

STEPS TOWARD THE POWER SPECTRUM OF MATTER. II. THE BIASING CORRECTION WITH σ_8 NORMALIZATION

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Received 1998 May 19; accepted 1999 February 17

ABSTRACT

We suggest a new method to determine the bias parameter of galaxies relative to matter. The method is based on the assumption that gravity is the dominating force which determines the formation of the structure in the universe. Because of gravitational instability, matter flows out of underdense regions toward overdense regions. To form a galaxy, the density of matter within a certain radius must exceed a critical value (Press-Schechter limit); thus galaxy formation is a threshold process. In low-density environments (voids) galaxies do not form and matter remains in primordial form. We estimate the value of the threshold density which divides the matter into two populations, a low-density population in voids and a clustered population in high-density regions. We investigate the influence of the presence of these two populations on the power spectrum of matter and galaxies. We find that the power spectrum of clustered particles (galaxies) is similar to the power spectrum of matter. We show that the fraction of total matter in the clustered population determines the difference between amplitudes of fluctuations of matter and galaxies, i.e., the bias factor. To determine the fraction of matter in voids and clustered population we perform numerical simulations. The fraction of matter in galaxies at the present epoch is found using a calibration through the σ_8 parameter. We find $\sigma_8 = 0.89 \pm 0.09$ for galaxies, $\sigma_8 = 0.68 \pm 0.09$ for matter, and $b_{\text{gal}} = 1.3 \pm 0.13$, the biasing factor of the clustered matter (galaxies) relative to all matter.

Subject headings: galaxies: formation — large-scale structure of universe

1. INTRODUCTION

The relative distribution of matter and light in the universe is an unresolved problem of fundamental importance in cosmology. Direct observational data on galaxies give information on the distribution of light; in contrast, most theoretical models simulate only the distribution of matter. That these two distributions may be different on galactic scales has been evident since the discovery of dark halos around galaxies (Einasto, Kaasik, & Saar 1974a; Ostriker, Peebles, & Yahil 1974). Several years later it became clear that differences between the distribution of light and matter exist on large scales as well. Jõeveer & Einasto (1978, hereafter JE78) demonstrated that galaxies and clusters of galaxies are distributed in filaments and the space between them is practically void of visible matter, whereas in numerical simulations of structure formation (Zeldovich 1978) low-density regions are not completely empty of matter. This difference between the distribution of galaxies and dark matter (DM) was quantified by Zeldovich, Einasto, & Shandarin (1982): in numerical simulations there exists a population of almost isolated particles in voids which has no counterpart in the observed distribution of galaxies. Einasto, Jõeveer, & Saar (1980, hereafter EJS80) showed that this difference can be explained if the evolution of the universe is primarily due to gravity. As demonstrated by Zeldovich (1970, hereafter Z70), the gravitational instability enhances the density contrast: matter flows out from low-density regions toward high-density ones until it collapses to form galaxies and systems of galaxies. This process is slow, and gravity is not able to evacuate voids

completely—there must exist some primeval matter in voids.

Bahcall & Soneira (1983) and Klypin & Kopylov (1983) demonstrated that the correlation function of clusters of galaxies has an amplitude larger than that of galaxies, and Kaiser (1984) explained this using the theory of high peaks in a Gaussian density field, introducing the term “biasing” to describe this fact. A similar relation holds for power spectra of clusters and galaxies, and we define the bias parameter b_c through the power spectra of all matter, $P_m(k)$, and that of the clustered matter, $P_c(k)$,

$$P_c(k) = b_c^2(k)P_m(k), \quad (1)$$

where k is the wavenumber in units of $h \text{ Mpc}^{-1}$ and the Hubble constant is expressed as $H_0 = 100 h \text{ km s}^{-1} \text{ Mpc}^{-1}$. According to this definition, the biasing parameter is a function of wavenumber k . The power spectrum is calculated by integrating the density contrast over the whole space under study; thus, the biasing parameter is a mean averaged over the space.

As the bias factor of galaxies relative to matter was not known it was considered as a free parameter suitable for bringing models of structure formation into agreement with observations of density fluctuations of galaxies. As an example we refer to the pioneering study of the standard cold dark matter model by Davis et al. (1985). Here a large biasing factor $b = 2.5$ was applied to make the model agree with observations. Actually the bias factor is a fundamental parameter characterizing the distribution of matter and galaxies and must be determined from data.

In this paper we concentrate on the problem of how the presence of primordial matter in voids affects the power spectrum. The idea we shall use of how to estimate the bias factor of clustered matter relative to all matter was suggested by Gramann & Einasto (1992, hereafter GE92). They assumed that the structure evolution of the universe is primarily due to gravity. The primordial matter contracts and

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eventually forms galaxies only in case when its density is high enough; in other words, the formation of galaxies is essentially a threshold phenomenon (EJS80; Einasto & Saar 1986). GE92 demonstrated that the power spectra of matter and clustered matter (galaxies) are similar in shape and that the relative amplitude of the power spectrum of galaxies (clustered population) depends on the fraction of matter in the clustered population. We consider as “clustered matter” all matter associated with galaxies, including dark halos of galaxies and clusters of galaxies. Einasto et al. (1994, hereafter E94) studied the evacuation of voids and estimated the biasing parameter of clustered matter relative to all matter.

Here we shall investigate the relation between the power spectra of clustered matter and all matter in more detail and derive a new estimate of the respective bias factor. The paper is organized as follows. In § 2 we consider the biasing as a physical phenomenon and compare our approach with other biasing studies. Thereafter we investigate the influence of the void matter on the power spectra of galaxies and matter, using numerical simulations. One problem with numerical simulations is the identification of the present epoch. We do this in § 3 using the σ_8 normalization. In § 4 we discuss our results. Section 5 gives the main conclusions of the study.

2. POWER SPECTRA OF GALAXIES AND MATTER

2.1. Physical Biasing

We shall assume here that the structure evolution of the universe is basically due to gravity and that initial density fluctuations are Gaussian and adiabatic (i.e., velocities of particles are in agreement with the density field). Because of gravitational instability, the evolution of matter in under- and overdense regions is different. Gravity attracts matter toward high-density regions, thus particles flow away from low-density regions and density in high-density regions increases until contracting objects collapse. As shown by Z70, the collapse occurs along caustics. Initially it was assumed that caustics are two-dimensional pancake-like structures. However, direct observational data (JE78; EJS80; Einasto, Klypin, & Shandarin 1983) show that the basic structural elements of the universe are strings and chains of galaxies and clusters. Numerical simulations of structure evolution have confirmed the formation of essentially one-dimensional structures if the effective power index of the spectrum on galactic scales is negative (Melott et al. 1983; Einasto et al. 1986a; Einasto & Saar 1986; Gramann 1988; Melott & Shandarin 1993; Pauls & Melott 1995; Brodbeck et al. 1998). Bond, Kofman, & Pogosyan (1996) demonstrated analytically that under very general assumptions the gravitational evolution leads to a network (web) of high-density filaments and low-density regions outside of the web. General physical considerations suggest that in high-density regions the gas is cooling, which creates favorable conditions for star formation (Rees & Ostriker 1977; Silk 1977). Thus, the gravitational character of the evolution leads to the formation of a filamentary web of galaxies and clusters of galaxies.

According to Press & Schechter (1974), the contraction occurs if the linear overdensity exceeds a factor of 1.68 in a sphere of radius r which determines the mass of the system or the galaxy. In a low-density environment the matter cannot contract and remains primordial. Hydrodynamical

simulations of galaxy formation by Cen & Ostriker (1992, 1999), Katz, Hernquist, & Weinberg (1992), Katz, Weinberg, & Hernquist (1996), and Weinberg, Katz, & Hernquist (1997) have confirmed that the galaxy formation is ineffective in a low-density environment. Gas cooling, as required for star formation, occurs only in high-density regions—near the centers of contracting clumps of primordial matter.

These considerations lead us to the conclusion that within the gravitational instability picture the formation of galaxies is a density threshold phenomenon (EJS80; Einasto & Saar 1986). The central problems are how to find the threshold density which divides the primordial matter in low-density regions and the clustered matter in high-density regions and how the division of matter into underdense and overdense populations influences the power spectra of galaxies and systems of galaxies.

2.2. Density Field of Galaxies and Matter

The true density field of the universe is a continuous function of coordinates since the dominating population is DM which consists of particles of small mass. Here we accept the current paradigm that DM (or at least, most of it) is nonbaryonic. The dynamical evolution of the universe can be simulated using N -body calculations. As the number of particles in simulations is limited, it is impossible to simulate the motion of all DM particles; in practice, the mass of particles in simulations is usually a fraction of the mass of a typical galaxy. Thus, in order to find the true density field of DM, the distribution of a discrete set of particles is to be smoothed. Galaxies also are discrete objects; the density field of galaxies can be calculated by smoothing too. Here the smoothing length is of prime importance.

Galaxies form mostly in small groups which are collected from primordial matter in a comoving volume of a megaparsec scale. Thus, to start such a collapse of primordial matter, regions are needed with at least the mean matter density when smoothed on megaparsec scales. Presently DM forms halos around galaxies in clusters and groups, and the characteristic scale of halos is the semiminor radius of these systems, also $\approx 1 h^{-1}$ Mpc (Einasto et al. 1984). In the present paper our goal is to find the *true density field of DM* as accurately as possible. We conclude that the density field has to be found from positions of galaxies or simulation particles with a $\sim 1 h^{-1}$ Mpc smoothing length. We call densities calculated with a small (1 Mpc scale) smoothing parameter as *local* ones, in contrast to *global* densities which are found using a large (10 Mpc scale) smoothing length.

2.3. Comparison with Conventional Biasing Studies

Our approach to the biasing phenomenon differs from the approach of most other investigators. Commonly the biasing parameter is defined as the ratio of the density contrast of galaxies and matter at location x ,

$$\delta_{\text{gal}}(x) = b\delta_m(x). \quad (2)$$

As there are no galaxies in voids, we expect $b = 0$ there. If galaxies trace the matter in high-density regions, then in these regions $b = 1$. In order to apply this formula and to find a mean value of the biasing parameter, the density field is conventionally smoothed with a rather large smoothing length ($\geq 8 h^{-1}$ Mpc; see Dekel & Lahav 1998; Blanton et al. 1998 and references therein for recent studies). Excessive smoothing mixes unclustered DM in voids and clustered

DM in high-density regions which makes the simple biasing phenomenon rather complicated.

Our experience with density smoothing has shown that large smoothing is useful if one wants to locate large high-density regions, such as superclusters of galaxies; see Figure 1b of Lindner et al. (1995) and Figure 2 of Frisch et al. (1995). In this case the true filamentary nature of galaxy systems is completely lost and we can only see the distribution of large over- and underdense regions. Visualizations of high-resolution simulations of structure evolution show that high-density regions of gas and DM form an almost coinciding and very thin filamentary web (for a recent study see Brodbeck et al. 1998), confirming older results obtained with lower resolution. Thus, if we are interested in the true density field of DM we have to use a small smoothing length.

2.4. The Influence of a Homogeneous Population

The population of clustered particles is obtained from the population of all particles by the exclusion of void particles. Now we shall analyze how the exclusion of void particles influences the power spectrum. The power spectrum is defined through the density contrast

$$\delta(x) = \frac{\varrho(x) - \bar{\varrho}}{\bar{\varrho}}; \quad (3)$$

here $\varrho(x)$ is the density at location x and $\bar{\varrho}$ is the mean density.

Consider an idealized density field, which consists of a fluctuating clustered component and a background of constant density, so that

$$\varrho_m(x) = \varrho_c(x) + \varrho_s(x); \quad (4a)$$

here subscripts m , c , and s are for all matter and its clustered and smooth components, respectively. The density contrast of the matter is

$$\delta_m = \frac{\varrho_m - \bar{\varrho}_m}{\bar{\varrho}_m};$$

or, applying equation (4a),

$$\delta_m = \frac{\varrho_c + \varrho_s - (\bar{\varrho}_c + \bar{\varrho}_s)}{\bar{\varrho}_c + \bar{\varrho}_s}.$$

Since $\varrho_s = \bar{\varrho}_s$,

$$\delta_m = \frac{\varrho_c - \bar{\varrho}_c}{\bar{\varrho}_m} = \delta_c \frac{\bar{\varrho}_c}{\bar{\varrho}_m}.$$

In the last equation $\bar{\varrho}_c/\bar{\varrho}_m$ is the fraction of matter in the clustered population, F_c , and we get

$$\delta_m = \delta_c F_c. \quad (4b)$$

A similar formula holds for the density contrast in Fourier space, and we obtain the relation between power spectra of matter and the clustered population

$$P_m(k) = \langle |\delta_m(k)|^2 \rangle = F_c^2 P_c(k), \quad (5)$$

where $\delta_m(k)$ is the Fourier component of the matter density contrast for a wavenumber k and the averaging is over the whole space under study. We see that for this ideal case the power spectra of matter and the clustered population are related by an equation similar to equation (1). Hence, we get

for the bias factor

$$b_c = \frac{1}{F_c}. \quad (6)$$

Equations (5) and (6) were derived by GE92. These equations show that the subtraction of a homogeneous population from the whole matter population increases the amplitude of the spectrum of the remaining clustered population. In this approximation biasing is linear and does not depend on scale. These equations have a simple interpretation. The power spectrum describes the square of the amplitude of the density contrast, i.e., the amplitude of density perturbations with respect to the mean density. If we subtract from the density field a constant density background but otherwise preserve density fluctuations, then amplitudes of *absolute* density fluctuations remain the same but amplitudes of *relative* fluctuations with respect to the mean density increase by a factor which is determined by the ratio of mean densities, i.e., by the fraction of matter in the new density field with respect to the previous one.

The density of the real void population is not constant, neither is it appropriate to attribute part of the DM particles in high-density regions to the smooth background, i.e., to the void population, which does not penetrate high-density regions. Thus, we have to ask, how are power spectra of matter and the clustered population related for a more realistic distribution of matter?

2.5. The Evolution of Low- and High-Density Regions in Simulations

To answer this question we performed numerical simulations of the evolution of matter. As the above formulae are identical in the two-dimensional and three-dimensional cases we used a two-dimensional simulation to obtain a better resolution. We used a particle mesh (PM) algorithm with 512^2 particles and cells and a double power-law initial power spectrum (eq. [3] in Einasto et al. 1999b, hereafter Paper I), a simple approximation of the observed spectra of galaxies and clusters of galaxies with a sharp maximum (Frisch et al. 1995), as seen in Figure 1 of Paper I. The power index on large scales (Harrison-Zeldovich region) was taken to be $n = 2$ and $m = -1$ on small scales; in the three-dimensional case these indices correspond to $n = 1$ and $m = -2$ on large and small scales, respectively. The turnover at k_0 was $L/4$, where $L = 512 h^{-1}$ Mpc is the box size. The corresponding linear scale is $l_{\max} = 128 h^{-1}$ Mpc. The present epoch was identified using an rms density dispersion of $\sigma_1 = 4$ on a scale of $1 h^{-1}$ Mpc, which corresponds approximately to a variance of $\sigma_8 = 0.9$ on a scale of $8 h^{-1}$ Mpc.

We find the density field using a top-hat smoothing on a $1 h^{-1}$ Mpc scale (cell size), and, by linear interpolation in both coordinates, we attribute a density value to each particle. Densities are expressed in units of the mean density. We assume that particles with different density labels can be used to represent void particles and galaxies (with their halos) of different morphological type and environment. We shall discuss this assumption and the relation between clustered particles and galaxies in more detail in the next subsection.

In accordance with arguments discussed above we call all particles with low-density values ($\varrho < \varrho_0$) *void particles*; all others are called *clustered particles*. Particles with high-

density values ($\varrho \geq \varrho_{cl}$) are associated with clusters or groups, and particles with intermediate-density values ($\varrho_0 \leq \varrho < \varrho_{cl}$) with field galaxies. In the real universe field galaxies are located around clusters and form filaments between clusters and groups. Here ϱ_0 and ϱ_{cl} are the threshold densities that divide void particles from clustered ones and particles associated with clusters from particles bound to field galaxies.

Of course, in the real universe the threshold between void and clustered particles is not sharp. If a clump of primordial matter is small enough, then it can contract and form a dwarf galaxy, even if the density, smoothed on $1 h^{-1}$ Mpc level, is smaller than ϱ_0 . Similarly, if a clump is large, it can remain in primordial form if the density, smoothed on $1 h^{-1}$ Mpc level, is greater than ϱ_0 . Such local irregularities make the threshold fuzzy. What matters is the mean value of the threshold. We investigate the influence of the fuzziness of the threshold density on the power spectrum of galaxies below.

2.6. The Distribution of Real and Simulated Galaxies

In order to apply results of numerical simulations to samples of real galaxies we must find the relationship between the distribution of real and simulated galaxies (i.e., particles in the clustered population).

Based on considerations by EJS80 on the different evolution of under- and overdense regions, Einasto & Saar (1986) divided particles in simulations into void and clustered populations using the mean density as the threshold density. In this case the topologies of simulated and real galaxy samples are in very good agreement (Einasto et al. 1986a). A similar agreement between simulated and real galaxy samples exists if one uses the correlation function test (Einasto, Klypin, & Saar 1986b) and the percolation and filling-factor tests for various density levels (Einasto et al. 1986a; Einasto & Saar 1986; Gramann 1988, 1990). This test was extended to the void diameter statistics by Einasto, Einasto, & Gramann (1989) and to the void probability function by Einasto et al. (1991), and by Gramann & Einasto (1991) and GE92 to the power spectrum analysis. A further step to check the distribution of simulations with the real universe was done by Gramann & Einasto (1991), Einasto et al. (1991), Frisch et al. (1995), and Lindner et al. (1995), where not only simulated galaxies but also simulated clusters were compared with real clusters. In all these studies a small smoothing length (about $1 h^{-1}$ Mpc) was used to determine the density field and to divide particles in simulations into the high- and low-density populations. These tests have shown that statistical properties of simulated galaxies and clusters are very close to properties of real galaxies and clusters. In other words, the division of matter into the low-density primordial population in voids and the clustered population with galaxies and clusters in high-density regions describes well the actual distribution of galaxies and clusters.

The next step in the comparison of simulations with the real universe was to investigate the possibility of using different threshold densities to approximate the distribution of galaxies of different morphology and luminosity. It is well known that bright galaxies are concentrated in central dense regions of groups and that faint companion galaxies are located in outskirts of groups (Einasto et al. 1974b). Dressler (1980) extended the density relationship to morphological types—elliptical galaxies are located mostly in

dense regions and spirals in less dense environments. Einasto et al. (1991) compared the void probability function for galaxy samples of different luminosity limit with simulated samples selected at various threshold density levels, and a similar comparison was made by GE92 using the power spectrum test. These studies have shown that void probability functions, correlation functions, and power spectra of simulated galaxies selected using various threshold density intervals approximate well the behavior of real galaxies of different luminosity and morphology.

Now we shall compare the distribution of simulations with galaxies using the two-dimensional simulation described above. The analysis of the density field in the Local Supercluster by E94 shows that the threshold density, ϱ_0 , which divides the nonclustered matter located in voids and clustered matter associated with galaxies, is approximately equal to the mean density. Hydrodynamical simulations of Cen & Ostriker (1992) also indicate that the galaxy population is located in regions of matter density above the average when smoothed on a $1 h^{-1}$ Mpc scale. A more detailed hydrodynamical simulation by Weinberg et al. (1997) shows that the distribution of gas particles of different temperatures is very different in regions of different local density: the heating and successive cooling of gas occurs only in overdense regions. We take the mean density as the threshold density, $\varrho_0 = 1$.

In Figure 1 we present the distribution of void, field, and cluster particles in a $(90 h^{-1} \text{ Mpc})^2$ region of the above simulation. We used a threshold density $\varrho_0 = 1$ to separate void particles from clustered ones and $\varrho_{cl} = 5$ to separate field particles from particles in clusters. Panels *a*, *b*, and *c* show the distribution of void, cluster, and field populations, respectively. For comparison we show in panel *d* the distribution of real galaxies in supergalactic coordinates in a sheet which crosses the Local Supercluster, the Coma, and the southern corner of the Hercules supercluster. We see that simulated particles in voids are distributed rather uniformly, while particles in the field are distributed along well-defined filaments and cluster particles form essentially spherical systems.

To check the possibility that ϱ_0 is different from 1 we have compared the distribution of particles in the density intervals $\varrho < 1$, $1 \leq \varrho < 1.5$, and $1.5 \leq \varrho < 5$; see Figure 1. Particles with $1 \leq \varrho < 1.5$ form filaments in less dense environments and are absent in voids. Their distribution resembles the distribution of dwarf galaxies which form weak filaments in supervoids, i.e., in voids defined by clusters of galaxies (see panel *d*; more detailed distributions are given in Fig. 5 of Lindner et al. 1995 and in Fig. 3 of Lindner et al. 1996). This comparison shows that the use of density threshold to select various simulated galaxies is well suited to discriminate cluster and field galaxies, and among field galaxies to locate weak and massive filaments.

This example shows that, at least for this simulation, the threshold density values used reproduce well the actual distribution of galaxies of different types. Our figure shows also that there exists no one-to-one relationship between the distribution of simulated particles selected in small threshold density intervals and real galaxies chosen in small luminosity intervals. The reason for the absence of a very close relationship is clear: dwarf galaxies are located also in clusters and in other high-density regions. However, mean statistical properties sensitive to the distribution of galaxies in a low-density environment (such as the void diameter

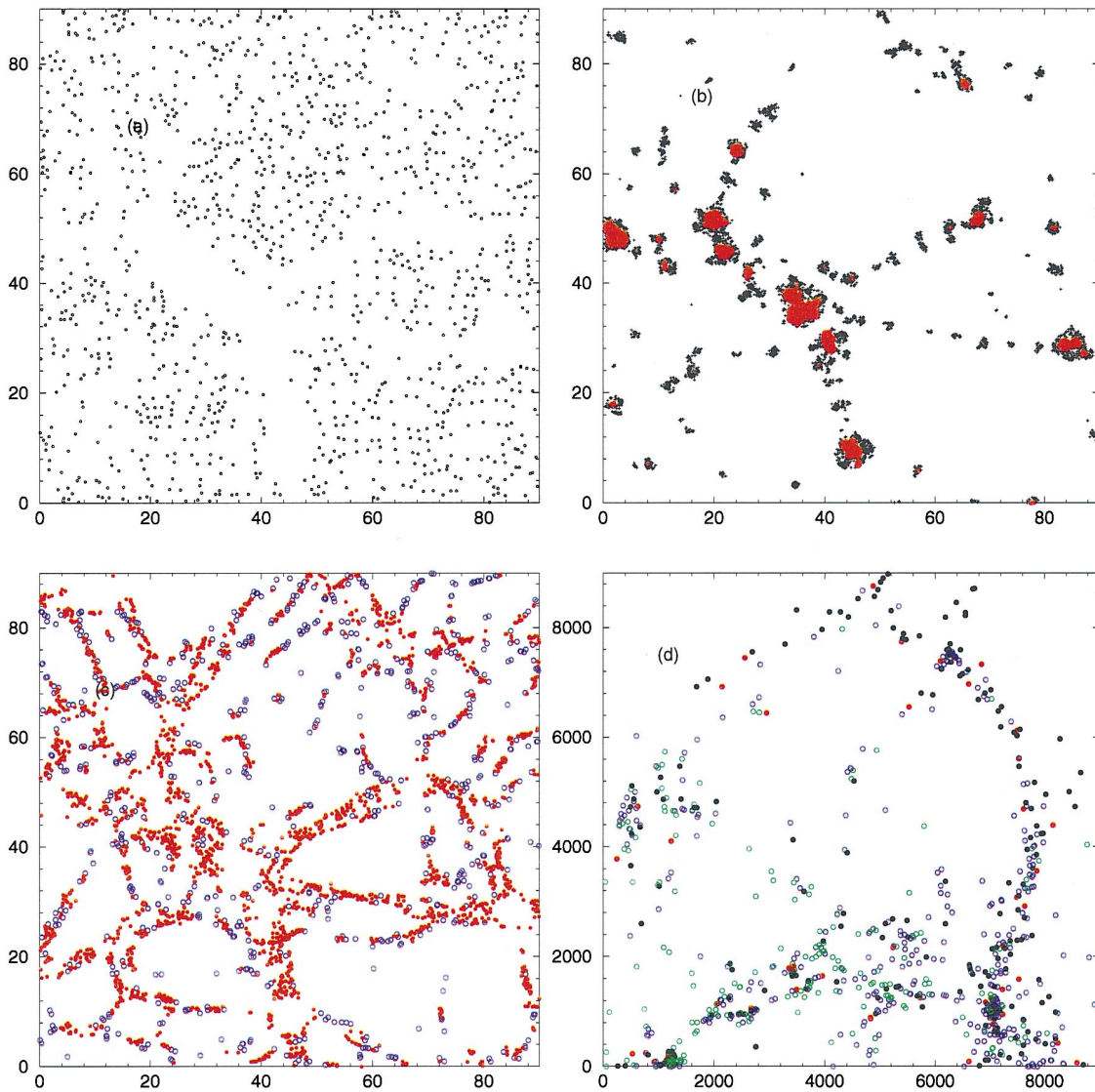


FIG. 1.—Distribution of simulated and real galaxies in a box of side length $90 h^{-1}$ Mpc. Panel *a* gives particles in voids ($q < 1$); panel *b* shows the distribution of simulated galaxies in high-density regions: galaxies in the density interval $5 \leq q < 20$ are plotted as black dots, and galaxies with $q \geq 20$ as filled (red) regions; panel *c* shows field galaxies in the density interval $1 \leq q < 1.5$ (open blue circles) and $1.5 \leq q < 5$ (red dots). Densities are expressed in units of the mean density of the universe. Panel *d* shows the distribution of galaxies in supergalactic coordinates in a sheet $0 h^{-1} \text{ Mpc} \leq X < 10 h^{-1} \text{ Mpc}$, and horizontal and vertical axes are supergalactic Y and Z , respectively; bright galaxies ($M_B \leq -20.3$) are plotted as red dots, galaxies $-20.3 < M_B \leq -19.7$ as black dots, galaxies $-19.7 < M_B \leq -18.8$ as open blue circles, and galaxies $-18.8 < M_B \leq -18.0$ as green circles (absolute magnitudes correspond to Hubble parameter $h = 1$). High-density regions are the Local, the Coma, and the Hercules superclusters in lower left-hand, lower right-hand, and upper right-hand corners, respectively. The long chain of galaxies between the Coma and Hercules superclusters is called the Great Wall; actually it is a filament.

distribution) are rather similar for galaxy samples of various limiting absolute magnitudes and for simulated galaxy samples using variable threshold density levels (Lindner et al. 1995, 1996; Frisch et al. 1995).

2.7. The Distribution of Matter and Galaxies in Different Environments

The previous analysis has shown a good agreement between the distribution of simulated and real galaxies. This analysis gives, however, no answer to the question, how accurately do galaxies follow the distribution of matter in high-density regions: groups, clusters, and superclusters. Direct observational data on the distribution of galaxies and matter are needed to clarify this problem.

The distributions of the density of galaxies and matter in groups were found to be essentially similar (Einasto et al. 1976; Vennik 1986; Zaritsky et al. 1993; David et al. 1994;

Pisani et al. 1995). Here we ignore the difference in concentration of bright and faint galaxies in groups; also we ignore the fact that within dark halos of galaxies the distribution of baryonic and dark matter is different. Since large differences occur only on submegaparsec scales, these differences do not influence the power spectrum on scales of interest for the present paper.

A comparison of the distribution of matter and light in clusters of galaxies is possible using several of the indicators of the matter distribution, e.g., gravitational lensing, X-ray-emitting gas distribution, or galaxy dynamics. These studies show that the matter and luminosity distributions are rather similar and that the concentration of light is more pronounced than that of matter, similar to the concentration of bright galaxies in groups (David et al. 1990; Carlberg 1994; Böhringer 1995; Squires et al. 1996; Carlberg et al. 1997; Markevitch & Vikhlinin 1997). Such small differ-

ences can influence the overall amplitude of the power spectrum as shown in the analysis of results of numerical simulations with different threshold densities and a sample of particles with positions shifted in high-density regions (for details see § 2.8); the shape changes only on scales comparable to the size of clusters (Fig. 2).

The comparison of the distribution of matter and light in superclusters is possible using numerical simulations. Simulations show that in large high-density regions (superclusters) more primordial matter contracts to form clusters of galaxies than in regions of lower density where systems of galaxies have lower richness. This effect raises the amplitude of the power spectrum of clusters, while the shape of the power spectrum changes only on smaller scales (see Fig. 2). To imitate this effect we have formed a sample (sample “Gal-120” discussed below) where only a fraction of particles in high-density regions is included. An observational argument in favor of the shape conservation of the power spectrum on large scales is given by the similarity of power spectra of clusters and galaxies in deep samples (see Fig. 3 of Paper I for a comparison of power spectra of clusters and that of the three-dimensional APM galaxy sample).

2.8. Simulation of Various Galaxy Populations

For further tests we have formed a number of samples of particles with various threshold density ϱ_0 and sampling rules. Table 1 gives the main parameters of these samples. N is the number of particles in samples; $F_c = N/N_{\text{tot}}$ is the fraction of particles in the clustered population (a particular sample in units of the number of particles in the sample of all matter); $b_F = 1/F_c$ is the biasing parameter calculated from equation (6); b_{mean} is the mean value of the biasing parameter found from the difference in the spectra of matter and the sample (a mean value of local differences in the wavenumber interval $0.01 < k \leq 1.0$); δ_{bk} is the relative error of the biasing parameter as a function of wavenumber (rms deviation of the local biasing parameter value from the mean value b_{mean} , in percent); δ_{ba} is the relative error of the biasing parameter b_{mean} with respect to the theoretical value $b_F = 1/F_c$, in percent.

Samples Gal-1, Gal-2, and Clust are defined by threshold densities, ϱ_0 , indicated in Table 1, such that all particles above the threshold density are included. In sample Gal-12 a fuzzy threshold density is randomly placed between densities 1 and 2, and in sample Gal-120 the threshold density

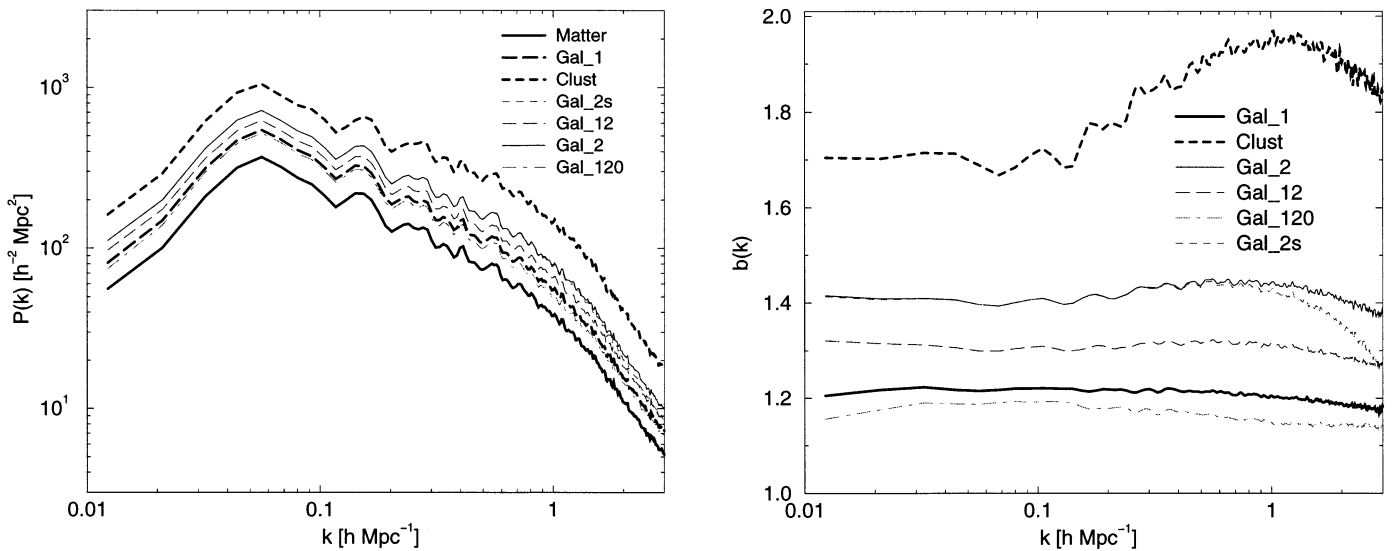


FIG. 2.—*Left-hand panel*: Power spectra of simulated galaxies. The solid bold line shows the spectrum derived for all test particles (the matter power spectrum); dashed and dotted bold lines give the power spectrum of all clustered particles (sample Gal-1) and clustered galaxies in high-density regions (sample Clust). Thin solid and dashed lines show the power spectra of samples of particles with various threshold densities and sampling rules (see Table 1 and text for details). *Right-hand panel*: The biasing parameter as a function of wavenumber, calculated from the definition in eq. (1). Samples and designations are as in the left-hand panel.

TABLE 1
BIASING PARAMETERS

Sample	ϱ_0	N	F_c	b_F	b_{mean}	δ_{bk} (%)	δ_{ba} (%)
Matter	0	262144	1.0000	1.000	1.000	0.0	0.0
Gal-1	1	219965	0.8391	1.192	1.212	0.4	1.7
Gal-120	1	215457	0.8219	1.217	1.167	1.1	4.1
Gal-12	1-2	202457	0.7723	1.295	1.374	0.8	6.1
Gal-2	2	185206	0.7065	1.415	1.432	1.1	1.2
Gal-2s	2	185206	0.7065	1.415	1.428	0.9	0.9
Clust	5	135730	0.5178	1.932	1.861	4.9	3.7

is 1, but only 90% of particles in the very high density region ($\varrho \geq 20$) are included, i.e., this sample imitates the deficiency of galaxies in superclusters. According to our simulations, particles in rich clusters of galaxies form 3%–5% of the total number of particles; if the relative number of galaxies in rich clusters is lower than in the field by a factor of up to 2 (in clusters at least half of the baryonic matter is in the form of hot X-ray gas, as indicated by direct observations and by the high value of the mass-to-luminosity ratio in clusters), then this corresponds to a $\sim 10\%$ decrease of the number of galaxies in the whole population of high-density regions. Finally, the sample Gal-2s contains all particles above a threshold density of $\varrho_0 = 2$, except that we added random shifts in an interval $[-0.25, 0.25] h^{-1}$ Mpc to the positions of particles in high-density regions ($\varrho \geq 5$). This sample imitates possible difference in the concentration of DM and galaxies in clusters (samples Gal-2 and Gal-2s correspond to visible and DM in the clustered population, respectively).

Figure 2 shows power spectra found for these samples. Obviously, all power spectra of samples of clustered particles are similar to the power spectrum of the matter but have a higher amplitude. From the difference in amplitude of the power spectra of these populations with respect to the power spectrum of matter we derived the biasing parameter as a function of wavenumber. The results are plotted in the right-hand panel of Figure 2. We see that for most samples the biasing parameter is almost constant. Only for the cluster sample and the sample with particle positions shifted in high-density regions (Gal-2s) the biasing parameter on small scales deviates from the value observed on large scales.

Table 1 shows that the biasing parameter b_{mean} , found from the difference in power spectra, is surprisingly close to the value expected from the number of particles in respective samples, b_F . These calculations show that equation (5), derived for the case of constant density of the void population, works well even in cases when we consider cluster galaxies. In this case the population of particles in low-density regions contains not only real void particles, but also particles which imitate field galaxies in filaments. In other words, equation (5) is very robust and insensitive to the distribution of the low-density population, and its relative error is a few percent. Only in the case of the sample Gal-120 where part of the galaxies in high-density regions have been removed is the biasing parameter about 5% lower than expected from equation (5). Similarly, in the case of the sample with a fuzzy threshold the mean biasing parameter value is about 5% higher than predicted from equation (5).

These data indicate that there is no evidence for the presence of large differences in the shape of the power spectra of matter and galaxies on scales of interest for the present study. Differences directly observed between cluster and galaxy samples, and predicted from small variations in the concentration of galaxies and matter in clusters and superclusters, are confined to smaller scales only, $k \geq 0.5 h \text{ Mpc}^{-1}$.

The principal result of our analysis is that power spectra of the population of all clustered particles and matter have similar shape. The difference in amplitude of power spectra of clustered particles and matter is given by the fraction of matter in the clustered population. The analysis done for various three-dimensional models of structure formation

since the study of GE92 has reached the same conclusion (Frisch et al. 1995). Sampling peculiarities which imitate various galaxy populations change the mean biasing parameter only very modestly.

3. THE AMPLITUDE OF DENSITY FLUCTUATIONS

The previous analysis has shown that the power spectrum of the clustered population (galaxies) can be reduced to the power spectrum of matter using a simple formula, equation (5), if we know the fraction of matter in voids and in high-density regions. In principle, the amount of matter in voids can be calculated using the velocity field and applying the methods of the restoration of the matter distribution (for a recent analysis see Freudling et al. 1998). The accuracy of peculiar velocity measurements is, however, not sufficient to get reliable results. For this reason we use a different approach here, and we derive the fraction of matter in voids and in the clustered population from numerical simulations. In doing so we assume that on large scales the evolution of the structure is determined by gravity alone and that simulated galaxy populations with appropriate threshold densities approximate real galaxy populations.

From numerical simulations it is straightforward to find the distribution of particles as a function of the local density of their environment. A simple counting of particles with associated density values exceeding the threshold ϱ_0 yields the fraction of matter in the clustered population, F_c . The problem is how to identify the present epoch in the simulation.

During the evolution of the universe matter flows from low-density toward high-density regions and the fraction of matter in the clustered population, F_c , grows. Simultaneously, the amplitude of the power spectrum increases; the mean amplitude can be expressed in terms of density fluctuations in a sphere of radius $r = 8 h^{-1}$ Mpc, σ_8 . We see that there exists a relation between F_c and σ_8 ; thus the epoch of the simulation can be measured in terms of the σ_8 parameter. If the present value of σ_8 of matter is known from other sources then the whole simulation can be calibrated.

E94 determined rms density fluctuations in the Local Supercluster on galactic scales, $\sigma_{1.2}$, and used this calibration to fix the present epoch of simulation. They found that for a wide class of models (with and without cosmological constant) the present fraction of matter in voids is almost independent of the model. The fraction of clustered matter is $F_c = 0.85 \pm 0.05$.

Here we modify the method of E94. The problem lies in the following: from simulations we know the amplitude of fluctuations of matter whereas from observations we have the amplitude of fluctuations of galaxies. These two quantities are related through a formula similar to equation (5). The quantity $\sigma^2(r)$ is proportional to $P(k)$, thus we get the relation between $(\sigma_8)_{\text{gal}}$ and $(\sigma_8)_m$,

$$(\sigma_8)_m = F_{\text{gal}}(\sigma_8)_{\text{gal}}. \quad (7)$$

Here we assume that $F_{\text{gal}} = F_c$. This formula holds under the same assumptions as equation (5). It is practically exact for the whole galaxy population (see the error analysis given in § 2.8).

Equation (7) gives one relation between F_c and $(\sigma_8)_m$; another relation can be found from numerical simulations (see below). By simultaneous solution of both relations we can find both parameters for the present epoch, F_c and

$(\sigma_8)_m$. Equation (6) yields then the bias factor of the clustered matter (galaxies).

3.1. The Amplitude of Galaxy Density Fluctuations

The rms amplitude of density fluctuations of galaxies, $[\sigma^2(r)]_{\text{gal}}$ in a sphere of radius r , is a direct observable. Usually it is determined from counts in cells or through the correlation function using a power-law approximation of the correlation function (Davis & Peebles 1983). For a recent determination of $(\sigma_8)_{\text{gal}}$ see Willmer, da Costa, & Pellegrini (1998).

Here we apply a different method to determine $[\sigma^2(r)]_{\text{gal}}$. The rms amplitude of density fluctuations in a sphere of radius r may be found by integrating the power spectrum:

$$\sigma^2(r) = \frac{1}{2\pi^2} \int_0^\infty P(k) W^2(kr) k^2 dk, \quad (8)$$

where $W(kr)$ is the window function. We shall use a top-hat window,

$$W(kr) = \frac{3(\sin kr - kr \cos kr)}{(kr)^3}. \quad (9)$$

The function $\sigma^2(r)$ is an integral representation of the power spectrum. Like all integral functions it is less dependent on local irregularities than the integrand. We shall determine this function from the observed power spectrum of all galaxies.

Observations allow us to determine the power spectrum in the wavenumber interval $0.03 \leq k \leq 1$ (Paper I). To apply equation (8) we have to extrapolate the observed spectrum to larger and smaller scales. For this extrapolation we shall use theoretical model spectra which fit the observed spectra. On small scales the observed nonlinear power spectrum was reduced to a linear one (see Einasto et al. 1999a, hereafter Paper III, for details). The observed power spectrum was determined for two populations, representing galaxy samples of the universe which include high-density regions and medium-density regions, $P_{\text{HD}}(k)$ and $P_{\text{MD}}(k)$, respectively.

In Figure 3 we show the function $\sigma(r)$ calculated for both variants of the power spectrum, $P_{\text{HD}}(k)$ and $P_{\text{MD}}(k)$. Theoretical spectra were calculated for a Hubble parameter $h = 0.6$, baryonic density parameter $\Omega_b = 0.04$, density parameter $\Omega_0 = 0.4$, and cosmological constant parameter $\Omega_\Lambda = 0.6$. In order to test how the value of σ , is affected by the extrapolation, we varied the density parameter and associated cosmological constant for a flat model (see the caption for Fig. 3 for details). We see that all variants of $\sigma(r)$ coincide around the scale $r = 8 h^{-1}$ Mpc. This proves that σ_8 is almost insensitive to the details of the extrapolation of the power spectrum on small and large scales and to the exact shape of the power spectrum around the maximum. A similar conclusion was obtained by White, Efstathiou, & Frenk (1993). For the galaxy power spectrum $P_{\text{HD}}(k)$ we obtain

$$(\sigma_8)_{\text{gal}} = 0.89 \pm 0.05, \quad (10)$$

which is rather close to the often used value of unity. The error of $(\sigma_8)_{\text{gal}}$ is determined by the error of the amplitude of the observed mean galaxy power spectrum, because of the scatter of power spectra of various samples. If we add the possible systematic error due to the uncertainty in the

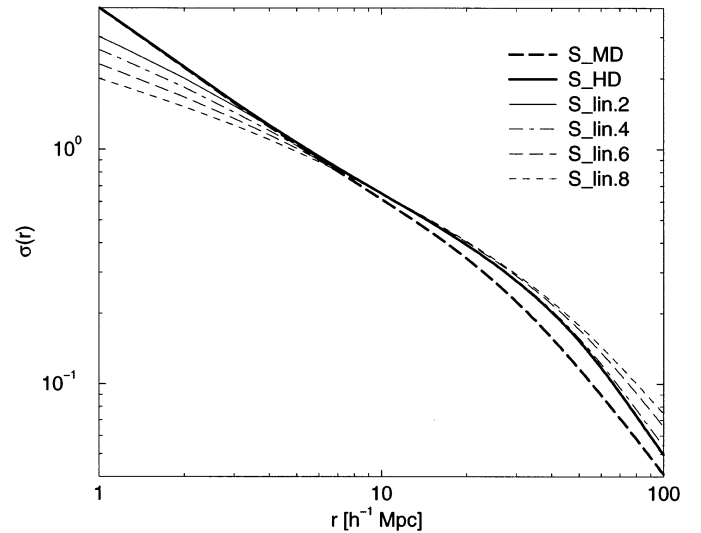


FIG. 3.—Integrated power spectrum, $\sigma(r)$. Bold solid and dashed lines give $\sigma(r)$ calculated from power spectra $P_{\text{HD}}(k)$ and $P_{\text{MD}}(k)$, respectively. Thin lines show linear extrapolations of $\sigma(r)$ calculated using flat CDM models with a cosmological constant parameter $\Omega_\Lambda = 0.2, \dots, 0.8$, Hubble constant $h = 0.6$, and baryonic density parameter $\Omega_b = 0.04$. All functions are reduced to matter using a bias factor $b = 1.18$ (E94).

overall normalization of the amplitude, we get for the 1σ error of $(\sigma_8)_{\text{gal}}$ 10%, i.e., ± 0.09 .

3.2. Reduction to Matter Power Spectrum

Equation (7) contains one observed quantity, $(\sigma_8)_{\text{gal}}$, and two unknowns, $(\sigma_8)_m$ and F_{gal} . To find a solution we need one more relation between these two unknowns. Here we use the fact that σ_8 grows with time and is suited to measure the epoch in numerical simulations. We recall that $F_c = F_{\text{gal}}$ is the total fraction of matter in high-density regions where the local density (smoothed on Mpc scale) exceeds the mean density. Initially, by definition of the mean density, we have $F_c = 0.5$. During the evolution matter flows from low-density regions toward high-density regions which evolve to clusters, groups, and galaxy filaments; thus F_c increases.

To determine the relation between F_c and σ_8 , we made three-dimensional simulations with a standard CDM model (SCDM), two realizations of a spatially flat model with cosmological constant (LCDM), and an open model (OCDM). Simulations have been made with the P³M code of Couchman (1991); simulations were run until $\sigma_8 \approx 1$. Model parameters are given in Table 2. Simulations have been made with two sets of initial positions of particles. In the first case particles were placed on a regular grid and then displaced via the Zeldovich approximation. In the second case initial particle positions have a homogeneous glasslike distribution; these positions are then used for displacing the particles according to the Zeldovich approximation, using the same random phases as in the first case. Homogeneous distributions as input for initial conditions are better because they inhabit no structure like a grid. Therefore, no remnants of the grid are seen as the simulation evolves.

As discussed above, we assume that in high-density regions ($q \geq q_0 = 1$) the distribution of the total matter density is identical with the distribution of the number density of galaxies (the density q_0 is expressed in mean density units). We calculate the density on the

TABLE 2
SIMULATION PARAMETERS

Model	Number of Particles	Number of Cells	L_{box} (h^{-1} Mpc)	Ω_0	Ω_Λ	h	$(\sigma_8)_{\text{max}}$
SCDM	128^3	128^3	200	1.0	0.0	0.5	1.16
LCDM1.....	128^3	128^3	280	0.3	0.7	0.7	1.07
LCDM2.....	128^3	128^3	280	0.3	0.7	0.7	1.00
OCDM	128^3	128^3	280	0.5	0.0	0.7	0.91

location of galaxies using an adaptive smoothing algorithm: it is determined from the outer radius of a sphere which contains 12 nearest neighbors to the galaxy. In systems of galaxies this number corresponds approximately to a smoothing scale comparable to the size of typical systems of galaxies—clusters, groups, and filaments.

For all time steps we calculated the integrated density distribution, i.e., the fraction of particles located in regions with density $F(\leq \varrho)$ for different ϱ . For the OCDM model, our results are shown in Figure 4a. They are very similar to those plotted in Figure 3 of E94. The fraction of matter in galaxies is given in Figure 4b, expressed in terms of σ_8 .

The relation between F_c and σ_8 , following from the equation (7), is also shown. For a given model of structure evolution, these two relations fix both parameters. The void evacuation is model dependent, thus we have a different solution for each model. In LCDM and OCDM models voids are emptied and matter flows into dense regions at surprisingly similar speeds while for the SCDM void evacuation occurs faster. Also we see that models with gridlike initial conditions yield systematically lower values for F_{gal} . As glasslike initial conditions give a smoother density field we prefer to use these models. For the SCDM model we obtain $F_{\text{gal}} = 0.83$ and $(\sigma_8)_m = 0.75$, for the LCDM1 model $F_{\text{gal}} = 0.78$ and $(\sigma_8)_m = 0.70$, for the LCDM2 model $F_{\text{gal}} = 0.73$ and $(\sigma_8)_m = 0.66$, and for the OCDM model $F_{\text{gal}} = 0.79$ and $(\sigma_8)_m = 0.72$.

We see that the amplitude fluctuation parameter, $(\sigma_8)_m$, depends slightly on the density parameter of the universe. Independent determinations (Ostriker & Steinhardt 1995; Bahcall, Fan, & Cen 1997; Bahcall & Fan 1998) favor a low-density universe; thus we prefer to use results obtained for LCDM and OCDM models which yield $F_{\text{gal}} = 0.75 \pm 0.08$ for the present epoch, in good mutual agreement. For the biasing parameter of galaxies relative to the matter, b_{gal} , we obtain

$$b_{\text{gal}} = 1/F_{\text{gal}} = 1.32 \pm 0.13. \quad (11)$$

We see that the fraction of matter in galaxies according to new models is smaller than that found by E94 which leads to a larger value of the biasing parameter. The difference is partly due to differences in models (most models used by E94 were standard CDM) and partly due to differences in the methods used in calculations and fixing the present epoch.

4. DISCUSSION

4.1. The Biasing of Galaxies Relative to Matter

We have investigated the biasing of galaxies relative to matter. We use the term “biasing” to denote the difference between the distribution of the whole matter and the matter associated with galaxies. Already in the early stages of cosmological studies it was clear that the evolution of

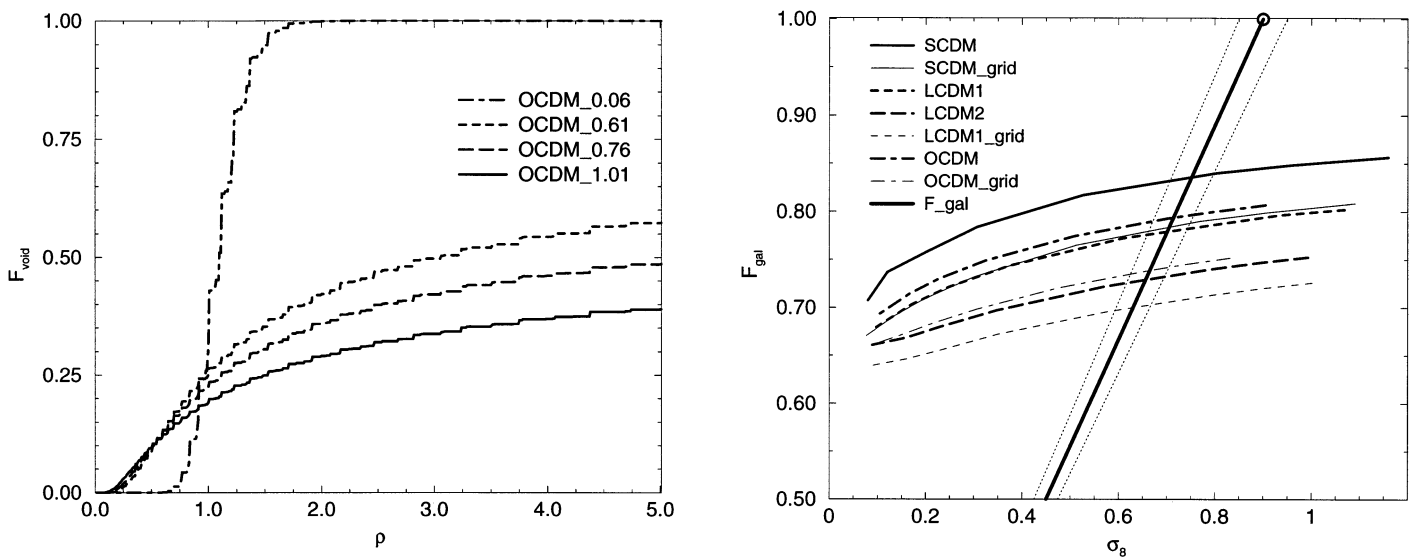


FIG. 4.—*Left-hand panel*: Evolution of the integrated density distribution for the OCDM model. The fraction of matter in low-density regions is plotted for different epochs, indicated by the σ_8 parameter. *Right-hand panel*: Relation between the fraction of matter in galaxies, F_{gal} , and σ_8 . The thick bold solid line shows the relation from eq. (5), and the bold solid, dashed, and dot-dashed lines give the relation obtained from numerical simulations of how voids are emptied in different cosmological models (see Table 2). Models with glasslike initial conditions are plotted as bold lines, and models with grid initial conditions as thin lines. The mean error of F_{gal} due to uncertainty in the threshold density level is ≈ 0.05 .

density perturbations in under- and overdense regions is completely different (Z70), which explains the presence of voids since matter in low-density regions cannot contract and form galaxies (JE78; EJS80). Thus, the discovery of voids was a clear indication for the dominating role of the gravity in the evolution of the universe on large scales. This leads us to our basic assumptions that the evolution of the structure on scales of interest is due to gravity and that density fluctuations are Gaussian and adiabatic. These assumptions have two important consequences. First, the evolution of under- and overdense regions is completely different: the density in underdense regions decreases (approximately exponentially) but never reaches zero; the density in overdense regions increases until the matter collapses (Z70; EJS80). Collapsed regions form a web of intertwined filaments and knots (JE78; Einasto et al. 1983; Melott et al. 1983; Bond et al. 1996; Cen & Simcoe 1997). Second, in order to form a galaxy or a system of galaxies, the clump of primordial matter must have a density exceeding some critical limit in a given volume (Press & Schechter 1974). For these reasons the galaxy formation is a density threshold phenomenon.

Dynamical studies of galaxies and clusters have shown that the dominating population in the universe is DM which forms halos around galaxies, groups, and clusters of galaxies. Practically all massive galaxies are located in groups or clusters; the brightest form main galaxies of groups, and other group members are dwarf companion galaxies (Einasto et al. 1974b; Zaritsky et al. 1993). The characteristic size of groups of galaxies is of the order $1 h^{-1}$ Mpc (Einasto et al. 1984), thus, in order to find the *true* density field of matter in the universe the discrete distribution of galaxies (and particles in numerical simulations) is to be smoothed using a smoothing scale of the order of the size of groups, i.e., about $1 h^{-1}$ Mpc (Einasto & Saar 1986).

Conventionally, the biasing is defined through the local density contrast, which is calculated from a smoothed density field using a smoothing scale of the order of $10 h^{-1}$ Mpc. Excessive smoothing mixes unclustered DM in voids and clustered DM in high-density regions which makes the simple biasing phenomenon rather complicated. To avoid excessive smoothing we define the biasing parameter using the difference in power spectra of populations, in our case of the population of all galaxies with respect to matter. We apply no additional smoothing, i.e., power spectra are calculated from particle positions, and the density field is calculated by interpolation of these positions on a grid which has a scale of the same order as real systems of galaxies. We use density also to find the population membership of particles, either void particle in low-density regions or simulated galaxy (with dark halo) in high-density regions. Here again we interpolate particle positions within the grid.

In the determination of the density at the location of particles we tacitly assume that DM is nonbaryonic and consists of particles of small mass. Under this assumption we can consider DM as a fluid which has a continuous density field. As shown by dynamical observations and numerical simulations, DM forms density enhancements around galaxies and in clusters of galaxies. The distribution of luminous matter differs from the distribution of DM on galactic and cluster scales. There exists no indication for large-scale segregation between luminous and dark matter in regions exceeding the critical threshold which divides void and clustered particles. We have investigated the influ-

ence of possible small differences in the spatial distribution of luminous and dark matter to power spectra of galaxies and matter. Our results show that on scales of interest for the present study these differences are small or negligible.

Our study demonstrates that the main difference between power spectra of galaxies and matter is the amplitude only, which is determined by the fraction of particles in high-density regions. Small differences in the spatial distribution and in the density threshold are the reason for cosmic scatter of the biasing parameter around the value defined by the fraction of particles in high-density regions; this error in the density threshold is of order of 10% or less. The density distribution (Fig. 4a) shows that a 10% error in the threshold density leads to a 3% error of the fraction of matter in the clustered population. Biasing parameter values $b \leq 1$ are possible only for samples with a very large deficit of particles in high-density regions (see Fig. 2 of Paper I). Such a large deficit is not supported by observation, as the mass-to-luminosity ratio of groups of galaxies is approximately the same as in clusters. A small deficit of luminous matter in rich clusters (high mass-to-luminosity value) is simulated in our test sample Gal-120: its power spectrum has a lower amplitude as defined by the number of particles, but only by a few percent. This adds a component to the cosmic scatter of the biasing parameter.

The main lesson from this study is that the gravitational origin of the structure evolution imposes strict limits on the biasing parameter. It is determined by the fraction of total matter in low- and high-density regions. As matter flows continuously away from low-density regions, the fraction of matter in high-density regions remains between 0.5 (the initial value) and 1 (the limit in the very far future) during the whole evolution of the structure. Thus, the corresponding biasing parameter of galaxies relative to matter lies between 2 (initial value) and 1 (limit in the future). These values apply if galaxies exactly follow the distribution of particles in high-density regions. Our analysis has shown that differences of the distribution of galaxies and matter in high-density regions may change these theoretical biasing parameter values by up to 10%.

The second lesson learned is that both the amplitude (i.e., the biasing parameter) and the shape of the power spectrum change very little because of possible disturbing effects. The amplitude is changed considerably only in the case when galaxy samples are incomplete in high-density regions. The shape is changed by differences in the concentration of matter and galaxies in groups, clusters, and superclusters, but this affects the power spectrum on small scales only (see Table 1 and Fig. 2), which is of less importance for the present study.

We have followed this approach to the biasing problem motivated by the discovery of the difference in the overall distribution of galaxies and matter (JE78; EJS80) and similarity in groups and clusters (Einasto et al. 1974a, 1976). It is a bit surprising that in the majority of studies on this subject the problem has been complicated by smoothing over large scales. The latter distorts the distribution of void particles and the clustered matter.

4.2. The Amplitude of Density Fluctuations

We have studied the evacuation of voids and the concentration of matter to high-density regions through numerical simulations. We used the rms density fluctuation on an $8 h^{-1}$ Mpc sphere, σ_8 , as a parameter which characterizes the

epoch of the simulation. We determined the present value of this parameter from the power spectrum of galaxies, $(\sigma_8)_{\text{gal}} = 0.89 \pm 0.09$. We find two relations for $(\sigma_8)_m$ and F_c and derive values for both parameters. We find the fraction of matter in the clustered population $F_c = 0.75 \pm 0.08$ and $(\sigma_8)_m = 0.68 \pm 0.09$. These data yield a value $b_{\text{gal}} = 1.32 \pm 0.13$ for the bias parameter of all galaxies relative to matter.

The amplitude of matter density fluctuations can be fixed on two different scales. *COBE* data measure the amplitude of density fluctuations on very large scales (e.g., Bunn & White 1997). On galactic scales the amplitude of density fluctuations is quantified by the σ_8 parameter. The problem here is how to link the distribution of galaxies to that of matter. To avoid this difficulty White, Efstathiou, & Frenk (1993) used an indirect method, based on the cluster abundance (number density). Cluster abundance depends on the amplitude of density fluctuations and density parameter, $\sigma_8 \Omega_0^{0.6}$. Previously, the σ_8 - Ω_0 relation was found as $\sigma_8 \sim 0.50 \Omega_0^{-0.41 \pm 0.02}$ (Eke, Cole, & Frenk 1996; Viana & Liddle 1996; Pen 1998; Cen 1998). Bahcall, Fan, & Cen (1997), Fan, Bahcall, & Cen (1997), and Carlberg et al. (1997) point out the fact that the evolution of cluster abundance depends strongly on σ_8 , making it possible to determine both parameters separately. Combining the observed abundance of local rich clusters (Bahcall & Cen 1992, 1993) with cluster evolution data yields the following parameters: $\Omega_0 = 0.3 \pm 0.1$, $\sigma_8 = 0.83 \pm 0.15$, and $b_{\text{gal}} = 1.2 \pm 0.2$. Using a similar method Eke et al. (1998) find $\Omega_0 = 0.44 \pm 0.2$, $\sigma_8 = 0.67 \pm 0.1$ for an open universe and $\Omega_0 = 0.38 \pm 0.2$, $\sigma_8 = 0.74 \pm 0.1$ for a flat universe with a cosmological constant. Bahcall & Fan (1998) find $\Omega_0 = 0.2^{+0.3}_{-0.1}$ based on similar arguments. A simple estimate of the cluster mass in the comoving volume from which that mass originated yields a mass-density estimate of the universe $\Omega_0 = 0.24 \pm 0.10$ (Carlberg et al. 1996). These results suggest that the standard CDM model can be excluded at a confidence level of more than 99%.

For comparison we note that *COBE* normalization provides $(\sigma_8)_m = 1.2 \pm 0.1$, 1.0 ± 0.1 , and 0.9 ± 0.1 for SCDM, LCDM, and OCDM, respectively. As $(\sigma_8)_m$ calculated from *COBE* normalization is an extrapolation from 1000 to 10 Mpc scales, based on certain models of structure evolution, the (dis)agreement between the direct determination of σ_8 and *COBE* extrapolation to this scale can be considered as evidence for the quality of the model used.

Within the errors our value of the σ_8 parameter coincides with those suggested by Carlberg et al. (1997), Bahcall, Fan, & Cen (1997), and Eke et al. (1998). A recent determination of the density parameter and σ_8 by Bahcall & Fan (1998) from the abundance of rich clusters of galaxies at high redshifts yields a rather high value, $\sigma_8 = 1.2^{+0.5}_{-0.4}$; however, the error is large. Since we used completely different input data, our determination is independent of previous ones. Moreover, our method uses directly observed power spectra and simple gravitational physics of void evacuation, thus the danger of the presence of large systematic errors is small. We may conclude that this parameter is now known rather reliably.

Using the integrated power spectrum we can also determine the excess power parameter, $EP = 3.4 \sigma_{25}/\sigma_8$, introduced by Wright et al. (1992). We find $EP = 1.42 \pm 0.05$ and $EP = 1.28 \pm 0.05$ from the power spectra $P_{\text{HD}}(k)$ and $P_{\text{MD}}(k)$, respectively. Both values are close to the value

$EP = 1.30 \pm 0.15$ found by Wright et al. A similar power spectrum shape parameter was introduced by Borgani et al. (1997).

The error of $(\sigma_8)_m$ depends on the errors of $(\sigma_8)_{\text{gal}}$ and F_{gal} . We have no reason to believe that the amplitude of the observed power spectrum of galaxies has a considerable systematic error. Thus, we can accept the quoted error of $(\sigma_8)_{\text{gal}}$ as a realistic one. The fraction of matter in galaxies derived from numeric simulations is less certain. But we can estimate upper and lower limits for this quantity. EJS80 and E94 have found analytic approximations for several simple scenarios for the evacuation of voids (planar and spherical void models). Negative and positive density fluctuations grow simultaneously, and the present fraction of matter in voids (and in the clustered population) depends on the effective epoch of structure formation (collapse), z_{form} . We find (see Fig. 7 of E94) that for the linear void and wall model we obtain $0.6 < F_c < 0.8$, if $1 < z_{\text{form}} < 5$, while spherical void models give $0.8 < F_c < 0.9$. Since a collapse at very early and very late epochs can safely be excluded, these simple models suggest that the present value of the fraction of matter in the clustered population must lie in the interval $0.6 < F_c < 0.9$. Using these limits we get $0.54 < (\sigma_8)_m < 0.8$. Our accepted value $(\sigma_8)_m = 0.68$ lies just in the middle of this interval. If we consider the limits derived from these simple analytic models as 3 σ errors, we get for 1 σ error 0.05, in good agreement with the error estimate calculated from formal errors of parameters used to find $(\sigma_8)_m$.

To conclude the discussion we stress again that $(\sigma_8)_m$ characterizes the rms amplitude of density fluctuations on galactic scales, similar to *COBE* observations which fix the amplitude of density fluctuations on a scale of ~ 1000 Mpc. Direct observables are the rms galaxy density fluctuations at the present epoch, $(\sigma_8)_{\text{gal}}$, and rms temperature fluctuations at the recombination epoch, respectively. In both cases, matter density fluctuations at the present epoch are calculated using theoretical models which involve simple physics, through the evacuation of voids and the growth of the amplitude of density fluctuations, respectively, for $(\sigma_8)_m$ and *COBE* cases. These methods to calibrate density fluctuations on different scales are complementary and independent, i.e., a correct model must pass both checks.

5. CONCLUSIONS

In cosmological studies the linear bias factor is often defined through the σ_8 parameter ($b \equiv 1/\sigma_8$; see Brodbeck et al. 1998). Actually, these quantities are independent parameters; the bias parameter characterizes the difference in amplitude of power spectra of galaxies and matter, and the σ_8 parameter the present amplitude of density fluctuations of matter on galactic scales. The main goal of this paper was to elaborate methods to determine these two parameters and to apply methods using actual data.

Our approach to the biasing phenomenon is based on the observation that there exist large voids in the galaxy distribution. This is a clear indication that the evolution of the structure in the universe is primarily due to gravity. Furthermore, we assume that primordial density fluctuations are Gaussian and adiabatic. We have shown that under these assumptions the formation of galaxies is a threshold phenomenon, i.e., that in underdense regions galaxies do not form at all and that in overdense regions galaxies and matter are distributed very similarly (ignoring differences on galactic scales).

Our first conclusion from the biasing analysis is that all matter in the universe is divided into two main populations, the unclustered primordial matter in voids and the clustered matter in high-density regions associated with galaxies. Our analysis and high-resolution hydrodynamical simulations of galaxy formation show that the threshold density which divides the unclustered matter in voids and the clustered matter associated with galaxies is approximately equal to the mean density of matter, if smoothed on scales comparable to the characteristic scale of groups of galaxies (about $1 h^{-1}$ Mpc). Using higher threshold densities it is easy to select galaxies located in filaments, while a still higher threshold density corresponds to galaxies in groups and clusters. Using intermediate threshold density intervals it is even possible to simulate statistically populations of galaxies of different luminosity.

We have investigated the influence of the density threshold on power spectra of galaxies and clusters of galaxies. The population of clustered particles is derived from the population of all particles by the exclusion of particles located in low-density environments. Our analysis shows that power spectra of galaxies and clusters are similar in shape to the power spectrum of all matter, the main difference being in the amplitude. The power spectrum describes the square of the amplitude of the density contrast, i.e., the amplitude of density perturbations with respect to the mean density. If we exclude from the sample of all particles a population of approximately constant density (void particles) and preserve all particles in high-density regions, then the amplitudes of *absolute* density fluctuations remain the same (as they are determined essentially by particles in high-density regions), but the amplitudes of *relative* fluctuations with respect to the mean density increase by a factor that is determined by the ratio of mean densities, i.e., by the fraction of matter in the new density field with respect to the previous one. Our analysis has shown that actual differences in amplitudes of power spectra of simulated galaxies with respect to the power spectrum of all matter are almost

exactly equal to differences calculated from the number of particles in respective samples, i.e., the fraction of matter in high-density regions, F_c . This fraction determines the biasing parameter of the sample of all galaxies with respect to matter.

We have determined the fraction of matter in high-density regions using numerical simulations of structure evolution for various cosmological models. Our analysis shows that the evolution is model dependent: in models with high cosmological density, voids are evacuated more rapidly and less matter is left in low-density regions. A problem with numerical simulation of the void evacuation is how to identify the present epoch in these simulations. We have done this using the calibration through the mean amplitude of density fluctuations in a sphere of radius $r = 8 h^{-1}$ Mpc, σ_8 . The parameter $(\sigma_8)_{\text{gal}}$ can be determined directly from the observed power spectrum of galaxies by integration, and it is related to the corresponding parameter for matter, $(\sigma_8)_m$, through an equation similar to the equation which relates amplitudes of power spectra of matter and galaxies, with the fraction of matter in galaxies (clustered matter). Simulations yield another relation between $(\sigma_8)_m$ and F_c , which allows us to determine both unknown parameters. We obtain $(\sigma_8)_{\text{gal}} = 0.89 \pm 0.09$ for galaxies, $(\sigma_8)_m = 0.68 \pm 0.09$ for matter, and $b_{\text{gal}} = 1.3 \pm 0.13$ for the biasing factor of the clustered matter (galaxies) relative to all matter.

We thank A. Chernin, M. Gramann, and D. Tucker for discussion and H. Andernach for suggestions to improve the text. We thank the referee, M. Vogeley, for constructive criticism. This work was supported by the Estonian Science Foundation (grant 2625). J. E. and A. S. were supported by the Deutsche Forschungsgemeinschaft in Potsdam; R. C. was supported by grants NAG5-2759 and AST 93-18185; A. S. was partially supported by the Russian Foundation for Basic Research under grant 96-02-17591.

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