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PERSISTENT COUNTERPARTS TO GAMMA-RAY BURSTS

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ABSTRACT

The recent discovery of persistent gamma-ray burst (GRB) counterparts at lower frequencies permits several important conclusions to be drawn. The spectrum of GRB 970508 is not consistent with an external shock origin for both the prompt GRB and the persistent emission, suggesting that at least the prompt radiation is produced by internal shocks. Comparisons among three GRBs with counterparts (or upper limits on them) establishes that GRBs are not all scaled versions of similar events. The angular size inferred from the apparent observation of self-absorption in the radio spectrum of GRB 970508 a week later implies that its expansion had slowed to semirelativistic speeds. This permits a remarkably low upper bound to be placed on its residual energy, suggesting either that radiation has been more than 99.7% efficient or that the initial outflow was strongly collimated. Observations of self-absorbed radio emission from future GRBs may permit direct measurement of their expansion and determination of their parameters and energetics. We estimate initial Lorentz factors of $\gamma_0 \sim 100$ for GRB 970228 and GRB 970508, and present a solution for the evolution of a blast wave with instantaneous cooling.

Subject headings: gamma rays: bursts — gamma rays: theory — radiation mechanisms: nonthermal — shock waves

1. INTRODUCTION

The recent discovery by *BeppoSAX* and coordinated visible, infrared, and radio observations of persistent emission from gamma-ray bursts (GRBs) has answered several important questions. Accurate coordinates of visible counterparts have led to conclusive evidence that GRBs are at cosmological distances, definitively resolving the oldest and most fundamental question in GRB astronomy. Yet other questions, such as the nature of the GRB emission mechanism, remain unanswered, and new questions have arisen. Most important, it is unclear how the persistent emission is produced and what its relation is to the prompt soft gamma-ray radiation that has defined GRBs since their discovery in 1973.

In this paper we examine two different classes of models for the persistent emission. In one class it reflects continuing gamma-ray activity, not directly observed because of the comparatively low sensitivity of GRB detectors to emission of low intensity but long duration. In the other class the persistent emission is the long-predicted afterglow produced when a relativistically moving shell sweeps up ambient matter and its large initial Lorentz factor decays as its energy is shared and radiated. We also examine two different classes of models for the prompt GRB emission. In one it is produced by "internal shocks"—interaction between different shells all moving at different but relativistic speeds. In the other class it is the result of an "external shock" produced when a relativistic shell collides with the ambient matter. We will argue that the presently available observations (chiefly of GRB 970111, GRB 970228, and GRB 970508) present strong evidence that the prompt GRB emission is that of internal shocks. The explanation of the persistent emission is less clear, with some evidence for continuing gamma-ray activity and some for afterglows; it is possible that both processes contribute with differing importance in different GRBs. We also present evidence that in at least one burst (GRB 970508) either the external shock is an extremely efficient radiator of the kinetic energy or the initial mass outflow and gamma-ray emission are strongly collimated.

In § 2 we summarize the two classes of models for persistent emission. If the prompt GRBs were produced by an external shock, then in a burst whose persistent emission is afterglow it should be possible to extrapolate the properties of the afterglow back to the prompt GRBs. In § 3 we present evidence that this cannot be done for GRB 970508, implying that its prompt emission was the result of internal shocks. This conclusion is particularly interesting because the prompt emission of GRB 970508 had a simple single-peaked time profile, consistent with an external shock; external shocks are not satisfactory sources of the gamma-ray emission even where permitted by the time history. We compare three bursts in § 4 and conclude that GRBs are not all scaled versions of similar events. In § 5 we discuss evidence for self-absorption at radio frequencies in GRB 970508, and estimate the size of the emitting region. From this we can infer either that radiation has been very efficient or that the initial expulsion of relativistic matter was strongly collimated. Collimation differing from burst to burst is one possible explanation of the comparisons presented in § 4. In § 6 we interpret the pulse shape and spectral history of GRB 970228 and GRB 970508 as the results of internal shocks followed by external shocks, and estimate their parameters. Section 7 presents simple estimates and scaling laws for the properties of afterglows produced when radiation is efficient, as suggested here. Section 8 contains a summary discussion and considers the prospects for determining the parameters of GRBs and their afterglows by using measurements of self-absorbed synchrotron radiation to measure their expansion. In the Appendix we rederive and explain the result for the self-absorbed intensity of a GRB given (with little explanation) by Katz (1994a).

2. WHAT MAKES PERSISTENT EMISSION?

Gamma-ray bursts are generally agreed to be associated with the expulsion of relativistic debris by a condensed

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object. The fundamental argument for this is the presence of gamma rays with energies greater than $m_e c^2$. If gamma-ray bursts are distant and luminous (the evidence for cosmological distances and great luminosities is now compelling), such gamma-rays can avoid destruction by γ-γ pair production only if they are narrowly collimated in a radially outward direction and their radius of emission is large enough. A sufficient, and probably necessary, condition for this to occur is that they be emitted by matter moving radially outward at relativistic speed with a Lorentz factor $\gg 1$.

2.1. *Afterglow Models*

The outward-moving matter will slow as a result of its interaction (generally assumed to be collisionless, for the collisional mean free path is extremely long) with any surrounding medium. Shocked matter will radiate, and as the Lorentz factor of the debris decreases, the radiation will shift to progressively lower frequencies and the corresponding timescales will lengthen.

Considerations of this kind led several authors (Katz 1994a, 1994b; Mészáros & Rees 1997; Sari & Piran 1997a; Vietri 1997; Waxman 1997) to predict that GRBs would be followed by an afterglow at frequencies gradually declining from soft gamma rays through X-rays to visible light and radio waves. It is possible to make many different assumptions concerning the efficiency of radiation, reconversion of postshock thermal energy to bulk kinetic energy, collimation of the initial debris, and development of the magnetic field. Mészáros & Rees (1993) and Mészáros, Rees, & Papathanassiou (1994), who integrated GRB spectra through the stages that are now called afterglow, and Paczyński & Rhoads (1993), who considered radio emission at very late times, demonstrate the possible variety and complexity of these assumptions. As a result, many different predictions of the quantitative development of the radiation spectrum with time are possible.

All such models in their basic form, in which only one set of assumptions is made, have the general property that characteristic timescales and peak intensities are power-law functions of frequency. The reason for this is that if there is no characteristic Lorentz factor defined between the Lorentz factor γ_0 of motion initially produced by the source and unity, then there is no opportunity for a break in the power-law dependence of any function on any variable between the values defined at these limits; such a break would define a new characteristic Lorentz factor, which by assumption is not present. Of course, it is possible to define more complex models in which this assumption does not hold (for example, radiation may be assumed efficient in some range of Lorentz factors, and inefficient outside that range). This possibility opens the door to a plethora of parameters and models of unmanageable complexity. Such models may fit the data, but it is hard to decide whether this is because they are physically correct or because it is easy to fit multiparameter functions of no physical significance; they will not be considered further.

Afterglow models generally predict that the intensity at a given frequency should rise until a characteristic time, and then decay (Katz 1994a, 1994b). This appears to have been observed for visible light from GRB 970508 (Galama et al. 1997b; Jaunsen et al. 1997; Schaefer et al. 1997), although the initial rise is defined by only a very few observations. The characteristic time is a decreasing function of frequency, short for gamma rays, longer for visible light, and still longer at infrared and radio wavelengths. Before this single maximum the intensity should rise monotonically, and afterward it should decay monotonically, without fluctuation, as a smooth (usually power-law, when well away from the maximum) function of time. This is so far consistent with nearly all the data (see, however, Kopylov et al. 1997), but the data are so sparse that this conclusion can hardly be said to have been proved.

2.2. *Continuing Emission Models*

In an alternative class of models (Katz, Piran, & Sari 1997) the persistent radiation at lower frequencies (X-rays, visible light, infrared, and radio waves) is produced by the same processes which make the initial GRB. The gamma-ray emission continues at lower intensity, perhaps maintaining a constant ratio to the lower frequency intensity.

There is little direct evidence for or against models of this class. Emission at frequencies below X-rays has never been observed simultaneously with a GRB for instrumental reasons; only high upper bounds can be set on its intensity. Analogously, weak continuing gamma-ray emission is nearly unobservable by a GRB detector, which is designed to trigger on intense short bursts. Continuing emission models agree with the spectral slope of persistent emission in I, V, and X-ray bands in GRB 970228 (Katz et al. 1997).

Some indirect evidence suggests that gamma-ray activity may continue long past the nominal duration (usually less than 100 s) of GRBs. High-energy gamma rays were observed 5000 s after GRB 940217 (Hurley et al. 1994). The "Gang of Four" spatially coincident (to within experimental accuracy) bursts of 1996 October 27–29 (Meegan et al. 1996; Connaughton et al. 1997) indicate repetitive activity from a single source, which can equivalently be described as a single burst lasting 2 days, with brief periods of intense emission within a longer period of much weaker or undetectably faint emission.

Models of this class predict that continuing gamma-ray activity should accompany the lower frequency radiation. In analogy to the behavior of the intensity during observed shorter GRBs, the intensity should fluctuate irregularly in all bands, although this may be hard to detect in faint objects for which observations require extended integrations.

3. EVIDENCE FOR INTERNAL SHOCKS IN GRB 970508

We wish to test the applicability of the models discussed in § 2 to GRBs. For a variety of reasons (satellite pointing constraints, instrumental sensitivity, Earth occultation, daylight, moonlight, etc.) observations of persistent emission by GRBs are sparse. In order to test continuing emission models, it is necessary to have data obtained simultaneously at several frequencies. The spectrum of GRB 970508 does not support these models.

GRB 970508 is therefore an excellent candidate for afterglow models. All the soft X-ray data and nearly all the visible data were obtained after the peak intensities in those bands, and therefore reflect the behavior of a high-energy tail to the particle distribution function (Wijers, Rees, & Mészáros 1997), and not the evolution of its characteristic energy or gross energetics. Fortunately, if there are sufficient data to define the peak flux $F_{\max\nu}$ at a frequency ν, this quantity can be determined without the need for simultaneous measurements across the spectrum or for frequent sampling on both sides of the maximum. Therefore, it may be easier to compare $F_{\max \nu}$ with theories than the instantaneous F_ν or than $F_\nu(t)$ at a single value of ν.

The data available for GRB 970508 are shown in Figure 1. The radio data are shown as a lower limit because (in the most recent available data; Frail et al. 1997b) the flux is still rising. The visible (Galama et al. 1997b; Jaunsen et al. 1997; Schaefer et al. 1997), hard X-ray (Costa et al. 1997b), and gamma-ray (Kouveliotou et al. 1997) maxima were observed. The soft X-ray flux (Piro et al. 1997) is a lower limit because its maximum was missed. Some specific afterglow models (Katz 1994a, 1994b; Mészáros & Rees 1997) predict $F_{\max \nu}$ to be independent of ν; these are evidently excluded.

These points are not consistent with a single power law. Simple afterglow models (§ 2.1) cannot describe the entire spectrum, regardless of the specific assumptions made. It is possible to resolve this problem in many ways—for example, by introducing two different regimes into the afterglow model, with suitably chosen parameters. We suggest, instead, that the brief measured hard X-ray and gamma-ray emission are not produced by the same process as the persistent emission at lower frequencies. Sari & Piran (1997a) argued that the gamma-ray emission of multipeaked GRBs could not be the result of an external shock but rather was the result of internal shocks. We suggest that this conclusion is also applicable to GRB 970508, resolving the discrepancy with afterglow models, which may explain the lower frequency emission. Because GRB 970508 was single peaked in gamma-ray intensity (Kouveliotou et al. 1997), the arguments of Sari & Piran (1997a) do not apply to it directly. Our argument for internal shocks in GRB 970508 then suggests that in *all* GRBs gamma-ray emission is produced by internal shocks, whatever the shape of their time profiles.

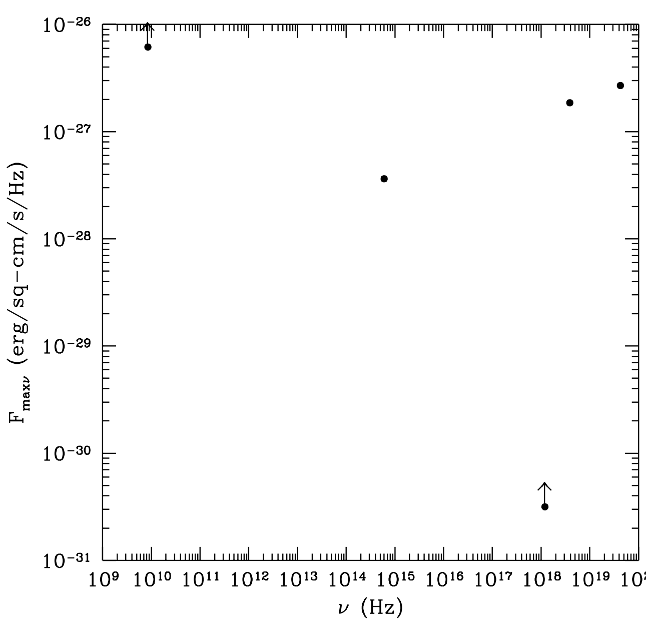

FIG. 1.—Peak fluxes in five bands for GRB 970508. The radio (8.46 GHz) and soft X-ray points are lower bounds, indicated by arrows, because the maxima in these bands were not observed. The radio, visible, and gamma-ray points are not consistent with a single power law. Data are from Kouveliotou et al. (1997) (gamma-rays), Costa et al. (1997b) (hard X-rays), Piro et al. (1997) (soft X-rays), Galama et al. (1997b) and Jaunsen et al. (1997) (V), and Frail et al. (1997b) (radio). The X-ray and gamma-ray points depend significantly on assumptions about the spectrum in those bands.

TABLE 1

RATIOS OF PEAK FLUXES IN FOUR BANDS: GRB 970508/GRB 970228

Gamma Ray	2–30 keV	V	Radio
0.06	1.3	~ 2	?

NOTE.—Data from Kouveliotou et al. 1997 (extrapolated to match *BeppoSAX* 40–600 keV band) and Hurley et al. 1997 (gamma rays), Costa et al. 1997a, 1997b (2–30 keV X-rays) and from Galama et al. 1997b, Jaunsen et al. 1997, and Groot et al. 1997 (V). The entry under V assumes that the characteristic timescale and postmaximum decay rate of GRB 970228 (whose maximum was not observed) is similar to that of GRB 970508; the data directly imply a V-band ratio less than 3.

4. GAMMA-RAY BURSTS DIFFER QUALITATIVELY

It is natural to ask whether all GRBs are scaled versions of similar events. External shock models predict that they are, because external shocks may be scaled by powers of their basic parameters: distance, total energy, initial Lorentz factor, and ambient gas density. Table 1 presents ratios of $F_{\max \nu}$ in four bands between GRB 970508 and GRB 970228. These two bursts have quite different spectral properties. A similar conclusion may be reached by comparing GRB 970508 to GRB 970111 (Table 2). The different spectral properties are again evident.

Tables 1 and 2 summarize the obvious fact that GRB 970508 was a comparatively weak burst in gamma rays. It was brighter in V than GRB 970228 and brighter at 1.43 GHz than GRB 970111, two bursts that were much stronger in gamma rays. The obvious explanation is the same as that reached in § 3: the gamma-ray emission is the result of a different process than that which produces the persistent emission at lower frequencies.

The weakness of GRB 970508 in gamma rays poses other problems. Its gamma-ray fluence (Kouveliotou et al. 1997) and redshift (Metzger et al. 1997) imply an energy output (if isotropic) of $\sim 3 \times 10^{51}$ ergs. This makes it difficult to argue that its comparatively low gamma-ray fluence results from unusually inefficient gamma-ray emission, because if it had its observed V magnitude and the same V/gamma-ray ratio as GRB 970228, the observed fluence would imply (assuming isotropy) the radiation of $\sim 10^{53}$ ergs of gamma

TABLE 2

RATIOS OF PEAK FLUXES IN FOUR BANDS: GRB 970508/GRB 970111

Gamma Ray	X-Ray	V	1.43 GHz
0.05	?	?	>0.2

NOTE.—Data from Kouveliotou et al. 1997 (extrapolated to match *BeppoSAX* 40–600 keV band) and Frail et al. 1997a (gamma rays) and from Frail et al. 1997a, 1997b and Galama et al. 1997a (radio). A lower bound is given at 1.43 GHz because the radio flux of GRB 970508 was still rising in the most recent available data.

rays. This is excessive; it is unlikely (Katz 1997; Kobayashi, Piran, & Sari 1997) that internal shocks are even 50% efficient!

One possible explanation is that GRB 970508 is, instead, an unusually efficient source of visible and radio emission. This would require that the efficiency of lower frequency afterglows varies greatly from burst to burst, which seems implausible; a priori, one might expect more variation in the efficiency of a complex process like internal shocks, which depend on the detailed temporal dependence of the Lorentz factor, than in that of a simple one like a blast wave, which depends on only a few parameters.

A more promising alternative is that the gamma-ray emission of GRB 970508 was beamed toward us, so that the total power radiated was much less than that implied by the assumption of isotropic radiation. Beaming could solve all energetic problems, but the flux ratios require that other bursts show an even greater degree of gamma-ray beaming in order to explain their comparatively (with respect to GRB 970508) greater strength in gamma rays than in visible or radio fluxes. Further evidence for beaming is discussed in the next section.

5. SELF-ABSORPTION, RADIATIVE EFFICIENCY, AND BEAMING IN GRB 970508

The observed (Frail et al. 1997b) radio spectrum of the persistent emission of GRB 970508 on 1997 May 15 was $\propto \nu^{1.1}$. This is an extraordinarily steep slope, and is naturally interpreted as the transition between optically thick and thin conditions in the frequency range observed. The expected self-absorbed spectrum (Katz 1994a) is $\propto \nu^2$ (differing from the familiar $\nu^{5/2}$ of self-absorbed synchrotron radiation because in relativistic shocks nearly all the low-frequency radiation is emitted by electrons with a single, and much higher, characteristic synchrotron frequency). The expected spectrum at frequencies between the self-absorption frequency and the characteristic frequency is $\propto \nu^{1/3}$.

A theory for self-absorption in GRBs was presented by Katz (1994a) and is given in more detail in the Appendix. The result is that, below the self-absorption frequency,

$$F_\nu = 2\pi\nu^2 m_p \zeta(1+z)\frac{R^2}{D^2}\,, \tag{1}$$

where R is the radius of the radiating shell, z its redshift, and $D = (2c/H_0)[1-(1+z)^{-1/2}]$ its proper distance. Here we include the factor $\zeta \equiv k_B T_e/(\gamma m_p c^2)$ describing the degree of electron equipartition in the plasma shock-heated to an internal energy per particle $\gamma m_p c^2$ and moving with Lorentz factor γ. Using the measured (Frail et al. 1997b) flux density on 1997 May 15 at 1.43 GHz as the optically thick value (this data point is only significant at the 2 σ level, but is consistent with the very significant extrapolation of higher frequency flux densities) and taking $H_0 = 60$ km s^{-1} Mpc^{-1} and $z = 0.835$ (Metzger et al. 1997) as the actual redshift yields $R \approx 4.1 \times 10^{16}\zeta^{-1/2}$ cm $\approx 1.2 \times 10^{17}$ cm for $\zeta = \frac{1}{9}$ (a likely upper limit, assuming equal energy in electrons, ions, and magnetic field). The usual result for the size after an interval t is $R \sim 2\gamma^2 ct$, which would yield $\gamma \approx 2$, for which the relativistic approximation is barely valid. Sari (1997) pointed out that because of the gradual deceleration of the shell the correct expression is $R \sim 8\gamma^2 ct$ (if $\gamma \propto r^{-3}$, as in the blast-wave model of Katz (1994a) and in § 7, this becomes $R \sim 14\gamma^2 ct$), implying that the expansion is nonrelativistic by the time of these observations, a week after the burst.

This may be hard to reconcile with the inferred minimum energy $\sim 10^{52}$ ergs because at a density of $n = 1$ cm^{-3} only 1.5×10^{28} g (1.4×10^{49} ergs of rest-mass energy) is contained within a sphere of $R = 1.3 \times 10^{17}$ cm. In order for the motion to have been only marginally relativistic, the kinetic energy must have been reduced to $\sim 10^{49} n_1$ ergs from its initial value, where $n_1 \equiv n/(1$ cm$^{-3})$, requiring a radiative efficiency $\epsilon > 99.7\%$, and probably at least 99.9%. Such efficient radiation is not possible from internal shocks for kinematic reasons (Kobayashi et al. 1997; Katz 1997), but may be in an external shock (Katz 1994a; Sari, Narayan, & Piran 1996; Sari 1997). The hypothesis of extremely efficient radiation may be tested if a complete energy budget is obtained for future GRBs, including the early stages of the external shock and the difficult spectral range of soft X-rays and the far-ultraviolet; the early stages of the afterglow must emit as much radiation as the GRB itself, and perhaps much more.

Alternatively, if the relativistic debris shell is initially produced only within a narrow solid angle Ω, its energy requirement is reduced. Once its motion becomes nonrelativistic the collimation is largely lost, as shock-heated matter expands in all directions. Then the inferred $\sim 10^{49} n_1$ ergs of kinetic energy may represent most of the actual burst energy, not requiring efficient radiative loss. The measured GRB fluence, now taken into the solid angle Ω, implies a radiated energy of $\approx 2.5 \times 10^{50}\,\Omega$ ergs. Then

$$\Omega \approx 0.04\,\frac{\epsilon}{1-\epsilon}\,n_1\,. \tag{2}$$

In internal shock models plausible values of $\epsilon \sim 0.5$ imply a collimation opening angle $\theta \sim 0.1 n_1^{1/2}$ radians.

This degree of collimation requires $\gamma_0 \gtrsim 1/\theta \approx 10 n_1^{-1/2}$, consistent with other estimates. It also argues against $n_1 \ll 10^{-3}$, such as would be found in galactic halos or intergalactic space, for this would lead to excessively small θ; even $n_1 = 10^{-3}$, as found in much of the volume of the Galactic disk, would imply $\gamma \gtrsim 300$.

6. POSSIBLE PARAMETERS OF GRB 970228 AND GRB 970508

6.1. *GRB 970228*

The gamma-ray (40–600 keV) light curve of GRB 970228 displays a strong and rapidly variable peak about 4 s long, followed by a much weaker and smoother peak that begins about 40 s later (Costa et al. 1997a). The first pulse is variable on a timescale of ~ 0.1 s (Hurley et al. 1997). The X-ray (2–30 keV) light curve of GRB 970228 displays two comparable peaks, corresponding in time to the gamma-ray peaks. The observed flux in the second X-ray peak is comparable to the flux obtained from backward extrapolation of the persistent X-ray emission light curve.

This suggests a simple interpretation: The first GRB pulse was produced by internal shocks, while the second pulse signals the beginning of an afterglow produced by an external shock when the debris sweeps up ambient gas. Behavior of this kind was suggested by Sari & Piran (1997a) and Katz (1997) who pointed out that internal shocks could not convert the total kinetic energy to radiation and that a significant fraction of the kinetic energy should be emitted later in other parts of the spectrum. This interpretation allows us to estimate some of the parameters of the event.

The overall duration of the first pulse corresponds to the duration of intense activity of the "inner engine" that emitted the ejecta. The variability on a timescale of ~ 0.1 s corresponds to variability in the "inner engine" and sets an (unsurprising) direct upper bound on its size of $\sim 3 \times 10^9$ cm.

The second pulse corresponds to the onset of significant radiation by the external shock. This takes place when the ejecta (with energy E, Lorentz factor γ_0, and proper mass $M = E/\gamma_0 c^2$) sweeps up an ambient mass M/γ_0 at

$$r = R_\gamma \equiv [3E/(4\pi n m_p c^2 \gamma_0^2)]^{1/3} \approx 1.2 \times 10^{18} \gamma_0^{-2/3} E_{52}^{1/3} n_1^{-1/3} \text{ cm} ,$$

where $E_{52} \equiv E/(10^{52}$ ergs). The corresponding time of arrival of photons from the onset of the external shock is

$$t_e \sim R_\gamma/(2c\gamma_0^2) \approx 2 \times 10^7 \gamma_0^{-8/3} E_{52}^{1/3} n_1^{-1/3} \text{ s} .$$

Comparison with the time delay of 40 s yields $\gamma_0 \approx 140(E_{52}/n_1)^{1/8}$. This is a reasonable value, but the dependence on E and n is so weak that no significant constraints can be placed on them. Fenimore, Epstein, & Ho (1993), Woods & Loeb (1995), and Piran (1996) give lower limits to $\gamma > 100$, but Sari & Piran (1997b) and Katz (1997) give also an upper limit to γ of the order of 10^3 for the observation of internal shocks.

6.2. *GRB 970508*

The observation of a visible maximum 1–2 days after GRB 970508 (Galama et al. 1997b; Jaunsen et al. 1997; Schaefer et al. 1997) suggests that at this time the characteristic synchrotron frequency of the afterglow passed through the visible spectrum. It should be possible to extrapolate this frequency backward in time to $t = t_e$, when $\gamma \approx \gamma_0$ and the afterglow begins.

In principle, this earlier part of the afterglow could have been observed by continuously monitoring GRB detectors. The fact that it was not seen in GRB 970508 (in contrast to the suggestion made above for GRB 970228) suggests that its peak frequency was too low for observation by GRB detectors, although we cannot be sure (without quantitative estimates of its radiative efficiency) how intense it would have been.

Interpreting the nondetection by GRB detectors to mean that the peak characteristic afterglow photon frequency $\nu < \nu_0$, we find

$$\gamma_0 \lesssim 60 \zeta_e^{-1/2} \zeta_B^{-1/8} n_1^{-1/8} (h\nu_0/100 \text{ keV})^{1/4} .$$

Although this may be less than lower limits estimated by consideration of γ-γ pair production (Fenimore et al. 1993; Woods & Loeb 1995; Piran 1996), these limits assume smaller numerical values for afterglow at larger radii than for prompt GRB emission, and are not directly applicable to a burst (like GRB 970508) for which no data exist on the high-energy spectrum.

At the time of the visible maximum we can estimate $\gamma \approx 4$ (eq. [7]). Extrapolation (using $\gamma \propto t^{-3/7}$; § 7) implies the blast wave will become only semirelativistic ($\gamma < 2$) after about a week, in agreement with the inference made in § 5.

The implied X-ray maximum occurred $\sim 10^3$ s after the burst, before the *BeppoSAX* follow-up observations (Piro et al. 1997). The X-ray emission measured simultaneously with the GRB was probably the low-frequency extrapolation of the gamma rays, produced by internal shocks, and was not the beginning of the afterglow.

7. BLAST WAVES WITH INSTANTANEOUS COOLING

Previous relativistic blast-wave models (Mészáros & Rees 1993; Katz 1994a, 1994b; Mészáros et al. 1994; Mészáros & Rees 1997; Vietri 1997; Waxman 1997) have generally assumed that only a small fraction of the energy is radiated, as in the Taylor-Sedov nonrelativistic blast-wave theory. The arguments of § 5 suggest that this may not be the case, at least in GRB 970508, but that radiation may be very efficient, as indicated by the estimated short electron radiation times (Katz 1994a; Sari et al. 1996). Alternatively, the angular spreading of an initial narrowly collimated outflow as its Lorentz factor degrades will reduce the energy per unit solid angle with increasing distance along the axis of the flow in a manner qualitatively resembling the effects of efficient radiation.

Here we develop a simple model for the evolution of a relativistic blast wave and its radiated spectrum, assuming that all the energy produced by the shock is instantly radiated. This is the limiting case opposite to that of negligible radiation, previously assumed. Strong coupling between the protonic and the electronic thermal energies is required.

We describe the interaction of a relativistic blast wave with the ambient matter by a series of inelastic collisions between the debris and previously swept-up mass with proper mass $M + m$ and Lorentz factor $\gamma(r)$ and infinitesimal shells of proper mass dm. The internal energy produced in each collision is instantaneously radiated. Conservation of energy and momentum yield[2]

$$\frac{d\gamma}{\gamma^2 - 1} = -\frac{dm(r)}{M + m(r)} , \quad (3)$$

which can be integrated:

$$\frac{\gamma + 1}{\gamma - 1} = \frac{\gamma_0 + 1}{\gamma_0 - 1}\left[1 + \frac{m(r)}{M}\right]^2 . \quad (4)$$

Then

$$\gamma(r) = \frac{(\gamma_0 + 1)[(r/L)^3 + 1]^2 + (\gamma_0 - 1)}{(\gamma_0 + 1)[(r/L)^3 + 1]^2 - (\gamma_0 - 1)} , \quad (5)$$

where $L \equiv [3M/(4\pi m_p n)]^{1/3} = [3E/(4\pi m_p c^2 n \gamma_0)]^{1/3}$. L corresponds to the radius within which the ambient mass equals the proper mass of the debris. L is smaller than the Taylor-Sedov radius $R_{\rm TS}$ within which the ambient rest-mass energy equals E and it is larger than R_γ; the ratios $R_{\rm TS}:L:R_\gamma = \gamma_0^{2/3}:\gamma_0^{1/3}:1$.

There are three limits for this solution:

$$\gamma(r) \approx \begin{cases} \gamma_0 & \text{for } r \ll R_\gamma , \\ (r/L)^{-3} & \text{for } R_\gamma \ll r \ll L , \\ 1 + 2(r/L)^6 & \text{for } L \ll r . \end{cases} \quad (6)$$

Comparison of this solution with the Lorentz factor when there is no significant cooling reveals, naturally, that this Lorentz factor is smaller. For our purposes the intermediate range is the most interesting, and we will use it in the following discussion. The $\gamma \propto r^{-3}$ dependence found here is also

[2] Eqs. (71) and (72) of Katz (1994a) are wrong.

that found if it is assumed that the blast energy is distributed uniformly, or in a way which scales with r, as r increases (Katz 1994a), and a similar assumption is made in the Taylor-Sedov nonrelativistic blast-wave theory.

This Lorentz factor is also the typical "thermal" Lorentz factor of the relativistic shocked protons. The thermal Lorentz factor of the relativistic electrons is, by definition of the equipartition factor ζ_e, $\gamma_e = \zeta_e(m_p/m_e)\gamma$. The characteristic synchrotron radiation frequency from the forward shock (the backward shock crosses the shell and is gone before the shell enters the $\gamma \ll \gamma_0$ regime) is (Piran 1994)

$$h\nu \approx 0.01 \text{ eV} \times \zeta_e^2 \zeta_B^{1/2} \gamma^4 n_1^{1/2}$$
$$\approx 0.01 \text{ eV} \times \zeta_e^2 \zeta_B^{1/2} \left(\frac{r}{L}\right)^{-12} n_1^{1/2} , \quad (7)$$

where the magnetic equipartition factor ζ_B is the ratio between the magnetic energy density and the "thermal" energy density of the shocked protons. For typical parameters the characteristic frequency is in the visible range when $\gamma \approx 4$, and the blast wave ceases to be relativistic when this frequency is in the infrared.

The energy emitted at radius r and Lorentz factor $\gamma(r)$ will be observed around a time

$$t(r) = \frac{r/c}{14\gamma^2(r)} = \left(\frac{L}{14c}\right)\left(\frac{r}{L}\right)^7 . \quad (8)$$

Here we have replaced the usual factor of 2 in the denominator by 14 to allow for deceleration (Sari 1997; see § 5).

Combining these results and setting the equipartition factors to unity, we find

$$h\nu \approx 0.00011 \text{ eV} \times \left(\frac{ct}{L}\right)^{-12/7} n_1^{1/2}$$
$$\approx 0.14 \text{ eV} \times \left(\frac{t}{1 \text{ day}}\right)^{-12/7} E_{52}^{4/7} \left(\frac{\gamma_0}{300}\right)^{-4/7} n_1^{-1/14} . \quad (9)$$

This gives a scaling law $t \propto \nu^{-7/12}$, which may be used to compare or predict the times of peak flux at different frequencies. Note also that $\gamma \propto t^{-3/7}$ and $r \propto t^{1/7}$.

A straightforward calculation gives the additional scaling law

$$F_{\max \nu} \propto \nu^{-5/12} . \quad (10)$$

This should be valid throughout the regime in which the shell is moving relativistically, roughly corresponding to ν from the infrared to X-rays. This law can, in principle, be compared with data like those shown in Figure 1, but in that figure there is only one significant point in the applicable frequency range.

8. DISCUSSION

The persistent counterparts of GRBs may offer as much complexity and variety as the classical GRBs themselves. Future observations will provide a catalog of their behavior, and will show whether this is true, or whether they may be described by a simple general model. Such a general description, with a few parameters to describe the possible collimation and degree of energy loss, as well as E, n, and γ_0, would support afterglow models, while complex "no two of them alike" behavior would support the suggestion of continuing emission. Afterglows are almost inevitable, and were predicted by many authors, but there is also evidence for continuing emission. Perhaps each may be important in different bursts.

The apparent discovery of self-absorption in GRB 970508 and the measurement of its redshift offer the prospect of detailed reconstruction of the history of a GRB blast wave. It may be possible to measure the blast wave radius directly as a function of time, and therefore to determine its slowing history. If radiation is efficient, then, by constructing an energy budget from the observed radiation, it would be possible to determine E directly. By comparing the radiated power to the slowing history of the blast wave, it would then be possible to determine n. It may also be possible to determine γ_0 (§§ 6 and 7).

If GRBs are collimated, as suggested for GRB 970508 in § 5 and as has been widely speculated on the grounds that their likely sources, coalescing compact binaries, are far from spherically symmetric, the analysis of afterglows will be much more complicated. They will have several more parameters, and will probably require numerical simulation. However, even the first clear demonstration of collimation would be significant.

In equation (8) most of the sensitivity of t to r comes from the γ^2 denominator; to a fair approximation, $t \propto \gamma^{-2}$. Combining this with $\nu \propto \gamma^4$ (eq. [7]) implies that t will generally be proportional to approximately the $-\frac{1}{2}$ power of ν. We expect this to hold for all afterglow models, whatever their detailed physics. For example, the present investigation found $t \propto \nu^{-7/12}$, and the very different model of Katz (1994a,1994b) found $t \propto \nu^{-5/12}$. If $\gamma \propto r^{-3/2}$, as assumed in many models, then $t \propto \nu^{-2/3}$. In simple synchrotron radiative cooling models without hydrodynamics, $t \propto \nu^{-1/2}$. This suggests that it may not be possible to distinguish among models on the basis of the time dependence of the characteristic frequency. However, this is not the only means available; for example, an energy budget will decide whether or not radiation is energetically important.

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APPENDIX

SELF-ABSORPTION

A quantitative calculation of the self-absorbed flux from a rapidly moving emitter is not trivial, because a retarded-time calculation must be performed to allow for the fact that different points on the radiating surface are observed at different radii and retarded times. Deceleration and limb darkening introduce further complications. Here we only demonstrate that for shock-heated emitters the Lorentz factor does not appear in the expression for the observed flux density.

In the comoving frame of the shocked radiating matter moving with Lorentz factor $\gamma \gg 1$ the temperature is given by $T = \zeta f m_p c^2/k_B$, where the Doppler shift factor $f = (1 + v \cos\theta/c)\gamma \approx 2\gamma$ (Katz 1994a took $\zeta = 1$ and used a variable γ_F which is here equal to f^2). In that frame the Rayleigh-Jeans brightness is given by

$$B_\nu = \frac{2\nu^2}{c^2} \zeta f m_p c^2 . \tag{A1}$$

The observer measures a blueshifted temperature $\zeta f^2 m_p c^2/k_B$, and hence observes a brightness

$$B'_\nu = 2\nu^2 \zeta f^2 m_p . \tag{A2}$$

However, the observer sees significant radiation only from a solid angle $\approx \pi r^2/(f^2 D^2)$ because of the relativistic beaming of radiation into a cone of angle $1/f$. The observed intensity is given by

$$F_\nu \approx 2\pi\nu^2 \zeta m_p \frac{r^2}{D^2} ; \tag{A3}$$

the powers of f cancel.

An alternative derivation begins by noting that the quantum occupation number n_ν is a Lorentz-invariant scalar. The intensity is

$$I_\nu = \nu^3 n_\nu . \tag{A4}$$

When Doppler shifting to the observer's frame, $I_\nu \rightarrow I_\nu f^3$. In the Rayleigh-Jeans limit the occupation number is

$$n_\nu = \frac{k_B T}{h\nu} . \tag{A5}$$

The temperature $T \propto \gamma \propto f$ (not because of a Doppler shift but from the shock jump conditions), so that $I_\nu \propto f^4$. This result must be divided by f^2 to allow for the fact that the observed radiation was emitted at a frequency lower by a factor f, and $I_\nu \propto \nu^2$. Finally, the observed effective radiating area is $\propto f^{-2}$, leading to the result that F_ν is independent of γ.

Each of these derivations uses the shock jump conditions. The final result cannot be obtained from Lorentz transformations of the radiation field alone.

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