

CURVATURE OF THE UNIVERSE AND OBSERVED GRAVITATIONAL LENS IMAGE SEPARATIONS VERSUS REDSHIFT

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ABSTRACT

In a flat, $k = 0$, cosmology with galaxies that approximate singular isothermal spheres, gravitational lens image separations should be uncorrelated with source redshift. But in an open, $k = -1$, cosmology, such gravitational lens image separations become smaller with increasing source redshift. The observed separations do become smaller with increasing source redshift, but the effect is even stronger than that expected in an $\Omega = 0$ cosmology. The observations are thus not compatible with the “standard” gravitational lensing statistics model in a flat universe. We try various open and flat cosmologies, galaxy mass profiles, galaxy merging and evolution models, and lensing aided by clusters to explain the correlation. We find the data are not compatible with any of these possibilities within the 95% confidence limit, leaving us with a puzzle. If we regard the observed result as a statistical fluke, it is worth noting that we are about twice as likely to observe it in an open universe (with $0 < \Omega < 0.4$) as we are to observe it in a flat one. Finally, the existence of an observed multiple-image lens system with a source at $z = 4.5$ places a lower limit on the deceleration parameter: $q_0 > -2.0$.

Subject headings: cosmology: miscellaneous — galaxies: clusters: general — galaxies: evolution — gravitational lensing — quasars: general

1. INTRODUCTION

The list of multiple-image gravitational lens systems has been growing steadily since the discovery of the first lens system (Walsh, Carswell, & Weymann 1979). At present, about 30 multiple-image systems are confirmed or very likely to be gravitationally lensed systems (see, e.g., Surdej & Soucail 1994; Keeton & Kochanek 1996). These lens systems can provide us with information about the universe as a whole and the mass distribution within.

Turner, Ostriker, & Gott (1984, hereafter TOG) did extensive studies on the statistical nature of gravitational lenses and their implications for cosmology and galaxy formation. One of the results of this work was that the mean image separations of lens systems have different dependences on source redshift in different cosmologies and that it may therefore be possible to measure the curvature of the universe directly. Gott, Park, & Lee (1989, hereafter GPL) explored the lens statistics in more general cosmologies where the cosmological constant Λ is not zero. They showed that the then-available data ruled out extreme closed models having an antipodal redshift of $z_{\text{antipode}} < 3.5$ and a deceleration parameter of $q_0 > -2.3$.

As the list of lenses grows, it has been applied to a variety of problems. One prominent application is to place limits on the cosmological constant. With the observed galaxy mass distribution and number density, a universe with a large cosmological constant should produce more multiple-image systems than are actually observed. This has placed steadily improving limits on the cosmological constant:

$\Omega_\Lambda \lesssim 0.95$ (Fukugita et al. 1992) or $\Omega_\Lambda \lesssim 0.66$ (Kochanek 1996), where $\Omega_\Lambda \equiv \Lambda/3H_0^2$ and H_0 is the Hubble constant. This limit is already strong enough to place telling constraints on an otherwise appealing cosmological model ($\Omega + \Omega_\Lambda = 1$, $k = 0$; see Ostriker & Steinhardt 1995 for a summary), where $\Omega = 8\pi\rho_0/3H_0^2$. In addition, Maoz & Rix (1993) investigated the effects of the mass distribution in E/S0 galaxies and concluded that the *Hubble Space Telescope* (HST) snapshot survey data requires E/S0 galaxies to have significant halos. Further studies of galaxy merger/evolution show that only some specific merger models can be rejected, and the above limit on Λ is not affected (Rix et al. 1994; Mao & Kochanek 1994). However, most applications of gravitational lensing statistics do assume specific mass (or velocity dispersion) distributions for lensing galaxies—e.g., a Schechter luminosity function and a luminosity-velocity relation—and specific number density distributions—e.g., a constant comoving density of galaxies.

In this work, we focus on the image separations versus source redshift of the current multiple-image lens systems to see whether it is consistent with the “standard” lensing statistics models. We find that the image separations are strongly negatively correlated with source redshift, which is incompatible with the “standard” lensing statistics model in a flat universe. We explore possible causes to see if this correlation can be explained. We also update the limit on the deceleration parameter q_0 with the current data.

2. OBSERVED MULTIPLE-IMAGE LENS SYSTEMS

The list of multiple-image quasi-stellar objects (QSOs) and radio sources has grown through systematic optical and radio surveys and through serendipitous discoveries.

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Keeton & Kochanek (1996; also see Surdej & Soucail 1994) summarize the data on the 29 relatively secure multiple-image lens systems. They classify these systems into three grades of secureness: class A for “I’d bet my life this is a lens,” class B for “I’d bet your life this is a lens,” and class C for “You should worry if I’m betting your life,” all of which (A, B, and C) show convincing spectral similarities and identical redshifts (Table 1 for references) and have separations that are either quite similar to 0957, which is surely a lens (having the time delay between its two images measured recently; Kundić et al. 1997), or smaller separations. Also, note that the largest separation lens system 2345 (in class C) now has more observational support for being a true lens—its lens has been found (Fischer et al. 1994). In this work, we use all 20 systems in this list (A, B, and C) that have a known source redshift. The system 2237+0305 (Huchra et al. 1985) is not included because in that system the source (QSO) was found after the lens. Such systems would have different statistical properties than systems where the source is discovered first. These 20 systems are listed in Table 1, and their maximum image separations, $\Delta\theta$, are plotted against source redshift, z_s , in Figure 1 (circles, class A; triangles, class B; crosses, class C). They show visually quite a strong negative correlation between $\Delta\theta$ and z_s . The major source of this correlation is a number of small redshift ($z_s \lesssim 2$), large separation ($\Delta \gtrsim 4''$) lens systems (2345, 1120, 0240, 0957, 1429) and large redshift ($z_s \gtrsim 3.5$), small separation ($\Delta \lesssim 1''$) lens systems (1208, J03.13). (This effect was noted by Gott 1997. The original data [seven QSOs] in GPL showed no significant correlation.)

Of course, there is always a possibility of contamination by “false” lenses, i.e., observing real physical pairs of QSOs, at wide separation at small source redshift due to quasar clustering (which might be larger at low source redshift). We can roughly estimate how many QSO physical pairs might be expected to show up as “false” lenses. Djorgovski (1991) lists three quasar pairs (or triplets) with arcminute-scale separations and $\Delta V_{\text{rest}} < 1000 \text{ km s}^{-1}$, where ΔV_{rest} is the redshift difference between quasars: QQ 0107–025 AB ($z = 0.954$ and $\Delta\theta = 77''$), QQ 1146+111 BC ($z = 1.012$

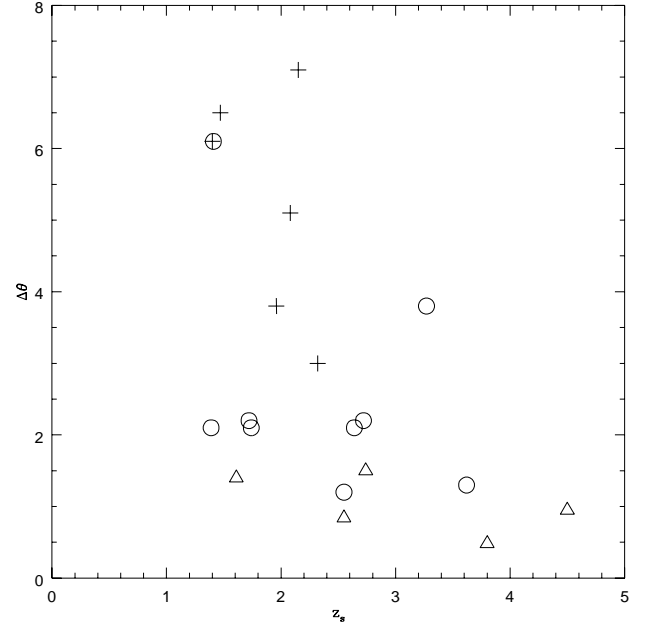


FIG. 1.—Maximum image separation $\Delta\theta$ vs. source redshift z_s of multiple-image gravitational lens systems. Circles denote class A (“I’d bet my life this is a lens”), triangles class B (“I’d bet your life this is a lens”), and crosses class C (“You should worry if I’m betting your life”), according to Keeton & Kochanek (1996).

and $\Delta\theta = 157''$), and Hoag 1, Hoag 2, Hoag 3 ($z = 2.049$ and $\Delta\theta = 121''$, $128''$, and $214''$). This number roughly agrees with the covariance function $w(\theta) \propto \theta^{-0.8}$ expected for gravitational clustering with the average comoving density of quasars of $\langle \rho \rangle \simeq 1000 \text{ Gpc}^{-3}$ and a correlation length of $r_0 \simeq 10 h^{-1} \text{ Mpc}$. From the power-law shape of $w(\theta)$, the existence of two QSO pairs within $128'' < \Delta\theta < 256''$ implies that we would expect to see roughly 0.06 QSO pairs in the interval $0'' < \Delta\theta < 8''$. Hence, the contamination would be unimportant if QSO pairs follow the covariance function expected for the hierarchical clustering. However, Djorgovski (1991) also lists

TABLE 1
MULTIPLE-IMAGE LENS SYSTEMS USED IN THIS WORK

Name	z_s	$\Delta\theta$	References
CLASS 1608+656.....	1.39	2.1	Myers et al. 1995
QJ 0240–343.....	1.4	6.1	Tinney 1995
0957+561.....	1.41	6.1	Walsh et al. 1979
1120+019.....	1.47	6.5	Meylan & Djorgovski 1989
CLASS 1600+434.....	1.61	1.4	Jackson et al. 1995
1115+080.....	1.72	2.2	Weymann et al. 1980
MG 1654+1346.....	1.74	2.1	Langston et al. 1989
1634+267.....	1.96	3.8	Djorgovski & Spinrad 1984
1429–008.....	2.08	5.1	Hewett et al. 1989
2345+007.....	2.15	7.1	Weedman et al. 1982
HE 1104–1805.....	2.32	3.0	Wisotzki et al. 1993
J03.13.....	2.55	0.84	Claeskens, Surdej, & Remy 1996
H1413+117.....	2.55	1.2	Magain et al. 1988
MG 0414+0534.....	2.64	2.1	Hewitt et al. 1992
0142–100.....	2.72	2.2	Surdej et al. 1987
LBQS 1009–0252.....	2.74	1.5	Surdej et al. 1994
2016+112.....	3.27	3.8	Lawrence et al. 1984
B1422+231.....	3.62	1.3	Patnaik et al. 1992
1208+1011.....	3.80	0.48	Bahcall et al. 1992; Magain et al. 1992
BRI 0952–0115.....	4.5	0.95	McMahon, Irwin, & Hazard 1992

NOTE.—(1) z_s : Redshift of the source. (2) $\Delta\theta$: The maximum image separation. (3) See Surdej & Soucail 1994 or Keeton & Kochanek 1996 for more references.

three QSO pairs with arcsecond-scale separation and $\Delta V_{\text{rest}} < 1000 \text{ km s}^{-1}$, which are about 2 orders of magnitude overabundant relative to the prediction of hierarchical clustering: PKS 1145–071 AB ($z = 1.345$ and $\Delta\theta = 4''.2$), 0151+048 AB ($z = 1.91$ and $\Delta\theta = 3''.3$), and QQ 1343+266 AB ($z = 2.030$ and $\Delta\theta = 9''.5$). Among these, only the 1145 AB pair has the required spectral similarities in the optical to be confused with a lens (Djorgovski et al. 1987) and is within the $4'' < \Delta\theta < 8''$ interval. If we regard 0957 and 2345 as the only proven gravitational lensed cases with $\Delta\theta > 4''$, the a priori probability that a given QSO pair with $4'' < \Delta\theta < 8''$ with similar spectra is a gravitational lens rather than a physical pair is 2 in 3 (because of the decided cases, 0957 and 2345 are lenses while 1145 is not). Hence, the probability that all three remaining systems (1120, 0240, 1429) in Table 1 within $4'' < \Delta\theta < 8''$ are just physical pairs (even if one disregards all other observations) is $1/3^3$, which is less than 5%. The probability that two specific systems (e.g., 1120 and 0240) are physical pairs is $1/3^2$, and the probability that one specific system is a physical pair is $1/3$. The quantitative aspects of possible contamination of a few “false” cases are discussed later. So, for the time being, we are treating all 20 cases in Table 1 as real gravitational lens systems.

3. GEOMETRY OF THE UNIVERSE

3.1. Flat and Open Cosmological Models

One of the most important cosmological parameters is the curvature of the universe. The Friedmann big bang models admit three solutions: (1) universes that are flat, $k = 0$, with a Euclidean three-space geometry R^3 at fixed epoch; (2) universes that are closed, $k = +1$, with an S^3 three-space geometry at fixed epoch; and (3) universes that are open, $k = -1$, with a hyperbolic H^3 three-space geometry at fixed epoch (Misner, Thorne, & Wheeler 1973). We would very much like to know whether our universe is flat, closed, or open, so direct measurement of the curvature is extremely important. Models with $\Omega + \Omega_\Lambda < 1$ are open ($k = -1$), models with $\Omega + \Omega_\Lambda = 1$ are flat ($k = 0$), and models with $\Omega + \Omega_\Lambda > 1$ are closed ($k = +1$).

Flat, $k = 0$, models (with $\Omega = 0.4$, $\Omega_\Lambda = 0.6$) are popular with many people (cf. Ostriker & Steinhardt 1995) because they could be produced naturally in any inflationary scenario where significantly more than 67 e -folds of inflation occur and (other than the theoretical problems with a finite Λ term) would require no fine tuning of parameters. But there are also open ($k = -1$) inflationary models. Open inflationary universes, as suggested by Gott (1982), are created naturally during the decay of an initial metastable inflationary state. Individual bubble universes are created that have an open geometry, with a negative curvature inherited from the bubble formation event. Inflation continues within the bubble for approximately 67 e -folding times, creating a universe with a radius of curvature $\exp(67)$ times larger than the wavelength of the microwave background photons and which is uniform except for quantum fluctuations (cf. Gott & Statler 1984; Gott 1986, 1997). The single-bubble open inflationary model (Gott 1982) has come under increased discussion recently because of a number of important developments. On the theoretical side, Ratra & Peebles (1994, 1995) have shown how to calculate the random quantum fluctuations in the H^3 hyperbolic geometry. This is very important since it allows predictions

of fluctuations in the microwave background. Bucher, Goldhaber, & Turok (1995a, 1995b) have done similar calculations, as well as Yamamoto, Sasaki, & Tanaka (1995). It is important to note that they have explained that the fine tuning in these models is only “logarithmic” and, therefore, not so serious. Linde (1995) has shown how there are reasonable potentials that could produce such open universes, indeed, different open bubble universes with different values of Ω .

The inflationary power spectrum with cold dark matter (CDM) (Bardeen, Steinhardt, & Turner 1983) has been amazingly successful in explaining the qualitative features of observed galaxy clustering including great walls and great attractors (see Geller & Huchra 1989; Park 1990a, 1990b; Park & Gott 1991). The amount of large-scale power seen in the observations suggests an inflationary CDM power spectrum with $0.2 < \Omega h < 0.3$ (Maddox et al. 1990; Saunders et al. 1991; Park et al. 1992; Shectman et al. 1995; Vogeley et al. 1994). A number of recent estimates of h have been greater than 0.55 (i.e., $h = 0.65 \pm 0.06$, Riess, Press, & Kirshner 1995; $0.68 \leq h \leq 0.77$, Mould & Freedman et al. 1996; $0.55 \leq h \leq 0.61$, Sandage et al. 1996; and $h = 0.67 \pm 0.06$ from the time delay of 418 days observed in 0957 [Kundić et al. 1996] using the best model by Grogin & Narayan 1996). Ages of globular cluster stars have a 2σ lower limit of about 11.6 billion yr (Bolte & Hogan 1995); if the age of the universe $t_0 \geq 11.6$ billion yr, we require $h < 0.56$ if $\Omega = 1$ and $\Omega_\Lambda = 0$ but a more acceptable $h < 0.65$ if $\Omega = 0.4$, $\Omega_\Lambda = 0$. Models with low Ω but $\Omega + \Omega_\Lambda = 1$ are also acceptable. With the COBE normalization there is also the problem that with $\Omega = 1$ and $\Omega_\Lambda = 0$, $(\delta M/M)_{8 h^{-1} \text{ Mpc}} = 1.1\text{--}1.5$, and this would require galaxies to be antibiased [since for galaxies $(\delta M/M)_{8 h^{-1} \text{ Mpc}} = 1$], and this would also lead to an excess of large separation gravitational lenses over those observed (Cen et al. 1994). These things have forced even enthusiasts of $k = 0$ models to move to models with $\Omega < 1$ but with a cosmological constant so that $\Omega + \Omega_\Lambda = 1$ and $k = 0$.

3.2. Gravitational Lensing Curvature Test

In this paper, we will discuss a curvature test based on gravitational lens image separations as a function of source redshift. Studies on statistics of lensing (TOG; GPL) show that if a source is lensed by a singular isothermal sphere (SIS) galaxy, randomly distributed in the universe with constant comoving density, the mean separation—averaged over all possible lenses at different distances—of multiple-images in a flat universe should be constant independent of source redshift (Fig. 2a, *solid line*). However, the mean separation will decrease with source redshift in an open universe (Fig. 2a, *dotted and dashed lines*) and increase in a closed universe (TOG; GPL). In an open universe the volume increases faster with redshift than in a flat universe, and the source is more likely to be lensed by lensing galaxies at larger distances, which produces smaller image separations, and vice versa for a closed universe. This applies to lensing by galaxies and/or clusters, both of which are well approximated by SIS. Also, this test is independent of individual values of Ω and Ω_Λ when the universe is flat ($\Omega + \Omega_\Lambda = 1$).

This is quite important because it is a pure curvature test that distinguishes a $k = 0$ cosmology from a $k = -1$ cosmology. We have tests for Ω : i.e., peculiar velocities are proportional to $\Omega^{0.6}/b$, where b is the bias parameter, and

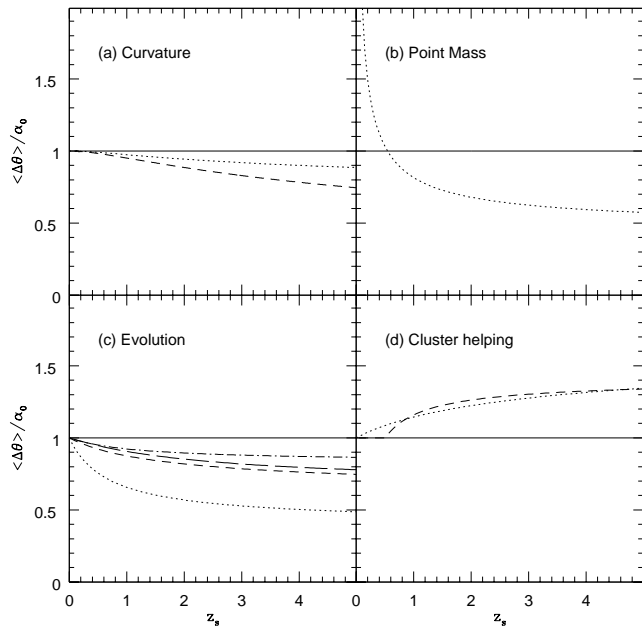


FIG. 2.—Mean separation of images as a function of source redshift for various possibilities: (a) Curvature: Flat universe ($\Omega + \Omega_\Lambda = 1$) (solid line), $\Omega = 0.4$ open universe (dotted), and empty universe, $\Omega = 0$ (dashed line), all with SIS lenses. (b) SIS lenses (solid line) vs. point mass lenses (dotted line, in arbitrary unit) in a $\Omega = 1$ flat universe. (c) Evolution: No mergers (solid line), Broadhurst et al. merger model (dotted line), cosmological infall of satellite galaxies (short-dashed line), mass accretion (dot-dashed line), all in a flat universe, and mass accretion (long-dashed line) in an $\Omega = 0.4$ open universe. (d) Lensing aided by a cluster at the same redshift as the galaxy (dotted line) and at the fixed redshift of 0.5 (dashed line) in a flat universe.

power on large scales in galaxy-scale clustering measures Ωh , where $h = H_0/100 \text{ km s}^{-1} \text{ Mpc}^{-1}$. But these tests do not distinguish between a model with $\Omega < 1$, $\Omega_\Lambda = 0$, which is open ($k = -1$), and a model with the same value of Ω but with $\Omega + \Omega_\Lambda = 1$, which is flat ($k = 0$).

How can we distinguish between the $\Omega = 0.3\text{--}0.4$, $\Omega_\Lambda = 0$, $k = -1$ models and the $\Omega = 0.3\text{--}0.4$, $\Omega_\Lambda = 0.6\text{--}0.7$, $k = 0$ models? They produce galaxy clustering and masses of groups and clusters that are virtually indistinguishable. Turner (1990) and Fukugita, Futamase, & Kasai (1990) showed that a flat $\Omega_\Lambda = 1$ model produces about 10 times as many gravitational lenses as a flat model with $\Omega = 1$. By comparing the observed number of lenses, Kochanek (1996) was able to set a 95% confidence lower limit of $0.34 < \Omega$ in flat models where $\Omega + \Omega_\Lambda = 1$ and a 90% confidence lower limit $0.15 < \Omega$ in open models with $\Omega_\Lambda = 0$. Thus, extreme Λ -dominated models are ruled out by producing too many gravitational lenses. Another possibility is future data on the cosmic background radiation for spherical harmonic modes from $l = 2$ to $l = 500$: an $\Omega = 1$, $\Omega_\Lambda = 0$ model reaches its peak value at $l \simeq 200$; an $\Omega = 0.3$, $\Omega_\Lambda = 0.7$ model reaches its peak value at $l \simeq 200$; while an $\Omega = 0.4$, $\Omega_\Lambda = 0$ model reaches its peak value at $l \simeq 350$ (Ratra et al. 1997). This can be measured by the *Microwave Anisotropy Probe (MAP)* and *COBRAS/SAMBA (Planck Surveyor)*, satellites which will measure this range with high accuracy. The test in this paper (gravitational lens separations as a function of source redshift) is also able, in principle, to differentiate between an $\Omega = 0.4$, $\Omega_\Lambda = 0$, $k = -1$ model and an $\Omega = 0.4$, $\Omega_\Lambda = 0.6$, $k = 0$ model.

3.3. Curvature Test Results

For our gravitational lensing curvature test, we first estimate the probability of producing the observed correlation by chance in a flat universe where the distribution of the separations is expected to be independent of source redshift. Since we do not assume any specific distribution of image separations at a given redshift, we use Spearman's rank correlation test, which tests the strength of the correlation between the ranks in the image separations and the corresponding ranks in source redshifts. The Student- t distribution gives the approximate probability for the random distribution to have stronger than a given correlation (Press et al. 1992). Whenever there are ties, midranks are used. We checked this probability against Monte Carlo simulations, and they agree well. The two-sided probability (of observing either a positive correlation or negative correlation as strong as that observed in Fig. 1) in a flat universe with SIS galaxies is $P = 0.012$ (Table 2). This confirms the visual impression that the distribution is significantly (negatively) correlated with source redshift. We also divide the sample into three redshift intervals, $[0, 2]$, $[2, 3]$, and $[3, \infty]$, and apply the Kolmogorov-Smirnov test. The distribution of separations in $[0, 2]$ and $[2, 3]$ are not significantly different. However, those in $[0, 2]$ and $[3, \infty]$ are statistically different with 95% confidence. If we are in a flat universe, this is a very special sample.

To see the possible effect of any “false” cases, we repeat the Spearman test for the data set where some cases are excluded intentionally. For example, if we exclude any two large separation ($4'' < \Delta\theta < 8''$) systems (except 0957, of course), the probability is small, $P \leq 0.051$. Excluding three systems, for example, 1120, 0240, 1429, increases the probability only to $P = 0.063$. Similarly even if the most favorable large separation and small separation cases are excluded (1120 and 1208), the probability is still small, $P = 0.029$. Only when two large separation and one small separation cases (1120, 0240, and 1208) are excluded is $P = 0.066$. On the other hand, if two of the largest redshift cases (0952 and 1208, both class B) are excluded, the probability becomes quite significant, $P = 0.098$. So we conclude that three or more largest separation “false” cases or two or more largest redshift “false” cases are needed to change the incompatibility of the observed data with the standard lensing statistics model in a flat universe at the $\sim 95\%$ confidence level.

TABLE 2

TWO-SIDED PROBABILITIES OF OBSERVING A CORRELATION AS STRONG AS THAT SEEN IN THE DATA IN VARIOUS MODELS, USING THE SPEARMAN RANK CORRELATION TEST

Models	Probability
Flat universe ($\Omega + \Omega_\Lambda = 1$)	0.012
Empty universe ($\Omega = 0$, $\Omega_\Lambda = 0$)	0.030
Open universe ($\Omega = 0.4$, $\Omega_\Lambda = 0$)	0.019
Point-mass lens in a flat universe	0.030
Merger model in a flat universe ^a	<0.05
Cosmological infall in a flat universe	<0.03
Mass accretion in a flat universe ^b	0.019
Mass accretion in an open universe ^c	0.025
Lensing aided by a cluster	<0.01

^a Broadhurst et al. merger model in a flat universe with $\Omega_\Lambda < 0.9$.

^b Flat universe with $\Omega = 1$, $\Omega_\Lambda = 0$.

^c Open universe with $\Omega = 0.4$, $\Omega_\Lambda = 0$.

If it is not just a statistical fluke, what could be responsible for this correlation? We first check if negative curvature can create this strong a trend. We try two open universes: an $\Omega = 0$, $\Omega_\Lambda = 0$ empty universe and an $\Omega = 0.4$, $\Omega_\Lambda = 0$ open universe. The mean image separation, $\langle \Delta\theta \rangle$, is calculated as a function of source redshift z_s (in Fig. 2a, a dashed line for $\Omega = 0$, $\Omega_\Lambda = 0$ and a dotted line for $\Omega = 0.4$, $\Omega_\Lambda = 0$). We then divide the observed image separations $\Delta\theta_i^{\text{obs}}$ by expected mean separation $\langle \Delta\theta \rangle(z_s)$. If the correlation is due to the curvature, these “corrected” separations should not show any correlation with z_s . However, Spearman tests indicate that in both the empty and $\Omega = 0.4$ open universes, significant correlations still exist between the ranks in the “corrected” separations and those in source redshifts, and the probability that the data could be randomly drawn from these empty and $\Omega = 0.4$ models is $P = 0.030$ and $P = 0.019$, respectively (Table 2). So although negative curvature lessens the strength of the correlation, it alone cannot fully explain the correlation. We also test for the possible effect of “false” cases in $\Omega = 0.4$ open universe. Exclusion of 1120 from the data set increases the probability to $P = 0.039$, and exclusion of 1120 and 1429 increases the probability to $P = 0.048$, while exclusion of 1120 and 0240 increases the probability to $P = 0.080$. Also, exclusion of 1208 (smallest separation) increases the probability to $P = 0.059$, just above 5% level, although the correlation still exists. This is higher than the probability for the flat universe because some of the negative correlation would be explained by the curvature of the universe.

It is also worth noting that if this is just a statistical fluke, we are about twice as likely to see it in an open universe (with $0 \leq \Omega \leq 0.4$) than in a flat universe (with $\Omega + \Omega_\Lambda = 1$).

4. POSSIBLE RESOLUTIONS

4.1. Mass Profile

Other factors that can affect the distribution of image separations include the density profile of the lenses. The density profile of a SIS produces a constant bending angle regardless of the impact parameter, and the distribution of image separations is independent of source redshift if lenses are uniformly distributed in a flat universe. If the density profile is steeper than a SIS, image separations decrease as source redshift increases (TOG; GPL). To access the effect of a steeper density distribution, we try the extreme case of point-mass lenses.

In a flat universe, the mean separation of images produced by point-mass lenses decreases by a factor of 0.82 from $z_s = 1.5$ to $z_s = 4.0$. We again calculate $\langle \Delta\theta \rangle(z_s)$ (Fig. 2b), normalize the observed image separation $\Delta\theta_i^{\text{obs}}$ with it, and test for any correlation. The probability of finding either a positive correlation or a negative correlation as large as observed in this model is $P = 0.030$ (Table 2). So even this most extreme density profile cannot explain the correlation.

4.2. Galaxy Merger and Infall

The next possibility is that of evolution of the lenses (galaxies). If the number density or the mass of the lenses changes over cosmic timescales, this introduces a dependence of image separations on source redshift: If the comoving number density increases with redshift, that is, more lenses per comoving volume at higher redshift, the mean separation decreases with source redshift. If the lens mass decreases with redshift, the mean separation again decreases with redshift.

Following GPL notation, we represent a Robertson-Walker metric as

$$ds^2 = -dt^2 + \frac{a^2(t)}{a_0^2} [a_0^2 d\chi^2 + a_0^2 S^2(\chi)(d\theta^2 + \sin^2 \theta d\phi^2)] , \quad (1)$$

where $S(\chi) = \chi$ for a flat universe, $S(\chi) = \sin(\chi)$ for a closed universe, and $S(\chi) = \sinh(\chi)$ for an open universe. The comoving distance χ is related to z through

$$\chi = \Delta \int_0^z [\Omega(1+t)^3 + (1-\Omega-\Omega_\Lambda)(1+t)^2 + \Omega_\Lambda]^{-1/2} dt . \quad (2)$$

Then $S(\chi)$ is equal to the proper motion distance times Δ , where $\Delta = |\Omega + \Omega_\Lambda - 1|^{1/2}$ in a closed or open universe and $\Delta = 1$ in a flat universe (see, e.g., Kochanek 1993). Here, Ω and Ω_Λ represent the values observed at the present epoch. The scale factor of the universe $a(t)$ has a present value of $a_0 = cH_0^{-1} \Delta^{-1}$, where c is the speed of light.

The probability of lensing, in the general case where lenses evolve, is given by

$$\tau = \pi a_0^3 n_0 \alpha_0^2 \int_0^{z_s} \frac{n(\chi_l)}{n_0} \left[\frac{\alpha(\chi_l)}{\alpha_0} \right]^2 \frac{S^2(\chi_s - \chi_l) S^2(\chi_l)}{S^2(\chi_s)} d\chi_l , \quad (3)$$

where $n(\chi_l)$ is the comoving density and $\alpha(\chi_l)$ the bending angle of the SIS lenses at the distance χ_l . The subscript “0” refers to values at present. The mean angular separation as a function of the comoving distance of the source, $\chi_s(z_s)$, is

$$\langle \Delta\theta \rangle = 2\alpha_0 \int_0^{z_s} \frac{n(\chi_l)}{n_0} \left[\frac{\alpha(\chi_l)}{\alpha_0} \right]^3 \frac{S^3(\chi_s - \chi_l) S^2(\chi_l)}{S^3(\chi_s)} \times d\chi_l \Big/ \int_0^{z_s} \frac{n(\chi_l)}{n_0} \left[\frac{\alpha(\chi_l)}{\alpha_0} \right]^2 \frac{S^2(\chi_s - \chi_l) S^2(\chi_l)}{S^2(\chi_s)} d\chi_l . \quad (4)$$

Merging between galaxies and the infall of surrounding mass onto galaxies are two possible processes that can change the comoving density of galaxies and/or their mass. The effects of galaxy merging or evolution have been studied by Rix et al. (1994) and Mao & Kochanek (1994). They focused on the lensing probability and the limits on the cosmological constant. Under the generic relation between the velocity dispersion and mass of early-type galaxies, they find merging and/or evolution do not significantly change the statistics of lensing.

We try three merger/infall models. The first merger model is that of Broadhurst, Ellis, & Glazebrook (1992), which was originally motivated by the faint galaxy population counts. The exact nature of excess of faint galaxy counts is uncertain at present. Excess counts at large redshift may indicate that one is just seeing pieces, like giant H II regions (Colley et al. 1996) (with appropriate K corrections) of already formed galaxies rather than galaxy mergers. In this case, the lensing statistics would be unaffected. We use the Broadhurst et al. model as simply an example of a rather severe merging scenario. This model assumes the number density of the lenses to be

$$n(\chi_l) = f(\delta t) n_0 , \quad (5)$$

where δt is the look-back time, and the velocity dispersion of the SIS lenses at χ_l is

$$\sigma(\chi_l) = [f(\delta t)]^{-\nu} \sigma_0, \quad (6)$$

where the parameter ν specifies the evolution of the velocity dispersion of lenses relative to the number density evolution. This form implies that if we had f galaxies at look-back time δt each with velocity dispersion σ , they would by today have merged into one galaxy with a velocity dispersion of $[f(\delta t)]^\nu \sigma$. The strength and time dependence (or redshift dependence) of merging is described by the function $f(\delta t)$:

$$f(\delta t) = \exp(QH_0 \delta t), \quad (7)$$

where H_0 is the Hubble constant and Q represents the merging rate. The look-back time δt is related to χ through

$$H_0 \delta t = \Delta^{-1} \int_0^x \frac{d\chi}{1+z}. \quad (8)$$

We take $Q = 4$ (following Broadhurst et al. 1992) and $\nu = 1/4$ (see Rix et al. 1994 for the discussion on the value of ν). This choice of parameters preserves the total probability of lensing and means that galaxies at $z = 2$ were more numerous by a factor $\sim e^2$ and that their velocity dispersion was smaller by $\sim e^{-1/2}$ than those at present with $\Omega = 1$.

Since this description of merging depends directly on time rather than the redshift, the function f depends on the individual values of Ω and Ω_Λ even in a flat universe. We take $\Omega = 1$ and $\Omega_\Lambda = 0$ as our exemplary flat universe. The mean separation as a function of source redshift is shown for this model in Figure 2c as a dotted line. We again test for the strength of the correlation between the ranks in the “corrected” separations (observed image separations divided by the predicted mean separations) and those in source redshifts. The Spearman test shows that the Broadhurst et al. merger model produces a probability of $P = 0.030$, proving that even this strong merging cannot explain the observed correlation. Most combinations of Ω and Ω_Λ in a flat universe have a steep dependence of $\langle \Delta \theta \rangle$ at small z_s only, and the normalized $\Delta \theta$ ranks of the observed are not significantly affected as long as $\Omega_\Lambda < 0.7$. However, the probability is $P = 0.051$ in an $\Omega = 0.1$, $\Omega_\Lambda = 0.9$ universe. Only the combination of severe merging and extremely large Λ (one which would cause severe difficulties with the total number of lenses as discussed earlier) marginally pushes the correlation below the 95% level.

We also try a less extreme merger model in which the total mass of the galaxies within a given comoving volume is conserved but the comoving number density of galaxies goes like $t^{-2/3}$ while the mass of an individual galaxy increases like $t^{2/3}$, where t is the cosmic time since the big bang. (This is what would be expected for cosmological infall [Gunn & Gott 1972] if galaxies grew by swallowing companion galaxies in an $\Omega = 1$ model. It would overestimate the mass increase in flat and open models with $\Omega < 1$.) We further assume the mass-velocity relation $M \propto \sigma^4$. This description also does not change the total lensing optical depth as a function of redshift. So

$$n(\chi_l) = n_0[1 - (\delta t/t_0)]^{-2/3}, \quad \sigma(\chi_l) = \sigma_0[1 - (\delta t/t_0)]^{1/6}, \quad (9)$$

where t_0 is the current age of the universe. Again various combinations of Ω and Ω_Λ are tested for a flat universe. The mean separation for $\Omega = 1$, $\Omega_\Lambda = 0$ universe is shown in

Figure 2c as a short-dashed line. This prescription of merging in any flat universe produces a probability of $P < 0.025$ in the Spearman test.

The third model we try is a mass accretion model in which the comoving density of the galaxies is constant but the mass increases with $t^{2/3}$ as in the cosmological infall model (as would occur if galaxies accreted gas by cosmological infall in an $\Omega = 1$ model). The total mass in galaxies thus increases with time and the total lensing optical depth is increased:

$$n(\chi_l) = n_0(\text{constant}), \quad \sigma(\chi_l) = \sigma_0[1 - (\delta t/t_0)]^{1/6}. \quad (10)$$

Although different combinations of Ω and Ω_Λ in flat universe give different $\Delta \theta(z_s)$, the difference is practically negligible (Fig. 2c, *dot-dashed* line). However, the open model produces a different $\Delta \theta(z_s)$ (Fig. 2c, *long-dashed* line) because the effect due to merging is increased to by that due to the curvature. The Spearman test for the flat universe has a probability of $P = 0.019$ while that for $\Omega = 0.4$, $\Omega_\Lambda = 0$ open universe $P = 0.025$. So even moderate mass accretion in an open universe can not produce the strong correlation seen in the data.

4.3. Clusters

Since large image separations in some lens systems ($\Delta \theta \gtrsim 5''$) are too large to be explained comfortably within the currently accepted galaxy mass distributions (Lee & Park 1994; Park 1996; Yoon & Park 1996), we expect these systems to be the result of galaxy lensing aided by a cluster as in the case of 0957. We investigate what kind of effects would be expected if lensing is aided by a cluster. The cluster is simply modeled as a sheet constant mass surface density (TOG).

When multiple images are produced by an SIS lens aided by a cluster, the lensing cross section is not affected but the image separation is widened (TOG),

$$\frac{\Delta \theta_{G+C}}{\Delta \theta_G} = \left(1 - \frac{\Sigma}{\Sigma_{cr}}\right)^{-1}, \quad (11)$$

where $\Delta \theta_{G+C}$ is the separation by the SIS plus cluster and $\Delta \theta_G$ that by the SIS alone, Σ is the surface mass density of the cluster, and $\Sigma_{cr} \equiv \Sigma_0 S(\chi_s)/[S(\chi_l)S(\chi_s - \chi_l)]$ is the critical surface mass density, where $\Sigma_0 \equiv c^2/(4\pi G a_0)$. Thus the total lensing probability is unchanged, but the mean image separation is

$$\langle \Delta \theta \rangle = 2\alpha_0 \int_0^{z_s} \frac{\Sigma_{cr}}{\Sigma_{cr} - \Sigma} \frac{S^3(\chi_s - \chi_l)S^2(\chi_l)}{S^3(\chi_s)} \times d\chi_l \int_0^{z_s} \frac{S^2(\chi_s - \chi_l)S^2(\chi_l)}{S^2(\chi_s)} d\chi_l. \quad (12)$$

If one attributes the large separation lenses seen at small source redshift to a cluster helping a galaxy, one might hope that the observed effect is due to a lack of clusters at large redshifts. Can this be due to an evolution of clusters with redshift? No. Because nearby clusters help lensing for all more distant sources and even more effectively as source redshift increases. If there were no distant clusters beyond some redshift z_i , then this would have the effect of causing an *increase* in image separation with increasing source redshift.

We assume two cases for the position of the cluster. For the first, we assume the same redshift for the cluster and the

lensing galaxy. The resulting mean image separation is shown in Figure 2d for an $\Omega = 1$, $\Omega_\Lambda = 0$ universe (*dotted line*). The mean separation increases with source redshift because adding the cluster effectively makes the mass distribution even more extended than SIS. For the second, the redshift of a cluster is at some fixed value smaller than source redshift. The mean image separation in the same flat universe for this case is shown in Figure 2d for the cluster redshift of 0.5 (*dashed line*). It is also an increasing function of source redshift. This is expected because Σ_{cr} for any cluster is always smaller for a higher redshift source regardless of the lens redshift. Therefore, for a given surface density, a cluster is closer to the critical surface density for more distant sources, and we expect larger image separations. This is just the opposite of the correlation seen in the data.

5. OTHER IMPLICATIONS

5.1. Test of the Curvature of the Universe

It was hoped that the dependence of image separations of lens systems on the redshift of the source may make it possible to test the curvature of the universe directly (TOG; GPL). However, the small number of the lens systems available makes this test very difficult (GPL). Here we examine how many multiple-image lens systems are required to reliably test the curvature of the universe. Since we are not sure that the observed distribution of the image separations, especially that of the large separation ones, is explained by a simple lensing model where the sources are lensed by a single galaxy following the Schechter luminosity function, we do not use any assumptions on the lensing galaxies and use only the observed image separation distribution as the intrinsic distribution we are likely to discover in the future.

Although the observed data may contain the curvature effect already, we assume that the observed distribution is just the intrinsic one before being affected by the curvature. We create N Monte Carlo multiple-images systems out of randomly shuffled image separations and source redshifts seen in the observed lens systems (and listed in Table 1). This shuffled data set will have the same histogram of separations as observed and the same histogram of observed redshifts—but the redshifts and separations will be by definition uncorrelated as would be true in a flat model with SIS lenses. Then the image separations of the simulated samples are multiplied by the mean image separation at the simulated redshift expected in various cosmologies. We then run the Spearman test on all simulated data sets to detect the existence of the negative correlation at above the 95% confidence level. We find that to distinguish the flat universe versus the empty universe at the 95% probability level requires 800 multiple-image systems. Proving a less extreme open universe like the $\Omega = 0.4$, $\Omega_\Lambda = 0$ universe at the same 95% confidence level requires a staggering ~ 1600 systems. This proves that pure curvature test from lens statistics is harder than originally expected mainly because the observed scatter in image separations is larger than initially expected. Yet it might well be within the reach of future sky surveys (Sloan Digital Sky Survey expects to discover ~ 100 new lenses in its spectroscopic survey, and about 1000 new lenses from its faint quasar candidate list based on their stellar type images but QSO-type colors. This is how many such lenses would be expected to be confirmed by later spectra from these candi-

dates using other telescopes. [SDSS Collaboration NASA Proposal 1997, The Black Book, D. York, P.I.]

5.2. New Limits on q_0

GPL discovered that in a $\Lambda \neq 0$ universe where the observer's antipode is within the particle horizon, a source just beyond the antipode is overfocused due to the lensing action of the universe as a whole and cannot create multiple images under most lensing mass distributions, e.g., SIS, SIS with external shear, and elliptical potential. Hence, the existence of ordinary multiple-image lens systems at various source redshifts up to some maximum in general constrains the antipode to be farther away than the largest observed redshift multiple-image lens system source (now at $z_s = 4.5$). (See GPL for details.) This limit on the antipodal redshift (now $z_{\text{antipode}} > 4.5$) revises the allowed parameter space in Ω versus q_0 (the unshaded region in Fig. 3). We also provide a graph (Fig. 4) for the lower limit on q_0 as a function of the antipodal redshift, so as new record-breaking (in z_s) lensed QSOs are discovered, the lower limit on q_0 can be revised upward accordingly.

6. SUMMARY AND DISCUSSION

We find that the currently observed multiple-image lens systems show a very strong negative correlation between the image separation and the redshift of the source in the sense that larger redshift sources have smaller separations. The probability of this occurring in a flat universe with standard nonevolving galaxies is only 1%.

Possible causes are investigated: the curvature of the universe, different mass profiles for lensing galaxies, merger or accretion of galaxies, and lensing aided by clusters. Although all of these except the lensing-aided-by-clusters model can create a negative correlation between the separa-

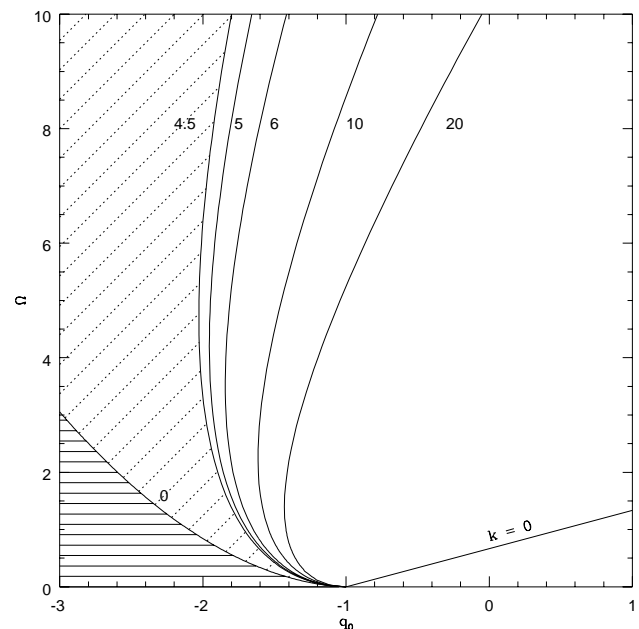


FIG. 3.—The lines of constant antipodal redshift in Ω vs. q_0 plane. The numbers denote the value of the antipodal redshift z_{antipode} . There is no big bang in the horizontally shaded region below $z_{\text{antipode}} = 0$. The horizontally and diagonally shaded regions are both excluded if $z_{\text{antipode}} > 4.5$, as must be the case since we see numerous multiply lensed QSOs up to and including one at $z = 4.5$ (see GPL). The solid line marked $k = 0$ represents flat universes, $\Omega + \Omega_\Lambda = 1$.

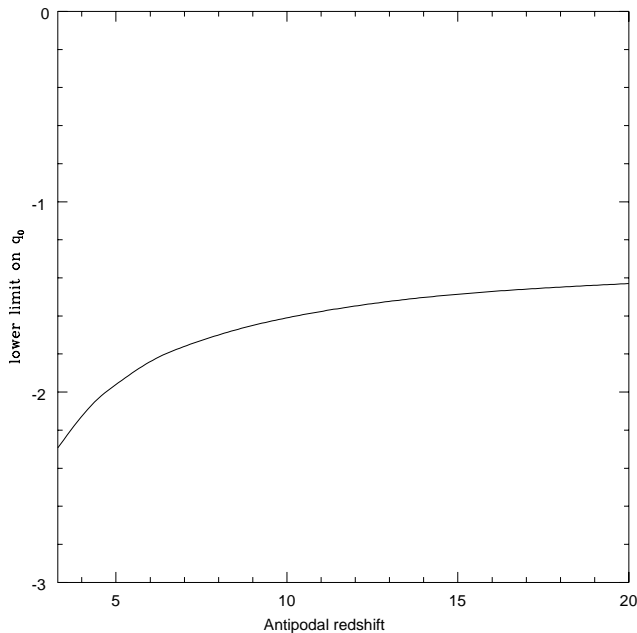


FIG. 4.—Lower limit on q_0 as a function of antipodal redshift

tions and source redshifts, none of them produce a negative correlation as strong as that seen in the data. This leaves us with a puzzle. If there is a cause (not explored in this work) that can explain the correlation, it has to have very strong evolutionary effects, especially between $z \sim 2$ and $z \sim 4$.

Interstellar dust in younger galaxies causing obscuration is not helpful. Obscuration might prevent us from seeing some high-redshift QSOs (see Malhotra, Rhoads, & Turner 1997 about the evidence for dusty gravitational lenses). But in analyzing the separation versus redshift question, we are dealing with only the ones we do see. If there is dust in the lensing galaxies, we would expect it to knock out small separation cases preferentially, and if dust increases with increasing redshift in lensing galaxies, as we would expect, then this would cause separations to increase slightly with increasing source redshift, which is the opposite of what we observe.

Are there any observational selection effects that would produce the effect? It is not easy to think of one. One of the small separation, large redshift cases (1208) was discovered with the *HST*, which is better able to discover small separation cases than ground-based telescopes. But of course the *HST* is equally well able to discover small source redshifts. The *HST* snapshot survey includes both small and large separation cases and both small and large redshift cases: 0957, 0142, 1115, 1413, 1208, and 1120 (Maoz et al. 1993). As a matter of fact, even this small number of systems shows a very strong correlation (a two-sided probability that they are drawn from a random data sample in a flat universe is $P = 0.036$). Many lenses are found by the VLA² where the source redshift is found only after the confirming spectra are

taken. The VLA can detect separations as small as $0''.3$ and QSOs at any redshift. Optical surveys simply stumble on cases and might miss some small separation cases but again would be expected to find large and small source redshifts equally well. Therefore, it is hard to think of a selection effect that would be biased against detection of large separation, large redshift quasars only and which would be present in *HST* observations, VLA observations, and ground-based optical observations.

One possible, yet unlikely, explanation of the observed correlation may be “false” gravitational lenses. We have shown that if three or more of the largest separation cases or two or more of the largest redshift (small separation) cases turn out not to be true lenses, the probability of having as strong a correlation as that seen in the data increases above 5% level in a flat universe. Smaller number of “false” cases would do the same for an open universe. However, most of the lens cases in Table 1 have been on the list for more than a few years without being disconfirmed. Also, even if the probability is above 5%, it has to be multiplied by the additional likelihood that the specific cases are “false” lenses and the final probability would be very small. So it seems unlikely that three or more large separation cases or two or more large redshift cases will turn out to be just physical pairs and at the same time the correlation is from random distribution. However, the likelihood would be larger in an open universe: A single “false” lens (1208) and $\sim 6\%$ of chance in an $\Omega = 0.4$, $\Omega_\Lambda = 0.0$ universe could produce a correlation as strong as that seen in the data. Needless to say, either stronger confirmation or disconfirmation of large separation cases and large redshift cases through future observations would be very helpful.

If this negative correlation is real (i.e., not just a statistical fluke, or an observational selection effect, or due to “false” lens contamination), we may have to revise various conclusions. For example the limit on Ω_Λ (Fukugita & Turner 1991; Fukugita et al. 1992; Kochanek 1996) may have to be weakened because its hypothesis (constant comoving density unevolving SIS lenses, flat universe) would be wrong. On the other hand, if the current sample is a statistical fluke, then the observed lens systems must constitute a nontypical subset of the parent population and the limit of Ω_Λ would also have to be weakened for the reason that our current sample is not a fair sample. The same thing can be said if the correlation is due to unknown observational selection effects or if the lens sample is contaminated with “false” cases. It is worth noting that if the negative correlation is just a statistical fluke, we would be twice as likely to observe such an anomaly in an open universe with $k = -1$ and $0 \leq \Omega \leq 0.4$ as we would be to observe it in a flat $k = 0$ universe with $\Omega + \Omega_\Lambda = 1$.

Finally, the existence of multiple-image systems up to a redshift of 4.5 places a limit on the deceleration parameter, $q_0 > -2.0$.

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