

STARQUAKE-INDUCED MAGNETIC FIELD AND TORQUE EVOLUTION IN NEUTRON STARS

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Received 1998 May 8; accepted 1998 July 8

ABSTRACT

The persistent increases in spin-down rate (offsets) seen to accompany glitches in the Crab and other pulsars suggest increases in the spin-down torque. We interpret these offsets as due to starquakes occurring as the star spins down and the rigid crust becomes less oblate. We study the evolution of strain in the crust, the initiation of starquakes, and possible consequences for magnetic field and torque evolution. Crust cracking occurs as equatorial material shears under the compressive forces arising from the star's decreasing circumference and as matter moves to higher latitudes along a fault inclined to the equator. A starquake is most likely to originate near one of the two points on the rotational equator farthest from the magnetic poles. The material breaks along a fault approximately aligned with the magnetic poles. We suggest that the observed offsets come about when a starquake perturbs the star's mass distribution, producing a misalignment of the angular momentum and spin axes. Subsequently, damped precession to a new rotational state increases the angle α between the rotation and magnetic axes. The resulting increase in external torque appears as a permanent increase in the spin-down rate. Repeated starquakes would continue to increase α , making the pulsar more of an orthogonal rotator.

Subject headings: dense matter — magnetic fields — pulsars: individual (Crab) — stars: magnetic fields — stars: neutron — stars: rotation

1. INTRODUCTION

The magnetic braking torque acting on an isolated neutron star would be steady in the absence of abrupt changes to the star's magnetic configuration. Most pulsars, however, do not slow in a regular fashion but suffer variations in their spin rates in the form of glitches and timing noise. Perhaps the most striking aspect of spin evolution is the persistent increases in spin-down rate that accompany glitches in the Crab pulsar (Lyne, Pritchard, & Smith 1993), PSR 0355+54 (Lyne 1987), and PSR 1830–08 (Shemar & Lyne 1996); see Table 1. In the Crab, these permanent offsets involve fractional changes in the spin-down rate of $\sim 10^{-6}$ – 10^{-4} (see Fig. 1). PSR 1830–08 has exhibited one offset of 8×10^{-4} , and a persistent offset might have followed the large 1986 glitch of PSR 0355+54. Similar offsets might also occur in the Vela pulsar (Link & Epstein 1997), giving rise to the small braking index of 1.4 ± 0.2 reported by Lyne et al. (1996).

It is striking that all observed offsets are of the same sign and correspond to *increases* in the spin-down rate. One interpretation of this phenomenon is that glitches are accompanied by sudden and permanent increases in the external torque acting on the star (Gullahorn et al. 1977; Demiański & Prószyński 1983; Link, Epstein, & Baym 1992; Link & Epstein 1997). Such torque changes could occur if either the direction or magnitude of the star's magnetic moment changes. Starquakes, occurring as the star spins down (Baym & Pines 1971; Ruderman 1976), would affect the external torque if they change the orientation of the magnetic moment with respect to the spin axis. If struc-

tural relaxation occurs asymmetrically about the rotation axis, due perhaps to magnetic stresses or asymmetric material properties, the star's spin and angular momentum vectors would become misaligned. As the star precesses and relaxes to a new rotational state, the magnetic moment would assume a new orientation with respect to the rotation axis, leading to a change in the external torque. In this paper, we study how the rigid neutron star crust relaxes its structure and consider possible consequences for evolution of the magnetic field and torque.

Alpar & Pines (1993; also Alpar et al. 1996) have suggested that the Crab's offsets result from a reduction in the moment of inertia on which the external torque acts. Such a change could occur either by a structural change of the star, e.g., the star becomes less oblate, or through a decoupling of a portion of the star's liquid interior from the external torque. If the moment of inertia decreases through structural readjustment, to conserve angular momentum, the star would always spin *more* rapidly than had the glitch not occurred (Link et al. 1992; Link & Epstein 1997). The large offsets following the 1975 and 1989 glitches in the Crab, however, eventually caused the star to spin *less* rapidly than had the glitch not occurred (Lyne et al. 1993). In principle, the Crab's spin-down rate offsets could be due to the decoupling of a portion of the star's superfluid interior from the external torque (Alpar & Pines 1993; Alpar et al. 1996), although quantitative agreement of this model with the data has yet to be demonstrated. Torque increases associated with the surface field structure appear to be the most straightforward explanation.

A neutron star relaxes its oblateness as it spins down, moving equatorial material toward the rotation axis and

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TABLE 1
GLITCH PROPERTIES

| Pulsar | Age (yr) | Glitch Year | $\Delta\Omega/\Omega$ | Permanent Offset $ \Delta\dot{\Omega}/\dot{\Omega} $ |
|------------------------------|-----------------|-------------------|-----------------------|--|
| Crab ^a | 10^3 | 1969 | 4×10^{-9} | -4×10^{-6} |
| | | 1975 | 4×10^{-8} | -2×10^{-4} |
| | | 1986 | 4×10^{-9} | -2×10^{-5} |
| | | 1989 | 9×10^{-8} | -4×10^{-4} |
| 0355 + 54 ^b | 6×10^5 | 1986 ^c | 4×10^{-6} | $-3 \times 10^{-3}?$ |
| 1830 - 08 ^b | 2×10^5 | 1990 | 2×10^{-6} | -8×10^{-4} |

^a Lyne et al. 1993.

^b Shemar & Lyne 1996.

^c The magnitude of this offset is quite uncertain, as the pulsar has not had time to completely recover from the glitch.

reducing the equatorial circumference. If the stellar crust is brittle, stresses lead to starquakes as the yield strength of the crustal material is exceeded (Baym & Pines 1971).² The actual response of solid neutron star matter at high pressure to shear is not well understood. Here we assume that the neutron star crust is brittle and explore the consequences of this assumption.³

2. GROWTH OF STRAIN IN A SPINNING-DOWN STAR

To study the qualitative features of the growth of strain in a neutron star as it spins down under its external torque, we model the star as an incompressible, homogeneous, self-gravitating elastic sphere of constant shear modulus. We take the initial configuration to be unstrained with rotation frequency Ω and investigate how the star deforms as the spin rate decreases by $-\delta\Omega$, where $0 < \delta\Omega \ll \Omega$. The initial rotation produces an equatorial bulge of relative height $\sim (R\Omega/v_k)^2 \ll 1$, where $v_k = (2GM/R)^{1/2}$ is the Keplerian velocity near the surface and R is the radius the star would have if it were spherical. A change $-\delta\Omega$ in the rotation rate produces a small decrease in the equatorial bulge of $\sim \Omega \delta\Omega R^2/v_k^2$. As the star deforms, matter originally at r is displaced to $r + u(r)$, where $u(r)$ is the displacement field. In a spherical coordinate system centered on the star, with $\theta = 0$ in the direction of the rotation vector, the displacement field is (Baym & Pines 1971)

$$u_r(r, \theta) = \frac{\Omega \delta\Omega R^2}{v_k^2} r \left(r^2 - \frac{8}{3} \right) (1 - 3 \cos^2 \theta), \quad (1)$$

$$u_\theta(r, \theta) = \frac{\Omega \delta\Omega R^2}{v_k^2} r (5r^2 - 8) \sin \theta \cos \theta. \quad (2)$$

Here r and u are in units of the stellar radius, and we have neglected terms of the order of $(c_s/v_k)^2 \lesssim 10^{-4}$, where c_s is the transverse sound speed. The displacement field is illustrated in Figure 2. The local distortion of the solid is

² Superfluid stresses in the crust (Ruderman 1976) could also drive crust cracking but will not be considered here.

³ Known materials exhibit ductile rather than brittle behavior under pressures comparable to their material shear moduli (Dubá et al. 1990). Nevertheless, deep-focus earthquakes are known to originate from regions of very high pressure (Green & Houston 1995). These faults are thought to be facilitated by densification phase changes; small regions of higher density nucleate as the material is stressed and act as a lubricant for shearing motion. Analogous processes might occur in the high-pressure material of the neutron star.

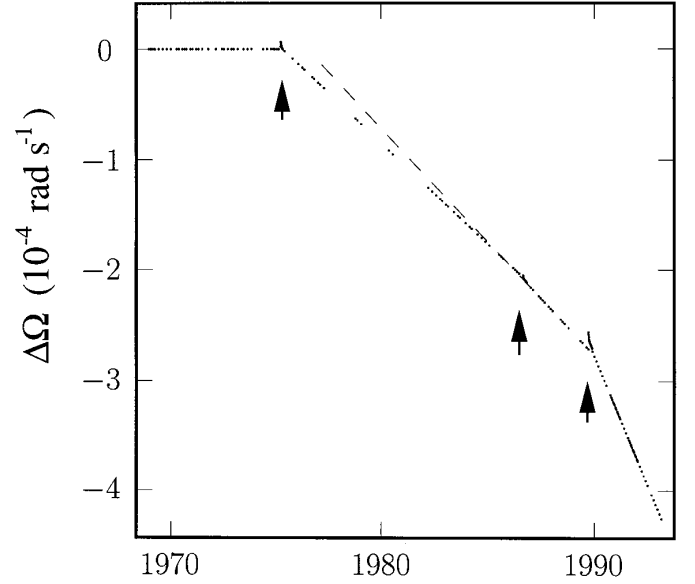


FIG. 1.—Twenty-five yr of spin history of the Crab pulsar showing apparent torque increases (adapted from Lyne et al. 1993). Shown are spin rate residuals relative to a model for data prior to the first glitch. Glitches are indicated by arrows. Following each glitch, the pulsar acquired a greater spin-down rate than before the glitch. The small glitch of 1986 was also followed by a small offset. The dashed line, which is parallel to the data following the 1986 glitch, shows the change in slope that occurred. These offsets appear to be permanent and cumulative.

described by the strain tensor (see, e.g., Landau & Lifshitz 1959)

$$u_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right). \quad (3)$$

In a local coordinate system in which this matrix is diagonal, the eigenvalues ϵ_{ii} represent compression ($\epsilon_{ii} < 0$) or

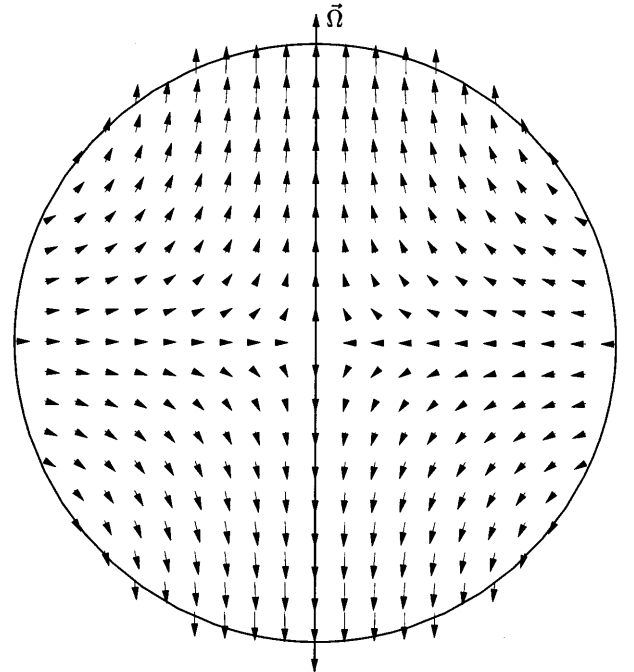


FIG. 2.—Displacement field in a spinning-down neutron star. A cross section through the center of the star is shown. The equatorial diameter decreases, while the polar diameter increases.

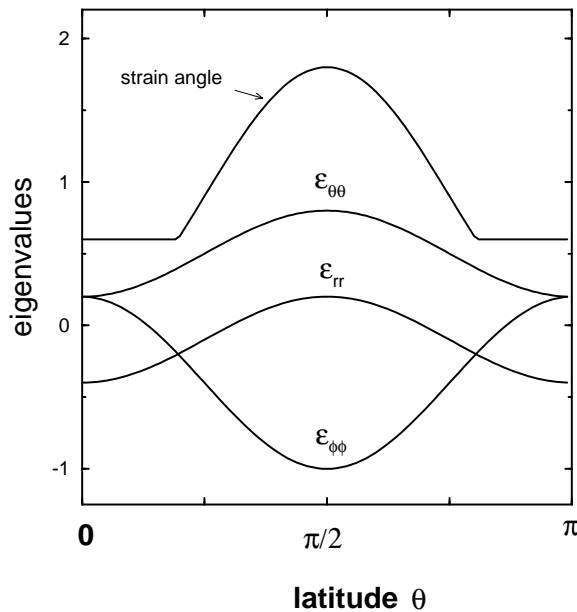


FIG. 3.—Strain eigenvalues and strain angle vs. latitude θ at the stellar surface for $B = 0$ (arbitrary units). The short-dashed curves give the strain angle; the other curves are the eigenvalues.

dilation ($\epsilon_{ii} > 0$) along the respective axes. Suppose the largest and smallest eigenvalues are ϵ_l and ϵ_s , respectively. The *strain angle* is $\epsilon_l - \epsilon_s$, and we refer to the plane containing the corresponding principal axes as the *stress plane*. For the uniform sphere, we find that the *stress plane* is tangential to the stellar surface near the equator with compression in the azimuthal direction and dilation toward the rotational poles (see Fig. 3). Near the poles, the stress plane switches to the radial-azimuthal plane, i.e., along cones.

3. STARQUAKES

At zero pressure, a stressed element of matter can break under shear or tension. At the pressure of the neutron star crust ($p \gg \rho c_t^2$), however, only shear fractures can occur. Compressed material strained at a critical angle shears along a plane as is shown in Figure 4 (Green & Houston 1995). The angle this plane takes with respect to the plane of maximum compression depends on the material's internal friction and is in the range 30° – 45° . The critical strain angle of neutron star material is not accurately known. Highly compressed terrestrial minerals can support strain angles of $\gtrsim 0.01$ before breaking (Billings 1972; Duba et al. 1990).

The geometry of a starquake is independent of the value of the critical strain angle for neutron star matter. Strain builds as the star slows and reduces its circumference. For the homogeneous sphere, the maximum strain is reached on the equator, as each material element is compressed by its neighbors on the equator (see Fig. 3). When the strain angle for an element reaches a critical value, the material breaks at the stellar surface along a fault that takes an angle of 30° – 45° with respect to the equator and propagates to higher latitudes (see Fig. 5). Movement of material from the equatorial bulge to higher latitudes decreases the star's oblateness. At higher latitudes the strain angle diminishes and the fault terminates (Hertzberg 1996). This picture of starquakes differs fundamentally from that of Baym &

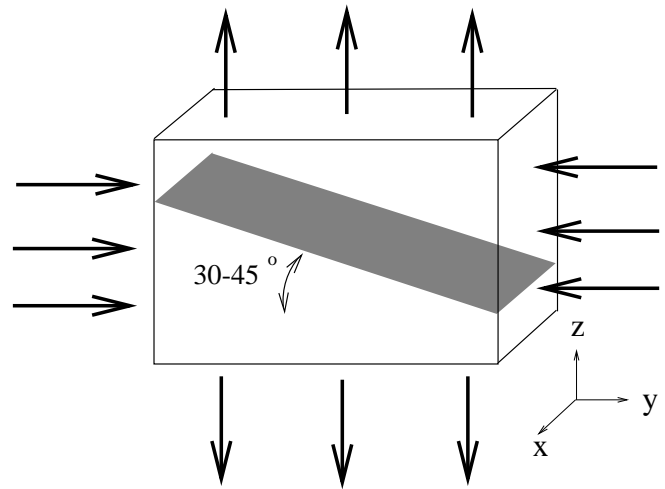


FIG. 4.—Breaking of a compressed block of matter. A block under horizontal compression and vertical tension shears along a plane as shown when the critical strain is reached. Shearing along a complementary plane flipped over with respect to the plane shown is equally likely for isotropic material. The y - z plane is the stress plane.

Pines (1971), who used a zero-pressure breaking criterion and found that the material breaks as tensile forces pull apart material in the equatorial plane. In our analysis, however, the enormous pressure in neutron star matter prevents tensile rupturing. For the homogenous model dis-

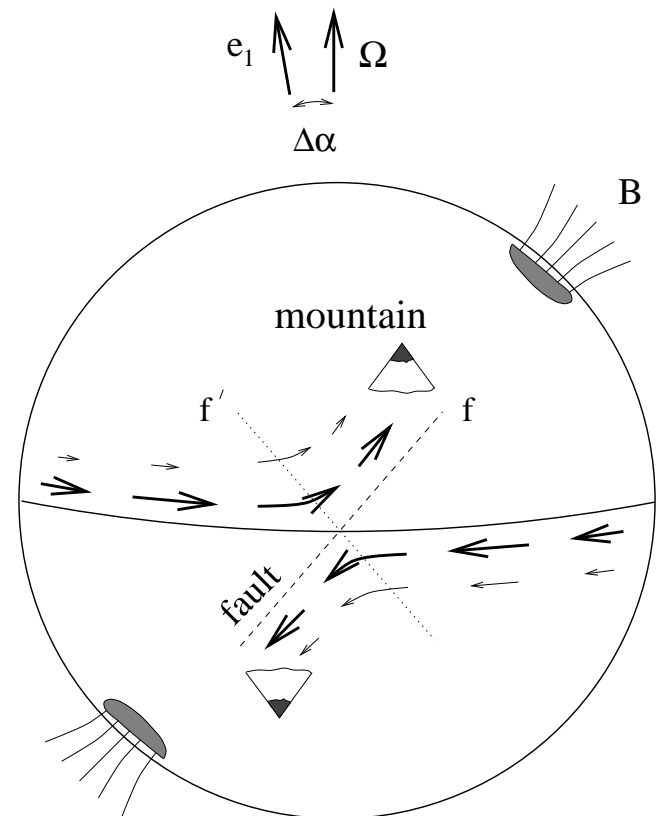


FIG. 5.—Starquake. In the absence of a magnetic field, the material is equally likely to begin breaking along faults f and f' anywhere on the equator. In the presence of magnetic stresses, fault f is more likely, creating "mountains" (indicated by the snow-capped peaks) and shifting the largest principal axis of inertia to a new direction e_1 (fixed in the star).

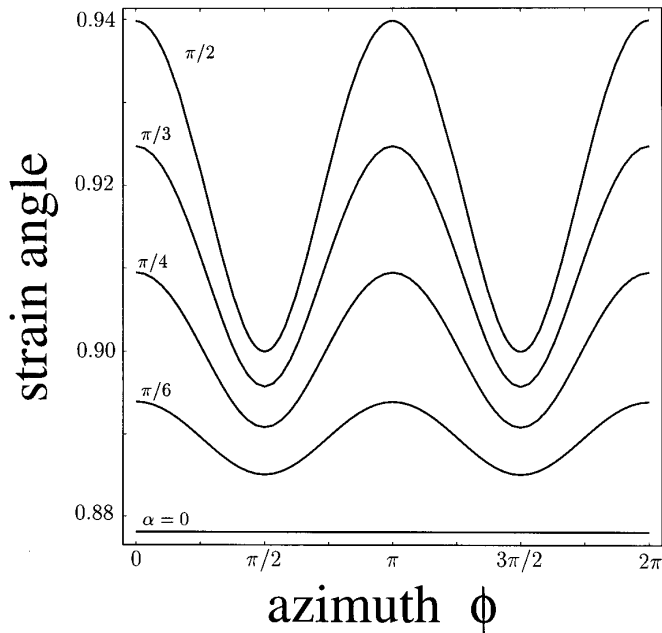


FIG. 6.—The strain angle of surface equatorial material in the presence of a magnetic field (arbitrary units). The curves correspond to different values of the angle α between the magnetic and rotation axes. The magnetic poles are at azimuthal angles $\pi/2$ and $3\pi/2$.

cussed here, the maximum strain is reached on the stellar surface. In the actual neutron star crust, the depth at which the material breaks and the radial extent of the fault will be determined by the dependences of the shear modulus and the critical strain angle on density.

For an axially symmetric star, the crust is equally likely to begin breaking at every point on the equator. In the presence of a magnetic field, however, the rotation-induced displacements of the star's crust distort the initial magnetic field $\mathbf{B}(\mathbf{r})$ that is anchored in the stellar crust thus generating magnetic stresses that affect the development of strain. The strength of the magnetic stress relative to the shear stress is $\sim \beta \equiv (v_A/c_t)^2$, where v_A is the Alfvén velocity. Through most of the crust the induced magnetic stresses are significantly smaller than the material shear stresses and can be treated perturbatively. For example, at moderate densities in the outer crust we have $\beta = 8 \times 10^{-4} (B/10^{12} \text{ G})^2 (\rho/10^{10} \text{ g cm}^{-3})^{-1} (c_t/10^8 \text{ cm s}^{-1})^{-2}$, while the magnetic stresses can exceed the material ones at low densities, below $\sim 10^7 \text{ g cm}^{-3}$.

The strain field that develops in magnetized material could be obtained by solving the elasticity equations and Maxwell's equations self-consistently, subject to boundary conditions at the star's surface. To illustrate how the star's magnetic field determines where starquakes originate, we consider the simpler problem of how the presence of a magnetic field affects the material strain in an infinite medium (one in which boundary effects are unimportant). In the absence of the magnetic field, the equilibrium state is given by

$$\frac{\partial \sigma_{ij}}{\partial x_i} + F_j = 0, \quad (4)$$

where F_j represents gravitational and centrifugal forces, and repeated indices are summed over here and in the following. In the presence of a magnetic field, the stress tensor changes

by $\delta \sigma_{ij}$. The displacement of the material induces a change in the field $\delta \mathbf{B}$, with a corresponding change in the Maxwell stress tensor T_{ij} of δT_{ij} . We take the unperturbed field \mathbf{B} to be dipolar, which gives no force on the undistorted matter, i.e., $\partial T_{ij}/\partial x_i = 0$. The new equilibrium is given by

$$\frac{\partial}{\partial x_i} (\delta \sigma_{ij} + \delta T_{ij}) = 0. \quad (5)$$

For an infinite medium, the solution is $\delta \sigma_{ij} = -\delta T_{ij}$. The magnetic field induces a change $\delta \mathbf{u}$ in the displacement field. For a field that is sufficiently small that this correction to the displacement field is small, the perturbation to the Maxwell stress tensor is

$$\delta T_{ij} = \frac{1}{4\pi} (\delta B_i B_j + B_i \delta B_j - B_k \delta B_k \delta_{ij}), \quad (6)$$

$$\delta \mathbf{B} = \nabla \times (\mathbf{u} \times \mathbf{B}). \quad (7)$$

Here \mathbf{u} is the displacement field in the absence of the magnetic field. For incompressible matter, the correction δu_{ij} to the strain tensor is

$$\delta u_{ij} = \frac{1}{2\mu} \left(\delta \sigma_{ij} - \frac{1}{3} \delta \sigma_{ll} \delta_{ij} \right), \quad (8)$$

where μ is the shear modulus. The eigenvalues of this tensor give the correction to the strain angle due to the induced magnetic forces. Using the displacement field of equation (1) in equation (7), we obtain an estimate of the corrections to the strain angle on the star's equator; the result is shown in Figure 6. Near the magnetic poles, the field gives the material extra rigidity and reduces the strain angle. Consequently, as the star spins down, the strain increases the fastest at the two equatorial points farthest from the poles. These results indicate that a starquake will begin at one of these two points when critical strain is reached. A more complete treatment would account for magnetic stresses at the stellar surface; we do not expect treatment of the boundary to alter our qualitative conclusions.

Isotropic material under compressive stress is as likely to break along the plane indicated in Figure 4 as along its complementary plane. In a neutron star, however, the magnetic field gives the material extra rigidity along shear planes that cross field lines. Cracking is thus favored along the plane that is most parallel to the field (fault f in Fig. 5).

4. EFFECTS ON THE SPIN-DOWN TORQUE

In the picture of starquakes described above, a slowing star reduces its equatorial circumference by shearing material across the equator, moving the material along the fault to higher latitudes (see Fig. 5). We now discuss how this crustal motion could affect the spin-down torque.

Quake-induced mass motion produces a small change in the orientation of the principal axes of the star's moment of inertia. The material motions create mass concentrations, or "mountains" ($\sim 100 \mu\text{m}$ high), at higher latitudes, breaking the axial symmetry of the star's mass distribution. Suppose that before the starquake the largest principal moment of inertia corresponds to a principal axis aligned with the rotation axis. As the mass motion occurs, the principal axis of inertia shifts away from the magnetic axis by $\Delta\alpha$, pointing in a new direction \mathbf{e}_1 (fixed in the star, see Fig. 5). The star's rotation and angular momentum vectors are

now misaligned and the rotation axis precesses about the angular momentum axis (Shaham 1977). Dissipative processes, such as the interaction of superfluid vortices with the nuclei of the inner crust or electron-vortex scattering in the core, damp the precessional motion (Sedrakian, Wasserman, & Cordes 1998), bringing the star to a new stable equilibrium in which the principal axis of inertia is again aligned with the star's angular momentum. As the precession damps, the angle between the star's rotation and magnetic axes α increases by $\Delta\alpha$.⁴ In some models of pulsar spin down, an increase in α leads to an increase in the spin-down torque. In the magnetic dipole model for pulsar slowdown, for example, an increase by $\Delta\alpha$ gives a *permanent* increase in the spin-down rate of $\Delta\dot{\Omega}/\dot{\Omega} = 2\Delta\alpha/\tan\alpha$. We suggest that the Crab's offsets arise from starquake-induced changes of the order of $\Delta\alpha \sim 10^{-3}$ (Link et al. 1992; Link & Epstein 1997).

The following dimensional argument gives an estimate of the magnitude of the shift of the principal axes. Suppose that between starquakes the star spins down by $\delta\Omega$ and reduces its equatorial circumference by ΔR . As the starquake occurs, crust material suddenly moves along the fault by a comparable amount. From equation (1), we estimate

$$\begin{aligned} \frac{\Delta R}{R} &\simeq \frac{10\pi R^2 \Omega \delta\Omega}{3v_k^2} \\ &= 4 \times 10^{-5} \left(\frac{\Omega}{200 \text{ rad s}^{-1}} \right)^2 \left(\frac{v_k}{10^{10} \text{ cm s}^{-1}} \right)^{-2} \\ &\quad \times \left(\frac{t_q}{14 \text{ yr}} \right) \left(\frac{t_{\text{age}}}{10^3 \text{ yr}} \right)^{-1}, \end{aligned} \quad (9)$$

where $t_q \equiv \delta\Omega/|\dot{\Omega}|$ is the average time between *large* quakes, $\dot{\Omega}$ is the spin-down rate, and $t_{\text{age}} \equiv \Omega/2|\dot{\Omega}|$ is the spin-down age. Assuming that quakes are coincident with glitches, $t_q \sim 14$ yr for the Crab pulsar. Matter accumulates in mountains over a length scale of lateral extent comparable to the fault length L . The fault length is difficult to estimate but should be roughly the length scale characterizing the strain field, larger than the crust thickness ($\sim 0.1R$) but smaller than R . On formation of mountains, e_1 changes its angle with respect to the rotation axis by an amount of the order of the ratio of the off-diagonal elements of the moment of inertia tensor to the bulge moment of inertia; the shift is of the order of $\Delta\alpha \sim (M_m/M_B)(L/R)$, where M_m is the mass of a mountain and M_B is the mass in the equatorial bulge. The mass in the bulge is of the order of $M_B \sim (R\Omega/v_k)^2 M_c$, where M_c is the mass of the crust. The fraction of crust mass that accumulates in a mountain is roughly the fraction of the star's area that moves with the fault, i.e., $M_m \sim (\Delta R L/R^2) M_c$. Hence, we obtain an upper limit of

$$\Delta\alpha \lesssim \frac{10\pi}{3} \frac{L^2 \delta\Omega}{R^2 \Omega} = 0.07 \left(\frac{L}{R} \right)^2 \left(\frac{t_q}{14 \text{ yr}} \right) \left(\frac{t_{\text{age}}}{10^3 \text{ yr}} \right)^{-1}. \quad (10)$$

For the Crab pulsar, Rankin (1990) obtains $\alpha = 86^\circ$ from radio data, and Yadigaroglu & Romani (1995) find $\alpha = 80^\circ$ from their analysis of gamma-ray and radio data. For a

fault length of $L = 0.1R$, the corresponding fractional changes in spin-down rate ($2\Delta\alpha/\tan\alpha$) are 10^{-4} ($\alpha = 86^\circ$) and 2×10^{-4} ($\alpha = 80^\circ$), which are comparable to the largest offsets seen.

We cannot yet apply equation (10) to PSRs 0355+54 and 1830-08; only one large glitch has been observed in each of these pulsars, so the quake intervals and typical offset magnitudes are unknown. Moreover, the magnitude of the permanent offset in PSR 0355+54, if any, is unknown, since the spin-down rate has continued to change following the large glitch of 1986.

Applying equation (10) to the Vela pulsar, assuming a quake interval equal to the glitch interval of ~ 3 yr, gives an upper limit of $\Delta\alpha \simeq 1.4 \times 10^{-3}$. Link & Epstein (1997) argued that smaller changes, $\sim 2 \times 10^{-4}$, could account for the low braking index of 1.4 ± 0.2 reported by Lyne et al. (1996). Persistent shifts of $\sim 10^{-4}$ would be difficult to see in Vela timing data, as the pulsar does not have time to recover rotational equilibrium between glitches; years after a glitch, $\dot{\Omega}$ typically differs from the average spin-down rate by $\sim 1\%$, which is much larger than the expected persistent shift resulting from a change in α .

5. SUMMARY

The persistent increases in spin-down rate (offsets) seen to accompany glitches in the Crab and other pulsars suggest sudden increases in the spin-down torque. Starquakes occurring as a neutron star spins down and readjusts its structure affect the spin-down torque exerted on the star by changing the magnetic field geometry or orientation. In this paper we have examined the evolution of strain in the crust of a spinning-down neutron star and the initiation of starquakes as the material reaches critical strain. Crust cracking occurs as equatorial material shears under the compressive forces arising from the star's decreasing circumference. The star decreases its oblateness as matter is moved to higher latitudes along a fault inclined to the equator. Magnetic stresses suppress shearing near the magnetic poles, especially shearing motions across the field lines. Starquakes are thus most likely to originate near the two points on the equator farthest from the magnetic poles and then propagate toward the magnetic poles.

Starquake-induced misalignment of the star's angular momentum and spin, associated with glitches, is a possible explanation for the spin-down offsets seen in the Crab pulsar. Following the misalignment, damped precession to a state of larger angle α between the magnetic and rotation axes could increase the external torque, giving a permanent increase in the spin-down rate. Repeated starquakes would continue to increase α , making the pulsar more of an orthogonal rotator.

The picture of starquakes developed here is based on a solution for a homogeneous, incompressible sphere. In a future publication, we will consider a more realistic model of a thin, stratified crust afloat on a liquid core.

We thank A. Olinto, P. Richards, and D. Longcope for valuable discussions. This work was performed under the auspices of the US Department of Energy and was supported in part by NASA EPSCoR grant 291748 and by IGPP at LANL.

⁴ Crust motion would also change the external torque by moving the magnetic poles with respect to the rotation axis. For the Crab pulsar, the associated change in α , however, is only $\sim \Delta R/R \sim 10^{-5}$, which is too small to account for the largest observed persistent shifts.

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