

GENERAL RELATIVISTIC EFFECTS ON THE SPECTRUM REFLECTED BY ACCRETION DISKS AROUND BLACK HOLES

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ABSTRACT

Making the usual assumption that the relatively cold matter within the central engine of an active galactic nucleus (or galactic black hole candidate) is in the form of a relativistic accretion disk, we compute the composite spectrum of the original disk plus a primary X-ray power-law source illuminating it from above, as well as the reflected emission from the disk. All special and general relativistic effects on both infalling photons and outgoing photons are considered in a Schwarzschild geometry. The strength, shape, and broadening of the reflected spectrum depend on the direction of the X-ray source relative to the disk and the observer's viewing angle. The reflected photons extract energy and angular momentum from the relativistically rotating accretion disk and are beamed in the direction of the disk velocity. The reflection hump could essentially disappear if viewed far from the symmetry axis because the X-ray photons are affected by gravity both approaching and leaving the disk. This may produce a difference between X-ray spectra for Seyfert 1 and Seyfert 2 galaxies. For a given observation angle, the reflection hump is most sensitive to the inclination of the source relative to the accretion disk. Thus the spectral shape may also shed light on the location of the primary X-ray source, which is probably either in a jet or in a corona; however, additional computations involving distributed sources will be necessary before detailed comparisons with observations are feasible.

Subject headings: accretion, accretion disks — black hole physics — galaxies: active — galaxies: Seyfert — radiative transfer — X-rays: general

1. INTRODUCTION

The observed energy spectra of most Seyfert galaxies and some X-ray binaries through the EUV to hard X-rays show many significant features: a quasi-blackbody thermal bump in the EUV and soft X-ray bands; a nonthermal power law in the hard X-ray band; a “reflection hump” from 10 keV to 100 keV; and fluorescent lines around 6.4 keV (e.g., Mushotzky 1982; Malkan 1983; Arnaud et al. 1985; Turner & Pounds 1989; Pounds et al. 1990; Matsuoka et al. 1990; Turner et al. 1990). Most of these features are usually interpreted as accretion disk-related phenomena. The “soft X-ray excesses” are observed to be variable, sometimes on short timescales ($t < 1000$ s for Seyfert galaxies; Lawrence et al. 1985; Kaastra & Barr 1990), suggesting they come from the region close to the central engine. The temperature ($T \sim 10^5$ K for Seyferts) and the luminosity of this emission imply that there exist large quantities of relatively cold matter within the innermost region of active galactic nuclei (AGNs) and in some X-ray binaries (e.g., Cygnus X-1 with $T \sim 10^6$ K).

Guilbert & Rees (1988) proposed that nonthermal X-rays may be absorbed by an ensemble of gas clouds in and around the central engine. If the clouds are dense enough to remain cold, they absorb much of the nonthermal power and reradiate it. A direct prediction of the model is the occurrence of spectral features such as absorption edges, fluorescent lines, and a Compton reflection hump in the X-ray spectrum.

Lightman & White (1988) gave an early detailed calculation of the reflected spectrum. They considered reprocess-

ing of the X-rays by absorption and scattering in the cold matter, and predicted a broad “hump” of emission peaking at ~ 30 keV. The “reflected” spectrum employing various geometries of cold matter has been extensively studied this decade and indicates a large covering factor ($\sim 2\pi$ sr) for this cold matter with respect to the dominant X-ray source (e.g., George & Fabian 1991; Ross & Fabian 1993; Matt, Fabian, & Ross 1993; Bond & Matsuoka 1993; Nandra & George 1994; Ghisellini, Haardt, & Matt 1994). The widths and profiles of the fluorescent iron lines of both galactic black hole candidates (e.g., Cygnus X-1) and AGNs are interpreted as arising from large Doppler and gravitational shifts (Fabian et al. 1989; Pounds et al. 1990; Matt et al. 1993, 1996). Along with the net shift of the lines, the line width provides firm evidence that the cold matter responsible for the X-ray spectral features is in the form of a disk (or clumpy disk) very close to the central engine of the source, and is probably associated with the accretion disk. Thanks to the higher energy resolution of the X-ray satellite *ASCA* at the Fe $K\alpha$ line, the detailed study of the shape of this broad iron line has become possible over the last few years. Some of the observed line profiles are found to be in good agreement with calculations of what is expected to arise from the inner regions of accretion disks around black holes (Mushotzky et al. 1995; Tanaka et al. 1995; Fabian et al. 1995; Nandra et al. 1997).

Additional studies of the energy spectra from AGNs and X-ray binaries are needed to test theoretical models for both kinds of objects. All viable models involve the efficient conversion of gravitational potential energy to radiation as

matter falls onto the central black holes or neutron stars through accretion disks. Full general relativistic (GR) effects, including redshift, bending, and focusing, have been considered for a long time in the study of the spectrum arising directly from the accretion disk (e.g., Cunningham 1975; Fabian et al. 1989; Fu & Taam 1990; Hollywood & Melia 1995), but they have not usually been included in calculating the reflected component. Matt et al. (1993) produced theoretical models for broadband X-ray spectra in which they considered detailed radiation transfer for a centrally illuminated disk; they included GR effects using the approach of Fabian et al. (1989) for the radiation emerging directly from the disk, but not for the ingoing radiation. Ross, Fabian, & Brandt (1996) analyzed smeared iron $K\alpha$ lines and edges by incorporating relativistic corrections, and Życki, Done, & Smith (1997) considered an approximation to GR effects (Chen, Halpern, & Filippenko 1989) when calculating the composite spectrum for the *Ginga* spectra of GS 2023+338; but again these authors did not consider the full GR effects on the reflected component, which would require including both infalling photons originating from the X-ray source as well as outgoing reprocessed photons.

Including full GR effects for both incoming and outgoing photons could be very important because the X-ray photons from a hard X-ray source have extra interactions with the strong gravity around the black hole: they are frequency shifted and focused both as they impact the relativistic disk and as they head away to the observer. This consideration is mostly relevant to the reflected component of the spectrum; therefore, the reflected spectrum carries additional information on the gravitational field in the central engine as well as the location of the primary X-ray source. In this paper, we consider full GR effects (in the Schwarzschild metric) for the disk spectra, as well as the spectrum produced by reflection of the nonthermal X-ray source above the disk. The key new aspect of this work is that we compute the effects of light bending for both incoming and outgoing photons, and we consider different locations for the X-ray source. But, in contrast to many of the references cited above, we do not perform careful radiation transfer within the disk, and therefore cannot discuss the details of line shapes; rather our results are relevant to the shape of the broad reflection hump. Very recently, Reynolds & Begelman (1997) independently considered illumination of the disk, including the region within the innermost stable orbit, by an X-ray source located on-axis and close to the disk. They also included the gravitational focusing of photons coming from that special geometry of an on-axis point source as well as a full GR treatment for the photons emitted from the accreting matter in order to determine the shape of the iron line profiles. Our techniques differ from theirs, and we assume a different geometry, one where the photons can originate from any direction (not necessarily along the axis). In § 2 we describe the method of the calculation and the numerical results; § 3 comprises conclusions and a discussion.

2. THE MODEL AND RESULTS

The high-energy spectral features of Seyfert galaxies are usually interpreted in terms of a model in which the primary X-ray power-law continuum is reprocessed/reflected in an accretion disk. Here we adopt the same picture, and, to simplify the computations dramatically, we assume the disk is illuminated by plane-parallel radiation from the source,

as has also been done by many other authors (e.g., Nandra & George 1994). Figure 1 shows the geometry considered; i_1 and i_0 are respectively the inclination angles of the observer and the X-ray source to the symmetry (disk angular momentum) axis, and Δ is the orientation of the source relative to the observer (defined so that $\Delta = 0$ if source and observer are on the same side of the axis).

One physical situation in which this approximation would hold nicely is if the X-ray illumination arises from the base of a jet that is located substantially above the black hole and disk; it is more interesting if that source can be significantly off-axis. We admit that this is an idealized geometry that is unlikely to be the most typical physical situation, but it allows us to concentrate on the new GR effects discussed in § 1. This assumption will not be realistic if the X-ray source is small in extent and very close to the inner portion of the disk, but the key effect that modifies the strength of the hump should still hold. Moreover, unlike the situation in the Newtonian case, the calculation of reflection of non-plane-parallel illumination is substantially more difficult to carry out in the GR case. If the X-ray source is widely distributed, e.g., in the form of a corona above and below the disk, the calculation becomes even more complex and model dependent; in that case, the final result is likely to resemble our results for an on-axis source ($i_0 = 0$).

In contrast to the case of flat spacetime, X-ray photons coming from the nonthermal source will be gravitationally shifted on their route to the disk and will be shifted again after the “reflection” toward the observer at infinity. The quasi-thermal photons coming directly from the accretion disk will be affected only once (this effect has been considered by many authors earlier; see § 1). Also, if the inclinations of both the source and observer are high, then external X-ray photons will experience double bending before they reach the observer, but the thermal photons are bent only once.

The approximation to the observed spectrum considered here contains three components: the nonthermal radiation coming directly from the source, the radiation from the accretion disk itself, and the reprocessed radiation by the accretion disk, which comprises effectively cold matter as far as the X-rays are concerned. This condition for cold disk matter implies that the disk should be radiating substantially below the Eddington limit ($L \lesssim 0.1L_{\text{Edd}}$) (Matt et al. 1993; Nandra et al. 1995), which is definitely satisfied for the specific model we consider. The power-law X-ray component in its rest frame is expressed by $I_\nu \propto E^{-\alpha} \exp(-E/E_{\text{cut}})$, where E_{cut} is the effective upper cutoff energy of

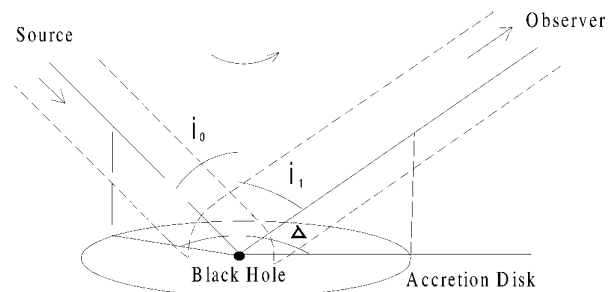


FIG. 1.—Geometry. Photons come from the source and are reprocessed and reflected by the accretion disk, which also emits directly. In both directions the photons are redshifted and follow geodesics in the field of the central black hole. The inclination of the observer to the symmetry axis is i_1 , and that of the source is i_0 . The phase of the source relative to the observer is Δ .

the spectrum, typically set to 100 keV. We compute the accretion disk spectrum in its rest frame using the standard prescription for thin α -viscosity accretion disks from Novikov & Thorne (1973); this spectrum is basically a sum of modified blackbodies. The “reflected” spectrum is computed similarly to Lightman & White (1988), assuming a covering factor of 0.5. This scenario assumes that the non-thermal source illuminates the cold matter and is then reprocessed by the matter through atomic absorption and electron scattering. To produce the hump, it is required that the radiation from the object should not be close to the Eddington limit (Nandra et al. 1995). To calculate the reflected components, we employ a program kindly provided by F. Haardt. We note that we do not perform the complex radiative transport necessary to resolve the shape of the iron line and edges; this has been considered in detail by other authors (e.g., Ross et al. 1996). We emphasize that our calculation is designed to show that significant effects on the reflection hump can arise from double gravitational light bending, something that has not been considered before.

Let I_{vem} be the local specific intensity at frequency ν_{em} at the disk and I_{vob} the intensity observed at frequency ν_{ob} at infinity. From the generalization of Liouville’s theorem (e.g., Misner, Thorne, & Wheeler 1973), their relation can be written

$$I_{\text{vob}} = g^3 I_{\text{vem}} f(\mu) \text{ ergs cm}^{-2} \text{ s}^{-1} \text{ Hz}^{-1} \text{ sr}^{-1}, \quad (1)$$

where $f(\mu)$ is a direction dependence function, which is determined by the local physics of scattering and will be described later. The total frequency shift at a point (r, φ_s) on the disk is defined as the ratio of the observed energy of a photon to that of the emitted energy (measured at the local frame), $g = E_{\text{ob}}/E_{\text{em}}$. This g factor includes both the gravitational shift and the Doppler shift (from disk rotation), and is given by (cf. Bao 1992)

$$g = \frac{\sqrt{(1 - v_s^2)(1 - 2/r)}}{1 - \Omega L_z}, \quad (2)$$

where $v_s = \Omega r(1 - 2/r)^{-1/2}$ is the velocity of the emitting matter measured in the local static frame, Ω is its angular velocity, and L_z is the component of its angular momentum about the z (symmetry) axis; note that the angular dependence of the redshift factor g is contained in L_z . Hereafter, spherical coordinates and the geometric units in which $G = c = 1$ will be used, and for simplicity, physical quantities are expressed as dimensionless ones.

We parameterize the scattering function $f(\mu) = (1 + a\mu)\mu^b$, where $\mu = \cos \hat{\theta}$, and $\hat{\theta}$, the angle between the normal to the disk and the direction of emission, is measured in the comoving frame of the emitting element. Different values of the coefficients a and b correspond to different local conditions in the emitting matter. For example: $a = 0$, $b = 0$, for the case where the emission is from optically thick matter (this is the simple case considered below); $a = 0$, $b = -1$, for the optically thin case; and $a \neq 0$, $b = 0$ for a general “limb darkening law.” We see that $\hat{\theta}$ can be expressed in terms of the total impact parameter B as

$$\cos \hat{\theta} = \left(\frac{B}{r}\right) g \sqrt{1 - \left(\frac{L_z}{B}\right)^2}. \quad (3)$$

It is easy to verify that $\hat{\theta} = i$ when r is very large and GR effects are not important.

Thus, the observed spectrum can be written as

$$F_{\text{vob}} = \int I_{\text{vob}} d\Pi \text{ ergs cm}^{-2} \text{ s}^{-1} \text{ Hz}^{-1}, \quad (4)$$

where Π is the solid angle the emitting matter subtends at the observer; this directly measures the bending or lensing effect. In terms of the total impact parameter B and the position angle ϕ' of the photon, $d\Pi = B dB d\phi' D^{-2}$, where D is the distance between the observer and the emitting matter; $d\Pi$ can be rewritten in terms of the coordinates of the disk, r and φ_s . The methods used to calculate the solid angle and the impact parameter are given in Bao, Hadrava, & Østgaard (1994); see also Hadrava, Bao, & Østgaard (1997).

The above equations can be used directly to calculate the observed spectrum of the thermal radiation from the accretion disk, but have to be modified to calculate the reflected spectrum. This is because thermal photons come from the disk directly, while the reflected X-ray photons come first from the source to the disk and then from the source to the observer. According to equations (1) and (4), the power-law intensity measured *on the disk* will be inversely proportional to the redshift factor to the third power and the solid angle at infinity, and therefore the reflected intensity measured by the observer at infinity will be proportional to the third power of the ratio of the redshift factor for the outgoing photon to that of the incoming photon and the ratio of the solid angle for the outgoing case to that for the incoming case. Of course, the impact parameters B of the incoming and outgoing X-ray photons are different and must be calculated independently. These features make the observed reflected spectrum quite complex, even for our simplifying assumptions of plane-parallel illumination and a non-rotating black hole.

In the following illustrative calculations, we have assumed a relativistic disk around a $10^6 M_\odot$ black hole with an accretion rate of $0.01 \dot{M}_{\text{Edd}}$ and $\alpha_{\text{vis}} = 0.01$. We have taken the ratio of the total power of the power law plus its reflected power to the total power from the thermal disk (integrated between $r = 6.1$ and 100 m) to be 11.26; this number takes the integrated range of energy to be from 0.05 to 1000 keV, assumes $\alpha = 0.9$ and $E_{\text{cut}} = 100$ keV for the power-law parameters, and is exact for $i_0 = i_1 = 1^\circ$. This ratio changes slightly for different i_0 and i_1 .

The best way to judge how the gravitational shifts alone affect the observed spectra is to consider the situation where the relativistic rotation of the accretion disk and the gravitational focusing have their minimum influences. This occurs when both the source and the observer have their inclinations, i_1 and i_0 , close to 0° . Figure 2 shows both the spectra directly from the disk as well as the reflected spectrum for this case, with GR effects included or excluded; note that we have plotted νF_ν (and not just F_ν) against $h\nu$, and this shifts the peak of the disk spectrum to higher frequencies. For the face-on view of the disk with the primary X-ray source being right above it, the spectrum from the disk in Schwarzschild spacetime is significantly softer than that observed in flat spacetime, because the pure gravitational shift is proportional to $(1 - 2/r)^{1/2}$. For the reflected spectrum with the primary X-ray photons returning in the same direction, the gravitational effects essentially cancel since we assume illumination by parallel rays, and the reflected spectra in GR and in flat spacetime are identical (to within the thickness of the lines in the figure).

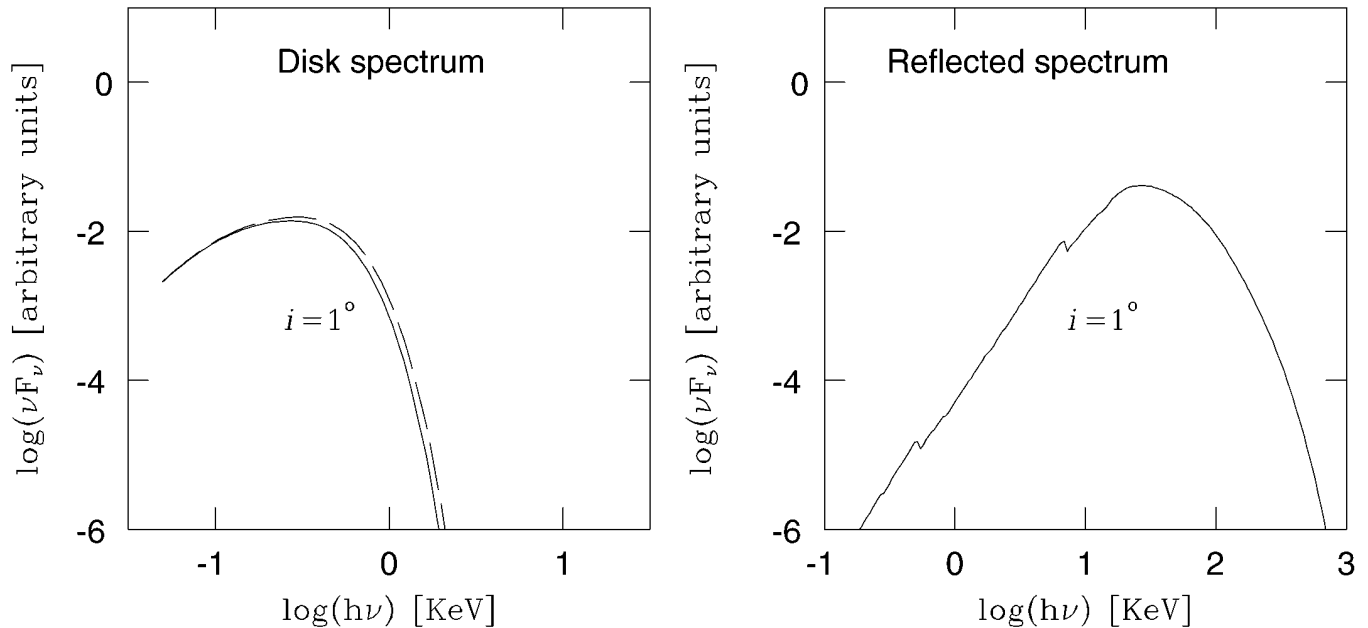


FIG. 2.—Accretion disk spectra (*left*) and the spectrum reflected by the disk (*right*). The inclination of the observer, i_1 , and that of the source, i_0 , are both 1° , and the orientation of the source relative to the observer is $\Delta = 0$. The relativistic accretion disk extends from an inner radius $r_{\text{in}} = 6.1r_g$ ($r_g = GM/c^2$) to the outer radius $r_{\text{out}} = 100r_g$. A black hole mass of $M = 10^6 M_\odot$, an accretion rate of 0.01 (the Eddington rate), and a viscosity parameter $\alpha_{\text{vis}} = 0.01$ are assumed. The X-ray source has a quasi-power-law spectrum given by $f_E \propto E^{-0.9} \exp(-E/100 \text{ keV})$, for $E \geq 0.05 \text{ keV}$. Solid curves refer to the spectra including all GR effects, while the dashed curve refers to the spectra observed locally.

The observed spectrum includes direct radiation both from the primary X-ray source and from the disk as well as reflected X-rays. Figure 3 presents the composite spectra with and without GR effects when both the source and the observer have high inclinations to the symmetry axis. Note that even this is not the extreme case for the gravitational effect on the spectrum, which depends on an average over the disk of the ratios of incident and reflected solid angles and redshift factors. Nonetheless, in this case gravitational focusing plays an important role, and we can see clearly that the reflection hump is amplified.

The location and orientation of the source relative to the observer modifies the observed spectrum. As noted above,

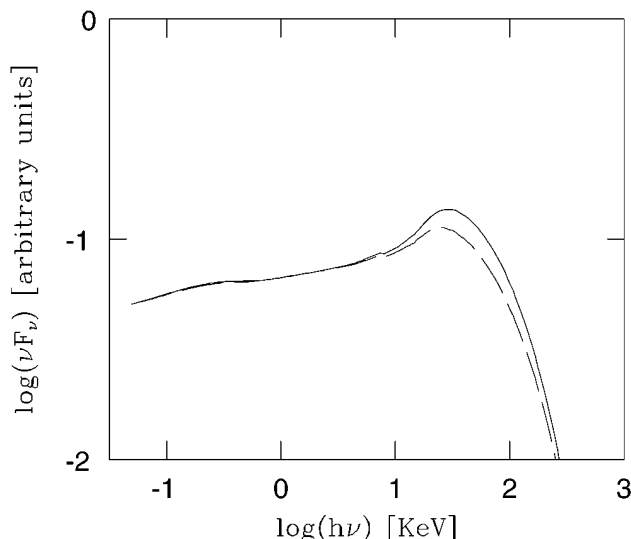


FIG. 3.—Composite spectra with the same disk and X-ray source parameters as in Fig. 2; again the solid curve refers to the spectrum including all GR effects, while the dashed curve refers to the spectrum observed locally. The inclination of the observer and the inclination of the source are now $i_1 = i_0 = 80^\circ$.

unlike the case in which the spectrum comes directly from an accretion disk, the amplitude of the observed reflected spectrum depends on the ratio of the redshift factors (which include rotational Doppler effects) of outgoing to incident photons, and the ratio of the average solid angles seen by the observer and by the disk.

The spectra for various values of inclination angles of the source (for a fixed observing angle of $i_1 = 40^\circ$) are displayed in Figure 4 (for $\Delta = 0^\circ$) and Figure 5 (for $\Delta = 180^\circ$). Phases $\Delta = 0^\circ$ (180°) correspond to the different situations where the source is in front of (behind) the black hole relative to the observer. Comparing the amplitudes of the humps between these two figures, we see that the reflection effect is clearly stronger when the source is on the same side of the black hole as is the observer, independent of i_0 . Examining the effects of various incident inclinations for the source, both Figure 4 and Figure 5 illustrate that as i_0 increases, the amplitudes of the reflected humps also increase. When i_1 is fixed, the observed solid angle is also fixed, but as i_0 increases, the incident solid angle decreases, so the ratio of the solid angles for each element of the disk increases, and thus the amplitudes of the reflected components grow. Since the disk spectrum itself passes through the gravitational field only once, its spectrum is independent of i_0 .

Thus we note that a moving X-ray source (in i_0 , Δ , or both) would give rise to spectral variability. Large motions of this type may not be probable, particularly if the power-law source arises near the base of a relativistic jet. On the other hand, if most of the X-rays (at least for a limited time) arise from a single flare in the corona, then such motion might be present; however, it is likely in this case that the plane-parallel approximation we have employed would break down and more complex ray-tracing algorithms would have to be used to obtain accurate results.

Finally, we examine how the observed spectrum depends on the inclination of the observer, i_1 . Figure 6 presents the spectrum for different inclinations of the observer for a fixed

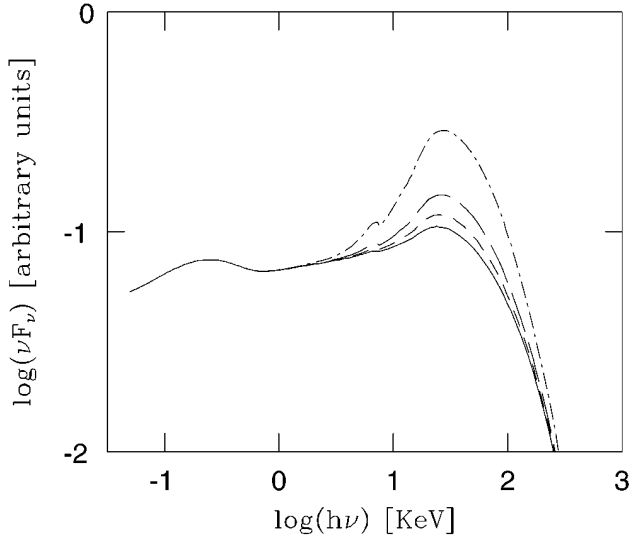


FIG. 4.—Composite spectra where the inclination of the observer is $i_1 = 40^\circ$ and the inclinations of the source are $i_0 = 1^\circ$ (solid line), 40° (short dashed line), 60° (long-dashed line) and 80° (dot-dashed line); for all curves, $\Delta = 0$. Source and disk parameters are as in Fig. 2.

source position of $i_0 = 40^\circ$. For the same reason explicated above, we see that as i_1 increases, the amplitude of the hump decreases.

3. CONCLUSIONS

We have illustrated complete composite spectra emerging from an accretion disk around a Schwarzschild black hole illuminated by an external plane-parallel X-ray source under various geometric situations, including more GR effects than had been considered in earlier works. One of the significant features of this calculation is the behavior of the reflection hump as a function of the viewing angle of the observer (Fig. 6). For high inclinations of the observer to the disk normal, we notice that the hump effectively disappears from the composite spectrum because of the dual passages near the black hole of the hard X-ray photons.

In the last decade, a general picture of the central engine of radio-quiet AGNs has been developed. The unified model for Seyfert 1 and Seyfert 2 galaxies (e.g., Antonucci

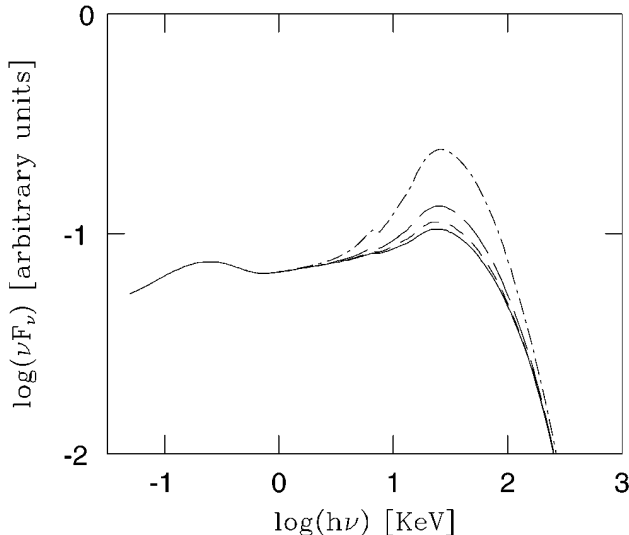


FIG. 5.—As in Fig. 4, except $\Delta = 180^\circ$ for all spectra

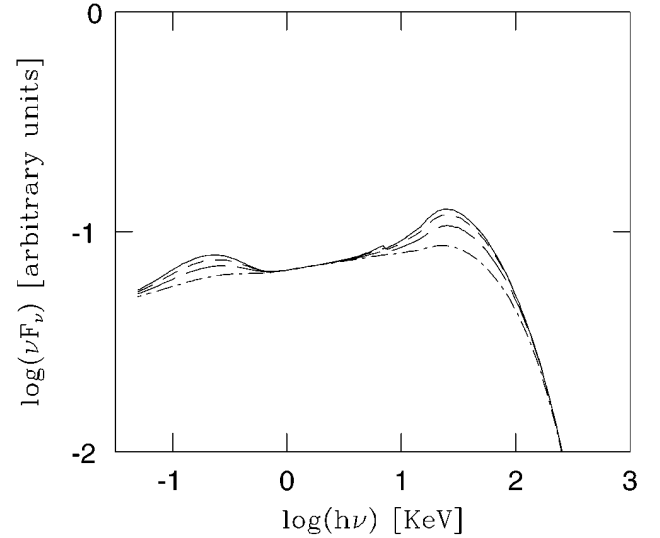


FIG. 6.—As in Fig. 4, except the inclination of the source is fixed at $i_0 = 40^\circ$ and the inclinations of the observer are $i_1 = 1^\circ$ (solid line), 40° (short-dashed line), 60° (long-dashed line), and 80° (dot-dashed line).

1993) argues that the different properties of the two classes are simply due to inclination effects (e.g., Lawrence & Elvis 1982; Antonucci & Miller 1985; Mushotzky, Done, & Pounds 1993). In this model, there are no differences in the production of the primary X-ray sources above the accretion disk. A geometrically and optically thick torus is supposed to surround the active nucleus and disk, absorbing much optical UV radiation and soft X-rays.

Within the unified schemes, Seyfert 1's and quasars have $i_1 \lesssim 45^\circ$ while Seyfert 2's and powerful radio galaxies have $i_1 \gtrsim 45^\circ$ (Antonucci 1993; Urry & Padovani 1995; Gopal-Krishna 1996), while i_0 is presumably independent of the observed "type" of the active galaxy. The geometry and location of a primary X-ray source is poorly understood, as indicated above; it could be in the base of a jet, but could also be produced in active regions in a corona above the disk. Observations made from near Earth provide a fixed i_1 for each galaxy. From purely geometric considerations, our model (cf. Fig. 6, plus the unified scheme) would imply weaker reflection humps for Seyfert 2's than for Seyfert 1's, but this may be difficult to confirm, as the spectral differences between Seyfert classes are probably dominated by absorption in the torus.

The azimuthal orientation (Δ) of the primary X-ray source influences the observed shape of the reflected spectrum. Figures 4 and 5 show the extreme cases. The humps in Figure 4 are larger than those in Figure 5, which is similar to the result obtained by Hadrava et al. (1997) when considering only pure reflection (without reprocessing) off an accretion disk. The reflected photons extract energy from the relativistically rotating accretion disk and are preferentially beamed into the direction of the disk's velocity. The efficiency of this process is dependent on the geometry of the system, i.e., the source's inclination and azimuthal location with respect to the fixed observer. Therefore, in principle, fitting this model to observed data could provide information about the location of the power-law X-ray source if it were localized. In most of the models computed earlier (see § 1) the X-ray sources are assumed to be right above the accretion disk, perhaps arising in a jet (i.e., $i_0 \approx 0^\circ$). From our calculation (Figs. 4 and 5), the strength of the

hump is very sensitive to the inclination of the source if other parameters are fixed, especially i_1 ; the larger i_0 , the stronger the hump. Therefore, if strong humps are revealed to be highly probable (once a statistically significant sample of Seyferts has been examined), and the preliminary conclusions drawn from our calculations hold up when more general cases are computed, one might then be able to argue against the X-ray sources emanating near the symmetry axes.

Although we discussed a special case, i.e., the disk is illuminated by a parallel beam of irradiation, the fact that nonthermal photons extract energy and angular momentum from the relativistically rotating disk (and thus enhance the hump) may be a common feature for the system regardless of where the source is located. An interesting question concerns the efficiency of the process. We have seen that the maximum efficiency corresponds to the case where the source is on the same side as the observer relative to the black hole and the minimum efficiency to the case where the source is on the symmetry axis. Even though a source is on the symmetry axis, as long as it subtends a nonzero solid angle as seen by various locations on the disk, some level of enhancement of the hump due to rotational energy extraction will occur. In the plausible case where there are many sources covering a wide range of the rotating disk (e.g., flares in a corona), the enhancement could reach a reasonable level. All of this depends on the specific geometry of the system and is worthy of further investigation.

Any changes in either i_0 or Δ will lead to temporal and spectral variability. We will consider these points in more detail in a subsequent paper (Bao & Wiita 1998, in preparation), although that effort will concentrate on the spectral variability that is induced if the accretion disk is clumpy in the inner regions. Earlier work on these clumpy "bright spot" models (e.g., Abramowicz et al. 1989; Wiita et al. 1991; Zhang & Bao 1991; Mangalam & Wiita 1993) has focused on temporal variability at single frequencies.

We note that the code we used ignores the angular dependence of photons during their processing in the disk, and this may not be a valid approximation for some cases; however, the main results we obtained should be robust with respect to this simplification, as they arise from GR effects which mostly act on the photons before and after they are reflected from the disk. Other important complications that remain to be treated before one should make detailed comparisons with data include the effects of a rotating black hole (Kerr geometry) and the different classes of disk spectra that can arise for much higher or lower accretion rates (e.g., Chakrabarti 1996; Narayan 1996). Detailed comparisons with data remain to be performed, as the radiative transfer in the disk should be treated in a more careful manner (e.g., Ross et al. 1996; Życki et al. 1997).

Despite the relative simplicity of our model, we have shown that including more GR effects in calculations of the reflection spectrum significantly affects the observed spectral shapes. Unlike the thermal disk spectrum, reflected X-ray photons must pass through strong gravitational fields both "coming and going." Gravitational shifts and bending can, in some cases, enhance the reflection hump, and in others, reduce its amplitude. As shown by Reynolds & Begelman (1997) for a different assumption about the geometry, these effects also can be very important for the iron line shape. Therefore, it will be highly desirable for subsequent attempts at comparing theoretical models with observations to include the GR effects we have highlighted here.

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