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CONTRIBUTIONS OF THE PLASMONS TO THE ENERGY DENSITY AND PRESSURE IN THE EARLY UNIVERSE. II. CORRELATION EFFECTS

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ABSTRACT

The contributions of correlation effects in the electron-positron plasma in the early universe to the thermodynamic quantities are calculated. These are found to be of the same order of magnitude as the plasmon contributions calculated in Paper I. Their contributions are of the order of 0.1% of the contributions of blackbody photons. Therefore, plasma effects in the electron-positron plasma in the early universe will not drastically alter the standard model of the evolution of the early universe. For the sake of future applications, we give detailed numerical results for the calculation of various thermodynamic quantities of the cosmic fluid in the early universe. The enhancement factor of thermonuclear reaction rates due to Debye screening is also calculated. This factor is again of the order of 0.1%.

Subject headings: early universe — nuclear reactions, nucleosynthesis, abundances — plasmas

1. INTRODUCTION

Recently, we have calculated the contributions of plasmons to the energy density and pressure of the cosmic fluid in the early universe (Itoh et al. 1997, hereafter Paper I). We have found that plasmon effects make contributions to the energy density and pressure of the order of 0.1% of the contributions of blackbody photons. Our work in Paper I has verified the assumption hitherto made that the plasmon effects are even smaller in the cosmic fluid in the early universe. In Paper I, we calculated the number density, energy density, and pressure of the plasmons. Entropy is also an important physical quantity in the evolution of the early universe (Dicus et al. 1982; Krauss & Romanelli 1990; Kernan & Krauss 1994; Krauss & Kernan 1994). Therefore, in this paper we will calculate plasmon contributions to the entropy of the cosmic fluid in the early universe.

In Paper I we estimated the correlation energy of the electron-positron plasma in the early universe, and we found that it is of the same order of magnitude as the plasmon contribution. Therefore, in this paper we will also calculate the correlation energy density and its contribution to the pressure and the entropy of the electron-positron plasma in the early universe. Although these quantities turn out to be rather small, we present detailed numerical results of our calculations for the sake of future applications of the present results. We also calculate the enhancement factor of the thermonuclear reaction rates due to Debye screening in the electron-positron plasma in the early universe. This factor again turns out to be rather small, of the order of 0.1%.

The present paper is organized as follows. In § 2 we calculate plasmon contributions to entropy. In § 3 we calculate the contributions of the correlation energy of the electron-positron plasma to the thermodynamic quantities of the plasma. The enhancement factor of the thermonuclear reaction rates due to Debye screening is calculated in § 4. Numerical results of the calculations are given in § 5. Concluding remarks are given in § 6.

2. PLASMON CONTRIBUTION TO ENTROPY

In this section we use a method developed in Paper I; for the sake of completeness it is also described fully here. Our method is based on the works of Silin (1960), Tsytovich (1961), Beaudet, Petrosian, & Salpeter (1967), Braaten & Segel (1993), and Itoh et al. (1996). For the calculation of pressure, we employ a different method from that presented in Paper I, one that includes correction terms.

In the electron-positron plasma of the early universe, there exist two kinds of plasmon modes: the transverse plasmon and the longitudinal plasmon. The former corresponds to the photon in the plasma, and the latter corresponds to the longitudinal wave of the electric field in the plasma. In vacuum, the photon dispersion relation is $\omega = ck$, where ω is the angular frequency, c is the velocity of light, and k is the wavenumber. In the plasma, the dispersion relations for the transverse and longitudinal plasmons $\omega_t(k)$ and $\omega_l(k)$ both approach in the long-wavelength limit the plasma frequency ω_p, given by Beaudet et al. (1967) as

$$\omega_p^2 = \frac{4\pi\alpha\hbar c}{m} \int \frac{mc^2}{E} \left(1 - \frac{p^2c^2}{3E^2}\right)(dn_- + dn_+) , \tag{1}$$

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where α is the fine-structure constant, m is the electron mass, p and E are the momentum and energy of the electrons or positrons, and dn_- and dn_+ are related to the number density of the electrons and positrons defined by

$$n_{\mp} = \int dn_{\mp} = \frac{2}{(2\pi\hbar)^3} \int \frac{d^3p}{\exp(E/k_B T \mp \nu) + 1}, \tag{2}$$

$$E = (p^2c^2 + m^2c^4)^{1/2} . \tag{3}$$

In equation (2), $\nu = \mu/k_B T$ is the chemical potential for an electron, including its rest-mass energy, expressed in units of $k_B T$. Since we consider electron-positron pairs in equilibrium with blackbody radiation photons (zero potential), the chemical potential for a positron is $-\nu$. We further define the functions

$$G_n^{\pm}(\lambda, \nu) = \lambda^{3+2n} \int_{\lambda^{-1}}^{\infty} \frac{x^{2n+1}(x^2 - \lambda^{-2})^{1/2}}{e^{x \pm \nu} + 1} dx , \tag{4}$$

$$\lambda = \frac{k_B T}{mc^2} = \frac{T}{5.9302 \times 10^9 \text{ K}} . \tag{5}$$

We also define

$$\omega_1^2 = \frac{4\pi\alpha\hbar c}{m} \int \frac{mc^2}{E} \frac{p^2c^2}{E^2} \left(\frac{5}{3} - \frac{p^2c^2}{E^2}\right)(dn_- + dn_+) . \tag{6}$$

We then obtain the following expressions (Beaudet et al. 1967):

$$\left(\frac{\hbar\omega_p}{mc^2}\right)^2 = \frac{4\alpha}{3\pi} (2G_{-1/2}^{+} + 2G_{-1/2}^{-} + G_{-3/2}^{+} + G_{-3/2}^{-}) , \tag{7}$$

$$\left(\frac{\hbar\omega_1}{mc^2}\right)^2 = \frac{4\alpha}{3\pi} (2G_{-1/2}^{+} + 2G_{-1/2}^{-} + G_{-3/2}^{+} + G_{-3/2}^{-} - 3G_{-5/2}^{+} - 3G_{-5/2}^{-}) . \tag{8}$$

As the wavenumber k increases, the dispersion relation for the longitudinal plasmon $\omega_l(k)$ in the Braaten-Segel approximation crosses the light cone $\omega = ck$ at a point k_c, given by

$$k_c^2 = \frac{4\pi\alpha\hbar}{mc} \int \frac{mc^2}{E} \left(\frac{E}{pc} \ln \frac{E + pc}{E - pc} - 1\right)(dn_- + dn_+) . \tag{9}$$

This satisfies the condition $\omega_p/c < k_c < \infty$, and represents the maximum wavenumber for which an undamped longitudinal plasmon can propagate. In the relativistic electron-positron plasma of the early universe, we can set the chemical potential of the electron $\nu = 0$, assuming that

$$n_- = n_+ \gg n_p , \tag{10}$$

n_p being the number density of the protons. Hereafter we will use this approximation.

Following Braaten & Segel (1993), we define a parameter v_* by

$$\frac{v_*}{c} = \frac{\omega_1}{\omega_p} . \tag{11}$$

It lies in the range $0 \le v_*/c < 1$, and can be intuitively interpreted as a typical velocity of electrons in the plasma. Values of v_*/c as a function of $\log_{10} \lambda$ are given in Table 1. The graph is shown in Figure 1 of Paper I. The dispersion relations for the undamped plasmons $\omega_t(k)$ and $\omega_l(k)$ in the Braaten-Segel approximation are obtained by solving the following equations, which depend on v_*:

$$\omega_t^2 = c^2k^2 + \omega_p^2 \frac{3\omega_t^2}{2v_*^2 k^2} \left(1 - \frac{\omega_t^2 - v_*^2 k^2}{\omega_t^2} \frac{\omega_t}{2v_* k} \ln \frac{\omega_t + v_* k}{\omega_t - v_* k}\right), \qquad 0 \le k < \infty , \tag{12}$$

and

$$\omega_l^2 = \omega_p^2 \frac{3\omega_l^2}{v_*^2 k^2} \left(\frac{\omega_l}{2v_* k} \ln \frac{\omega_l + v_* k}{\omega_l - v_* k} - 1\right), \qquad 0 \le k \le k_c. \tag{13}$$

The maximum wavenumber for the undamped longitudinal plasmon is

$$k_c = \left[\frac{3c^2}{v_*^2} \left(\frac{c}{2v_*} \ln \frac{c + v_*}{c - v_*} - 1\right)\right]^{1/2} \frac{\omega_p}{c} . \tag{14}$$

Here we evaluate the damping of the longitudinal plasmon for $k > k_c$. Tsytovich (1961) has calculated the damping of the longitudinal plasmon due to absorption by electrons and positrons. Following his discussion, we calculate the imaginary part

TABLE 1

VARIOUS THERMODYNAMIC QUANTITIES AS FUNCTIONS OF $\log_{10} \lambda$

$\log_{10} \lambda$	v_*/c	$\hbar k_c'/(mc)$	$\hbar k_c''/(mc)$	κ	$\beta_t \times 10^3$	$\beta_l \times 10^3$	$\beta_{corr} \times 10^3$	$\gamma_t \times 10^3$	$\gamma_l \times 10^3$	$\gamma_{corr} \times 10^3$	$\delta_t \times 10^3$	$\delta_l \times 10^3$	$\delta_{corr} \times 10^3$
−1.5......	0.3677	0.0000	0.0000	0.0000	0.001	0.000	0.000	0.005	0.000	0.000	0.002	0.000	0.000
−1.4......	0.4053	0.0000	0.0000	0.0000	0.001	0.000	0.000	0.005	0.000	0.000	0.002	0.000	0.000
−1.3......	0.4451	0.0000	0.0000	0.0004	0.001	0.000	0.000	0.005	0.000	0.000	0.002	0.000	0.000
−1.2......	0.4869	0.0000	0.0000	0.0027	0.001	0.000	0.000	0.005	0.000	0.000	0.002	0.000	0.000
−1.1......	0.5302	0.0001	0.0001	0.0116	0.001	0.000	0.000	0.005	0.000	0.000	0.002	0.000	0.000
−1.0......	0.5745	0.0004	0.0004	0.0362	0.001	0.000	0.000	0.006	0.000	0.000	0.002	0.000	0.000
−0.9......	0.6192	0.0012	0.0011	0.0869	0.004	0.000	0.000	0.014	0.000	0.000	0.006	0.000	0.000
−0.8......	0.6636	0.0031	0.0029	0.1700	0.015	0.000	0.002	0.046	0.002	0.000	0.023	0.001	0.001
−0.7......	0.7070	0.0069	0.0065	0.2822	0.047	0.001	0.007	0.142	0.010	0.001	0.071	0.003	0.006
−0.6......	0.7490	0.0134	0.0128	0.4120	0.118	0.003	0.023	0.348	0.034	0.004	0.176	0.011	0.018
−0.5......	0.7889	0.0237	0.0230	0.5436	0.241	0.010	0.053	0.700	0.087	0.010	0.356	0.029	0.042
−0.4......	0.8261	0.0388	0.0380	0.6629	0.414	0.021	0.096	1.193	0.178	0.021	0.609	0.061	0.077
−0.3......	0.8601	0.0605	0.0596	0.7615	0.624	0.041	0.147	1.781	0.317	0.038	0.913	0.110	0.120
−0.2......	0.8901	0.0900	0.0891	0.8369	0.847	0.068	0.198	2.397	0.495	0.057	1.234	0.175	0.163
−0.1......	0.9158	0.1299	0.1299	0.8913	1.062	0.105	0.242	2.983	0.722	0.077	1.542	0.260	0.201
0.0	0.9370	0.1827	0.1818	0.9289	1.253	0.144	0.276	3.498	0.944	0.095	1.814	0.344	0.231
0.1	0.9539	0.2521	0.2521	0.9540	1.414	0.190	0.302	3.924	1.207	0.111	2.041	0.445	0.254
0.2	0.9670	0.3449	0.3449	0.9706	1.542	0.243	0.320	4.262	1.491	0.123	2.222	0.555	0.271
0.3	0.9768	0.4637	0.4637	0.9812	1.642	0.294	0.332	4.520	1.761	0.133	2.362	0.661	0.282
0.4	0.9839	0.6225	0.6225	0.9881	1.717	0.354	0.340	4.712	2.072	0.139	2.466	0.784	0.290
0.5	0.9891	0.8263	0.8263	0.9925	1.773	0.413	0.345	4.851	2.368	0.144	2.542	0.902	0.295
0.6	0.9926	1.0911	1.0911	0.9952	1.813	0.475	0.348	4.950	2.670	0.147	2.597	1.023	0.298
0.7	0.9951	1.4400	1.4400	0.9970	1.841	0.544	0.351	5.019	3.006	0.150	2.636	1.160	0.300
0.8	0.9967	1.8879	1.8879	0.9981	1.861	0.612	0.352	5.066	3.327	0.151	2.662	1.291	0.302
0.9	0.9979	2.4612	2.4612	0.9988	1.875	0.677	0.353	5.099	3.630	0.152	2.681	1.415	0.303
1.0	0.9986	3.2130	3.2130	0.9992	1.884	0.751	0.353	5.120	3.973	0.153	2.693	1.557	0.303
1.1	0.9991	4.1877	4.1877	0.9995	1.890	0.830	0.354	5.135	4.330	0.153	2.701	1.705	0.304
1.2	0.9994	5.4345	5.4345	0.9997	1.894	0.906	0.354	5.144	4.667	0.153	2.707	1.846	0.304
1.3	0.9996	7.0452	7.0452	0.9998	1.897	0.985	0.354	5.150	5.016	0.154	2.710	1.993	0.304
1.4	0.9997	9.1246	9.1246	0.9999	1.899	1.069	0.354	5.154	5.377	0.154	2.713	2.146	0.304
1.5	0.9998	11.7759	11.7759	0.9999	1.900	1.148	0.354	5.157	5.713	0.154	2.714	2.289	0.304

of the frequency, γ, defined by

$$\gamma = \frac{\mathrm{Im}\ \epsilon_l}{\partial(\mathrm{Re}\ \epsilon_l)/\partial\omega}\,, \tag{15}$$

where ϵ_l is the longitudinal dielectric function. Tsytovich (1961) has calculated Im ϵ_l for the Boltzmann distribution of electrons. Here we calculate Im ϵ_l for the Fermi-Dirac distribution with chemical potential $\mu = 0$ as

$$\mathrm{Im}\ \epsilon_l = \frac{8\alpha k_B T}{(\hbar ck)^3} \sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n} \sinh \frac{n\hbar\omega}{2k_B T} \left[\frac{(mc^2)^2}{1-\omega^2/(ck)^2} + \frac{2}{n} k_B T \kappa_0 + \frac{2}{n^2}(k_B T)^2\right] \exp\left(-\frac{n\kappa_0}{k_B T}\right) \tag{16}$$

where

$$\kappa_0 = \left[\frac{(mc^2)^2}{1-\omega^2/(ck)^2} + \frac{(\hbar ck)^2}{4}\right]^{1/2} . \tag{17}$$

In order to evaluate Re ϵ_l, we use the longitudinal dielectric function of Tsytovich (1961), written in the present case as

$$\mathrm{Re}\ \epsilon_l = 1 - \frac{4\alpha}{\pi\hbar^2 k^2} \int_0^\infty dp\, \frac{pc}{E} \frac{1}{e^{E/k_B T}+1} \left\{-2p + \frac{E\hbar\omega - E^2 - \hbar^2 K^2/4}{\hbar ck} \ln \left|\frac{E\hbar\omega - \hbar^2 K^2/2 + \hbar c^2 pk}{E\hbar\omega - \hbar^2 K^2/2 - \hbar c^2 pk}\right|\right\}$$
$$+ \left\{\frac{E\hbar\omega + E^2 + \hbar^2 K^2/4}{\hbar ck} \ln \left|\frac{E\hbar\omega + \hbar^2 K^2/2 + \hbar c^2 pk}{E\hbar\omega + \hbar^2 K^2/2 - \hbar c^2 pk}\right|\right\}, \tag{18}$$

where

$$K^2 = \omega^2 - c^2 k^2 . \tag{19}$$

We compute Re ϵ_l numerically for given values of λ and k. We then seek a solution ω that satisfies Re $\epsilon_l(k, \omega) = 0$. This procedure gives the longitudinal plasmon dispersion relation $\omega_l = \omega_l(k)$. We confirm that for $k < k_c$, the dispersion relation in equation (13) of Braaten & Segel is a good approximation. However, as $k(>k_c)$ increases, we encounter such a critical value k_c' that for $k > k_c'$ there no longer exists a solution ω to the equation Re $\epsilon_l(k, \omega) = 0$. This is the definite endpoint of the longitudinal plasmon dispersion curve. At this point $\partial[\mathrm{Re}\ \epsilon_l(k, \omega)]/\partial\omega|_{k=k_{c'}} = 0$, and thus the imaginary part of the frequency given by equation (15) becomes infinitely large. We shall impose a stronger condition for the existence of the longitudinal plasmon,

$$\frac{\gamma}{\omega} \le \frac{1}{10} . \tag{20}$$

The choice of the factor 1/10 is of course not unique. However, the value γ/ω is a rapidly varying function of k near the critical value k_c'. Thus we can with reasonable accuracy determine the endpoint of the well-defined longitudinal plasmon dispersion curve k_c'' that satisfies the condition of equation (20). This point can be readily seen from Table 1. For $\log_{10} \lambda \ge -1.0$ where the contributions of the longitudinal plasmons are of any significance, k_c'' practically coincides with k_c', thereby showing the essential independence of the thermodynamic quantities from the exact choice of the critical wavenumber k_c' or k_c''. In Figures 2–3 of Paper I we have shown the dispersion curves for the transverse and longitudinal plasmons corresponding to the conditions $\nu = 0$ and $\lambda = 1.0$, 10. The endpoints of the longitudinal plasmon dispersion curves correspond to the condition of equation (20).

The number densities of the transverse and longitudinal plasmons are given by

$$n_t = \frac{2}{(2\pi)^3} \int_0^\infty \frac{4\pi k^2\, dk}{\exp(\hbar\omega_t/k_B T) - 1} , \tag{21}$$

$$n_l = \frac{1}{(2\pi)^3} \int_0^{k_c''} \frac{4\pi k^2\, dk}{\exp(\hbar\omega_l/k_B T) - 1} . \tag{22}$$

Since the chemical potential of the plasmons is zero, the free energy of the transverse plasmons is calculated (see Lifshitz & Pitaevskii 1980) as

$$F_t = k_B T \frac{2V}{(2\pi)^3} \int_0^\infty \ln\left[1 - \exp\left(-\frac{\hbar\omega_t}{k_B T}\right)\right] 4\pi k^2\, dk . \tag{23}$$

The free energy of the longitudinal plasmons is similarly calculated as

$$F_l = k_B T \frac{V}{(2\pi)^3} \int_0^{k_c''} \ln\left[1 - \exp\left(-\frac{\hbar\omega_l}{k_B T}\right)\right] 4\pi k^2\, dk . \tag{24}$$

We note that the k integral extends to infinity in the case of transverse plasmons, whereas it is limited by the cutoff wavenumber k_c'' in the case of longitudinal plasmons. The entropy is calculated from the free energy by the relationship $S = -\partial F/\partial T$, the energy is calculated by the relationship $E = F + TS$, and the pressure is calculated by the relationship $P = -\partial F/\partial V$. Thus, for the energy density $W = E/V$, pressure P, and entropy density $s = S/V$ of the transverse and longitudinal plasmons we obtain the expressions

$$W_t = \frac{2}{(2\pi)^3} \int_0^\infty \frac{\hbar\omega_t 4\pi k^2\, dk}{\exp(\hbar\omega_t/k_B T) - 1} , \tag{25}$$

$$W_l = \frac{1}{(2\pi)^3} \int_0^{k_c''} \frac{\hbar\omega_l 4\pi k^2\, dk}{\exp(\hbar\omega_l/k_B T) - 1} , \tag{26}$$

$$P_t = \frac{2}{(2\pi)^3} \frac{1}{3} \int_0^\infty \frac{\hbar k(d\omega_t/dk) 4\pi k^2\, dk}{\exp(\hbar\omega_t/k_B T) - 1} , \tag{27}$$

$$P_l = \frac{1}{(2\pi)^3} \frac{1}{3} \int_0^{k_c''} \frac{\hbar k(d\omega_l/dk) 4\pi k^2\, dk}{\exp(\hbar\omega_l/k_B T) - 1} - \frac{k_B T}{6\pi^2} k_c''^3 \ln\left\{1 - \exp\left[-\frac{\hbar\omega_l(k_c'')}{k_B T}\right]\right\} , \tag{28}$$

$$s_t = \frac{2}{(2\pi)^3} \frac{1}{T} \int_0^\infty \frac{\hbar\omega_t 4\pi k^2\, dk}{\exp(\hbar\omega_t/k_B T) - 1} + \frac{2}{(2\pi)^3} \frac{1}{3T} \int_0^\infty \frac{\hbar k(d\omega_t/dk) 4\pi k^2\, dk}{\exp(\hbar\omega_t/k_B T) - 1} , \tag{29}$$

$$s_l = \frac{1}{(2\pi)^3} \frac{1}{T} \int_0^{k_c''} \frac{\hbar\omega_l 4\pi k^2\, dk}{\exp(\hbar\omega_l/k_B T) - 1} + \frac{1}{(2\pi)^3} \frac{1}{3T} \int_0^{k_c''} \frac{\hbar k(d\omega_l/dk) 4\pi k^2\, dk}{\exp(\hbar\omega_l/k_B T) - 1} - \frac{1}{6\pi^2} k_c''^3 \ln\left\{1 - \exp\left[-\frac{\hbar\omega_l(k_c'')}{k_B T}\right]\right\} . \tag{30}$$

We rewrite these equations as

$$n_t = \left[\frac{2\zeta(3)}{\pi^2}\left(\frac{k_B T}{\hbar c}\right)^3\right] \frac{1}{2\zeta(3)} \lambda^{-3} \int_0^\infty \frac{(\hbar k/mc)^2\, d(\hbar k/mc)}{\exp[\lambda^{-1}(\hbar\omega_t/mc^2)] - 1} \equiv (1 - \alpha_t)\left[\frac{2\zeta(3)}{\pi^2}\left(\frac{k_B T}{\hbar c}\right)^3\right] , \tag{31}$$

$$n_l = \left[\frac{2\zeta(3)}{\pi^2}\left(\frac{k_B T}{\hbar c}\right)^3\right] \frac{1}{4\zeta(3)} \lambda^{-3} \int_0^{\hbar k_c''/mc} \frac{(\hbar k/mc)^2\, d(\hbar k/mc)}{\exp[\lambda^{-1}(\hbar\omega_l/mc^2)] - 1} \equiv \alpha_l\left[\frac{2\zeta(3)}{\pi^2}\left(\frac{k_B T}{\hbar c}\right)^3\right] , \tag{32}$$

$$W_t = \left[\frac{\pi^2}{15}\frac{(k_B T)^4}{(\hbar c)^3}\right] \frac{15}{\pi^4} \lambda^{-4} \int_0^\infty \frac{(\hbar\omega_t/mc^2)(\hbar k/mc)^2\, d(\hbar k/mc)}{\exp[\lambda^{-1}(\hbar\omega_t/mc^2)] - 1} \equiv (1 - \beta_t)\left[\frac{\pi^2}{15}\frac{(k_B T)^4}{(\hbar c)^3}\right] , \tag{33}$$

$$W_l = \left[\frac{\pi^2}{15}\frac{(k_B T)^4}{(\hbar c)^3}\right]\frac{15}{2\pi^4}\lambda^{-4}\int_0^{\hbar k''/mc}\frac{(\hbar\omega_l/mc^2)(\hbar k/mc)^2\, d(\hbar k/mc)}{\exp\,[\lambda^{-1}(\hbar\omega_l/mc^2)] - 1} \equiv \beta_l\left[\frac{\pi^2}{15}\frac{(k_B T)^4}{(\hbar c)^3}\right], \tag{34}$$

$$P_t = \left[\frac{\pi^2}{45}\frac{(k_B T)^4}{(\hbar c)^3}\right]\frac{15}{\pi^4}\lambda^{-4}\int_0^{\infty}\frac{(\hbar k/mc)^3[d(\hbar\omega_t/mc^2)/d(\hbar k/mc)]d(\hbar k/mc)}{\exp\,[\lambda^{-1}(\hbar\omega_t/mc^2)] - 1} \equiv (1-\gamma_t)\left[\frac{\pi^2}{45}\frac{(k_B T)^4}{(\hbar c)^3}\right], \tag{35}$$

$$P_l = \left[\frac{\pi^2}{45}\frac{(k_B T)^4}{(\hbar c)^3}\right]\left(\frac{15}{2\pi^4}\lambda^{-4}\int_0^{\hbar k_c''/mc}\frac{(\hbar k/mc)^3[d(\hbar w_l/mc^2)/d(\hbar k/mc)]d(\hbar k/mc)}{\exp\,[\lambda^{-1}(\hbar w_l/mc^2)] - 1}\right.$$
$$\left. - \frac{15}{2\pi^4}\lambda^{-3}\left(\frac{\hbar k_c''}{mc}\right)^3 \ln\left\{1 - \exp\left[-\frac{\hbar\omega_l(k_c'')}{k_B T}\right]\right\}\right) \equiv \gamma_l\left[\frac{\pi^2}{45}\frac{(k_B T)^4}{(\hbar c)^3}\right]. \tag{36}$$

$$s_t = \left(1 - \frac{3\beta_t + \gamma_t}{4}\right)\left[\frac{4\pi^2}{45}k_B\left(\frac{k_B T}{\hbar c}\right)^3\right] \equiv (1-\delta_t)\left[\frac{4\pi^2}{45}k_B\left(\frac{k_B T}{\hbar c}\right)^3\right], \tag{37}$$

$$s_l = \left(\frac{3\beta_l + \gamma_l}{4}\right)\left[\frac{4\pi^2}{45}k_B\left(\frac{k_B T}{\hbar c}\right)^3\right] \equiv \delta_l\left[\frac{4\pi^2}{45}k_B\left(\frac{k_B T}{\hbar c}\right)^3\right]. \tag{38}$$

We note that a correction term is added to the expression for P_l from the corresponding expression in Paper I. In equation (31), $\zeta(3)$ designates Riemann's zeta function, $\zeta(3) = 1.202057$. Since the timescale needed to establish equilibrium between electrons and positrons is shorter than the expansion timescale of the universe for $\lambda \geq 10^{-1.5}$, this condition has also been imposed.

3. CORRELATION EFFECTS

As discussed in Paper I, we expect that the average interaction energy of the electron-positron plasma is of the same order of magnitude as the plasmon contributions. In this section, therefore, we will calculate the contributions of the correlation effects in the electron-positron plasma to the thermodynamic quantities.

We start from the expression for the correlation energy of the electron-positron plasma (Lifshitz & Pitaevskii 1980),

$$E_{\rm corr} = -\tfrac{1}{2}(n_+ + n_-)Vk_D\hbar c\alpha\,, \tag{39}$$

where k_D is the Debye wavenumber for the electron-positron plasma, given by (Tsytovich 1961)

$$k_D^2 = -4\pi\hbar c\alpha\frac{2}{(2\pi\hbar)^3}\int d^3p\left(\frac{dn_p^-}{dE} + \frac{dn_p^+}{dE}\right), \tag{40}$$

$$n_p^\pm = \frac{1}{\exp\,(E/k_B T \mp \nu) + 1}. \tag{41}$$

In the above formula, we set $\nu = 0$, considering the case $n_- = n_+ \gg n_p$. We then obtain the Debye wavenumber for the electron-positron plasma with $\nu = 0$ as

$$k_D^2 = \frac{8}{\pi}\alpha\left(\frac{k_B T}{\hbar c}\right)^2\int_{\lambda^{-1}}^{\infty}(2x^2 - \lambda^{-2})(x^2 - \lambda^{-2})^{-1/2}\frac{dx}{e^x + 1}. \tag{42}$$

From the general thermodynamic relationship

$$\frac{E}{T^2} = -\frac{\partial}{\partial T}\left(\frac{F}{T}\right), \tag{43}$$

we calculate the free energy due to correlation effects (Debye screening) as

$$F_{\rm corr} = \frac{1}{2}\hbar c\alpha VT\int_0^T(n_+ + n_-)k_D\frac{dT'}{T'^2}. \tag{44}$$

In deriving equation (44), we have chosen an integration constant such that the conditions $F_{\rm corr} = 0$ and $S_{\rm corr} = 0$ are satisfied for $T = 0$. In equation (44), n_+ and n_- are given by

$$n_\pm = \frac{1}{\pi^2}\left(\frac{mc}{\hbar}\right)^3 G_0^\pm(\lambda, 0)\,, \tag{45}$$

$G_n^\pm(\lambda, \nu)$ being defined by equation (4).

The correlation energy density, $W_{\rm corr}$, is given by

$$W_{\rm corr} = \frac{E_{\rm corr}}{V}$$
$$= -\left[\frac{\pi^2}{15}\frac{(k_B T)^4}{(\hbar c)^3}\right]\frac{30\sqrt{2}}{\pi^{9/2}}\alpha^{3/2}\lambda^{-3}G_0^-(\lambda, 0)\left[\int_{\lambda^{-1}}^{\infty}(2x^2 - \lambda^{-2})(x^2 - \lambda^{-2})^{-1/2}\frac{dx}{e^x + 1}\right]^{1/2} \equiv -\beta_{\rm corr}\left[\frac{\pi^2}{15}\frac{(k_B T)^4}{(\hbar c)^3}\right]. \tag{46}$$

From the relationship $P = -(\partial F/\partial V)_T$, the pressure due to correlation effects is calculated as

$$P_{\mathrm{corr}} = -\left[\frac{\pi^2}{45}\frac{(k_{\mathrm{B}}T)^4}{(\hbar c)^3}\right]\frac{90\sqrt{2}}{\pi^{9/2}}\alpha^{3/2}\lambda^{-3}\int_0^{\lambda} G_0^-(\lambda', 0)\left[\int_{\lambda'^{-1}}^{\infty}(2x^2-\lambda'^{-2})(x^2-\lambda'^{-2})^{-1/2}\frac{dx}{e^x+1}\right]^{1/2}\frac{d\lambda'}{\lambda'} \equiv -\gamma_{\mathrm{corr}}\left[\frac{\pi^2}{45}\frac{(k_{\mathrm{B}}T)^4}{(\hbar c)^3}\right]. \tag{47}$$

From the relationship $S = -(\partial F/\partial T)_V$, the entropy density due to the correlation effects is calculated as

$$\begin{aligned} s_{\mathrm{corr}} &= \frac{S_{\mathrm{corr}}}{V} \\ &= -\left(\frac{3\beta_{\mathrm{corr}}+\gamma_{\mathrm{corr}}}{4}\right)\left[\frac{4\pi^2}{45}k_{\mathrm{B}}\left(\frac{k_{\mathrm{B}}T}{\hbar c}\right)^3\right] \\ &\equiv -\delta_{\mathrm{corr}}\left[\frac{4\pi^2}{45}k_{\mathrm{B}}\left(\frac{k_{\mathrm{B}}T}{\hbar c}\right)^3\right]. \end{aligned} \tag{48}$$

Thus the total radiation energy density, including the correlation energy of the electron-positron pairs, is written as

$$\begin{aligned} W &= W_t + W_l + W_{\mathrm{corr}} \\ &= (1-\beta_t+\beta_l-\beta_{\mathrm{corr}})\left[\frac{\pi^2}{15}\frac{(k_{\mathrm{B}}T)^4}{(\hbar c)^3}\right]. \end{aligned} \tag{49}$$

The corresponding pressure and entropy density are written as

$$\begin{aligned} P &= P_t + P_l + P_{\mathrm{corr}} \\ &= (1-\gamma_t+\gamma_l-\gamma_{\mathrm{corr}})\left[\frac{\pi^2}{45}\frac{(k_{\mathrm{B}}T)^4}{(\hbar c)^3}\right], \end{aligned} \tag{50}$$

$$\begin{aligned} s &= s_t + s_l + s_{\mathrm{corr}} \\ &= (1-\delta_t+\delta_l-\delta_{\mathrm{corr}})\left[\frac{4\pi^2}{45}k_{\mathrm{B}}\left(\frac{k_{\mathrm{B}}T}{\hbar c}\right)^3\right]. \end{aligned} \tag{51}$$

4. ENHANCEMENT OF THERMONUCLEAR REACTION RATES

The enhancement factor of thermonuclear reaction rates is calculated by Salpeter & Van Horn (1969) and also by Itoh and his collaborators (Itoh, Totsuji, & Ichimaru 1977; Itoh et al. 1979; Itoh, Kuwashima, & Munakata 1990). However, the enhancement factor of thermonuclear reaction rates in the early universe is not calculated in their papers. In this section we will address ourselves to this problem.

As we have discussed in the previous sections, the number density of charged particles in the early universe is dominated by electron-positron pairs. Therefore, the screening of the Coulomb interaction in the early universe is due exclusively to electron-positron pairs. Thus, in the following discussion we take into account the contributions of the electron-positron pairs only. Let us consider thermonuclear reactions of two species of nuclei, with charges $Z_1 e$ and $Z_2 e$. Since the electron-positron plasma in the early universe is a weakly interacting plasma (the Debye screening length being much greater than the mean separation of the electrons and positrons), the present condition corresponds to the weak-screening regime considered by Salpeter & Van Horn (1969). In this regime, the enhancement factor f of the thermonuclear reaction rate is given by

$$f = \exp\left(\frac{Z_1 Z_2 k_{\mathrm{D}}\hbar c\alpha}{k_{\mathrm{B}}T}\right), \tag{52}$$

where k_{D} is the Debye wavenumber as given in equation (42). From this we obtain

$$\begin{aligned} f &= \exp\left\{Z_1 Z_2\left(\frac{8}{\pi}\right)^{1/2}\alpha^{3/2}\left[\int_{\lambda^{-1}}^{\infty}(2x^2-\lambda^{-2})(x^2-\lambda^{-2})^{-1/2}\frac{dx}{e^x+1}\right]^{1/2}\right\} \\ &= \exp(1.28\times10^{-3}Z_1 Z_2\kappa), \end{aligned} \tag{53}$$

$$\kappa \equiv \left[\frac{6}{\pi^2}\int_{\lambda^{-1}}^{\infty}(2x^2-\lambda^{-2})(x^2-\lambda^{-2})^{-1/2}\frac{dx}{e^x+1}\right]^{1/2}. \tag{54}$$

In the high-temperature limit $\lambda^{-1}\to 0$, we obtain $\kappa = 1$, and thereby $f \to \exp(0.00128 Z_1 Z_2)$. Thus we find that the enhancement of thermonuclear reaction rates due to Debye screening in the electron-positron plasma in the early universe is of the order of 0.1%.

5. RESULTS AND IMPLICATIONS

The results of the numerical computation of the terms β_t, β_l, β_{corr}, γ_t, γ_l, γ_{corr}, δ_t, δ_l, δ_{corr}, and κ are shown in Table 1. They are also shown in Figures 1, 2, 3, and 4 as functions of $\log_{10}\lambda$. The thermodynamic quantities of the longitudinal plasmons are

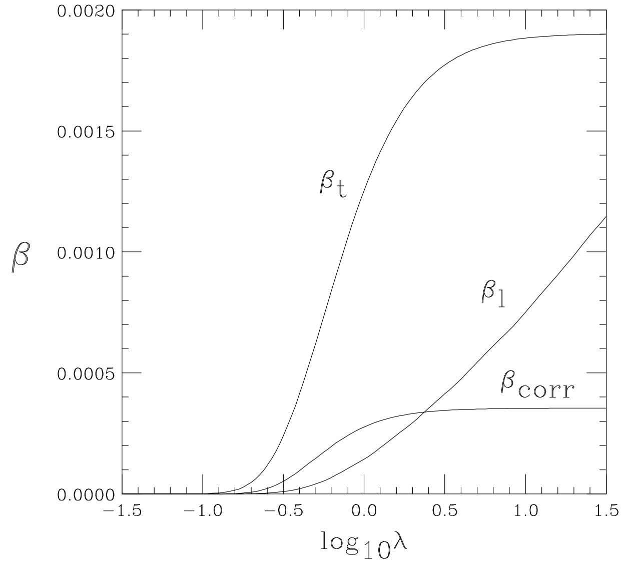

FIG. 1.—Terms β_t, β_l, and $\beta_{\rm corr}$ as functions of $\log_{10} \lambda$

calculated using the dispersion relations of Braaten & Segel (1993), with a termination at k_c''. This procedure is considered to be reasonably accurate.

We find that the plasma effects make contributions of the order of 0.1% of blackbody photons to the thermodynamic quantities of the cosmic fluid in the early universe. Therefore, plasma effects will not drastically alter the standard model of the evolution of the early universe. The results of the calculations of the thermodynamic quantities presented in this paper can be readily applied to calculations of the evolution of the early universe that include the plasma effects.

The enhancement factor of thermonuclear reaction rates due to Debye screening in the electron-positron plasma in the early universe is found to be of the order of 0.1%. Thus, the enhancement of the thermonuclear reaction rates can be legitimately neglected in calculations of the primordial nucleosynthesis.

6. CONCLUDING REMARKS

We have calculated the contributions of plasma effects to the thermodynamic quantities of the cosmic fluid in the early universe. We have found that plasma effects make contributions of the order of 0.1% of the blackbody photons. This finding consolidates the soundness of the assumption made by standard calculations of the evolution of the early universe that neglect plasma effects. The detailed numerical results calculated in this paper for the thermodynamic quantities of the cosmic fluid in the early universe that include plasma effects will be readily applied to calculations of the evolution of the early universe that take plasma effects into account.

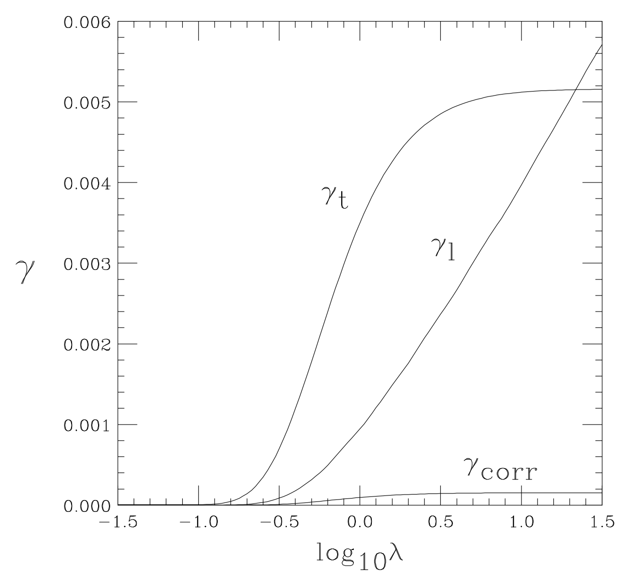

FIG. 2.—Terms γ_t, γ_l, and $\gamma_{\rm corr}$ as functions of $\log_{10} \lambda$

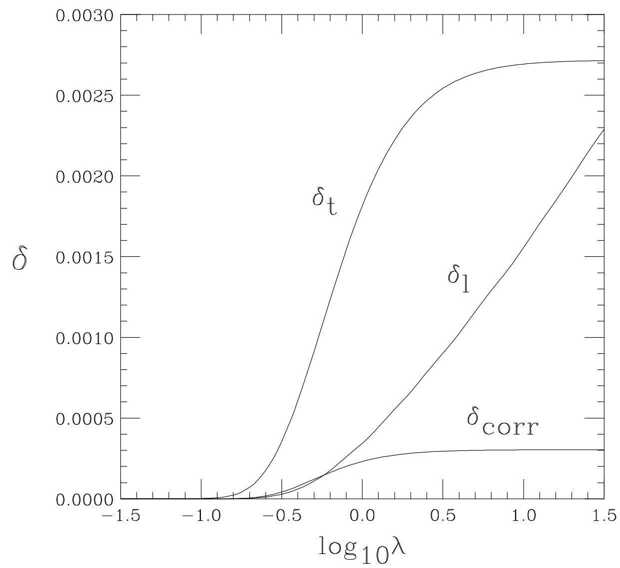

FIG. 3.—Terms δ_t, δ_l, and $\delta_{\rm corr}$ as functions of $\log_{10} \lambda$

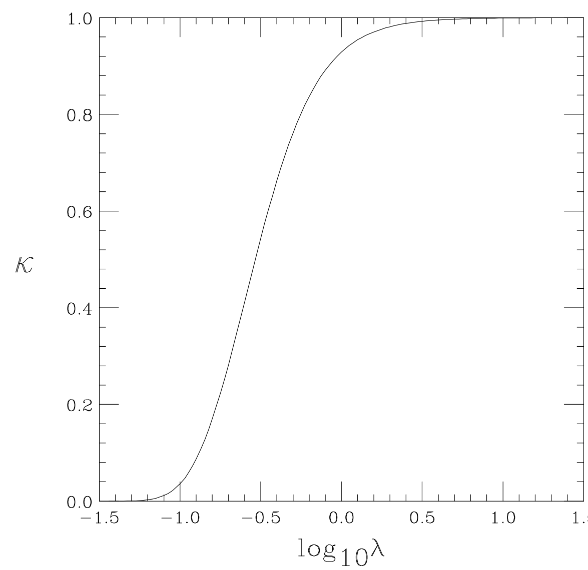

FIG. 4.—Parameter κ, defined by eq. (54), as a function of $\log_{10} \lambda$

It has also been found that the enhancement of thermonuclear reaction rates due to Debye screening in the electron-positron plasma in the early universe makes a negligible contribution, of the order of 0.1%. Thus in this case as well, the plasma effects are safely neglected. All these facts are derived from the factor $\alpha^{3/2}$ that enters into the plasma effects.

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