THE ASTROPHYSICAL JOURNAL, 486:60–74, 1997 September 1

IMPROVED CALIBRATION OF COSMIC DISTANCE SCALE BY CEPHEID PULSATION PARALLAXES

G. P. DI BENEDETTO
Istituto di Fisica Cosmica, CNR, via Bassini, 15, 20133 Milano, Italy; pdibene@ifctr.mi.cnr.it
Received 1996 November 19; accepted 1997 March 27

ABSTRACT

I report on a modern empirical realization of the geometric Baade-Wesselink (BW) method aimed at providing accurate distances to Cepheid variable stars. The method is carefully calibrated using high-precision observational data now available from spectroscopic and interferometric techniques. The distance-dependent linear radius R of a Cepheid, derived from the period P according to the period-radius (PR) relation, is matched to the distance-independent angular diameter Φ inferred from the surface brightness-color (SC) correlation using the infrared color $(V-K)$ as brightness indicator. The resulting pulsation parallax is found to be largely insensitive to most of known biasing effects, notably the reddening and metallicity, because of the application of the magnitude-color combination $(V, V-K)$ in deriving stellar angular sizes.

The reliability of the actual BW realization is critically checked using 25 galactic Cepheids in clusters with available high-precision photometry, reddenings and distances by the zero-age main-sequence (ZAMS) fitting approach. I find that 20 of these stars show an average residual $(\mathrm{BW} - \mathrm{ZAMS}) = -(0.01 \pm 0.03)$ mag with a scatter $\sigma = \pm 0.12$ mag, whereas the remaining five stars deviate by up to 5 σ. The fairly good agreement with 20 primary calibrators gives high confidence on the reliability of the method which can provide now an independent route to the problem of cosmic distance scale calibration. According to the overall residual, the actual uncertainty on the absolute galactic distance scale is suggested to be ± 0.04 mag, i.e., a factor of 2 smaller than that currently quoted for the Pleiades distance modulus which limits the ZAMS calibration.

The BW approach is then applied to the extragalactic range by including the Magellanic Clouds Cepheids (LMC and SMC) with available high-precision photometry in V and K passbands. An accurate PR relation is recalibrated by a composite fit to galactic and extragalactic Cepheids. The relation shows a dispersion as low as $\sigma = \pm 0.11$ mag nearly close to the scatter affecting BW distances to calibrating galactic Cepheids. It is argued that this scatter is likely due to the asymptotic spread set by the finite width of the instability strip. The BW distances to the LMC and SMC are also determined to be $\mu_0(\mathrm{LMC}) = (18.58 \pm 0.024)$ mag and $\mu_0(\mathrm{SMC}) = (19.00 \pm 0.025)$ mag or $d(\mathrm{LMC}) = (52.0 \pm 0.6)$ kpc and $d(\mathrm{SMC}) = (63.2 \pm 0.7)$ kpc with uncertainties *not* including the contribution due to the absolute distance scale calibration. The Cepheid-based distance to the LMC shows close agreement with the geometric expansion parallax of the SN 1987A in LMC given by $d(\mathrm{SN\ 1987A}) = (51.1 \pm 1.5)$ kpc (Panagia et al. 1996).

The BW method is also calibrated by including the Johnson-Cousins color $(V-I)$ relevant in the *Hubble Space Telescope* (*HST*) observations of Cepheids. By using sets of SMC, LMC, and galactic Cepheids with high-precision V, I, and K photometry, it is shown that the angular sizes predictable by the magnitude-color combination $(V, V-I)$ can be affected by the metallicity. However, the induced effects on BW distances are found to be as small as 0.03 mag up to the metal content of the SMC. Furthermore, it is demonstrated that the current *HST* approach to the extragalactic distances using a linear combination of period-luminosity (PL) relations in V and I passbands yields the same observational relation as that of the BW realization in the magnitude-color combination $(V, V-I)$. This achievement plays a major role in improving the extragalactic distance scale set by Cepheid data from *HST* observations. First, all distances spanning from the galactic to extragalactic range can be now sampled by the same BW relation strongly supported by the fundamental results of several observational techniques. Second, all distances will result to be affected by the metallicity as the BW data, i.e., as the SC correlation applied for inferring the Cepheid angular sizes. Third, all distances will suffer from random errors as the almost reddening-free BW data which cancel out the extinction on a star-by-star basis, notably the differential reddening internal to the parent galaxies. In order to show this relevant improvement, the BW distance to the Virgo galaxy M100 is determined to be $\mu_0(\mathrm{M100}) = (31.03 \pm 0.06)$ mag or $d(\mathrm{M100}) = (16.1 \pm 0.5)$ Mpc with a random error lowered by about a factor of 3 with respect to that derived according to the PL relations by the *HST* procedure.

Subject headings: Cepheids — distance scale — open clusters and associations: general — stars: distances — stars: oscillations

1. INTRODUCTION

Cepheid variable stars were recognized long ago to be the cornerstone to the extragalactic distance scale calibration (Sandage 1972), providing in addition keys to our understanding of many astrophysical phenomena. Thus, the requirement for accuracy in these fundamental standard candles has increased steadily in recent years. At present, the luminosities of galactic Cepheids can be calibrated with uncertainties up to about 10% (Feast & Walker 1987) which would imply individual distances as accurate as 5%. In contrast, even the nearest extragalactic Cepheids in the well-studied Magellanic Clouds seem to be unable to achieve the same individual accuracies, mostly because of difficulties caused by reddening and metallicity differences affecting the galactic and extragalactic range. Indeed, the luminosity laws currently available for predicting Cepheid distances, i.e., the period-luminosity-color (PLC) and the period-luminosity (PL) correlations, show a quite different behavior against those systematic effects. While the PLC relations are known to minimize both the reddening and the spread set by the finite width of the instability strip and then are ideal indicators of individual distance, they are unfortunately more or less sensitive to the metallicity through the color term, according to model predictions (Stothers 1988). In contrast, the PL relations are selectively sensitive to reddening effects as a function of wavelength, notably to the differential reddening internal to parent galaxies, and have a considerable intrinsic spread because of discarding the theoretically necessary third parameter but are expected to be practically insensitive to abundance effects (Chiosi, Wood, & Capitanio 1993). Thereby, to avoid somewhat significant model-dependent systematic corrections due to metallicity which is known to be different from galaxy to galaxy, the PL relations are currently preferred as distance indicators, although their calibration as well as the average distance results become relatively more uncertain due to the greater real scatter, thus affecting the overall extragalactic distance ladder. How to recover the advantages inherent in the PLC relations as accurate distance indicators remains an open problem which necessarily involves the absolute calibration of the Cepheid-based distance scale.

The present paper is an attempt to bring together some of the most accurate observational results now available from spectroscopic and interferometric techniques with the aim to provide new consistent and unbiased determinations of Cepheid distances. The pulsation parallax approach is exploited as a realization of the geometric BW method, i.e., one of the modern paths to galactic and extragalactic Cepheid-based distances (Jacoby et al. 1992). Previous preliminary investigations have already proved the potential accuracies of the BW approach as well as some of the difficulties encountered in developing it (Di Benedetto 1994, 1995). The emphasis now will be on the BW distances to individual Cepheids up to the ultimate accuracy set by the finite width of the instability strip and related improvements achievable in the extragalactic Cepheid-based distances by *HST* observations.

2. CEPHEID DATABASE

The photometric database adopted as fiducial throughout this paper draws widely upon the published literature. A sample of 51 galactic Cepheids with periods in the range $0.4 < \log P < 1.9$ is available with high-precision K photometry (Laney & Stobie 1993a). The list includes as well the B, V photometry and the color-excess E_{B-V} collected from several sources of optical data. For a subset of 43 stars, there is the additional coverage of the I band in the Cousins system (Caldwell & Coulson 1987). Among these Cepheids, 40 stars have BW radii recently determined using infrared photometry most suitable to minimize biasing effects (Laney & Stobie 1995) and 25 stars are known to be members of open clusters with well-determined ZAMS distance moduli (Feast & Walker 1987; Welch et al. 1985; Laney & Stobie 1994). These 25 primary calibrators are listed in Table 1. All galactic Cepheids throughout this paper will be dereddened according to a total-to-selective absorption $R_V = A_V/E_{B-V}$ given by

$$R_V = (3.06 \pm 0.09) + 0.25(B-V)_0 + 0.05E_{B-V} . \quad (1)$$

There are 46 LMC Cepheids and 60 SMC Cepheids with available B, V, K photometry and periods in the range $0.9 \le \log P \le 2.0$. The quality of K data is not equally good, since the errors may reach values up to about 0.2 mag. This uncertainty, however, can be reduced to less than 0.1 mag for a smaller sample of 27 LMC and 36 SMC stars which, in addition, include the I-band photometry in the Cousins magnitude system (Martin, Warren, & Feast 1979; Caldwell & Coulson 1984; Welch et al. 1987). These Cepheids will be adopted below as calibrating stars and their photometric data are listed in Tables 2 and 3. Several subsets of data will be further adopted to discuss potential sample-dependent variations. These include 22 LMC and 32 SMC Cepheids with reduced scatter in the photometric data; 20 LMC and 28 SMC stars with periods in the range $0.9 \le \log P \le 1.8$ relevant for extragalactic applications; 18 LMC and 19 SMC stars with errors on K data much less than 0.1 mag (Laney & Stobie 1986). All visual magnitudes of Magellanic Cloud Cepheids will be dereddened by a total-to-selective absorption $A_V/E_{B-V} = 3.30$, i.e., the standard value for the interstellar medium, and an average total (foreground + intrinsic) color-excess E_{B-V} derived from Cepheids themselves, as discussed in § 4.1.

All photometric magnitudes will be referred to intensity means over the pulsational cycle of the light curves. The K magnitudes will be normalized to the Johnson magnitude system. This merely implies a correction of $+0.02$ mag to K data currently referred to CIT or SAAO photometric systems. Moreover, since the data in the I band are currently done as magnitude means $\overline{(V-I)}$ or intensity means $\langle V-I \rangle$ of simultaneous pairs, the following approximations will be applied for yielding the intensity means magnitudes $\langle I \rangle$:

$$\langle V \rangle - \langle I \rangle \approx \overline{(V-I)} - 0.026 , \quad \text{LMC} ;$$

$$\langle V \rangle - \langle I \rangle \approx \langle V-I \rangle - 0.019 , \quad \text{Galactic, SMC} ,$$

with the small offsets derived from known photometric data (Coulson, Caldwell, & Gieren 1985).

3. PULSATION PARALLAXES TO CEPHEID VARIABLE STARS

3.1. *Formulation and Realization of the Method*

The realization of the geometric BW method (Baade 1926; Wesselink 1946) derives the distance to a Cepheid variable by matching a distance-independent angular diameter variation $\Phi(t)$ to a distance-dependent linear radius variation $R(t)$. Since the angular and linear size variations

TABLE 1

CALIBRATING GALACTIC CEPHEIDS IN CLUSTERS

Star	log P[a]	V[a]	$(B-V)$[a]	$\langle V-I\rangle$[b]	K[a]	E_{B-V}[a]	μ_0[c]	$\Delta\mu_0$[d]	W[d]
EV Sct	0.490	10.131	1.182	1.472	7.028	0.588	10.88	−0.13	1
SZ Tau	0.498	6.530	0.852	0.996	4.311	0.295	8.79	−0.64	0
QZ Nor........	0.578	8.866	0.908	...	6.622	0.265	11.13	−0.39	0
CF Cas	0.688	11.140	1.180	1.414	8.010[e]	0.590	12.39	+0.07	1
CV Mon	0.731	10.306	1.337	1.646	6.576	0.720	11.26	−0.17	1
V Cen	0.740	6.823	0.872	1.036	4.508	0.285	9.13	+0.09	1
CS Vel	0.771	11.688	1.345	...	8.018	0.728	12.55	+0.15	1
V367 Sct	0.799	11.604	1.844	...	6.662	1.241	11.28	+0.10	1
U Sgr	0.829	6.685	1.091	1.267	3.952	0.438	9.03	−0.07	1
DL Cas	0.903	8.970	1.150	1.364	5.930[e]	0.490	11.24	−0.08	1
S Nor	0.989	6.426	0.945	1.030	4.161	0.171	9.81	−0.05	1
TW Nor	1.033	11.670	2.000	2.361	6.391	1.209	11.43	+0.45	0
V340 Nor......	1.053	8.375	1.151	...	5.586	0.314	11.13	+0.22	1
VY Car	1.277	7.460	1.164	1.215	4.804	0.249	11.59	−0.18	1
RU Sct.........	1.295	9.465	1.701	2.041	5.071	0.930	11.56	+0.08	1
RZ Vel.........	1.310	7.089	1.129	1.267	4.308	0.294	11.15	−0.12	1
WZ Sgr	1.339	8.023	1.404	1.523	4.565	0.435	11.22	+0.06	1
SW Vel	1.370	8.121	1.151	1.317	5.233	0.346	12.11	+0.08	1
T Mon	1.432	6.123	1.168	1.175	3.525	0.171	11.10	−0.41	0
KQ Sco	1.458	9.810	1.936	2.173	4.945	0.886	12.32	−0.35	0
U Car	1.589	6.281	1.178	1.266	3.521	0.303	11.42	−0.12	1
RS Pup	1.617	7.028	1.434	1.579	3.633	0.466	11.25	+0.18	1
SV Vul.........	1.654	7.243	1.465	1.527	3.920	0.440	11.79	+0.08	1
GY Sge	1.713	10.208	2.215	...	4.597	1.140	12.61	−0.10	1
S Vul...........	1.838	8.968	1.898	...	4.599	0.680	13.20	−0.12	1

[a] From Laney & Stobie 1993a, 1993b.
[b] From Caldwell & Coulson 1987.
[c] From Feast & Walker 1987 and Laney & Stobie 1994.
[d] From present work (see § 3.3 and Fig. 5).
[e] From Welch et al. 1985.

should agree in form and phase between each other, the true distance modulus at any instant t along the cycle of the pulsating variable is given by

$$\mu_0(\mathrm{BW}) = 5\log[\langle R\rangle + r(t)] - 5\log[\langle\Phi\rangle + \phi(t)] - 0.159\,, \quad (2)$$

where $\langle R\rangle$, r (radii in solar units) are the mean linear size and the displacement from this mean, respectively, whereas $\langle\Phi\rangle$, ϕ (diameters in milliarcseconds) are the corresponding angular quantities. Spectroscopic and interferometric techniques are required in order to measure the linear and angular size variations, respectively, and then to realize the full potential of the BW approach. However, only galactic Cepheids are expected to be within the reach of these direct techniques (Davis 1993), whereas indirect methods should be currently adopted for estimating the distance to extragalactic Cepheids. In this respect, the BW method requires the calibration of two independent photometric relationships related to fundamental stellar properties:

$$5\log[\langle\Phi\rangle + \phi(t)] = S_M(t) - m(t) = [\alpha + \beta C(t)] - m(t)\,, \quad (3)$$

$$5\log[\langle R\rangle + r(t)] = [S_M(t) + \mu_0 + 0.159] - m(t) = [\gamma + \beta C(t)] - m(t)\,, \quad (4)$$

where S_M is the surface brightness in the m magnitude system; C is a color index; α, β, γ are constants to be determined according to suitable sets of calibrating stars. The reliability and accuracy of such indirect results depend *crucially* on the quality of the SC correlations available and then on the choice of the magnitude-color combination. As it will be shown in the next section, the accurate observational results now available from interferometric techniques clearly enhance the combination $(V, V-K)$ as being free of many systematics for inferring stellar angular sizes over a wide spectral range, notably including the Cepheid domain. This combination also plays a relevant role in determining reliable mean angular sizes by using only two phase points out of the whole cycle of the Cepheid variable. Indeed, by assuming the magnitude $\langle V\rangle$ as the reference in the light curve implies a choice of the radius close to the maximum or minimum values $(\langle R\rangle + r_{\max})$ and $(\langle R\rangle + r_{\min})$, respectively (Balona 1977). Since these peak values are known to be nearly related to the minimum and maximum magnitudes $K_{\min}$ and $K_{\max}$, respectively, in the infrared light curve (Laney & Stobie 1993b), the above equations can be rewritten as

$$5\log\langle\Phi\rangle = \alpha + \beta(\langle V\rangle - \langle K\rangle)_0 - \langle V\rangle_0 + \epsilon\,, \quad (5)$$

$$5\log\langle R\rangle = \gamma + \beta(\langle V\rangle - \langle K\rangle)_0 - \langle V\rangle_0 + \epsilon = s\log P + z\,, \quad (6)$$

$$\epsilon \approx \beta[\langle K\rangle - (K_{\max} + K_{\min}/2)] - 2.5\log[(\langle R\rangle + r_{\max})(\langle R\rangle + r_{\min})/(\langle R\rangle^2]\,,$$

where the unreddened magnitudes yield now true angular and linear radii. The equation (6) is also written in the current form of a PR relation which allows us to provide the radius of the variable star through the independent and potentially very accurate observable P.

The quantity ϵ is readily estimated according to the available light curves of galactic Cepheids together with their radial displacements (Laney & Stobie 1992, 1995). Figure 1 shows the results achieved by adopting the slope $\beta = 1.30$ (see next section). As it can be seen, the values of ϵ do not

TABLE 2

LARGE MAGELLANIC CLOUD CEPHEIDS

HV	log P[a]	V[a]	$(B-V)$[a]	$\overline{(V-I)}$[a]	K[b]
12700......	0.911	14.85	0.73	0.75	13.12
2854.......	0.936	14.64	0.72	0.78	12.90
2733.......	0.941	14.69	0.62	0.76	13.05
12816......	0.960	14.51	0.57	0.72	12.992
2864.......	1.041	14.70	0.79	0.89	12.771
2260.......	1.114	14.86	0.89	1.00	12.753
997........	1.119	14.52	0.85	0.95	12.42
2352.......	1.134	14.17	0.75	0.86	12.286
2580.......	1.229	13.96	0.75	0.86	11.971
2836.......	1.246	14.62	1.01	1.13	12.117
2793.......	1.283	14.09	1.01	1.08	11.821
1013[c]......	1.382	13.83	1.04	0.96	11.456
12815......	1.418	13.46	0.95	1.03	11.332
1023.......	1.425	13.74	1.04	1.06	11.501
1002[c]......	1.484	12.94	0.75	0.76	11.04
899[c].......	1.492	13.43	0.94	1.10	11.40
2294[c]......	1.563	12.68	0.81	0.99	10.77
879........	1.566	13.35	1.03	1.06	11.064
909[c].......	1.575	12.74	0.80	1.01	10.89
2257.......	1.592	13.06	0.96	1.04	10.82
2338.......	1.625	12.78	0.94	0.95	10.67
877........	1.655	13.38	1.20	1.19	10.824
900........	1.677	12.78	0.92	0.92	10.64
953........	1.680	12.28	0.87	0.91	10.28
2369.......	1.684	12.60	0.96	1.05	10.39
2827[d]......	1.897	12.30	1.24	1.08	9.844
5497[d]......	1.995	11.95	1.20	1.15	9.47

[a] From Martin et al. 1979.
[b] From Laney & Stobie 1986 and Welch et al. 1987.
[c] Omitted in $N = 22$ Cepheids.
[d] Omitted in $N = 20$ Cepheids.

TABLE 3

SMALL MAGELLANIC CLOUD CEPHEIDS

HV	log P[a]	B[b]	V[b]	$\langle V-I \rangle$[b]	K[a]
1338[c].......	0.929	15.614	15.160	0.625	13.849
2017........	1.057	15.383	14.713	0.804	12.94
1610........	1.066	15.254	14.669	0.777	13.02
1365[c].......	1.094	15.692	15.016	0.806	13.266
2227........	1.096	15.557	14.791	0.866	12.85
1744........	1.101	15.156	14.556	0.763	12.96
1873........	1.112	15.607	14.882	0.841	12.91
2225........	1.119	15.587	14.782	0.924	12.78
2202........	1.120	15.083	14.412	0.790	12.72
1335........	1.158	15.425	14.773	0.805	12.95
2088........	1.164	15.409	14.660	0.860	12.85
1695[c].......	1.164	15.444	14.711	0.922	12.50
2233........	1.181	14.498	13.935	0.722	12.43
854.........	1.203	14.844	14.245	0.771	12.64
1954........	1.223	14.425	13.840	0.744	12.176
1342........	1.254	14.787	14.206	0.717	12.569
1884........	1.258	15.354	14.474	0.915	12.43
817.........	1.276	14.468	13.854	0.774	12.171
11211[c]......	1.330	14.623	13.835	0.896	11.853
2209........	1.355	14.164	13.549	0.736	11.858
1430........	1.380	15.109	14.300	0.949	12.23
2205........	1.405	14.916	14.070	0.917	11.88
847.........	1.432	14.737	13.916	0.930	11.879
863.........	1.462	14.045	13.338	0.837	11.435
823.........	1.504	14.679	13.769	0.973	11.629
840.........	1.519	14.397	13.580	0.941	11.563
865.........	1.523	13.801	13.121	0.830	11.238
2064........	1.527	14.579	13.726	0.958	11.655
2231........	1.564	14.456	13.517	1.001	11.30
11182.......	1.593	14.707	13.698	1.018	11.454
2195........	1.621	13.757	13.001	0.850	11.104
837.........	1.630	14.117	13.229	0.948	11.165
824[d]........	1.818	13.193	12.367	0.906	10.364
11157[d]......	1.839	14.033	12.943	1.067	10.597
834[d]........	1.867	13.043	12.207	0.891	10.233
829[d]........	1.943	12.738	11.911	0.892	9.972

[a] From Laney & Stobie 1986 and Welch et al. 1987.
[b] From Caldwell & Coulson 1984.
[c] Omitted in $N = 32$ Cepheids.
[d] Omitted in $N = 28$ Cepheids.

exceed the limit of ± 0.03 mag around the ridge-line, independently of the Cepheid period P. Ignoring ϵ would lead to a systematic error affecting individual determinations of $\langle \Phi \rangle$ and then of μ_0. However, its value can be reduced to the negligible level of $|\epsilon| < 0.01$ mag by averaging ensemble of data, as it is currently done in distance estimates, and then it will be ignored in applying the equations (5), (6).

According to the equations (2), (5), and (6), the BW distance modulus to a Cepheid predictable by P, V, K photometry and reddening data can be explicitly written as

$$\mu_0(\mathrm{BW}) = (s \log P + z) - [(\alpha + 0.159) + \beta(V-K) - V] - A_V[1 - \beta(E_{V-K}/A_V)] , \quad (7)$$

where the brackets indicating intensity means are now omitted. The uncertainties involved in calibrating the independent zero points z and α will affect all BW distances given below. For accurate results, the small contribution due to the reddening term should be retained and evaluated. For a given visual absorption A_V, the data depend somewhat on the adopted infrared color-excess (E_{V-K}/A_V). Several color excesses have been proposed with values given by

$$(E_{V-K}/A_V) = 0.839 + (0.148/R_V) ,$$
$$= (1/1.1) ,$$
$$= 2.826/R_V ,$$

(respectively, Cardelli, Clayton, & Mathis 1989; Laney & Stobie 1993; Wegner 1994). For a total-to-selective absorption $R_V = 3.10$, the R_V-dependent excesses from CCM and W lead to the same results as those from LS. For more straightforward comparisons, the color excess suggested by LS according to grain model investigations (Whittet & van Breda 1978) will be adopted below as a conservative value being independent of the absorption R_V.

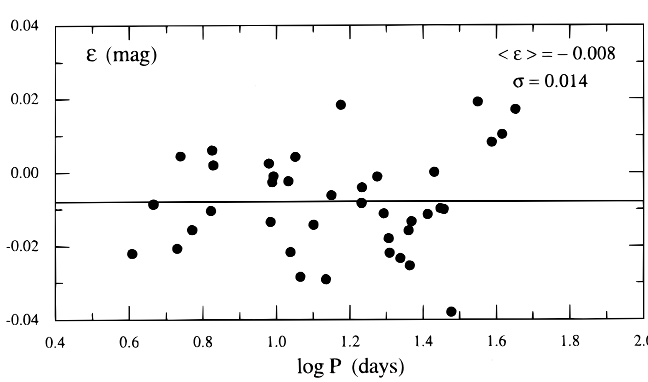

FIG. 1.—Systematic error affecting individual BW distances to galactic Cepheids vs. stellar period. The error is due to Cepheid angular sizes inferred by V, K photometry and a two-phase points approach out of the whole stellar cycle.

3.2. *Calibration of Surface Brightness-Color Correlation*

The surface brightness of a star of intrinsic magnitude m_0 and angular diameter Φ is defined by

$$S_M = m_0 + 5 \log \Phi ,$$

where the zero point can be chosen such that $S_M = m_0$ mag when $\Phi = 1$ mas. The surface brightness is currently calibrated from observational data of nonvariable late-type stars. These have been a suitable target for most of the recently developed interferometric techniques because of their large apparent angular sizes. Therefore, many reliable measurements of stellar diameters have become available with accuracies better than 5% from the modern observational Michelson interferometry (Davis & Tango 1986; Di Benedetto & Rabbia 1987; Mozurkewich et al. 1991). Notably, most of those were also obtained at near-infrared wavelengths where the systematic effect of limb darkening currently related to model-dependent corrections is expected to be as negligible as $\approx 3\%$, allowing measurements of "true" stellar photospheres. According to these improved sets of angular diameters, the empirical correlations calibrated as a function of observable photometric quantities have also steadily reduced the uncertainties in predicting stellar parameters (Di Benedetto 1993). Figure 2 displays the relevant improvement in calibrating the surface brightness scale for $m = V$ and $m = K$ as a function of the same color-index $(V-K)$ adopted as brightness indicators. The set of calibrating diameters includes four dwarfs and 18 giants with accuracies better than 5% and 13 supergiants with accuracies better than 15%, covering the wide spectral range from A to M. A much tighter linear correlation over the whole range clearly appears in the magnitude-color diagram $(V, V-K)$ rather than in $(K, V-K)$, resulting in apparent data with a smaller dispersion induced by several sources of biasing effects. Other magnitude-color combinations using optical photometry do increase both the nonlinear effects as well as the scatter due to random and systematic errors.

The purely observational diagram $(S_V, V-K)$ of nonvariable stars also plays a relevant role in achieving results for Cepheid variables. First, it covers the color range of these stars as well as their luminosity class. Second, it includes practically unreddened stars as well as highly reddened ones. Third, the magnitude-color combination $(V, V-K)$ is expected to be largely insensitive to metallicity and gravity effects. For metallicity as poor as $[\mathrm{Fe/H}] = -0.5$, the overall photometric term predicting stellar size via the equation (5) with $\beta = 1.30$ (see below) is estimated to change by much less than 0.01 mag with respect to that of metal-normal content over a range of gravities and colors $0.75 < \log g < 2.25$ and $1.3 < (V-K) < 3.4$, respectively, according to model-atmosphere results (Bell & Gustafsson 1989). The negligible sensitivity to gravity effects is notably important, since it leads to a well defined single-valued SC correlation with the same functional form for one and the same Cepheid. Since, in addition, the quasi-static approximation seems to be reasonably satisfied for Cepheid variables (Gautschy 1987), it would be expected that the SC correlation of nonvariable stars might also represent the "true" photospheric size variation of Cepheids during their pulsation cycle. This assumption can be checked now, by using the high-precision V, K photometry and radial displacement data from BW techniques along each pulsation cycle of GAL Cepheids (Laney & Stobie 1992, 1995). Indeed, it becomes possible to draw changes in surface brightness as a function of the apparent color index $(V-K)$, in order to determine the best slope β in equation (4) for representing the brightness variations of Cepheids, according to a procedure already outlined (Thompson 1975). Figure 3 displays these variations along the pulsation cycle of the cluster Cepheid U Sgr by using the available simultaneous photometry $(V, V-K)$ and radial displacement data

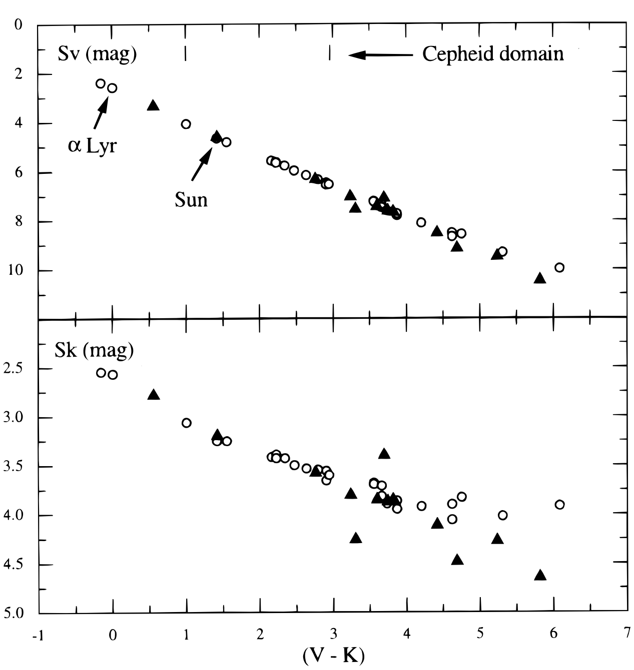

FIG. 2.—Observed visual (S_V) and infrared (S_K) surface brightness for an angular diameter of 1 mas vs. $(V-K)$ color index. The diameter measurements corrected for limb-darkening effects are from interferometric techniques of nonvariable stars at visual and infrared wavelengths. *Open circles*: dwarf and giant stars with $\Delta S \leq 0.11$ mag. *Solid triangles*: supergiant stars with $\Delta S \leq 0.33$ mag.

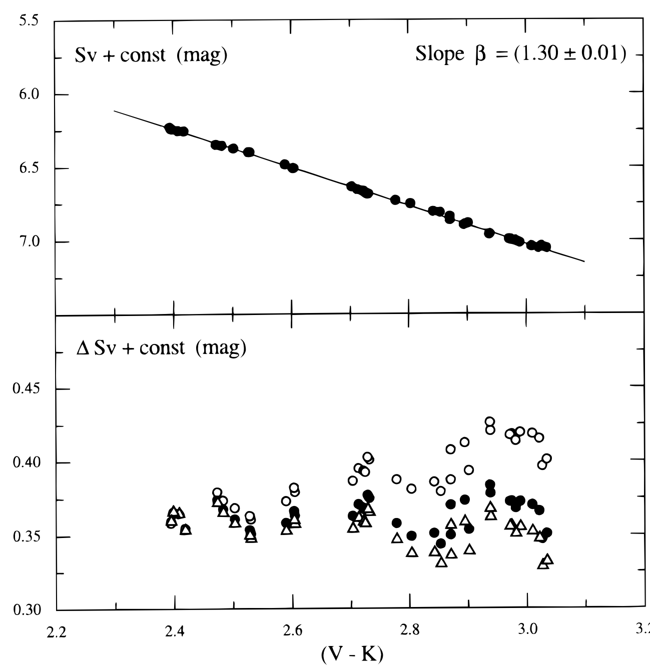

FIG. 3.—Visual surface brightness variations vs. $(V-K)$ color index along the pulsation cycle of the cluster Cepheid U Sgr. *Top*: brightness variations according to radial displacement data from BW techniques. *Bottom*: brightness residuals between SC correlations of different slope β and BW data. *Solid circles*: $\beta = 1.30$ (U Sgr). *Open triangles*: $\beta = 1.33$ (nonvariable supergiants). *Open circles*: $\beta = 1.22$ (nonvariable dwarfs and giants). The residuals are normalized at $(V-K) \approx 2.4$.

(Welch 1994). The upper plot allows us to calibrate the best slope $\beta = (1.30 \pm 0.01)$ representing the photospheric radial displacements of the actual Cepheid. The goodness of this slope in removing brightness variations along the cycle is observed in the lower panel of Figure 3 which draws residuals between BW data and photometric correlations of different slopes. These residuals compare quite well with those achievable by using the slope $\beta = (1.33 \pm 0.01)$ already derived for nonvariable supergiants (Di Benedetto 1993), giving evidence for a likely constant slope β representing the surface brightness correlation of all supergiants over the Cepheid color domain.

The constant α in equation (5) defining the absolute calibration of the visual surface brightness scale cannot be evaluated without further information. It depends on the distance of the star as well as on the interstellar absorption and could be determined from the calibrating Cepheids listed in Table 1. In this case, the value of α would be related to the calibration of the galactic distance scale, which, in turn, relies on heavily reddened stars. However, the most direct and independent approach is to derive the zero-point α according to the accurate angular diameters yielding the brightness data of Figure 2. This method enables as well a more fundamental realization of the BW method, in contrast with several previous realizations (Barnes & Moffett 1985; Fernley, Skillen, & Jameson 1989; Hindsley & Bell 1989). Figure 4 shows the values of α derived according to the slope $\beta = 1.30$ for stars with $(V-K) < 4.0$. Two supergiants with largest errors are not included in the plot. The mean value of α is obtained by averaging only data of practically unreddened dwarfs and giants within the color range matching that of Cepheids. There appears, however, clear evidence for several other stars, notably six supergiants, to be well represented by the same zero point, and this would support the value of α as being independent of the luminosity class within the Cepheid color domain. Thereby, the single-valued SC correlation for Cepheids is likely to be

$$S_V = V_0 + 5 \log \Phi = (2.762 \pm 0.01) + 1.30(V-K)_0 , \quad (8)$$

This improved correlation for inferring Cepheid angular sizes supersedes that applied in previous investigations

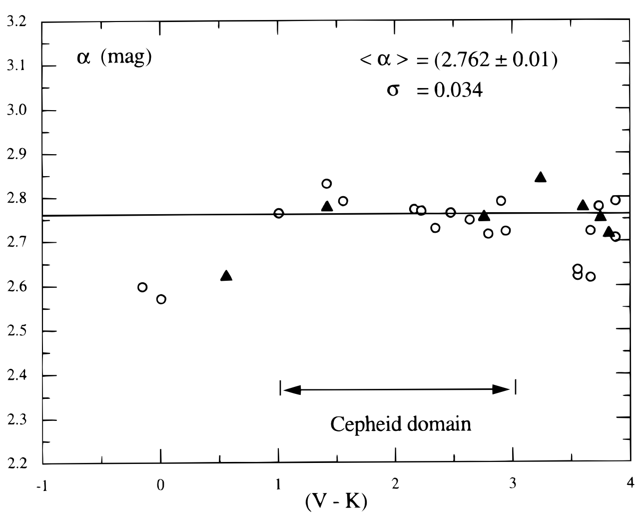

FIG. 4.—Absolute calibration of visual surface brightness for a slope $\beta = 1.30$ vs. $(V-K)$ color index. Angular diameter measurements are from Fig. 2. The solid line represents the mean zero-point by averaging data from dwarfs and giants within the Cepheid domain.

using the surface brightness parameter $F_V = 4.2207 - 0.1\ S_V$ (Di Benedetto 1994, 1995) and will be adopted throughout this paper as "universal" reference, i.e., independent of metallicity, to a level much less than 0.01 mag according to model-atmosphere predictions.

3.3. *Improved Calibration of Galactic Distance Scale*

Several realizations of the spectroscopic BW technique using magnitude-color combinations at optical wavelengths have yielded the sets of calibrating linear radii of galactic Cepheids (Balona 1977; Coulson, Caldwell, & Gieren 1986; Gieren, Barnes, & Moffett 1989). More recently Laney & Stobie (1995) published a sample of 40 BW radii derived by using infrared color indices less sensitive to the limiting factors critically affecting optical BW radii. The corresponding PR relation with coefficients reported in Table 4 is adopted now as the most reliable ridgeline correlation to be combined with the SC relation equation (8). Then, according to the equation (7), the BW distances to galactic Cepheids are given by

$$\mu_0(\mathrm{BW}) = V + 3.75 \log P - 1.30(V-K) + (2.434 \pm 0.04) + 0.18A_V , \quad (9)$$

where the zero-point error derived according to $\sigma/(N-1)^{1/2}$ is dominated by the uncertainties in the radial displacement curves of the BW technique. We have omitted the standard errors of the slopes being not of direct relevance for the present purposes. The small zero-point uncertainty clearly shows that the individual BW data achievable by using high-precision photometry are potentially very accurate. However, they assume that the expected intrinsic spread $\sigma_w(5 \log R)$ around the PR ridgeline due to the finite width of the Cepheid instability strip is $\sigma_w \ll 0.04$ mag and this might not be the case. The value of σ_w is not available at this stage, but will be derived below from empirical data.

The reliability of the actual BW method can be checked by comparing the results from equation (9) with the 25 ZAMS calibrating distance moduli listed in Table 1 assumed as fiducial. They have individual uncertainties of ± 0.04 mag and are consistent with a Pleiades distance modulus of (5.57 ± 0.08) mag (Feast & Walker 1987). We begin to check the internal consistency of the independent approaches by determining the residuals $\Delta\mu_0 = (\mathrm{BW} - \mathrm{ZAMS})$. Compounding the uncorrelated uncertainties of individual data, the overall scatter of these residuals is expected to be $\sigma \approx 0.06$ mag. The residuals are included in Table 1 and displayed in Figure 5. As it can be seen, 20 stars show values of $|\Delta\mu_0| < 3$–$4\ \sigma$, whereas five calibrators deviate by more than 5 σ. The fair consistency of 20 individual data, several of those with the largest reddenings, would suggest a wrong assignment of cluster distance to the remaining five calibrating Cepheids. In any case, if their weight is set to $W = 0$, the average value of the residuals for the 20 calibrators is found to be remarkably close to zero, i.e., $\langle \Delta\mu_0 \rangle = -(0.01 \pm 0.03)$ mag. This result is well within the uncertainty of ± 0.08 mag set by the absolute calibration of the galactic distance scale (Pleiades distance modulus), suggesting the most likely value to be ± 0.04 mag according to an overall calibration by the ZAMS and BW distances. It should be also emphasized that including the five discrepant calibrators would lead to an average value $\langle \Delta\mu_0 \rangle = -0.06$ mag, i.e., the systematic error likely affecting previous calibrations of distance scale by ZAMS.

TABLE 4

GALACTIC AND EXTRAGALACTIC DISTANCE SCALE CALIBRATION BY THE BW METHOD

NUMBER OF STARS						LMC		SMC	
GAL	LMC	SMC	s^a	z^b	σ^c	$\mu_0{}^d$	σ^c	$\mu_0{}^d$	σ^c
40[e]......	0	0	3.75	5.355	0.23	...	...	...	...
	27	36	3.66	5.461		18.585	0.12	19.006	0.18
	22	32	3.65	5.472		18.579	0.11	19.002	0.14
	20	28	3.68	5.437		18.579	0.12	18.997	0.15
	18	19	3.69	5.425		18.582	0.11	19.026	0.18
20[f]......	0	0	3.77	5.342	0.13	...	...	...	...
	27	36	3.66	5.468		18.593	...	19.014	...
	22	32	3.65	5.480		18.587	...	19.009	...
	20	28	3.68	5.445		18.587	...	19.006	...
	18	19	3.69	5.434		18.590	...	19.034	...
15[f]......	0	0	3.66	5.483	0.11	...	...	...	...
	27	36	3.61	5.539		18.594	...	19.016	...
	22	32	3.60	5.550		18.589	...	19.010	...
	20	28	3.62	5.528		18.591	...	19.009	...
	18	19	3.61	5.539		18.583	...	19.018	...

[a] Slope of PR relation from composite fit to the sets of Cepheids.
[b] Zero point of PR relation from the fit to calibrating galactic Cepheids.
[c] Standard deviation derived from the fit.
[d] BW distance moduli dereddened by average reddenings from Cepheids themselves (see § 4.1).
[e] Calibrating BW radii of galactic Cepheids (Laney & Stobie 1995).
[f] Calibrating distance moduli of galactic Cepheids in clusters (see Table 1).

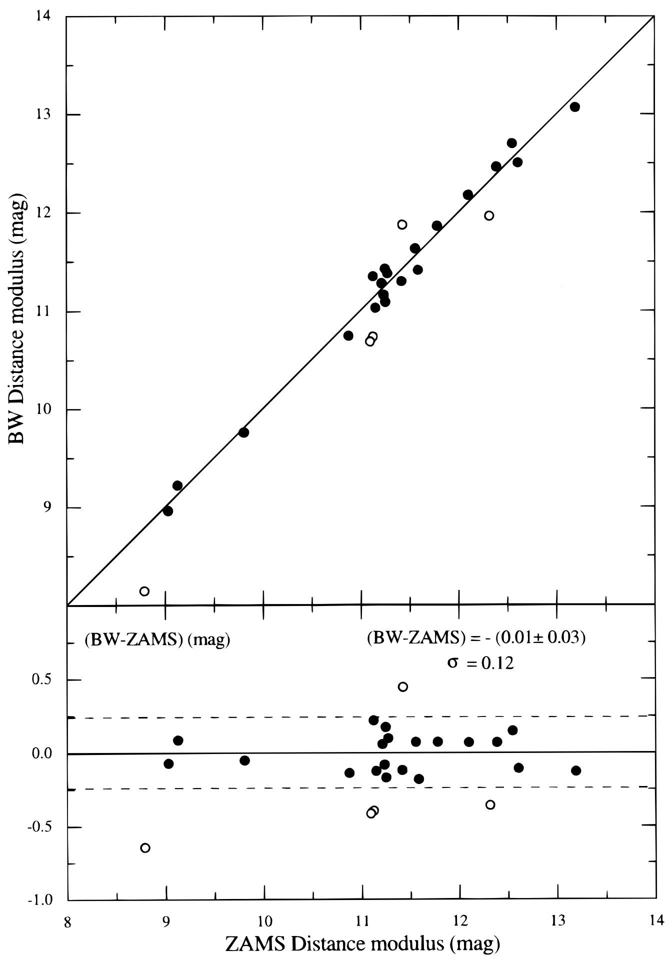

FIG. 5.—*Top*: BW vs. ZAMS distances of calibrating galactic Cepheids in clusters. The solid line is the locus of equal distances. *Bottom*: plot of residuals with dashed lines at ± 0.24 mag reflecting the 2 σ scatter around the solid line. *Open circles*: Cepheids falling more than 3 σ away from the solid line (see Table 1).

Then, it can be concluded that, the BW distances by the equation (9) calibrated according to independent sets of updated observational data provided by spectroscopy, interferometry, and photometry are strongly supported by the ZAMS fitting approach to the heavily reddened Cepheids in clusters up to the uncertainty of ± 0.04 mag. Furthermore, since the results drawn in Figure 5 give high confidence on the individual BW distances achievable through the equation (9), it becomes possible to increase the number of calibrating objects by including all galactic Cepheids with available P, V, K, and R_V data. Conversely, the set of primary distance calibrators in clusters listed in Table 1 can be transformed in a sample of calibrating linear radii for galactic Cepheids. These secondary calibrators will be adopted as fiducial for more detailed discussion below.

3.4. *Improved Calibration of Extragalactic Distance Scale*

According to the results of pulsation theory (Fricke, Stobie, & Strittmatter 1971), the linear radii achievable from a PR relation by measuring the Cepheid period P are expected to be slightly sensitive to the chemical composition of host Galaxies and essentially independent of the temperature of the star. Then, the equation (9) might also be applied in a straightforward manner to obtain distances to any set of extragalactic Cepheids with available P, V, K, and absorption data. However, these extragalactic Cepheids currently lie at a constant distance and have periods longer than the galactic ones. Then, for more accurate individual distances, it seems convenient to recalibrate a composite ad hoc PR relation. This can be done by using the additional sets of LMC and SMC Cepheids for determining the overall PR slope and the galactic Cepheids (primary and/or secondary calibrators) for calibrating the PR zero point (Di Benedetto 1994). Table 4 shows the results achieved from unweighted least-squares fits in the (log R, log P)-plane. The free parameters determined by the fit are the slope s and zero-point z of the PR relation

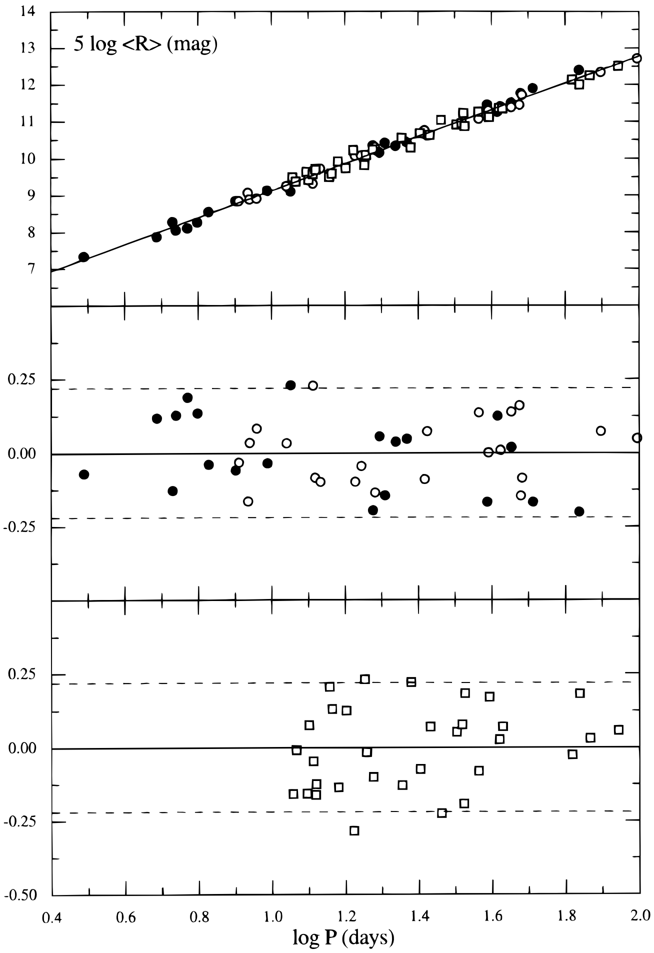

FIG. 6.—*Top*: calibration of period-radius relation by a composite fit to extragalactic and galactic Cepheid data. The solid ridgeline represents the best unweighted fit. *Middle and bottom*: residuals from the ridgeline. *Solid circles*: 20 calibrating galactic Cepheids in clusters. *Open circles*: 22 LMC Cepheids scaled to the ridgeline by μ_0(BW) = 18.58 mag. *Open squares*: 32 SMC Cepheids scaled to the ridgeline by μ_0(BW) = 19.00 mag. The dashed lines, drawn at ± 0.22 mag, reflect the 2 σ scatter around the zero line due to the finite width of the instability strip.

together with the true distance moduli to LMC and SMC. The quoted standard deviations sigmas are derived from the fits themselves.

The first result is achievable by comparing the standard deviations sigmas. Several sources of error can potentially contribute to the overall dispersion affecting each ensemble of Cepheids. These include an expected asymptotic spread set by the intrinsic positioning of a Cepheid within the finite width of the instability strip as well as the observational errors due to the radial velocity measurements (BW radii), photometry, variations of distance across the sample of Cepheids, differential reddening. The dispersion σ (5 log R) = 0.23 from 40 BW radii closely reflecting the high intrinsic scatter of the spectroscopic technique (Laney & Stobie 1995) cannot be of direct concern, since it significantly exceeds all other values. The values of σ related to LMC are found to be closely consistent among each other, despite the quite different quality of K data of each sample; this is further confirmed by the SMC data at an increased level of the average scatter. More important, there is now clear evidence for the scatter of the primary galactic calibrators to be practically the same as that of the extragalactic LMC Cepheids. Since the above mentioned sources of observational errors affecting the actual samples of galactic and extragalactic Cepheids are quite different and independent, it seems reasonable to conclude that the dispersion $\sigma_w = 0.11$ mag, significantly larger than that estimated from the photometric errors ($\ll 0.07$ mag), should be the result of temperature induced variations in the (5 log R, log P)-plane due to the finite width of the Cepheid instability strip. The same ultimate dispersion should also affect the BW distance moduli derived according to the equation (7). It is worth noting that the observed dispersion in Figure 5 is nearly close to the fundamental limit σ_w, as one should expect given that this scatter is the dominant source affecting individual BW determinations. Moreover, the increased values of σ affecting the samples of SMC Cepheids are likely to be ascribed to depth/tilt geometry effects along the line of sight (Caldwell & Coulson 1986). These might explain why the high-precision sample of 19 SMC Cepheids does increase the overall scatter with respect to that less accurate of 32 SMC Cepheids. Indeed, as shown in Table 3, this larger sample does not include three stars of high-precision K-photometry which seem to contribute significantly to the back-to-front scatter.

The second result concerns the coefficients (s, z) of the composite PR relation. There appears to be no practical difference between the slopes derived by using the 40 BW radii and 20 galactic calibrators. The corresponding zero points, compared at a constant slope, are also consistent to less than ± 0.01 mag which closely reflects the degree of consistency between the independent calibrations by BW and ZAMS, as already shown in Figure 5. The data derived from the additional subset of 15 galactic calibrators with available B, V, I, K photometry show the sensitivity of the slope s on the sample size variation. The calibration by 40 BW radii is likely to be preferred since it is free of the intermediate step requiring an SC calibration, whereas the slope from the 22 LMC and 32 SMC Cepheids is adopted as final value according to the reduced scatter of the corresponding solutions. Then, the most accurate PR relation calibrated by the sets of SMC, LMC and galactic Cepheids becomes

$$5 \log R = s \log P + z = 3.65 \log P + (5.472 \pm 0.11) , \tag{10}$$

where the scatter around the PR ridgeline is set by the fundamental limit σ (5 log R) = σ_w = 0.11 mag or 5% of accuracy in predicting the radius R from the period P. According to the equations (8) and (10), the equation (7) yields the individual BW distance moduli to extragalactic and/or galactic Cepheids of known P, V, K, and A_V data by the improved relation

$$\mu_0(\mathrm{BW}) = V + 3.65 \log P - 1.30(V-K) + (2.551 \pm 0.11) + 0.18 A_V , \tag{11}$$

which supersedes the equation (9). Of course, these individual BW data will be affected by the observational errors in addition to the quoted limiting uncertainty and the overall scatter σ should be estimated by fitting the data themselves, as will be shown below.

The third result concerns the true distance moduli to the Magellanic Clouds as primary calibrating extragalactic sources. Although the reddening is found to affect negligibly the BW distances, they are dereddened by the average data E_{B-V}(LMC) = 0.07 and E_{B-V}(SMC) = 0.00 derived from Cepheids themselves (see § 4.1). As it can be seen, the BW solutions for each source are found to be strikingly consistent among each other to less than ± 0.01 mag, whatever is either the set of calibrating galactic Cepheids or the extragalactic sample size or its internal accuracy or its range of covered periods up to log $P = 2.0$. The sets of 22 LMC and 32 SMC Cepheids show the smallest observational scatter σ and are likely to be preferred. They lead to the BW distances of μ_0(LMC) = (18.58 $\pm$ 0.024) mag and μ_0(SMC) = (19.00 $\pm$ 0.025) mag or d(LMC) = (52.0 $\pm$ 0.6) kpc and d(SMC) = (63.2 $\pm$ 0.7) kpc with uncertainties *not* including the above contribution of ± 0.04 mag due to the absolute galactic distance scale calibration.

Figure 6 shows the very tight PR relation plotted star-by-star for the 32 SMC, 22 LMC and 20 galactic Cepheids. The LMC and SMC Cepheid data are scaled to those of galactic stars according to their BW distances. Then, the Cepheid-based distance scale calibration can be extended to include the extragalactic domain with the actual BW distance moduli to LMC and SMC chosen as fiducial. Of course, these distances are less fundamental than those of galactic Cepheids, since they rest on the further assumption of both PR and SC correlations as being largely insensitive to metallicity according to the stellar pulsation and atmosphere theories, respectively. This assumption avoids any model-dependent correction which would imply an accurate knowledge of the systematic effect involved. On the other hand, the strong support to the actual extragalactic BW distances by the equation (11) comes now from the distance to LMC which compares remarkably well with the distance measured by the geometric expansion parallax of the SN 1987A in LMC via *HST* observations. This recently improved estimate is given by d(SN 1987A) = (51.1 $\pm$ 1.5) kpc (Panagia et al. 1996).

4. CEPHEID DISTANCES BY PL AND PLC RELATIONS

Galactic and extragalactic distances can be derived from calibrated PL and/or PLC relations. The PL relations are widely accepted as being practically unbiased by metallicity according to recently reiterated theoretical investigations (Iben & Renzini 1984; Stothers 1988; Chiosi et al. 1993) and to an empirical test (Freedman & Mador 1990). Madore & Freedman (1991) reviewed the modern techniques for determining the extragalactic distances and reddenings according to PL relations. These will be now revisited taking into account of the actual BW results. For the present purpose, I will be considering photometry in the B, V, I, K bands and tackling again the problem of the calibration of PL and PLC relations with the major attention to the Johnson-Cousins V and I passbands relevant in the *HST* observations of Cepheids (Freedman et al. 1994; Saha et al. 1994).

4.1. *Distances by PL Relations*

Madore & Freedman (1991) present the multiwavelength PL relations used for determining the distances and reddenings of Cepheids in external galaxies. Specifically, all PL relations have been derived from self-consistent sets of data based on the same stars in order to eliminate sample-dependent variations in the solutions. A sample of LMC Cepheids with periods in the range $0.2 < \log P < 1.8$ was chosen as fiducial (Madore 1985) together with adopting ab initio a true distance modulus μ_0(LMC) = 18.50 mag for an average total (foreground + internal) line-of-sight reddening E_{B-V}(LMC) = 0.10 mag. Then, all fits have been carried out for such a set of stars defined by the simultaneous availability of photometry at B, V, and I wavelengths. Finally, all other Cepheid-based distances have been scaled and dereddened according to a total-to-selective absorption $R_V = 3.30$ along with the law of extinction determined by Cardelli et al. (1989). For specific applications to *HST* observations of Cepheids, the procedure outlined in Freedman et al. (1994) is currently applied to the Cousins I and Johnson V passbands to derive distances and reddenings to Cepheids in external galaxies.

For the self-consistent subsets of 22 LMC and 32 SMC Cepheids of Table 4 we are able to check the consistency of distance data derived independently by the BW and PL relations. In keeping with the BW results of Table 4, the absolute galactic calibration should be incorporated in the PL data by taking into account that all the actual relations, except PL (B), are expected to be largely insensitive to metallicity according to either the stellar evolution theory (PL relations) or the stellar pulsation and atmosphere theories (BW). To do this, we can adopt either the subset of 38 BW radii as secondary calibrators or the set of 15 calibrating galactic Cepheids in clusters with available B, V, I, K photometry. The calibration of PL relations can be carried out according to the same procedures already done for the PR relation above. Table 5 summarizes the results from the composite least-squares fits in the (M_X, log P)-plane defined by

$$M_X = X - \mu_X = -S_X \log P - Z_X \,, \qquad (12)$$

where M_X is the absolute magnitude and μ_X the apparent distance modulus in the band $X = B, V, I, K$. The sample of 22 LMC Cepheids is selected from the larger list of Madore (1985), but it does not include stars with periods $\log P < 0.9$ lacking their K-photometry for BW data. Hence, to avoid any bias in a slope fitted to the actual data, due to incompleteness at short periods, the slopes have been fixed to the values already derived by Madore & Freedman (1991). Then, each fit determines the galactic PL zero point, the apparent distance moduli to LMC and SMC and the dereddened ones according to either the currently adopted average reddenings for these extragalactic sources (Caldwell & Coulson 1985) or the values yielded by Cepheids themselves through photometry in the V and I bands (see below). The coefficients of the PR relation along with the BW distances to LMC and SMC are also reported for a more straightforward comparison. The quoted standard deviations sigmas are derived from the fits themselves.

The achieved results in Table 5 call for the following discussion:

1. The systematic of PL relations at wavelengths ranging from the blue to the near-infrared can be readily derived. The observed dispersion σ is of major concern. Ignoring the higher values set by the secondary BW calibrators (too high observational uncertainties due to BW technique), the 15 primary galactic calibrators are likely to show the intrinsic scatter due to temperature-induced variations in brightness across the instability strip, according to the discussion in § 3.4. Indeed, this scatter is seen to decrease with increasing wavelength up to the asymptotic value σ (5 log R) = 0.11 mag observed to affect the PR relation. Furthermore, there would be evidence for a quite large component increasing

TABLE 5

GALACTIC AND EXTRAGALACTIC DISTANCE SCALE CALIBRATION BY PL RELATIONS

				LMC (N = 22)				SMC (N = 32)			
RELATION	S[a]	Z[b]	σ[c]	μ[d]	μ_0[e]	μ_0[f]	σ[c]	μ[d]	μ_0[e]	μ_0[f]	σ[c]
PL (B)......	2.43	0.934	0.32	18.87	18.56	18.54	0.38	18.91	18.65	18.91	0.38
		0.948	0.23	18.88	18.58	18.61		18.92	18.66	18.92	
PL (V)......	2.76	1.333	0.28	18.81	18.58	18.56	0.28	19.01	18.81	19.01	0.28
		1.354	0.19	18.83	18.60	18.62		19.03	18.83	19.03	
PL (I).......	3.06	1.760	0.26	18.71	18.57	18.56	0.20	19.00	18.88	19.00	0.22
		1.798	0.17	18.74	18.61	18.62		19.04	18.92	19.04	
PL (K)......	3.42	2.322	0.23	18.60	18.58	18.58	0.13	19.00	18.98	19.00	0.16
		2.340	0.11	18.62	18.59	18.60		19.02	19.00	19.02	
PR[g]	3.65	5.472	0.23	18.54	18.58	18.58	0.11	19.00	19.04	19.00	0.14
		5.494	0.11	18.56	18.60	18.60		19.02	19.06	19.02	

[a] PL slope fixed to the value from Madore & Freedman 1991.

[b] PL zero point determined by 38 BW radii as secondary calibrators (upper values) and by 15 galactic Cepheids as primary calibrators (lower values) with available B, V, I, K photometry.

[c] Standard deviation derived from the fit.

[d] Apparent distance moduli.

[e] Distances dereddened by average reddenings E_{B-V}(LMC) = 0.07 mag and E_{B-V}(SMC) = 0.06 mag (Caldwell & Coulson 1985).

[f] Distances dereddened by average reddenings from Cepheids themselves (see § 4.1).

[g] Calibration by BW distances (see Table 4).

significantly the scatter of the extragalactic LMC and SMC Cepheids with respect to the intrinsic level set by the galactic Cepheids. It is easy to show that this additional component increases with decreasing the wavelength and is likely due to the differential reddening along the line of sight of the extragalactic sources. Accordingly, it does increase the intrinsic scatter of PL relations quite sensitive to reddening, whereas it disappears in the BW data.

2. According to the standard *HST* procedure, the average color-excess E_{V-I} estimated by the average apparent V and I distance moduli from equation (12) may be explicitly written as

$$E_{V-I} = A_V - A_I = \mu_V - \mu_I \,,$$
$$= \langle (V-I) - (S_I - S_V) \log P \rangle - (Z_I - Z_V) \,, \quad (13)$$

where the angle brackets indicate ensemble averages over the set of Cepheids. By adopting an extinction law, the equation (13) is currently solved in the unknown variable E_{B-V}. To be consistent with the *HST* procedure, I shall adopt the extinction $\rho = (A_I/A_V) = [0.773 - (0.587/R_V)]$ with $R_V = 3.30$ (Ferrarese et al. 1996). Of course, the reddening data remain closely related to these absorption ratios as well as to the absolute galactic calibration of PL relations through the zero points Z_V and Z_I. However, it is worth noting that the actual extinction gives rise to the same color-excesses as the galactic law of Cepheids $E_{B-V} = E_{V-I}/1.34$ (Dean, Warren, & Cousins 1978) and that the galactic zero points now reside on two improved and quite independent sets of calibrators both providing very consistent results. Since the error $\sigma/(N-1)^{1/2}$ on the zero point of each PL relation is nearly the same for either of the calibrations, we assume the final unweighted average zero points $Z_V = 1.344$ and $Z_I = 1.779$. These are slightly different from those currently adopted in the *HST* data reduction, i.e., $Z'_V = 1.40$ and $Z'_I = 1.81$ (Ferrarese et al. 1996), implying somewhat lower estimates of color excesses from the actual absolute calibration according to the equation (13).

3. The apparent LMC distance moduli systematically decrease from B to K as one might expect being the corresponding photometric data selectively reddened. According to equation (13), the average color-excess of LMC would be $E_{B-V}(\text{LMC}) \approx 0.07$ mag quite consistent with the current value. In contrast, the apparent SMC data show to be in striking good agreement among each other, except the value from the B band, leading to an average color-excess $E_{B-V}(\text{SMC}) \approx 0.00$ mag. This excess differs significantly from the average value of $E_{B-V}(\text{SMC}) \approx 0.06$ mag (Caldwell & Coulson 1985) currently adopted to deredden previous data, but it seems to be quite consistent with *IUE* results, as reported by Bessell (1991). Of course, the total (foreground + internal) reddening of the Magellanic Clouds estimated from Cepheids themselves could be biased by metallicity effects. Indeed, the solution derived by applying the equation (13) implies the ridgeline of all period-color (PC) relations in the color $(V-I)$ to be independent of metallicity. For the actual sets of LMC and SMC Cepheids, this independence seems to be supported by the fact that even by solving with the average apparent V and K distance moduli would lead to the same average color excesses as those from V and I photometry.

4. According to the reddenings estimated from Cepheids themselves, all dereddened distances quoted for the same source would be much more consistent among each other, notably with PL (K) and BW data largely insensitive to reddening and metallicity. But the SMC distance from the B band would remain lower by about 0.1 mag. On the other hand, the SMC is known to be a metal-poor Galaxy with a degree of metallicity differing by a factor of about 4 with respect to the solar metallicity (Russell & Bessell 1989). Hence, there would be evidence for all PL distance indicators to be unbiased by the metallicity of SMC to a level much less than 0.1 mag, except the PL relation in the B-band. In contrast, such a conclusion would be hard to be supported using the older value of reddening. As previously found (Di Benedetto 1995), this led to the wrong conclusion of PL relations as being severely biased by metallicity.

5. According to the apparent V and I distance moduli from the equation (12) and reddening from Cepheids themselves by the equation (13), the *HST* procedure derives a true average distance modulus from any set of extragalactic

Cepheids given by

$$\mu_0(\mathrm{PL}) = [(\mu_V + \mu_I)/2] - [(A_V + A_I)/2] , \quad (14)$$
$$= [(\mu_V + \mu_I)/2] - [(1+\rho)/2(1-\rho)](\mu_V - \mu_I) ,$$
$$= \langle V + a \log P - b(V-I)\rangle + c ,$$
$$= \langle V + 3.501 \log P - 2.47(V-I)\rangle + 2.413 , \quad HST \text{ calibration} ,$$
$$= \langle V + 3.501 \log P - 2.47(V-I)\rangle + 2.418 , \quad \text{actual calibration} ,$$

where $b = 1/(1-\rho)$, $a = S_V + b(S_I - S_V)$ and $c = Z_V + b(Z_I - Z_V)$. The final observational relationships show that the linear combination of PL (V) and PL (I) relations yields the same average distance as that of a PLC relation in the magnitude-color combination (V, $V-I$) with the coefficients closely related to those of the PL relations and to the extinction ratio $\rho = (A_I/A_V)$. However, in the actual context, it would be quite difficult to take full advantage by the PLC structure of the distance solutions. Indeed, the results achievable according to the absorption term set either to zero or to the value determined by the equation (13) will in general differ significantly from one another, clearly confirming their remarkable sensitivity to reddening effects, as expected for PL relations rather than for PLC ones. Furthermore, the overall dependence of the distance modulus on the extinction ratio ρ will critically affect the final uncertainty of the average results due to cumulative photometric errors on the apparent V and I moduli.

It should be also noted that the quite small difference $\Delta\mu_0 = c - c' = 0.005$ mag between the zero points of the equation (14) using either the actual galactic calibration set by $Z_V = 1.344$ and $Z_I = 1.779$ or the *HST* extragalactic calibration set by $Z'_V = 1.40$ and $Z'_I = 1.81$ leads to distances strikingly consistent between each other. Then, it would seem to be quite surprising to find a discrepancy as significant as 0.09 mag in the true distance modulus to the LMC, i.e., 18.59 mag as suggested by the actual data of Table 5 and 18.50 mag as assumed ab initio by the *HST* data reduction according to the calibrations of Madore & Freedman (1991). However, the equation (12) shows that the *HST* data reduction does not necessarily require the explicit knowledge of the absolute distance to LMC but only of its apparent modulus for each passband. For example, the apparent V distance modulus $\mu_V = 18.83$ mag, assumed ab initio in the Madore & Freedman calibration, is found to be quite consistent with the value of Table 5 just to within the above difference $\Delta\mu_0$. Instead, the average absorption $A_V = 0.33$ mag seems to be a wrong assignment of overall extinction for LMC and it is likely responsible for the observed discrepancy. In fact, according to the PL results reported in Table 5 which rely on the improved galactic calibration and on the self-consistent determination of *all* extragalactic reddenings, the average line-of-sight reddening of LMC for a total-to-selective absorption $R_V = 3.30$ is likely to be $E_{B-V} \approx 0.07$ mag, i.e., $A_V \approx 0.23$ mag, which yields a true distance modulus of ≈ 18.59 mag by any of the PL relations. Now, this result is strongly supported to within 0.01 mag by the independent and almost reddening-free BW distance to the LMC discussed in § 3.4 and reported in Table 5.

4.2. *Distances by PLC Relations*

Up to now, current approaches to the PLC relations aimed at solving for the coefficients applying multilinear regression techniques to LMC Cepheid data (Martin et al. 1979; Caldwell & Coulson 1986). There are some disadvantages in using these techniques. First, they can lead to somewhat different values of the free parameters depending on the way the data itself were analyzed statistically (Feast & Walker 1987). In these circumstances, it becomes difficult to derive meaningful coefficients of the PLC relations, notably the slope of the color term (Brodie & Madore 1980) which closely determines the PLC sensitivity to metallicity. This effect on the zero point is of great significance for reliable distances and must be currently derived by applying model-dependent corrections. Second, the need to introduce a color term for reducing the spread of a PL relation becomes questionable for extragalactic applications, since neither the differential reddening across a galaxy nor the true color deviations within the strip, i.e., the intrinsic scatter due to the finite width of the Cepheids instability strip, are available in advance. Then, we can severely underestimate the errors affecting individual PLC distances, as shown by comparing the equations (9) and (11) and by the above discussion on PL relations.

In contrast with previous PLC approaches, the geometric BW method suggests the empirical reliable path to yield the intrinsic structure of the PLC relations for Cepheids. Hence, it becomes important to calibrate it by including other colors as brightness indicators, notably the Johnson-Cousins color ($V-I$). Since the PR relation is already established, only the SC correlation, i.e., its coefficients (α, β) of the equation (3), has to be suitably calibrated in order to predict Cepheid angular radii by using the actual color C. Since any color C is also known to be more or less sensitive to chemical abundances, their effects should be determined at least for the ensemble of LMC and SMC Cepheids with well known degree of metallicity. The most obvious step is to exploit the two surface brightness relations adopting the correlation equation (8) as *unbiased reference* for ensemble of Cepheids with well-matched range of periods (Di Benedetto 1995). Table 6 summarizes the results of calibration of the colors ($V-I$) and ($B-V$) by using the subsets of 28 SMC, 20 LMC, and 32 galactic Cepheids covering the same common range of periods $0.9 \leq \log P < 1.8$ which includes the most relevant extragalactic domain. The solutions are obtained by assuming the slope β to be the same as that already available from BW techniques [according to the eqs. (5) and (6)] and adopting the zero-point α as the free parameter to be determined. Since α is expected to change as a function of the average intrinsic color of each Cepheid distribution, i.e., of the degree of metallicity, it has been referred to the 32 metal-normal GAL Cepheids with available B, V, I, K photometry, whereas ($\alpha + \Delta\alpha$) is the zero point of each sample of metal-poor Magellanic Cloud Cepheids. The zero-point shift $\Delta\alpha$ is also determined according to model-atmosphere results with the available stellar parameters. As it can be seen, the shifts in the color ($B-V$) for SMC Cepheids are found to be quite consistent with the theoretical ones of roughly equal degree of metallicity, provided that the photometry is dereddened by the average color excess derived from Cepheids themselves according to equation (13). In contrast, the older reddening would decrease significantly the shifts, leading to the wrong

TABLE 6

SURFACE BRIGHTNESS-COLOR CALIBRATIONS

C_0	β[a]	α[b]	σ[c]	LMC ($N = 20$) $\Delta\alpha$[d]	LMC $\Delta\alpha$[e]	LMC σ[c]	SMC ($N = 28$) $\Delta\alpha$[d]	SMC $\Delta\alpha$[e]	SMC σ[c]	MODEL[f] ($\Delta\alpha$)
$(B-V)_0$	2.15	3.304	0.10	0.10	0.10	0.10	0.30	0.19	0.08	0.29–0.32
$(V-I)_0$	2.88	2.749	0.08	0.007	0.007	0.09	0.030	0.026	0.08	<0.02

[a] Assumed slope from BW techniques (Balona 1977; Coulson et al. 1986).
[b] Calibration by 32 galactic Cepheids with periods $0.9 \le \log P < 1.8$. The zero point is derived according to $\alpha = S_V - \beta C_0$ with S_V from eq. (8).
[c] Standard deviation derived from the fit.
[d] Zero-point shift according to average reddenings from Cepheids themselves (see § 4.1).
[e] Zero-point shift according to average reddenings from previous data (see Table 5).
[f] Model parameters: $T = 4500$–5500 K; $\log g = 1.50$; $[A/\mathrm{H}] = -0.5$ (Bell & Gustafsson 1989).

conclusion of a PLC (V, $B-V$) relation as being much less affected by metallicity than expected by model-atmosphere results (Di Benedetto 1995). The shifts in the color ($V-I$) are found to be, in any case, as negligible as ≈ 0.03 mag up to the metallicity of SMC. Since the relevant dependence on metallicity of a BW relation is expected to be related to the SC correlation, the actual shifts represent as well reliable effects on the zero point of the corresponding PLC relations. It is worth noting that these effects show close agreement with recent theoretical predictions (Chiosi et al. 1993) and are found to be significantly weaker than those determined by a previous investigation (Caldwell & Coulson 1986). Of course, we can deal with the magnitude-color combination (V, $V-I$) as being practically free of metallicity to the extent that the combination (V, $V-K$) is free of metallicity. Then, according to the actual galactic calibration, the SC correlation in the Johnson-Cousins color ($V-I$) within the Cepheid color domain $0.6 \le (V-I)_0 \le 1.1$ (see Fig. 7) becomes

$$S_V = V_0 + 5 \log \Phi = (2.749 \pm 0.02) + 2.88(V-I)_0 \,. \quad (15)$$

Adopting the PR relation equation (10), the individual BW (or PLC) distance moduli to the extragalactic and/or galactic Cepheids of known P, V, I, and A_V data takes the form of

$$\begin{aligned} \mu_0(\mathrm{BW}) &= V + 3.65 \log P - 2.88(V-I) + (2.564 \pm 0.11) \\ &\quad - A_V[1 - 2.88(E_{V-I}/A_V)] \,, \\ &= V + 3.65 \log P - 2.88(V-I) + (2.564 \pm 0.11) \\ &\quad + 0.17 A_V \,, \end{aligned} \quad (16)$$

where the value $(E_{V-I}/A_V) = (1-\rho) = 0.405$ has been adopted to be consistent with the *HST* data reduction. The small reddening term shows that the actual BW distances cancel out the extinction on a star-by-star basis as the BW data given by the equation (11).

The Figure 7 shows the PC relation along with the residual plots $\Delta\mu_0 = \mu_0 - \mu_{\mathrm{REF}}$ of individual distance moduli through equation (16), as a function of period for 36 SMC, 34 LMC, and 43 galactic Cepheids with photometry available in the V and I passbands, including seven short-period LMC Cepheids (Walker 1987). The dotted lines drawn in the PC diagram determine the most likely range of colors for reliable BW data within the domain $0.9 \le \log P \le 1.8$. The solid lines in the residual plots represent the best BW reference μ_{REF} set by the distances determined according to the equation (11). This reference is derived on a star-by-star basis for galactic Cepheids using reddening data available for individual stars (Laney & Stobie 1993a), whereas an average distance modulus dereddened according to the average reddening from equation (13) is adopted for LMC and SMC Cepheids (see Table 4 and Fig. 6). The dashed lines, drawn at ± 0.22 mag, reflect the finite width of

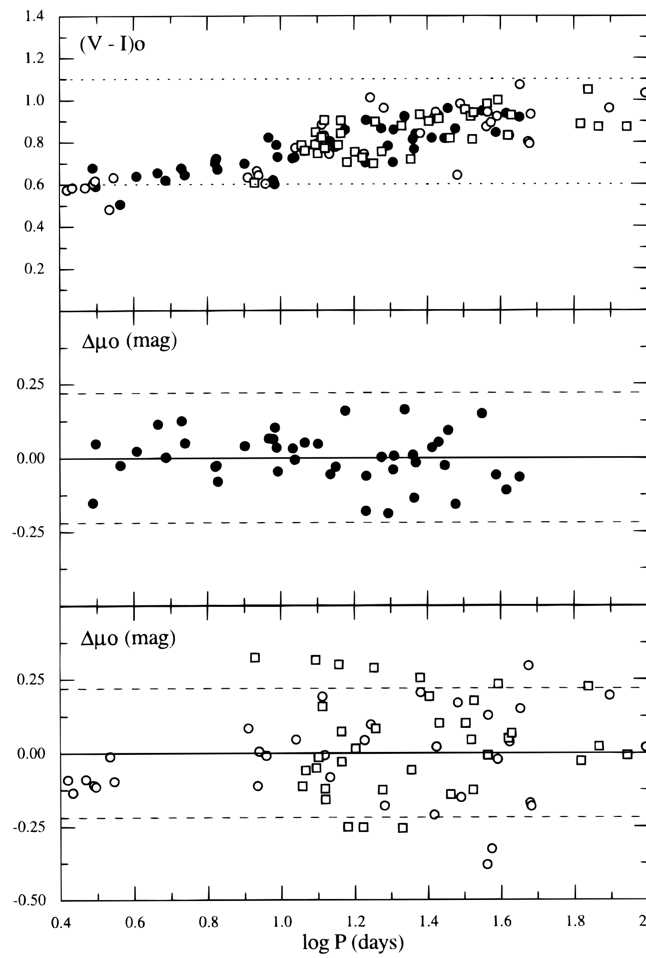

FIG. 7.—Calibration of BW method in the Johnson-Cousins color ($V-I$). *Top*: period-color relation for 36 SMC, 34 LMC, 43 galactic Cepheids. The dotted lines limit the range of allowed colors according to periods within the extragalactic domain $0.9 \le \log P < 1.8$. *Middle and bottom*: plots of residuals ($\mu_0 - \mu_{\mathrm{REF}}$) with distance moduli from eqs. (11) and (16). Symbols are the same as in Fig. 6.

the Cepheid instability strip, and thus the intrinsic 2 σ scatter envelope around the best fitting zero-point ridgeline. Now, the distribution of galactic Cepheids is found to be well within the 2 σ scatter, in contrast with the data of Figure 6. Indeed, all 43 galactic stars, used as secondary calibrators, cancel out on a star-by-star basis the large contribution of scatter due to the PR relation. Then, a lower dispersion should be expected according to the high-precision photometric data, provided that they have been accurately dereddened star by star. This, notably, implies the individual estimates of E_{V-I} given by the equation (13) to be quite consistent with the independent reddenings available for galactic Cepheids. Instead, the star-by-star representation of extragalactic Cepheids, including all sources of scatter affecting distance results, notably additional stars with respect to the plots of Figure 6, shows a dispersion slightly exceeding now the 2 σ envelope. Notice also the evidence for somewhat systematic deviations of the shortest period LMC Cepheids.

According to the individual data from the equation (16) dereddened now by the average reddening from equation (13), the average BW distance modulus from any set of extragalactic Cepheids can be written as

$$\mu_0(\mathrm{BW}) = \langle V + a_0 \log P - b(V-I)\rangle + c_0 ,$$
$$= \langle V + 3.527 \log P - 2.47(V-I)\rangle + 2.386 , \quad (17)$$

where $b = 1/(1-\rho)$, $a_0 = s + (b-\beta)(S_I - S_V)$, and $c_0 = (q - \alpha - 0.159) + (b - \beta)(Z_I - Z_V)$. The equation (17) shows that the PLC coefficients become dependent on the extinction ratio ρ, but this dependence is found to be negligible, since the distances by using either the equation (16) with the absorption term set to zero or the more accurate equation (17) will in general differ little from one another and this is simply a restatement of the result that a PLC relation is largely insensitive to reddening. More important, the observational relationship (17) allows now to fully exploit the PLC structure of the distance solutions derived by the linear combination of the PL (V) and PL (I) relations according to the equation (14). Indeed, the comparison of the equation (14) with (17) leads to the following remarkable conclusions:

1. The relationships have exactly the same color coefficient and yield an average difference $\mu_0(\mathrm{PL}) - \mu_0(\mathrm{BW}) = -0.026\langle \log P\rangle + \delta$ where δ is equal to 0.027 or 0.032 depending on the adopted absolute calibration of the PL relations. In any case, the difference is found to be less than 0.01 mag for any set of Cepheids whose average period matches the extragalactic range $0.9 \le \log P < 1.8$. The immediate concern is that the PL distances by the *HST* procedure come forward as BW distances in the magnitude-color combination (V, $V-I$), and then they will be affected by the reddening and metallicity in the same way as the BW data.

2. The error budget in the PL distances should be the same as that of the corresponding BW distances, and then it can be reliably achieved from the fits on the individual BW data which include all of the random fluctuations, as clearly shown in Figure 7. Now, the PL distances by the *HST* procedure are to be considered as almost reddening-free solutions which, like the BW data, cancel out the extinction on a star-by-star basis, notably the differential reddening internal to parent galaxies.

In order to show the actual improvement in reducing Cepheid data from *HST* observations, the individual BW distances to the Virgo galaxy M100 are determined by using the equations (13) and (16). The published photometry V^{ph} and I^{ph} is chosen as fiducial (Ferrarese et al. 1996) and Table 7 reports the relevant data for the BW determinations. The range of allowed colors and periods set by the actual calibration is applied along with the dispersion σ_0 in order to select the number of useful Cepheids for determining the average BW modulus. The overall scatter $\sigma_0 = 0.21$ mag is derived from the observational errors according to the relation

$$\sigma_0 = \sqrt{(1.47\sigma_V)^2 + (2.47\sigma_I)^2 + \sigma_w^2} , \quad (18)$$

where $\sigma_V = \sigma_I = 0.06$ mag are the uncorrelated random errors on V and I magnitude of the *HST* data and $\sigma_w = 0.11$ mag is the intrinsic scatter due to the width of the instability strip. As it can be seen, a well-determined solution $\mu_0(\mathrm{BW}) = (30.98 \pm 0.06)$ mag is achievable by omitting only the two stars falling more than 8 σ_0 away from the average value, although up to 21 stars, shown as open symbols in the lower panel of Figure 8, might contribute in various degree to increase the scatter σ_0 up to the largest dispersion σ derived from the fits themselves. Table 7 also reports the BW distance $\mu_0(\mathrm{BW}) = (31.00 \pm 0.06)$ mag determined by the same sample size as that selected in the *HST* data reduction for the final assignment of $\mu_0(\mathrm{PL}) = (30.99 \pm 0.17)$ mag (Ferrarese et al. 1996; see Table 8). As expected from the above discussion, the difference between the results is found to be quite small, i.e., $\mu_0(\mathrm{PL}) - \mu_0(\mathrm{BW}) = -0.013$ mag being $\langle \log P\rangle = 1.54$, but the BW distance is considerably more accurate since it benefits from the almost reddening-free approach of a BW realization. Adopting the most recent *HST* calibration of the photometric zero point, the distance modulus should be increased by (0.050 ± 0.007) mag (Ferrarese et al. 1996; see note added in manuscript) leading to the BW distance $d(\mathrm{M100}) = (16.1 \pm 0.5)$ Mpc for the sample size of 50 Cepheids reported in Table 7.

TABLE 7

DISTANCE MODULI TO VIRGO GALAXY M100 BY THE BW METHOD

Constraint	Number of Cepheids	σ[a]	μ_0[b]
None	52	0.52	30.92 ± 0.07
$<8\ \sigma_0$[c]	50	0.43	30.98 ± 0.06
$<3\ \sigma_0$[c]	41	0.29	30.96 ± 0.05
$0.60 \le (V-I) \le 1.10$[d]	41	0.53	30.91 ± 0.08
$<8\ \sigma_0$[c]	39	0.42	30.98 ± 0.07
$<3\ \sigma_0$[c]	32	0.26	30.99 ± 0.05
$0.90 \le \log P < 1.80$[d]	49	0.50	30.94 ± 0.07
$<8\ \sigma_0$[c]	48	0.43	30.98 ± 0.06
$<3\ \sigma_0$[c]	40	0.33	30.98 ± 0.05
$20 < P < 70$[e]	42	0.38	31.00 ± 0.06

[a] Standard deviation derived from the fit.

[b] Average true distance modulus according to the observational relation (11′).

[c] From photometric errors and width of instability strip: $\sigma_0 = 0.21$ mag from eq. (18).

[d] Range of allowed values from the calibration of the BW method in the $(V-I)$ color.

[e] Selected sample in the *HST* data reduction (Ferrarese et al. 1996, Table 8).

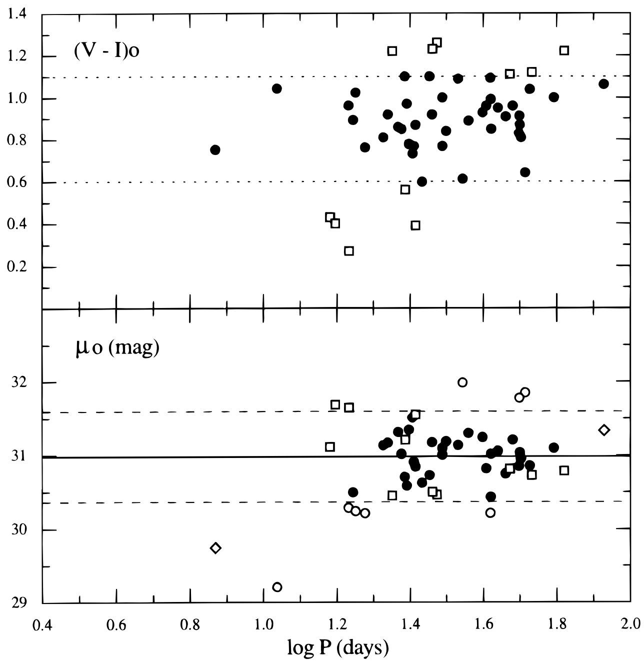

FIG. 8.—Determination of BW distance modulus to Virgo galaxy M100. *Top*: period-color relation. The dotted lines limit the allowed colors marked by solid circles. *Bottom*: individual distance moduli. The solid line is the adopted average distance modulus. The dashed lines, drawn at ± 0.62 mag, reflect the $3\sigma_0$ scatter from the photometric errors and the finite width of the instability strip. *Open symbols*: stars selected by the constraints listed in Table 7.

As a final remark, it is worth noting that the average reddening derived according to equation (13) might result $E_{V-I} < 0$ (Saha et al. 1994). This would mean that most of the observed individual colors lie systematically below the expected PC ridgeline set by the calibration. In this case, the large color coefficient in equation (16) could lead to cumulative systematic deviations towards longer distances. Avoiding this effect implies the omission of all Cepheids whose color does not satisfy the range of calibration drawn in Figure 7. Among several sources of error, the metallicity might really contribute to biasing the average color of the Cepheid sample. Then, the actual selection could suggest one empirical path for avoiding metallicity effects on the extragalactic distances.

5. CONCLUSIONS

Fundamental determinations of Cepheid distances remain an open problem of challenging for direct techniques. I believe that the suggested empirical realization of the BW method matching independent results from carefully calibrated PR and SC correlations provided by spectroscopic and interfermetric techniques, respectively, has proved to be the most accurate modern path to the cosmic distance scale calibration by Cepheids distance markers. Its major achievement is to reproduce the high-precision ZAMS distance to highly reddened calibrating galactic Cepheids in clusters as well as the individual extragalactic distances to LMC Cepheids up to the asymptotic dispersion of the PR relation set by the finite width of the instability strip.

Also, the almost reddening-free structure of the BW relations has been exploited to clear up several previous disappointing results. Then, reliable and accurate BW distances can be currently obtained to individual extragalactic Cepheids. I have taken care in avoiding model-dependent corrections which would imply an accurate knowledge of the systematic effects involved, but some a priori assumptions seem to be unavoidable. These include the PR relation and the SC correlation in the (V, $V-K$) combination as being practically independent of metallicity according to well established theoretical results.

The remarkable good agreement of BW data with the independent ZAMS distances for stars as critical as the calibrating galactic Cepheids, supports our confidence that reliable BW distances can also be achievable combining the actual well calibrated SC correlations with radial velocity measurements available for other field stars.

I would like to thank the referee of the paper, Wendy Freedman, for comments which led to a clearer presentation of the results.

REFERENCES

Baade, W. 1926, Astron. Nachr., 228, 359
Balona, L. A. 1977, MNRAS, 178, 231
Barnes, T. G., & Moffett, T. J. 1985, in IAU Colloq. 82, Cepheids: Theory and Observations, ed. B. F. Madore (Cambridge: Cambridge Univ. Press), 53
Bell, R. A., & Gustafsson, B. 1989, MNRAS, 236, 653
Bessell, M. S. 1991, A&A, 242, L17
Brodie, J. P., & Madore, B. F. 1980, MNRAS, 191, 841
Caldwell, J. A. R., & Coulson, I. M. 1984, SAAO, Circ. No. 8, 1
———. 1985, MNRAS, 212, 879
———. 1986, MNRAS, 218, 223
———. 1987, AJ, 93, 1090
Cardelli, J. A., Clayton, G. C., & Mathis, J. S. 1989, ApJ, 345, 245
Chiosi, C., Wood, P. R., & Capitanio, N. 1993, ApJS, 86, 541
Coulson, I. M., Caldwell, J. A. R., & Gieren, W. P. 1985, ApJS, 57, 595
———. 1986, ApJ, 303, 273
Davis, J. 1993, in IAU Symp. 158, Very High Angular Resolution Imaging, ed. J. G. Robertson & W. J. Tango (Dordrecht: Kluwer), 135
Davis, J., & Tango, W. J. 1986, Nature, 323, 234
Dean, J. F., Warren, P. R., & Cousins, A. W. J. 1978, MNRAS, 183, 569
Di Benedetto, G. P. 1993, A&A, 270, 315
———. 1994, A&A, 285, 819
———. 1995, ApJ, 452, 195
Di Benedetto, G. P., & Rabbia, Y. 1987, A&A, 188, 114
Feast, M. W., & Walker, A. R. 1987, ARA&A, 25, 345
Ferneley, J. A., Skillen, I., & Jameson, R. F. 1989, MNRAS, 237, 947
Ferrarese, L. et al. 1996, ApJ, 464, 568
Freedman, W. L., & Madore, B. F. 1990, ApJ, 365, 186
Freedman, W. L., et al. 1994, ApJ, 427, 628
Fricke, K., Stobie, R. S., & Strittmatter, P. A. 1971, MNRAS, 154, 23
Gautschy, A. 1987, Vistas Astron., 30, 197
Gieren, W. P., Barnes, T. G., & Moffett, T. J. 1989, ApJ, 342, 467
Hindsley, R. B., & Bell, R. A. 1989, ApJ, 341, 1004
Iben, I., Jr., & Renzini, A. 1984, Phys. Rep., 105, 329
Jacoby, G. H., et al. 1992, PASP, 104, 599
Laney, C. D., & Stobie, R. S. 1986, MNRAS, 222, 449
———. 1992, A&AS, 93, 93
———. 1993a, MNRAS, 263, 921
———. 1993b, MNRAS, 260, 408
———. 1994, MNRAS, 266, 441
———. 1995, MNRAS, 274, 337
Madore, B. F. 1985, in IAU Colloq. 82, Cepheids: Theory and Observations, ed. B. F. Madore (Cambridge: Cambridge Univ. Press), 166
Madore, B. F., & Freedman, W. L. 1991, PASP, 103, 933
Martin, W. L., Warren, P. R., & Feast, M. W. 1979, MNRAS, 188, 139
Mozurkewich, D., et al. 1991, AJ, 101, 2207
Panagia, N., Gilmozzi, R., Sonneborn, G., Cassatella, A., Fransson, C., Kirshner, R. P., Lundqvist, P., & Wamsteker, W. 1996, in the Extragalactic Distance Scale (STScI) in press
Russell, S. C., & Bessell, M. S. 1989, ApJS, 70, 865
Saha, A., Labhardt, L., Schwengeler, H., Macchetto, F. D., Panagia, N., Sandage, A., & Tammann, G. A. 1994, ApJ, 425, 14

Sandage, A. 1972, QJRAS, 13, 202
Stothers, R. B. 1988, ApJ, 329, 712
Thompson, R. F. 1975, MNRAS, 172, 455
Walker, A. R. 1987, MNRAS, 225, 627
Wegner, W. 1994, MNRAS, 270, 229
Welch, D. L. 1994, AJ, 108, 1421
Welch, D. L., McAlary, C. W., Madore, B. F., McLaren, R. A., & Neugebauer, G. 1985, ApJ, 292, 217
Welch, D. L., McLaren, R. A., Madore, B. F., & McAlary, C. W. 1987, ApJ, 321, 162
Wesselink, A. J. 1946, Bull. Astron. Inst. Netherlands, 368, 91
Whittet, D. C. B., & van Breda, I. G. 1978, A&A, 66, 57