The Astrophysical Journal, 482:442–447, 1997 June 10

LITHIUM DEPLETION IN FULLY CONVECTIVE PRE–MAIN-SEQUENCE STARS

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Received 1996 September 11; accepted 1997 January 7

ABSTRACT

We present an analytic calculation of the thermonuclear depletion of lithium in contracting, fully convective, pre–main-sequence stars of mass $M \lesssim 0.5\ M_{\odot}$. Previous numerical work relies on still uncertain physics (atmospheric opacities and convection, in particular) to calculate the effective temperature as a unique function of stellar mass. We assume that the star's effective temperature, $T_{\rm eff}$, is fixed during Hayashi contraction and allow its actual value to be a free parameter constrained by observation. Using this approximation, we compute lithium burning analytically and explore the dependence of lithium depletion on $T_{\rm eff}$, M, and composition. Our calculations yield the radius, age, and luminosity of a pre–main-sequence star as a function of lithium depletion. This allows for more direct comparisons with observations of lithium-depleted stars. Our results agree with those numerical calculations that explicitly determine stellar structure during Hayashi contraction. In agreement with Basri, Marcy, & Graham, we show that the absence of lithium in the Pleiades star HHJ 3 implies that it is older than 100 Myr. We also suggest a generalized method for dating Galactic clusters younger than 100 Myr (i.e., those with contracting stars of $M \gtrsim 0.08\ M_{\odot}$) and for constraining the masses of lithium-depleted stars.

Subject headings: nuclear reactions, nucleosynthesis, abundances — stars: evolution — stars: interiors — stars: pre–main-sequence

1. INTRODUCTION

Lithium depletion in gravitationally contracting, fully convective stars of mass $M < M_{\odot}$ (Hayashi & Nakano 1963; Bodenheimer 1965) and $M = M_{\odot}$ (Weymann & Moore 1963; Ezer & Cameron 1963) has been studied for over 30 years. Motivated by lithium observations of main-sequence stars in young clusters and the halo (e.g., Soderblom 1995), many have calculated lithium depletion for stars of $M \gtrsim 0.5\ M_{\odot}$ both before and during the main sequence (Vauclair et al. 1978; D'Antona & Mazzitelli 1984; Proffitt & Michaud 1989; VandenBerg & Poll 1989; Swenson, Stringfellow, & Faulkner 1990; Deliyannis, Demarque, & Kawaler 1990). For stars with $M \gtrsim 0.5\ M_{\odot}$, most lithium burns after convection has halted in the stellar core. Accurate predictions of lithium depletion then depend on the temperature at the bottom of the retreating convective zone. The location of the convective/radiative boundary and the amount of mixing across it depend on opacity, treatment of convection, and rotation; proper handling of these effects remains an open question.

Lower mass ($M \lesssim 0.5\ M_{\odot}$) stars are fully convective during lithium burning, which occurs before the star reaches the main sequence.[1] The effective temperature $T_{\rm eff}$ of a fully convective star determines the contraction rate and is found by matching the entropy at the interior to that at the photosphere (see Stahler 1988 for a review of Hayashi contraction). For these low-mass stars with $T_{\rm eff} \lesssim 4000$ K, the opacities are still uncertain, and there are still debates about the treatment of convection. These uncertainties have motivated many numerical calculations of lithium depletion (Pozio 1991; Nelson, Rappaport, & Chiang 1993; D'Antona & Mazzitelli 1994, hereafter DM94; Chabrier, Baraffe, & Plez 1996). The differing input physics results in different $T_{\rm eff}$ for the same stellar mass. For example, DM94 found that, depending on the opacities used, $T_{\rm eff}$ ranges from 3350 to 3640 K for a 0.2 $M_{\odot}$ star. Most calculations agree that $T_{\rm eff}$ remains approximately constant during the fully convective contraction phase.

In this paper, we present a different approach to calculating pre–main-sequence (or pre–brown dwarf) lithium depletion in gravitationally contracting, low-mass ($M \lesssim 0.5\ M_{\odot}$) stars. Rather than calculate $T_{\rm eff}$, we calculate the dependence of lithium depletion on $T_{\rm eff}$. Given $T_{\rm eff}$, M, and mean molecular weight μ, we can reproduce the results of prior works. Moreover, our approach allows the inferred effective temperature to be used directly in analyzing lithium depletion observations. Efficient convection throughout the star allows it to be modeled as a fully mixed $n = 3/2$ polytrope. Our analytic calculations then yield the age, radius, and luminosity at a given level of lithium depletion. Our results (eqs. [11] and [12]) can be used to constrain the mass and age of a star from its lithium abundance. We apply these same techniques to depletion of beryllium and boron in Ushomirsky et al. (1997).

2. CONTRACTION AND LITHIUM BURNING IN FULLY CONVECTIVE PRE–MAIN-SEQUENCE STARS

A number of authors have undertaken detailed numerical evolutionary calculations of contracting pre–main-sequence stars (see Burrows & Liebert 1993 for a review). Low-mass stars ($M \lesssim 0.5\ M_{\odot}$) remain completely convective at least through the end of lithium burning (DM94). For an ideal gas ($P = \rho N_{\rm A} k_{\rm B} T/\mu$, where $N_{\rm A}$ is Avogadro's number), the adiabatic relation is $P \propto \rho^{5/3}$, so that an $n = 3/2$ polytrope describes the stellar structure. During later stages of contraction, electron degeneracy modifies the equation of state by reducing the central temperature below its nonde-

[1] The interstellar lithium abundance is so low ($N_{\rm Li}/N_{\rm H} \sim 10^{-9}$) that the energy released by its fusion does not slow stellar contraction appreciably. Throughout this paper, we only consider ^{7}Li, since ^{6}Li always depletes first and is less abundant in the local interstellar medium (Lemoine, Ferlet, & Vidal-Madjar 1995).

generate value. However, partial degeneracy does not alter the polytropic structure. The central density and temperature are

$$\rho_c = 8.44\left(\frac{M}{M_\odot}\right)\left(\frac{R_\odot}{R}\right)^3 \text{ g cm}^{-3} ,$$

$$T_c = 7.41 \times 10^6\left(\frac{\mu_{\rm eff}}{0.6}\right)\left(\frac{M}{M_\odot}\right)\left(\frac{R_\odot}{R}\right) \text{ K} , \qquad (1)$$

where

$$\mu_{\rm eff} \equiv \frac{\rho N_{\rm A} k_{\rm B} T}{P} \le \mu \qquad (2)$$

accounts for the deviation in electron pressure from that of an ideal gas. We neglect Coulomb and ionization corrections to the equation of state. Except during deuterium burning, gravitational contraction powers the star's luminosity, $L = 4\pi R^2 \sigma_{\rm SB} T_{\rm eff}^4 = -(3GM^2/7R^2)(dR/dt)$, which is independent of the degree of degeneracy.

We define the time coordinate t under the assumption that $T_{\rm eff}$ is constant during contraction from a formally infinite radius. Therefore, this time differs from chronological age because of the deuterium-burning phase and the initial radius on the theoretical stellar birth line (Stahler 1988). Lithium depletion, however, occurs long (10–100 Myr) after these events, so that if the effective temperature used is correct at the time of lithium depletion, t differs only slightly from chronological age. Gravitational contraction then gives the stellar radius and luminosity as functions of time as

$$\frac{R}{R_\odot} = 0.850\left(\frac{M}{0.1\ M_\odot}\right)^{2/3}\left(\frac{3000\text{ K}}{T_{\rm eff}}\right)^{4/3}\left(\frac{\rm Myr}{t}\right)^{1/3} , \qquad (3)$$

$$\frac{L}{L_\odot} = 5.25 \times 10^{-2}\left(\frac{M}{0.1\ M_\odot}\right)^{4/3}\left(\frac{T_{\rm eff}}{3000\text{ K}}\right)^{4/3}\left(\frac{\rm Myr}{t}\right)^{2/3} . \qquad (4)$$

We define the contraction timescale in terms of the central temperature,

$$t_{\rm cont} \equiv -\frac{R}{dR/dt} = 115\left(\frac{3000\text{ K}}{T_{\rm eff}}\right)^4\left(\frac{0.1\ M_\odot}{M}\right) \times \left(\frac{0.6}{\mu_{\rm eff}}\right)^3\left(\frac{T_c}{3 \times 10^6\text{ K}}\right)^3 \text{ Myr} = 3t , \qquad (5)$$

which we then compare with the timescale for lithium destruction due to the reaction ^{7}Li(p, α) ^{4}He. Over the range of temperatures $[T_6 \equiv (T/10^6\text{ K}) < 6]$ appropriate for this work, the reaction rate is $N_{\rm A}\langle\sigma v\rangle = Sf_{\rm scr} T_6^{-2/3} \exp(-aT_6^{-1/3})$ cm^3 s^{-1} g^{-1}, where $S = 6.4 \times 10^{10}$, $a = 84.72$ (Caughlan & Fowler 1988), and $f_{\rm scr}$ is the screening correction factor (Salpeter & Van Horn 1969). Raimann (1993) recently discussed new experimental results at low energies ($\approx$11–13 keV) and updated S to 7.2×10^{10}.

Lithium is depleted when the local nuclear destruction time, $t_{\rm dest} \equiv m_p/X\rho\langle\sigma v\rangle$ (X is the hydrogen mass fraction, and m_p is the proton mass), becomes comparable to $t_{\rm cont}$. Although the full calculation involves integrating over the star (§ 3), one obtains a flavor of the calculation by evaluating $t_{\rm dest}$ at the center of the star (with $f_{\rm scr} = 1$ and $\mu_{\rm eff} = \mu$) and equating it with $t_{\rm cont}$. This gives a relation for the central temperature $T_{c6} \equiv T_c/10^6$ K at the time of depletion,

$$\frac{a}{T_{c6}^{1/3}} = 32.9 + \ln(S) - 3\ln\left(\frac{M}{0.1\ M_\odot}\right) - 4\ln\left(\frac{T_{\rm eff}}{3000\text{ K}}\right) - 6\ln\left(\frac{\mu}{0.6}\right) + \frac{16}{3}\ln T_{c6} . \qquad (6)$$

For example, if $T_{\rm eff} = 3500$ K and $M = 0.5\ M_\odot$, then ^{7}Li depletes when $T_{c6} = 3.04$. The temperature is only slightly different for other masses. For these temperatures, the reaction rate is extremely temperature sensitive ($\propto T^{20}$), so that $t_{\rm dest}$ decreases rapidly as the temperature increases. Hence, the transition from no depletion to full depletion occurs rapidly as long as the central temperature reaches these values. This does not occur for very low masses ($\lesssim 0.06\ M_\odot$) because electron degeneracy pressure dominates the equation of state (Pozio 1991; Magazzù, Martín, & Rebolo 1993; Nelson et al. 1993). Also, stars with $M \gtrsim 0.5\ M_\odot$ develop radiative cores prior to depletion. We only discuss stars safely between these limits. Since this depletion occurs on a contraction timescale (10–100 Myr), the central temperature during depletion is higher [$\approx$(3–4) $\times$ 10^6 K] than the temperature needed (2.4×10^6 K) at the base of a main-sequence star's convective zone to deplete ^{7}Li while the star is on the main sequence.

The narrow range of burning temperatures allows the solution of equation (6) to be approximated as a power law. For the nondegenerate case, T_c is proportional to $M^{1/8} T_{\rm eff}^{1/6} \mu^{1/4}$ at the time of depletion, and $t_{\rm depl}$ is proportional to $T_{\rm eff}^{-7/2} M^{-5/8}$. The radius at depletion, $R \propto M^{7/8}\mu^{3/4} T_{\rm eff}^{-1/6}$, is relatively insensitive to $T_{\rm eff}$ but nearly proportional to mass. These scalings, which are modified only slightly in the full calculation (§ 3), provide an intuitive picture of lithium burning and a means to evaluate observations (§ 4) once the prefactors are known.

3. FULL CALCULATION OF LITHIUM DEPLETION

Since lithium burns near the center of the star, depleting lithium throughout the star requires mixing lithium-poor fluid outward and lithium-rich fluid inward. For efficient convection, the mixing timescale ($\sim$10–100 yr) is much shorter than both $t_{\rm cont}$ and $t_{\rm dest}$; convective mixing keeps the lithium-to-hydrogen ratio, f, fixed throughout the star as the total lithium content decreases. Thus, we write the global depletion rate as

$$M\frac{df}{dt} = -\frac{Xf}{m_p}\int_0^M \rho\langle\sigma v\rangle dM . \qquad (7)$$

Changing to spatial variables and using the thermonuclear rate defined earlier, we obtain

$$\frac{d}{dt}\ln f = -\frac{4\pi X}{N_{\rm A} m_p M}\int_0^R r^2\rho^2 S f_{\rm scr} T_6^{-2/3} \exp\left(-\frac{a}{T_6^{1/3}}\right) dr . \qquad (8)$$

The temperature sensitivity of the nuclear reaction allows us to expand T and ρ about their central values. Integrating the lowest order terms (see Ushomirsky et al. 1997 for a discussion of the small errors introduced by this approximation) yields

$$\frac{d}{dt}\ln f = -18.0\left(\frac{X}{0.70}\right)\left(\frac{0.6}{\mu_{\rm eff}}\right)^3\left(\frac{0.1\,M_\odot}{M}\right)^2 \times S f_{\rm scr}\, a^7 \alpha^{-17/2}\left(1-\frac{21}{2\alpha}\right)e^{-\alpha}\,, \quad (9)$$

where $\alpha \equiv a T_{c6}^{-1/3}$ is a convenient representation of the central temperature. Using equation (5) and the fact that $d\alpha/dR = \alpha/3R$, we characterize the stellar state by α and integrate from $\alpha = \infty$ and initial abundance f_0 to find the depletion W as a function of α,

$$W \equiv \ln\left(\frac{f_0}{f}\right) = 7.70\times 10^{15} S f_{\rm scr}\, a^{16}\left(\frac{X}{0.70}\right) \times \left(\frac{0.6}{\mu_{\rm eff}}\right)^6\left(\frac{3000\ {\rm K}}{T_{\rm eff}}\right)^4\left(\frac{0.1\,M_\odot}{M}\right)^3 g(\alpha)\,, \quad (10)$$

where $g(\alpha) = \alpha^{-37/2}e^{-\alpha} - 29\Gamma(-37/2, \alpha)$, and $\Gamma(-37/2, \alpha)$ is an incomplete gamma function. This transcendental equation is similar to that found from the simple arguments in § 2. For a given depletion W, the solution $\alpha(W)$ determines T_c. The radius, luminosity, and age then follow from equations (3)–(5). Our treatment of degeneracy and screening in equation (10) is approximate in the sense that we neglect the slight variation of $\mu_{\rm eff}$ and $f_{\rm scr}$ with α. Instead, we evaluate their values at the time of depletion to solve the resulting transcendental equation. We discuss this process in more detail in a forthcoming paper (Ushomirsky et al. 1997).

Although solving the depletion in equation (10) numerically is straightforward, an analytic fitting formula for the time (and therefore the radius and luminosity from eqs. [3] and [4]) as a function of depletion is also helpful. The age at a given depletion (we use Raimann's 1993 rates; using Caughlan & Fowler's 1988 rates changes $t_{\rm depl}$ by no more than a few percent) for 2000 K $< T_{\rm eff} <$ 4000 K, 0.075 $M_\odot <$ M $<$ 0.5 $M_\odot$, and $0.65 < X < 0.75$ is

$$t_{\rm depl} = 54.1\left(\frac{0.1\,M_\odot}{M}\right)^{0.715}\left(\frac{3000\ {\rm K}}{T_{\rm eff}}\right)^{3.51}\left(\frac{0.6}{\mu}\right)^{1.98} \times\left(\frac{W}{\ln 2}\right)^{0.121}\left[1+0.117\left(\frac{0.1\,M_\odot}{M}\right)^{6.39} \times\left(\frac{T_{\rm eff}}{3000\ {\rm K}}\right)^{0.828}\left(\frac{0.6}{\mu}\right)^{11.8}\left(\frac{W}{\ln 2}\right)^{0.204}\right]\ {\rm Myr}\,. \quad (11)$$

The first line of equation (11) follows from the simple arguments of § 2, while the expression in brackets accounts for the onset of degeneracy in lower mass stars. A simpler fit,

$$t_{\rm depl} = 50.7\left(\frac{0.1\,M_\odot}{M}\right)^{0.663}\left(\frac{3000\ {\rm K}}{T_{\rm eff}}\right)^{3.50}\left(\frac{0.6}{\mu}\right)^{2.09} \times\left(\frac{W}{\ln 2}\right)^{0.124}\ {\rm Myr}\,, \quad (12)$$

is obtained if degeneracy is not important during depletion (0.2 $M_\odot \lesssim M \lesssim$ 0.5 $M_\odot$). These fits agree with Chabrier et al. (1996, Table 1) for masses 0.08 $M_\odot \le M \le$ 0.5 $M_\odot$ at 50% (99%) depletion to better than 4% (8%), 7% (17%), and 12% (25%) for the radius, luminosity, and age, respectively.[2] We also reproduce the results in Tables 5–8 of DM94 for 0.07 $M_\odot \le M \le$ 0.3 $M_\odot$ to better than 3%, 7%, and 15% for the radius, luminosity, and age, respectively.

The strong temperature dependence of the burning rate makes T_c at depletion very insensitive to our approximations. The central temperature is thus the most accurately determined quantity. Indeed, our T_c at a given depletion level deviate from those reported in Tables 5–8 of DM94 by less than 2% (Chabrier et al. 1996 did not report T_c). Our neglect of Coulomb and ionization effects (Ushomirsky et al. 1997) introduces a small error in mapping T_c to R (roughly, $T_c \propto M/R$). These errors then propagate into the age and luminosity determination at a given depletion level. Since the age is proportional to T_c^3 (see eq. [5]), the relative deviations in t are roughly 3 times larger than those in R. Moreover, any changes in $T_{\rm eff}$ during contraction cause additional uncertainty (typically up to 5%; see Ushomirsky et al. 1997) in calculating how long a star takes to contract to a certain radius.

In summary, the absolute error of age determination in our calculation is no more than 20%–25% under any conditions, and the average discrepancy with published results is about half that amount. In light of these uncertainties, it is important to note that, because the age at a level of depletion is $t \propto 1/T_{\rm eff}^{3.5}$, the typical 5% observational errors in $T_{\rm eff}$ lead to uncertainties in age that are comparable to the deviations of our results from the detailed numerical ones. Our results therefore are adequate for observational work, and they have the added advantage of enabling the observer to use the derived $T_{\rm eff}$ and L in conjunction with lithium observations to constrain the stellar mass. This is in contrast to using detailed evolutionary tracks to infer the mass from the star's location on the H-R diagram. As we show in the following section, the inferred mass can be in conflict with lithium depletion. In addition (insofar as $T_{\rm eff}$ is nearly constant during contraction), our results can serve as benchmarks in comparing and evaluating detailed numerical calculations.

4. COMPARISON WITH OBSERVATIONS OF LITHIUM-DEPLETED PRE–MAIN-SEQUENCE STARS

The radii of pre–main-sequence stars are typically inferred from $T_{\rm eff}$ and L, but (except in binaries) masses must be estimated by relating colors to a grid of computed evolutionary tracks. There are potentially significant uncertainties, both observational and theoretical, associated with inferring masses and radii of stars from these observations. On the other hand, the presence or absence of lithium is an indirect indicator of the central temperature of the star. Since lithium does not deplete until a characteristic central temperature ($T_c \sim [3–4] \times 10^6$ K) is reached, the lithium abundance can be used to constrain the mass to radius ratio (see eqs. [3] and [11]). Age determination is, in some sense, secondary to measuring the radius.

Since the central temperature is proportional to M/R, the additional constraint provided by the lithium abundance measurements (that are sensitive to the derived $T_{\rm eff}$) is most easily exploited by plotting stellar radii versus masses. Figure 1 is a compilation of published mass and radius values of pre–main-sequence stars where lithium abun-

[2] Percentages reported here are *maximum* deviations of our results (eqs. [11] and [12]) from the stated references.

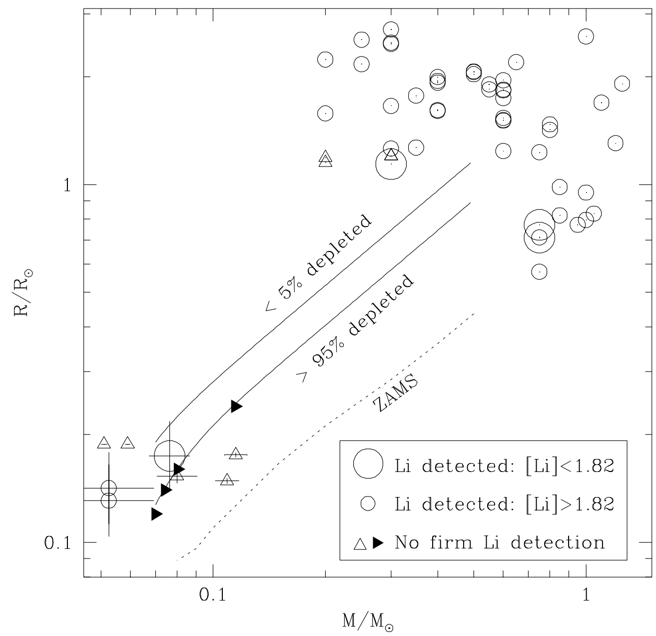

FIG. 1.—Compilation of published values (Burrows, Hubbard, & Lunine 1989; Magazzù, Martín, & Rebolo 1991, 1993; Martín et al. 1994b; Rebolo et al. 1996) of M, R, and $N_{\rm Li}$ for pre–main-sequence stars. Small circles represent stars reported to have $[{\rm Li}] \equiv 12 + \log (N_{\rm Li}/N_{\rm H}) > 1.82$, which corresponds to an abundance of more than 5% of the Population I value ($[{\rm Li}] = 3.1$). Large circles correspond to stars with $[{\rm Li}] < 1.82$. Triangles, both filled and empty, indicate stars with no firm lithium detection: either no lithium line is detected, or only an upper bound is reported for $N_{\rm Li}$. Where applicable, we have indicated reported uncertainties in mass and radius. The two solid lines are contours of constant depletion calculated from eq. (11) with $\mu = 0.6$ and $X = 0.7$ and with an effective temperature law $T_{\rm eff} = 4000\ (M/M_\odot)^{1/7}$ K; the position and shape of the depletion contours are insensitive to these choices. The solid lines end at $M = 0.5\ M_\odot$; above this mass stellar cores are not convective at the depletion time (DM94). At lower left are a series of points (*filled right-pointing triangles*) for which no lithium was detected and no mass was reported (see Table 1). Taking the nondetection of lithium to mean that these stars are more than 95% depleted, we solve for their minimum masses. The dotted line is the theoretical zero-age main sequence for $0.08\ M_\odot \leq M \leq 0.5\ M_\odot$ (DM94).

dances have been estimated or constrained. We divide the stars into high (*small circles*), low (*large circles*), and undetectable (*triangle*) lithium content bins (see Fig. 1 legend). Also plotted are lines of constant depletion, which correspond to nearly constant central temperature, i.e., $R \propto M$. These lines are relatively insensitive to the effective temperature and divide the graph into an undepleted, a depleting, and a depleted region. The narrowness of the depleting region results from the reaction's strong temperature sensitivity. We do not continue our lines to high masses where the stars develop radiative cores prior to depletion, nor to low masses where the stars become strongly degenerate.

A few stars appear depleted but lie above the depletion lines. Since central burning cannot have caused such a low lithium abundance, either the star was born with an anomalously low abundance or the reported masses and/or radii are in error. A challenge to our assumption of complete mixing would be a detection of lithium in a star lying beneath the depletion lines. We have found no such measurement for $M < 0.5\ M_\odot$. We also display in Figure 1 those stars for which no lithium is detected and no mass is reported (*filled triangles*). We interpret the lack of lithium to imply that more than 95% has burned and use the inferred radii to constrain their masses (see Table 1). Determinations of $T_{\rm eff}$ from the observed colors are still highly uncertain and depend on the temperature scale and model atmosphere used. We have not attempted to convert observed colors to a single temperature scale, but have just used the quoted results. However, our figure depends only on the value of $T_{\rm eff}$ and not the physics needed to infer it.

TABLE 1

RADIUS AND MASS OF LITHIUM-DEPLETED PRE–MAIN-SEQUENCE STARS

	OBSERVED		INFERRED		
STAR	$T_{\rm eff}$ (K)	$\log (L/L_\odot)$	$R/R_\odot$	$M_{\rm min}/M_\odot$	REFERENCES
HHJ 10	3120	−2.8	0.14	0.074	1, 2, (6)
HHJ 36	2825	−2.5	0.24	0.11	3, 2, (6)
LHS 248	2775	−2.5	0.24	0.11	4, (1)
GL 406	2600	−3.0	0.16	0.078	5, (7)
GL 569B......	2773	−3.1	0.12	0.069	5, (7)

NOTE.—The 5% uncertainties in $T_{\rm eff}$ correspond to a 10% uncertainties in radii and uncertainties of no greater than 10% in the minimum masses. The lithium nondetection is reported in the reference in parentheses. For stars with two references, the first is for effective temperature; the second is for luminosity. It is important to note that we have not calibrated the effective temperatures to a common scale. To quote our anonymous referee, "the pedigree of any temperature derived from observations cannot be ignored." The systematic errors between different $T_{\rm eff}$ scales can be as much as 10%.

REFERENCES.—(1) Martín, Rebolo, Magazzù 1994a; (2) Stauffer, Liebert, & Giampapa 1995; (3) Steele et al. 1995; (4) Bessel & Stringfellow 1993; (5) Burrows et al. 1989; (6) Oppenheimer et al. 1997; (7) Magazzù et al. 1993.

Figure 1 also clearly shows that there is a dearth of observations of pre–main-sequence stars near the depleting region with masses $0.1\ M_\odot < M < 0.5\ M_\odot$. *Observations in this mass range are important, as they would test the mixing assumption and may yield, independently of atmospheric physics,* $T_{\rm eff}$ *as a function of mass.*

As mentioned above, there are significant uncertainties in relating the observed colors to effective temperatures for low-mass stars and in using the inferred $T_{\rm eff}$ and evolutionary models to determine ages of stars. Nevertheless, we can still bound the stellar age and mass without relying on exact knowledge of $T_{\rm eff}$, as long as L is well determined and lithium is unobserved[3] down to some detection limit W_0. For a trial value for the age of the star, t, the contraction equation (4) yields $M \propto t^{1/2}/T_{\rm eff}$. Substituting this relation into the depletion formula, equations (11) or (12), we obtain the time $t_{\rm depl}$ at which this star is depleted to W_0. For consistency with the lack of lithium, we demand $t \geq t_{\rm depl}$, with equality yielding the lower limit on age for a given $T_{\rm eff}$. For high-mass stars, t is proportional to $T_{\rm eff}^{-2.1}$, and the upper bound on $T_{\rm eff}$ determines the minimum age. For lower mass stars, $t(T_{\rm eff})$ always has a minimum (because of the effects of degeneracy),

$$t_{\rm min} = 58.3 \left(\frac{10^{-2.5}\ L_\odot}{L} \right)^{0.922} \left(\frac{0.6}{\mu} \right)^{2.52} \left(\frac{W_0}{\ln 2} \right)^{0.0769}\ {\rm Myr}\ , \tag{13}$$

which is the lower bound on age independent of $T_{\rm eff}$ for $-3.50 < \log (L/L_\odot) < -2.32$. Because the minimum in $t_{\rm depl}$ is due to the effects of degeneracy, this technique is independent of $T_{\rm eff}$ only for low-mass stars; this is fortunate because for these stars $T_{\rm eff}$ is more uncertain.

[3] Similarly, a detection of lithium coupled with a lower bound on $T_{\rm eff}$ yields the maximum age.

We apply equation (13) to the lithium-depleted star HHJ 3 in the Pleiades [log $(L/L_\odot) = -2.78 \pm 0.05$; Basri, Marcy, & Graham 1996]. If the absence of ^{7}Li in its spectrum implies that this star is more than 99% depleted, then our calculations imply that it must be older than 100 Myr for normal composition ($X = 0.7$), regardless of its effective temperature or mass (i.e., regardless of an evolutionary track.) Basri et al. (1996) reached the same conclusion by using the evolutionary tracks of Nelson et al. (1993). That our estimates agree is a consequence of the age constraint (in this low-mass range) depending mostly on L and being nearly independent of $T_{\rm eff}$. As noted by Basri et al. (1996), this age agrees with the age of the Pleiades inferred from main-sequence turnoff *with* convective overshoot (Mazzei & Pigatto 1989; Maeder & Meynet 1991). In contrast, to share the 60–70 Myr age of the Pleiades inferred from main-sequence turnoff *without* convective overshoot (Mazzei & Pigatto 1989), HHJ 3 must have either a hydrogen content $X \sim 0.5$ (very unlikely) or a luminosity 50% higher than that reported.

If both L and $T_{\rm eff}$ are known, the absence (presence) of lithium sets a meaningful lower (upper) limit on both the mass and the age of an individual star. This method had its first practical application in the work of Basri et al. (1996), who used it to find a new age for the Pleiades and argued that the Pleiades' faintest members are substellar objects. Instead of plotting isochrones and evolutionary tracks on an L–$T_{\rm eff}$ diagram, we find it more convenient to directly relate the two unknown quantities, mass and age, on an M–t plot (Fig. 2). On this plot, the observed luminosity is represented by a line with positive slope found from the contraction relation (eq. [4]), while the limit on lithium abundance is represented by a line with negative slope inferred from the depletion formula (eq. [11]). For a given $T_{\rm eff}$, the star must lie in a swath of the M–t plane defined by the estimated luminosity range. The lithium abundance constraint intersects this swath, setting limits on age and mass.

As an example, the hatching in Figure 2 denotes the allowed regions of the Pleiads HHJ 3 and Calar 3 (which shows lithium and appears undepleted) (Basri et al. 1996; Rebolo et al. 1996). Assuming that the stars are coeval, Calar 3 must have a mass less than 0.075 $M_\odot$, and HHJ 3 must have a mass greater than 0.08 $M_\odot$, if the effective temperatures (Steele et al. 1995) are correctly measured. The recent detection of lithium in the Pleiades star PPl 15 [log $(N_{\rm Li}/N_{\rm H}) \geq -10.84$; Basri et al. 1996], which has $T_{\rm eff} = 2800 \pm 150$ K and log $(L/L_\odot) = -2.80 \pm 0.10$ (Rebolo et al. 1996), provides a better upper bound on the age of the Pleiades than Calar 3. In particular, we find that the maximum age of PPl 15 (and hence of the Pleiades) is 145 Myr, while its mass is constrained to lie in the range 0.07–0.09 $M_\odot$.

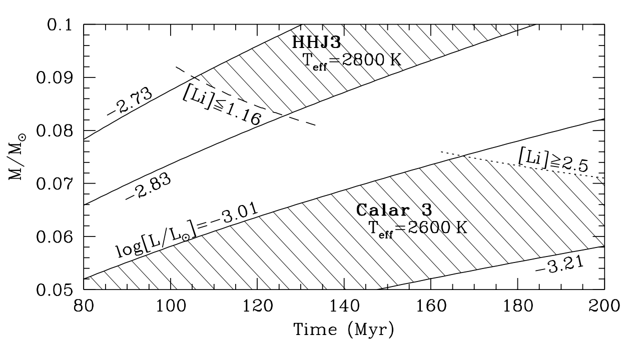

FIG. 2.—An example of using the formulae for depletion time (eq. [11]), radius (eq. [3]), and luminosity (eq. [4]) to bound the age of the Pleiades. For both HHJ 3 and Calar 3, we plot a contour of constant lithium abundance (dashed for HHJ 3; dotted for Calar 3) and two constant luminosity contours (*solid lines*). These constraints bound for each star a region of M–t space (*shaded areas*). HHJ 3 has an effective temperature $T_{\rm eff} = 2800$ K (Steele et al. 1995), a luminosity $-2.83 < \log (L/L_\odot) < -2.73$ (Basri et al. 1996), and [Li] < 1.16. For Calar 3, the region of M–t space is bounded by $T_{\rm eff} = 2600$ K, $-3.21 < \log (L/L_\odot) < -3.01$, and [Li] > 2.5. The overlap in t of the shaded areas sets a minimum age (105 Myr) and maximum age (not shown) of the Pleiades. Within this age range, each star then has a minimum and maximum allowed mass (Rebolo et al. 1996).

5. CONCLUSIONS

Assuming that a contracting star is fully mixed and that the time it spends prior to Hayashi contraction is negligible compared to typical depletion times, we have derived simple analytical relations (eqs. [3], [4], [11], and [12]) for the radius, luminosity, and age of a star as a function of lithium depletion. These formulae demonstrate the dependence of the time of lithium depletion on the mass, composition, and effective temperature of the star. Reasonable agreement with other published theoretical calculations supports the use of our results to evaluate observations.

We outline a method for using the observed $T_{\rm eff}$ and L of both depleted and undepleted stars in a cluster to constrain its age. This method (complementary to that used by Basri et al. 1996 to date the Pleiades) is also applicable to clusters of age 10–100 Myr, where higher mass stars (up to 0.5 $M_\odot$) are presently depleting lithium. These stars are relatively luminous ($L \gtrsim 10^{-2}\ L_\odot$) when depleting lithium and are hence more easily observed. For example, a 0.3 $M_\odot$ star will have depleted 50% of its lithium at about 16 Myr, when it has a luminosity 0.04 $L_\odot$ (for $T_{\rm eff} = 3300$ K). Because stars with a range of masses will deplete simultaneously (the mass of a 99% depleted star is approximately 1.4 times the mass of a 50% depleted star), observations of stars within this mass range will constrain the cluster's age and the relative ordering of stellar masses. In a young cluster, the most stringent bounds are obtained by observing the dimmest depleted star and the brightest undepleted star.

We have assumed that a fully convective star mixes material from the core to the photosphere faster than it contracts. Until a fully convective pre–main-sequence star is observed that has both lithium and a central temperature hotter than the maximum burning temperature (4.4×10^6 K for a 0.5 $M_\odot$ star), this approximation goes unchallenged. In this sense, we have calculated the earliest time for depletion in the contracting star. Incomplete mixing can only delay the time of depletion to a given level.

We thank foremost the graduate students in UC Berkeley's IDS 252 (Spring 1996) class who initially worked on this problem, especially Vadim Hilliard and Peter Schroeder. We also thank Isabelle Baraffe, Gibor Basri, Gilles Chabrier, Lynne Hillenbrand, Geoff Marcy, Chris McKee, and John Monnier for helpful discussions. We thank the anonymous referees for constructive comments. L. B. acknowledges support as an Alfred P. Sloan Foundation fellow. E. F. B. was supported by a NASA GSRP Graduate Fellowship under grant NGT-51662, and C. D. M. was supported by an NSF Graduate Research Fellowship. G. U. thanks the Fannie and John Hertz Foundation for fellowship support.

REFERENCES

Basri, G., Marcy, G. W., & Graham, J. R. 1996, ApJ, 458, 600
Bessel, M. S., & Stringfellow, G. S. 1993, ARA&A, 31, 433
Bodenheimer, P. 1965, ApJ, 142, 451
Burrows, A., Hubbard, W. B., & Lunine, J. I. 1989, ApJ, 345, 939
Burrows, A., & Liebert, J. 1993, Rev. Mod. Phys., 65, 301
Caughlan, G. R., & Fowler, W. A. 1988, At. Data Nucl. Data Tables, 40, 283
Chabrier, G., Baraffe, I., & Plez, B. 1996, ApJ, 459, L91
D'Antona, F., & Mazzitelli, I. 1984, A&A, 138, 431
———. 1994, ApJS, 90, 467 (DM94)
Deliyannis, C. P., Demarque, P., & Kawaler, S. D. 1990, ApJS, 73, 21
Ezer, D., & Cameron, A. G. W. 1963, Icarus, 1, 422
Hayashi, C., & Nakano, T. 1963, Prog. Theor. Phys., 30, 460
Lemoine, M., Ferlet, R., & Vidal-Madjar, A. 1995, A&A, 298, 879
Maeder, A., & Meynet, G. 1991, A&AS, 89, 451
Magazzù, A., Martín, E. L., & Rebolo, R. 1991, A&A, 249, 149
———. 1993, ApJ, 404, L17
Martín, E. L., Rebolo, R., Magazzù, A. 1994a, ApJ, 436, 262
Martín, E. L., Rebolo, R., Magazzù, A., & Pavlenko, Ya. V. 1994b, A&A, 282, 503
Mazzei, P., & Pigatto, L. 1989, A&A, 213, L1
Nelson, L. A., Rappaport, S., & Chiang, E. 1993, ApJ, 413, 364
Oppenheimer, B. R., Basri, G., Nakajima, T., & Kulkarni, S. R. 1997, AJ, 113, 296
Pozio, F. 1991, Mem. Soc. Astron. Italiana, 62, 171
Proffitt, C. R., & Michaud, G. 1989, ApJ, 346, 976
Raimann, G. 1993, Z. Phys. A, 347, 73
Rebolo, R., Martín, E. L., Basri, G., Marcy, G. W., & Zapatero-Osorio, M. R. 1996, ApJ, 469, L53
Salpeter, E. E., & Van Horn, H. M. 1969, ApJ, 155, 183
Soderblom, D. R. 1995, Mem. Soc. Astron. Italiana, 66, 347
Stahler, S. W. 1988, PASP, 100, 1474
Stauffer, J. R., Liebert, J., & Giampapa, M. 1995, AJ, 109, 298
Steele, I. A., Jameson, R. F., Hodgkin, S. T., & Hambly, N. C. 1995, MNRAS, 275, 841
Swenson, F. J., Stringfellow, G. S., & Faulkner, J. 1990, ApJ, 348, L33
Ushomirsky, G., Matzner, C. D., Brown, E. F., Bildsten, L., Hilliard, V., & Schroeder, P. 1997, in preparation
VandenBerg, D. A., & Poll, H. E. 1989, AJ, 98, 1451
Vauclair, S., Vauclair, G., Schatzman, E., & Michaud, G. 1978, ApJ, 223, 567
Weymann, R., & Moore, E. 1963, ApJ, 137, 552