dominated by the error associated with the normalization of the PS mass function; since the number density of massive clusters is strongly sensitive to the exponential term in equation (2), small changes in σ_8 can produce order-of-magnitude changes in the number density. The difference between flat and open cosmological models is primarily due to the increase in lensing probability with the addition of Ω_Λ. However, this difference is small, and when errors in observations are also considered, it is impossible to study the possible existence of a cosmological constant by using lensed arc statistics. Therefore, taking a conservative approach, we combine the upper curve valid for flat cosmologies with the lower curve defined by open models to combine the 95% confidence range in the predicted number of lensed sources.

2.2. *Background Sources*

In order to obtain reliable predictions on the number of lensed arcs, it is important that both the background source evolution and effects such as "magnification bias" (Kochanek 1991) be included in the calculation. Our description of background sources comes from the Hubble Deep Field (HDF; Williams et al. 1996). We use the HDF redshift and magnitude distribution and the luminosity function from Sawicki, Lin, & Yee (1997). Such an approach allows us to reliably account for the true redshift distribution of background sources, instead of an empirical distribution or a constant redshift, while also accounting for intrinsic evolutionary effects, which has shown to be important for lensing predictions (e.g., Hamana & Futamase 1997).

By using the probability that a source at redshift z is strongly lensed $[\tau(z, \Omega_m, M_{\rm min})]$ and the number of unlensed background sources between rest-frame luminosity L and $L + dL$ and between redshifts z and $z + dz$ $[\Phi(L, z)dL\,dz]$, we can write the number of lensed galaxies, $d\bar{N}$, in that luminosity and redshift interval as (see also Maoz et al. 1992)

$$\frac{d\bar{N}(L, z)}{dz} = \tau(z, \Omega_m, M_{\rm min}) \times \int \left[\Phi\left(\frac{L}{A}, z\right)\frac{dL}{A}\right] f(A, L, z) q(A) dA\ . \quad (16)$$

Here, the integral is over all allowed values of A, the amplification of the brightest lensed image; $q(A)$ is the probability distribution of amplifications; and $f(A, L, z)$ is the probability of observing the brightest image given A, L, and z. Our assumption that the lenses are nonsingular isothermal spheres implies that the minimum amplification, $A_{\rm min}$, is a function of β. In general, the probability distribution of amplifications can be written as

$$q(A)dA = 2A_{\rm min}^2 A^{-3}\,dA\ . \quad (17)$$

In Figure 2, we show $A_{\rm min}$ as a function of β, which is calculated following Cheng & Krauss (1999). In practice we use a fitting function that returns $A_{\rm min}$ for a given value of β, with an accuracy of better than 0.1% at all interested values of β in the present calculation. For simplicity, we assume that $f(A, L, z)$ is a step function, $\Theta(m_{\rm lim}, A)$, so that a lensed image with apparent magnitude brighter than $m_{\rm lim}$ is detected. For a given value of the core radius and the veloc-

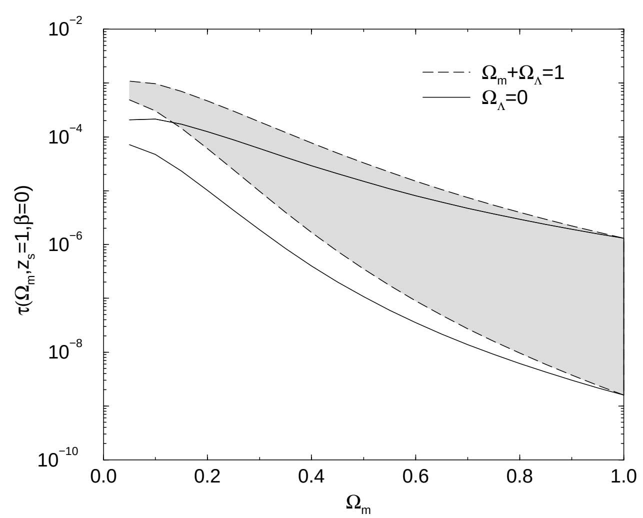

FIG. 1.—Optical depth for strong lensing for a background source at a redshift of 1, due to foreground massive clusters. Shown are the 95% confidence ranges for both flat (*dot-dashed lines*) and open (*solid lines*) cosmologies with and without a cosmological constant. Given the large uncertainty associated with the optical depth and the small difference between flat and open cosmological models, it is unlikely that lensed arc statistics can be used to reliably place limits on the cosmological constant.

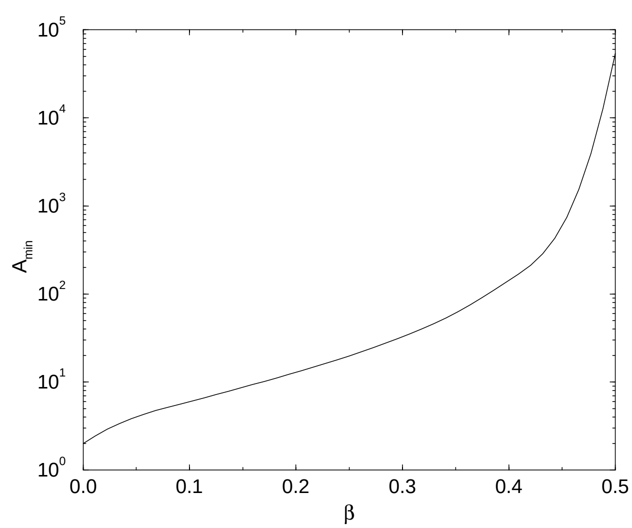

FIG. 2.—Minimum amplification vs. β, the ratio of core radius to critical radius at the redshift of the lensing cluster.

ity dispersion, β is determined from equation (11). For massive clusters discussed here with velocity dispersions of the order $\gtrsim 1000$ km s^{-1} and at redshifts ~ 0.2, $\beta \lesssim 0.07$ and $A_{\rm min} \lesssim 5$. Compared with the SIS model, the addition of a small core radius produces only slight changes in the lensing probability. As described in Kochanek (1996a), the effect of a core radius is to increase the magnification bias while increasing the effective lensing cross section; the overall effect is that the presence of a core radius is not significantly different from that of an SIS model.

We assume that the brightness distribution of background galaxies at any given redshift is described by a Schechter function (Schechter 1976), in which the comoving density of galaxies at redshift z and with luminosity between L and $L + dL$ is

$$\phi(L, z)\, dL = \phi^*(z) \left[\frac{L}{L^*(z)} \right]^{\alpha(z)} e^{-L/L^*(z)}\, dL \,, \qquad (18)$$

where, as before, both L and L^* are measured in the rest frame of the galaxy. Following Cooray, Quashnock, & Miller 1999, we can write the expected number $\bar{N}$ of lensed sources as

$$\bar{N} = \sum_i \tau(z_i, \Omega_m, M_{\rm min}) \int_2^\infty A^{-1-\alpha(z_i)} e^{L_i/L^*(z_i)} e^{-L_i/AL^*(z_i)} \times \Theta(m_{\rm lim}, A) \frac{2}{(A-1)^3}\, dA \,, \qquad (19)$$

where the sum is over each of the background galaxies. The index i represents each galaxy; hence, z_i, L_i, and m_i are, respectively, the redshift, rest-frame luminosity, and apparent magnitude of the ith galaxy.

Since L_i for an individual galaxy is unknown because of uncertain K-corrections, following Cooray et al. (1999), we estimate the total average bias by weighting the integral in equation (19) by a normalized distribution of luminosities L_i drawn from the Schechter function appropriate for the redshift z_i of galaxy i. We calculated the magnification bias for individual redshift intervals for which the Schechter function parameters are available in Table 1 of Sawicki et al. (1997). In principle, the uncertainties in the Schechter function parameters at a given redshift can affect the calculation of the bias, but in practice only the uncertainty in the power-law slope α has a significant effect. The effect of varying α on the lensing statistics was discussed in Cooray et al. (1999) for lensing statistics involving foreground galaxies in the Hubble Deep Field, which is also valid for the present case involving galaxy clusters; the general effect, which is due to uncertainties tabulated in Sawicki et al. (1997), is that the constraints on cosmological parameters vary by less than 5%, when α is in general varied by the quoted 1 σ uncertainties in Sawicki et al. (1997). Here, we take a conservative approach and allow the largest possible bias, so that the expected number is overestimated by an amount as suggested above. The only effect of this approach is to increase slightly our upper limit on Ω_m.

In Figure 3, we show the expected number of lensed arcs toward foreground massive clusters with total mass greater than $M_{\rm min} = 7.5 \times 10^{14}\ h^{-1}\ M_\odot$ and assuming a zero core radius for the lensing potential. We define an arc as a lensed source that is amplified by a factor equal to or greater than 10. To make a direct comparison to both observations and prior predictions, we impose a limiting V-band magnitude

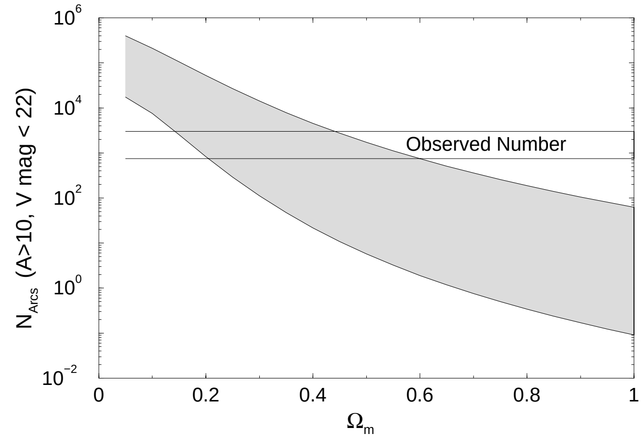

FIG. 3.—Expected number of lensed arcs on the whole sky with amplifications greater than 10 and V-band magnitudes brighter than 22 toward foreground massive clusters. The shaded range shows the 95% confidence upper and lower limits on the expected number of lensed arcs, while the horizontal lines show the range of current observed numbers. We use the lower limit on the current observed number to impose an upper limit on Ω_m.

of 22. Our numbers can be compared directly with previous estimates, especially those of Bartelmann et al. (1998). This study predicted ~ 2400 arcs in an open universe with $\Omega_m \sim 0.3$ and ~ 36 arcs in an Einstein–de Sitter universe. The number expected in a flat universe with $\Omega_m \sim 0.3$ and $\Omega_\Lambda \sim 0.7$ was ~ 280. Our estimates for an Einstein–de Sitter universe range from ~ 0.1 to 60, while for a $\Omega_m \sim 0.3$ universe (independent of Ω_Λ), they are ~ 50–7000 (with the higher end allowed by the cosmological constant). The primary reason for a lower number of arcs with Ω_Λ in the study by Bartelmann et al. (1998) was their assumption that clusters are different in universes with a cosmological constant, such that their concentration is lower. Based on numerical simulations performed by the Virgo Consortium, however, Thomas et al. (1998) studied a series of clusters in four different cosmologies, including an open model with $\Omega_m = 0.3$ and a flat model with a cosmological constant of $\Omega_\Lambda = 0.7$. The authors concluded that clusters do not exhibit differences between open and flat cosmologies with and without a cosmological constant and that cluster structures cannot be used to discriminate between the two possibilities. If Thomas et al. (1998) are correct, then the inclusion of a cosmological constant is not expected to change cluster mass profiles to an extent that would affect the gravitational lensing rate. In any case, such systematic effects are unlikely to be nearly as large as the current uncertainty in σ_8, which dominates the present calculation on the lensing rate. Ignoring this case, our predictions are generally consistent with Bartelmann et al. (1998). As we have demonstrated in Figure 1, lensed arc statistics are unlikely to provide useful limits on the cosmological constant. The same is true for alternatives to the cosmological constant, such as scalar field and quintessence models that have recently been introduced (e.g., Steinhardt, Wang, & Zlatev 1998).

3. CONSTRAINTS ON Ω_m

In order to derive a limit on Ω_m based on the number of lensed arcs, we require knowledge on the observed number of such lensing events. Current surveys of clusters are based on their X-ray luminosities rather than masses. For an example, the luminosity cutoff of the follow-up Extended

Medium-Sensitivity Survey (EMSS) cluster arc survey by Le Fèvre et al. (1994) is $8 \times 10^{44}\ h^{-2}$ ergs s^{-1}, measured in the EMSS band of 0.3–3.5 keV. Converting this to a total mass by following through a recently derived L-T relation (Arnaud & Evrard 1999) in addition to the above M-T relation suggests a reliable lower limit on the mass of $7.5 \times 10^{14}\ h^{-1}\ M_{\odot}$. We have taken the lowest limit on mass by considering all cosmologies—since the M-T relation and L estimates are different under varying cosmologies. The current observed arc statistics (e.g., Le Fèvre et al. 1994; Luppino et al. 1999), when converted to a whole-sky number, suggest that the number of arcs toward above-defined massive clusters and with amplifications greater than 10 down to a V-band–limiting magnitude of 22 is between 1500 and 2500 (e.g., C99; Bartelmann et al. 1998). Ignoring the upper value, which is likely to be unreliable, we use the lower estimate to derive an upper limit on Ω_m. Since the lower estimate is based on the observed number, this allows us to put a reliable upper limit on Ω_m. We also vary this lower limit to study its effects on our constraints.

In order to derive a constraint on Ω_m, we adopt a Bayesian approach, and take a uniform prior for Ω_m between 0 and +1. This is primarily due to the fact that we do not yet have a precise determination of Ω_m, and, based on various theoretical arguments, we do not wish to consider cosmologies in which this quantity lies outside the interval [0, 1]. Since the prior for Ω_m is uniform, the posterior probability density is simply proportional to the likelihood.

The likelihood $\mathscr{L}$—a function of Ω_m—is the probability of the data, given Ω_m. The likelihood for n observed arcs (at redshifts z_j) when $\bar{N}$ is expected is given by Cooray et al. (1999):

$$\langle \mathscr{L}(n) \rangle = \prod_{j=0}^{n} \tau(z_j) e^{-\bar{N}} \left\{ 1 + \sigma_\tau^2 \left[\frac{\bar{N}^2}{2} - n\bar{N} + \frac{n(n-1)}{2} \right] \right\} . \tag{20}$$

We have taken into account the uncertainty in the predicted lensing rate by introducing σ_τ, which is the fractional 1 σ error on τ, and then mariginalizing the likelihood over the variance of it. In order to constrain Ω_m, we also need the redshifts z_j of the observed arcs. Using published data on individual lensing clusters, we obtained a median redshift for the lensed arcs of $\bar{z} \sim 1.6$. As we find below, changing this mean redshift to a reasonably different value does not change our constraints on the Ω_m greatly.

When the observed lower limit is compared with predictions, we find that $\Omega_m \lesssim 0.62$ at the 95% confidence when there is no core radius (SIS). When we include a core radius of 35 h^{-1} kpc, the upper limit on Ω_m decreases to 0.56 at the 95% confidence level; the change in the upper limit on Ω_m is only a minor effect. If the core radius were to be as large as 100 h^{-1} kpc, then the derived upper limit on Ω_m could be as low as 0.29 at the 95% confidence level. However, such a large core radius for the cluster dark matter profile is ruled out, leaving the possibility for only a much smaller core radius of the order 30–40 h^{-1} kpc. When the effective median redshift of lensed arcs is changed to a lower number, as ~ 1, the upper limit increases to 0.58 from 0.56, while when the redshift is increased to a value of 3.0 from 1.6, the upper limit on Ω_m at the 95% confidence decreases to 0.52. When we increase the lower limit from 1500 to 2000, our upper limit on Ω_m with a model involving a core radius of 35 h^{-1} kpc decreases to 0.52 from 0.56. This is primarily due to the fact that the expected number of lensing events varies by orders of magnitude when Ω_m is changed from 1 to 0, with the variation in the expected number larger at the lower end of Ω_m values. For such a small core radius, the limit on Ω_m is consistent with current estimates based on other cosmological probes, such as Type Ia supernovae and galaxy cluster abundances. We note that our limit on Ω_m does not mean that the universe is open without a cosmological constant, but rather that lensed arc statistics are not sensitive enough to the cosmological constant to see its effects above the current uncertainties. In general, the upper limit on Ω_m with a cosmological constant is slightly higher when compared with an open model. However, this difference is rather small (approximately a few percent; see Fig. 1) and cannot be distinguished using current observations on cluster number counts and lensed arcs.

3.1. *Uncertainties and Systematic Effects*

Using a lower limit on the observed number of lensed arcs, we have derived an upper limit on Ω_m. A major uncertainty is likely to come when estimating a lower limit on the observed number of arcs, since it is based only on optical follow-up observations of EMSS clusters (e.g., Henry et al. 1992). As a reliable approach, we have taken the lower limit allowed by the observed number of lensed arcs toward this sample. In reality, the true number is likely to be higher, but the lower limit allows us to safely consider upper limits on cosmological parameters, especially the cosmological mass density. The present observational number of lensing events is unlikely to be improved unless large samples of clusters are followed up at optical wavelengths. Several attempts are currently underway (e.g., Luppino et al. 1999); however, all such surveys are still based on the EMSS sample. It is likely that the optical follow-up observations of additional cluster catalogs, such as the *ROSAT* Bright Cluster Survey (BCS; Ebeling et al. 1998), can greatly improve our knowledge on the lensing statistics from galaxy clusters, allowing better constraints on the cosmological parameters.

In addition to current low-number statistics, other uncertainties are likely to come from the conversion of observations, such as cluster X-ray luminosity, to mass. However, at each step, we have considered various estimates such that the predicted number of lensed arcs is overestimated; this approach allows us to consider a reliable limit on Ω_m, whose upper limit may have been systematically increased by our procedure. We have also investigated the effect of a core radius on arc statistics. As found, for luminous optical arcs with amplifications greater than 10, the effect of a core radius on our prediction of the number of lensing events is minimal. The upper limit varies only from 0.62 to 0.56 at the 95% confidence when a reasonable core radius of size 35 h^{-1} kpc is introduced. Increasing a core radius as high as 100 h^{-1} kpc reduces the upper limit by a factor of ~ 2; however, such a large core radius is ruled out by current observations of gravitational lensing of clusters (e.g., the nonexistence of radial arcs at large distances from the cluster center).

4. SUMMARY AND CONCLUSIONS

Using a lower limit on the observed number of lensed arcs due to clusters, we have calculated an upper limit on Ω_m. Due to large uncertainties in the predicted number of lensed sources, primarily dominated by the error in σ_8, we are unable to place limits on the cosmological constant.

However, after considering possible known errors and carefully taking into account various estimates such that the upper limit on Ω_m is not reduced, we conclude that $\Omega_m \lesssim 0.62$ at the 95% confidence.

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REFERENCES

Arnaud, M., & Evrard, A. E. 1999, MNRAS, 305, 631
Bahcall, N. A., & Fan, X. 1998, ApJ, 504, 1
Barbosa, A., Bartlett, J. G., Blanchard, A., & Oukbir, J. 1996, A&A, 314, 13
Bartelmann, M., Huss, A., Colberg, J. M., Jenkins, A., & Pearce, F. A. 1998, A&A, 330, 1
Carroll, S. M., Press, W. H., & Turner, E. L. 1992, ARA&A, 30, 499
Cheng, Y.-C. N., & Krauss, L. M. 1999, MNRAS, submitted (astro-ph/9810393)
Cooray, A. R. 1999a, A&A, 342, 353
———. 1999b, A&A, 341, 653 (C99)
Cooray, A. R., Quashnock, J. M., & Miller, M. C. 1999, ApJ, 511, 562
Ebeling, H., Edge, A. C., Böhringer, H., Allen, S. W., Crawford, C. S., Fabian, A. C., Voges, W., & Huchra, J. P. 1998, MNRAS, 301, 881
Falco, E., Kochanek, C. S., & Muñoz, J. A. 1998, ApJ, 494, 47
Fukugita, M., Futamase, T., Kasai, M., & Turner, E. L. 1992, ApJ, 393, 3
Guerra, E. J., & Daly, R. A. 1998, ApJ, 493, 536
Hamana, T., & Futamase, T. 1997, MNRAS, 286, L7
Henry, J. P., Gioia, I. M., Maccacaro, T., Morris, S. L., Stocke, J. T., & Wolter, E. 1992, ApJ, 386, 408
Hinshaw, G., & Krauss, L. M. 1987, ApJ, 320, 468
Kochanek, C. S. 1991, ApJ, 379, 517
———. 1996a, in IAU Symp. 173, Astrophysical Applications of Gravitational Lensing, ed. C. S. Kochanek & J. N. Hewitt (Dordrecht: Kluwer), 13
———. 1996b, ApJ, 466, 638
Lauer, T. R. 1985, ApJ, 292, 104
Le Fèvre, O., Hammer, F., Angonin, M. C., Gioia, I. M., & Luppino, G. A. 1994, ApJ, 422, L5
Lineweaver, C. H. 1998, ApJ, 505, L69
Luppino, G. A., Gioia, I. M., Hammer, F., Le Fèvre, O., & Annis, J. A. 1999, A&AS, 136, 117
Maoz, D., Bahcall, J. N., Doxsey, R., Schneider, D. P., Bahcall, N. A., Lahav, O. & Yanny, B. 1992, ApJ, 394, 51
Peacock, J. A., & Dodds, S. J. 1994, MNRAS, 267, 1020
Pen, U.-L. 1998, ApJ, 498, 60
Perlmutter, S., et al. 1998, Nature, 391, 51
Press, W. H., & Schechter, P. 1974, ApJ, 187, 425 (PS)
Quast, R., & Helbig, P. 1999, A&A, 344, 721
Riess, A. G., et al. 1998, AJ, 116, 1009
Sawicki, M. J., Lin, H., & Yee, H. K. C. 1997, AJ, 113, 1
Schechter, P. 1976, ApJ, 203, 297
Steinhardt, P. J., Wang, L., & Zlatev, I. 1998, Phys. Rev. D, submitted (astro-ph/9812313)
Thomas, P. A., et al. 1998, MNRAS, 296, 1061
Tyson, J. A., Kochanski, G. P., & Dell'Antonio, I. P. 1998, ApJ, 498, L107
Viana, P. T. P., & Liddle, A. R. 1998, MNRAS, 303, 535
Williams, R. E., et al. 1996, AJ, 112, 1335
Wu, X.-P., Fang, L.-Z., & Xu, W. 1998, A&A, 338, 813
Wu, X.-P., & Hammer, F. 1993, MNRAS, 262, 187
Wu, X.-P., & Mao, S. 1996, ApJ, 463, 404