# GALACTIC GAMMA-RAY BACKGROUND AS A CONSTRAINT ON MILLISECOND PULSARS 

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#### Abstract

A large number of millisecond pulsars are believed to exist in the Galaxy. Their magnetic fields are generally $2-4$ orders of magnitude smaller than those of the normal pulsars. These pulsars are thought to have been resurrected from the run-down old pulsars. We apply the mechanism proposed by Scharlemann et al. (1978) for particle acceleration along open field lines in the magnetospheres of these pulsars and find that particles can become highly relativistic. We then calculate the $\gamma$-ray flux of curvature radiation from the accelerated particles. Since the number of millisecond pulsars in the Galaxy is quite uncertain, we survey the literature to get some estimates of the number of millisecond pulsars in the plane of the Galaxy and in the globular clusters. We use these numbers to work out the integrated high-energy $\gamma$-ray flux from millisecond pulsars in the Galaxy as a function of the Galactic latitude and longitude. We discuss the results in the light of the $\operatorname{COS} B$ observations of the background $\gamma$-radiation, which is the resultant diffuse emission when the contribution of the known point sources has been subtracted from the total emission, and the flux calculated by Strong et al. from the interaction of $\mathrm{H}_{\mathrm{I}}$ and $\mathrm{H}_{2}$ with cosmic rays and inverse Compton emission. We are able to place useful constraints on the number of millisecond pulsars in the disk and in the globular clusters.


Subject headings: diffuse radiation - gamma rays: observations - pulsars: general -
radiation mechanisms: nonthermal

## 1. INTRODUCTION

It is now a well-established fact that our Galaxy is a dominant source of high-energy $\gamma$-rays. The observations made by SAS 2 (Fichtel et al. 1975; Hartman et al. 1979) and COS B satellites (Paul et al. 1978; Mayer-Hasselwander et al. 1980) reveal, in some detail, the longitude and latitude dependence of the Galactic background $\gamma$-radiation in various energy ranges. Many processes such as decay of neutral pions produced in collisions between cosmic-ray nucleons and interstellar gas nuclei, bremsstrahlung from cosmic-ray electrons in the Coulomb fields of nuclei, and inverse Compton scattering of microwave background and starlight photons from the cosmic-ray electrons have been suggested as contributing to this background (Higdon \& Lingenfelter 1976; Harding 1981; Bhattacharya \& Srinivasan 1991). In this paper we explore the possibility of millisecond pulsars contributing to high-energy $\gamma$-ray background. We then use the $\operatorname{COS} B$ observations to put constraints on the number of millisecond pulsars in the plane of the Galaxy as well as in the globular clusters. We believe this to be a useful exercise, since the number of millisecond pulsars is uncertain at the moment and pulsar surveys are underway to ascertain this number.

This new class of pulsars, called millisecond pulsars, has been recognized only recently from its existence as a separate group at the bottom left of the period-magnetic field diagram. The first millisecond pulsar was discovered by Backer et al. (1982), and since then many more have been discovered in the plane of the Galaxy and in the globular clusters (Wolszczan et al. 1990; Manchester et al. 1991; for a review and references see Phinney \& Kulkarni 1994). By virtue of their short period of rotation, it is expected that they might be very strong sources of $\gamma$-rays (Usov 1983; Bhatia, Misra, \& Panchapakesan 1992b). Efforts are already underway to observe millisecond pulsars at $\gamma$-ray energies. There are already reports that very high energy
$\gamma$-rays from two of these pulsars have been detected (Chadwick et al. 1985, 1987). Kniffen et al. (1992) have reported the detection of $\gamma$-rays of energy greater than 100 MeV from the 102 ms pulsar PSR 1706-44.

Most of the millisecond pulsars discovered to date have magnetic fields in the range $10^{8}-10^{10} \mathrm{G}$. The four millisecond pulsars found in the Galactic disk (listed in Srinivasan 1989) have magnetic fields of the order of $10^{8} \mathrm{G}$, which is 4 orders of magnitude smaller than the field of the usual pulsars. Since a neutron star with a sufficiently weak magnetic field, accreting matter from a surrounding Keplerian disk, can be spun up to millisecond periods (Alpar et al. 1982), it is believed that these millisecond pulsars have been spun up by accretion of matter from their companions in binary systems. This belief receives support from the fact that many millisecond pulsars have been found in binary systems (Wolszczan et al. 1990; Anderson et al. 1990; Kulkarni et al. 1991; Manchester et al. 1991; Anderson 1992, Phinney 1992; D'Amico et al. 1993; Deich et al. 1993; Lyne et al. 1993; Thorsett, Arzoumanian, \& Taylor 1993).

There is no general agreement on the number of millisecond pulsars in the Galaxy. Since these pulsars are thought to be "recycled" pulsars, the belief is that they are to be found in the Population II component of the Galaxy, which includes 200-odd globular clusters (Graham-Smith 1992). Already around 50 millisecond pulsars have been observed in the Galaxy (about a dozen in the disk, and the remainder in the globular clusters) (Ray 1993; Phinney \& Kulkarni 1994).

According to the generally accepted scenario (see, e.g., Graham-Smith 1992), at the time a neutron star is formed it has a magnetic field of $\sim 10^{12} \mathrm{G}$ and 2 rotation period of the order of a second or less. The normal pulsar activity causes the magnetic field of the star to decay and its time period to lengthen. This active phase may last for $10^{7}-10^{8}$
yr , at the end of which the magnetic field might have decayed to $\sim 10^{8} \mathrm{G}$, and the period might have increased to several seconds. (The decay of the pulsar magnetic field is controversial at the moment. Arguments in favor of and against field decay have been presented in the literature, with no clear-cut consensus emerging. This has led some workers to suggest that there may in fact be two pulsar distributions with two different mean field values, one centered on $10^{11.5} \mathrm{G}$ and the other centered on $10^{8.5} \mathrm{G}$. For a review of the various arguments regarding field decay see Phinney \& Kulkarni (1994).) Pulsar activity is no longer possible, and the pulsar is dead. The accretion of matter from the companion star may resurrect it. This accretion may continue for a time span of $10^{6}-10^{7}$ yr (Van den Heuvel 1984). The accretion causes the star to contract and as a consequence to spin up to periods of the order of milliseconds.

It has been shown (Radhakrishnan et al. 1981; Alpar et al. 1982) that the resurrected pulsar will have a period in milliseconds given by the relation

$$
\begin{equation*}
P \sim B_{8}^{6 / 7}\left(\dot{M} \times 10^{8} / M_{\odot} \mathrm{yr}^{-1}\right)^{-9 / 7} \mathrm{~ms}, \tag{1}
\end{equation*}
$$

where $\dot{M}$ is the mass accretion rate and the maximum value of $\dot{M}$ is the Eddington rate given by $M_{\odot} / 10^{8}$ per year, and $B_{8}$ is the star's magnetic field in units of $10^{8} \mathrm{G}$. During the process of accretion the accreting star might emit X-rays. Hence it was natural to consider low-mass X-ray binaries (LMXBs) as the progenitors of this variety of pulsars. Considering the selection effects of pulsar surveys, Kulkarni \& Narayanan (1988) estimated that the number of millisecond pulsars in the Galaxy may be of the order of $10^{6}$. This number seemed in consonance with the Princeton-Arecibo survey (Stokes et al. 1986). However, it was found that the birthrate of LMXBs found in the Galaxy is a factor of 20 smaller than the birthrate of millisecond pulsars. The matter was further examined by Narayan et al. (1989) in the light of more recent surveys, and they concluded that the birthrate of millisecond pulsars was an order of magnitude larger than that of LMXBs. So alternative evolutionary scenarios for the millisecond pulsars, such as tidal capture and accretion-induced collapse of white dwarfs, had to be considered.

A survey of the southern Galactic plane was undertaken by Johnston \& Bailes (1991). In the light of this survey, Johnston \& Bailes (1991) investigated the disk population of the millisecond pulsars. Using the improved distance and flux estimates of the pulsar PSR $1855+09$, they found that the number of bright millisecond pulsars like PSR $1855+09$ was about 2500 , rather than the conservative upper limit of 20,000 for similar pulsars. This, they suggested, removed the discrepancy between the birthrate of millisecond pulsars and that of LMXBs. They also argued that the number of steep spectrum pulsars like PSR $1957+20$ could be highly uncertain, anywhere between 12,700 and $2 \times 10^{5}$, resurrecting once again the problem of birthrates of LMXBs and millisecond pulsars.

Johnston \& Bailes (1991) expressed the hope that the Parkes all-sky survey for millisecond pulsars would either remove the discrepancy between the birthrates of millisecond pulsars and LMXBs or find many new millisecond pulsars. Clearly, at the present time the total number of millisecond pulsars in the disk is quite uncertain. As a working hypothesis, we have adopted a number $10^{4}$ to calculate the integrated $\gamma$-ray flux from the disk population.

Comparison with the $\operatorname{COS} B$ observations and the calculations based on the model of Strong et al. (1988) then allow us to suggest limits on this number.

The estimates of the globular cluster population of millisecond pulsars are as uncertain as those of the disk population. Kulkarni, Narayan, \& Romani (1990); estimated the total number of millisecond pulsars to be $\sim 2000 / f_{b}$ (total number $\sim 10^{4}$ ), where $f_{b}$ is the beaming factor, estimated to be between 0.2 and 1.0 (Wijers \& van Paradijs 1991). Using the radio luminosity of the cores of three globular clusters as guides, Wijers \& van Paradijs (1991) estimated the number of millisecond pulsars in globular clusters to be only 300-700. However, the observation of Manchester et al. (1991) of 11 millisecond pulsars in one globular cluster, 47 Tucanae, alone, changed the picture drastically. These authors remark that since they could identify these pulsars with great difficulty, the globular clusters must be harboring many more millisecond pulsars yet to be discovered. They suggested that the number of millisecond pulsars may therefore be a few hundred per globular cluster rather than $\sim 10$. Spergel (1991) argued that if there are indeed so many millisecond pulsars in the globular clusters, the relativistic particle winds generated by them will be able to drive all gas from the globular clusters and would explain the unsuccessful attempts to detect any gas in them. Working with an improved sample size, Michel (1993) estimated the total number of globular cluster millisecond pulsars to be similar to that given by Kulkarni et al. (1990). Tavani (1991) suggested that on theoretical grounds we should expect a large number of hidden pulsars, in addition to the isolated pulsars and the interacting binary pulsars of the type PSR $1957+20$ and PSR 1744-24A which ablate their companions and undergo periodic eclipses. The hidden pulsars would be completely enshrouded in the material evaporating from their irradiated companions, and their radio emission would be absorbed in the surrounding "bubble." There are no firm estimates of the total number of globular cluster pulsars of any type. Using SIGMA observations, Barret et al. (1993) derive an upper limit of $10^{3}$ for the PSR $1957+20$ type pulsars in one globular cluster, 47 Tuc, alone. Inclusion of the "hidden" variety may make this number larger. Recently, Michelson et al. (1994) have suggested an upper limit of $10^{4}$ globular cluster millisecond pulsars using the EGRET measurement of the high-energy $\gamma$-ray emission from nearby globular clusters. Their argument is based on the magnetospheric efficiency of emission of $\gamma$-rays, which is defined as the ratio of the $\gamma$-ray luminosity of a pulsar to its total spin-down energy. They assume that the mechanism of $\gamma$-ray emission that operates in the millisecond low magnetic field pulsars is the same that operates in the fast-spinning young pulsars with high magnetic fields. Whether the extension of the process that operates at high magnetic fields to low magnetic fields is possible or not is a moot point. However, this should not affect Michelson et al.'s (1994) estimate, because, as pointed out below, the energy that comes out in the form of $\gamma$-rays from a pulsar is largely model independent. In this situation of uncertainty, to develop our argument we have assumed the number of pulsars per globular cluster to be $5 \times 10^{2}$.

## 2. DERIVATION OF THE INTEGRATED FLUX OF $\gamma$-RAYS

The exact nature of the magnetic field of a pulsar is still a matter of argument. However, the starting point of any kind of modeling for magnetospheric activity of a pulsar is the
assumption that the magnetic field of a pulsar may be approximated by a dipole field. Although at the surface of the star the field may be a mixture of dipole and higher multipoles, far from the stellar surface the assumption of a dipolar field may be a very reasonable one. At this stage it is usual to assume, following Goldreich \& Julian (1969), that the pulsar magnetosphere may be divided into two regions: a "closed" or corotating region where the electric field is wholly perpendicular to the magnetic lines of force and therefore unable to accelerate particles, and an "open" region comprising magnetic lines near the polar caps of the star where the electric field has a nonzero component along the field lines that can accelerate the particles to high energies. The charged particle acceleration along the open field lines is an essential feature of any model of pulsar activity.

There are two types of models of particle acceleration, those which consider the particles bound strongly to the stellar surface and therefore not free to flow along the magnetic lines, and those in which the particles have negligible binding energy and are free to flow. In the former category is a model proposed by Ruderman \& Sutherland (1975) in which polar "gaps" develop near the polar caps of the star due to the lack of particle outflow. The large potential difference built up across these gaps is responsible for the particle acceleration. Since there is now evidence that the particles are very weakly bound to the stellar surface (Jones 1985), the models that consider particles flowing freely along the open magnetic lines of force are obviously preferable. In the model developed by Scharlemann et al. (1978), the particles are accelerated to high energies as they move freely along the lines of force. The energetic particles can then emit $\gamma$-rays (Bhatia, Misra, \& Panchapakesan 1992a, 1992b; Harding, Ozernoy, \& Usov 1993). This model appears more appropriate to us because it takes into account the radiation reaction on the charged particle emitting curvature radiation when the pulsar period is less than a certain period denoted by $P_{\mathrm{RR}}$, which is the regime of the millisecond pulsar periods. However, the energy delivered by the energetic particles to the pulsar magnetosphere in the two types of acceleration models is roughly the same. Therefore the $\gamma$-ray luminosity expected from the two sets of models is largely model independent.

According to Scharlemann et al. (1978), the curvature of magnetic field lines permits a pulsar with negligible surface binding energy to accelerate particles to extreme relativistic energies. The relativistic electrons bound to the curved field lines will emit curvature photons of very high energies. At these energies the radiation reaction is important. It has been shown by these authors that the radiation reaction is important when the period of a pulsars is given by the relation

$$
\begin{equation*}
P<P_{\mathrm{RR}}=8.108\left(B_{8} R_{6}\right)^{3 / 7} \mathrm{~ms} \tag{2}
\end{equation*}
$$

where $R_{6}$ is the radius of the neutron star in units of $10^{6} \mathrm{~cm}$. The particles are accelerated along the open field lines. In the pulsars with periods given by equation (2) the electrons acquire energies with Lorentz factor given by

$$
\begin{equation*}
\gamma_{e}=1.8 \times 10^{6} r^{1 / 8} R_{6}^{6 / 8} P_{\mathrm{ms}}^{-3 / 8} B_{8}^{1 / 4} \tag{3}
\end{equation*}
$$

where $P_{\mathrm{ms}}$ is the period in milliseconds and $r$ is the distance from the stellar surface to the point where the radiation is emitted. The model of Scharlemann et al. (1978) is based on the steady flow of nonneutral plasma along the narrow tube of curved field lines. Equation (3) for the Lorentz factor is
valid in the magnetosphere of a pulsar between the stellar surface and the speed-of-light cylinder. Since, according to these authors, most of the acceleration takes place at large radii where the field will be dipolar, we calculate the Lorentz factor in the vicinity of the light cylinder. So we put $r=c / \Omega$, where $\Omega$ is the angular frequency of rotation of the pulsar, and get

$$
\begin{equation*}
\gamma_{e}=1.23 \times 10^{7} R_{6}^{3 / 4} P_{\mathrm{ms}}^{-1 / 4} B_{8}^{1 / 4} \tag{4}
\end{equation*}
$$

The limiting value of the period, from equation (2), below which the Lorentz factor given by equation (4) is valid, is found to be $\sim 10 \mathrm{~ms}$. For the lower limit on the period of rotation, we find that the fastest millisecond pulsar observed has a period of 1.6 ms . A pulsar rotating faster than this rate would probably not be stable. So we shall be dealing with pulsars with periods lying between 1.6 and 10 ms .

The total instantaneous power emitted by an electron as curvature radiation at a frequency $v$ is given by (Ginzburg \& Syrovatsky 1965; Blumenthal \& Gould 1970)

$$
\begin{equation*}
P(v)=\frac{\sqrt{3} e^{3} B_{r}}{m c^{2}} \frac{v}{v_{c}} \int_{v / v_{c}}^{\infty} d \xi K_{5 / 3}(\xi) . \tag{5}
\end{equation*}
$$

Here $v_{c}$ is the so-called critical frequency of the curvature radiation corresponding to the energy $E_{c}$ given by

$$
\begin{equation*}
E_{c}=\frac{3}{2} \frac{\hbar c \gamma_{e}^{3}}{R_{c}} \tag{6}
\end{equation*}
$$

where $R_{c}=(r c / \Omega)^{1 / 2}=c / \Omega$ is the radius of curvature of the magnetic lines of force. Substituting for $\gamma_{e}$ from equation (4), we can rewrite $E_{c}$ as

$$
\begin{equation*}
E_{c}=1.7 \times 10^{-2} P_{\mathrm{ms}}^{-14 / 8} R_{6}^{18 / 8} B_{8}^{3 / 4} \mathrm{ergs} . \tag{7}
\end{equation*}
$$

In equation (5) $B_{r}$ is the strength of the magnetic field at the point where the radiation is emitted. Taking this point to be in the region of the light cylinder and assuming that the magnetic field is dipolar in nature, we may rewrite equation (5) as

$$
\begin{equation*}
P(v)=\sqrt{3} \times 10^{26}\left(e^{3} B_{8} \Omega^{3} / m c^{5}\right) F\left(v / v_{c}\right) \tag{8}
\end{equation*}
$$

where $B_{8}$ is the surface magnetic field and the function $F\left(v / v_{c}\right)$ stands for $\left(v / v_{c}\right) \int_{v / v_{c}}^{\infty} K_{5 / 3}(\xi) d \xi$. This function has been plotted and tabulated in the literature (Ginzburg \& Syrovatsky 1965; Blumenthal \& Gould 1970).

The number of photons emitted by one electron in 1 s between frequencies $v_{1}$ and $v_{2}$ is given by $\int[P(v) / h v] d v$, so that the rate of emission of photons in the energy range from $E_{1}$ to $E_{2}$ may be written as

$$
\begin{equation*}
\dot{N}_{E_{1}-E_{2}}=\frac{8 \pi^{3}}{27} \frac{\sqrt{3} e^{3}}{h m c^{2}} 10^{5} B_{8} R_{6}^{3} P_{\mathrm{ms}}^{-3} \int_{E_{1}}^{E_{2}} \frac{F(E)}{E} d E \tag{9}
\end{equation*}
$$

where energies are expressed in units of $E_{c}$. The total number of $\gamma$ photons emitted by a millisecond pulsar in 1 s can now be written as

$$
\begin{equation*}
\dot{N}_{\gamma}=\dot{N}_{E_{1}-E_{2}} \dot{N} \tau \tag{10}
\end{equation*}
$$

where $\dot{N}$ is the net rate at which the charged particles are emitted from the star at the polar cap and is given by the relation

$$
\begin{equation*}
\dot{N}=\frac{\Omega^{2} B R^{3}}{2 c e} \tag{11}
\end{equation*}
$$



FIG. 1.-Relationship between the Galactocentric and the Galactic systems of coordinates
where $R$ is the radius of the star. In equation (10), $\tau$ is the time during which an electron emits radiation. A reasonable measure of this time is $\left(m c^{2} \gamma_{e} / \ell\right)$, where $\ell$ given by

$$
\begin{equation*}
\ell=\frac{2}{3} \frac{e^{2} c \gamma_{e}^{4}}{R_{c}^{2}}, \tag{12}
\end{equation*}
$$

or

$$
\begin{equation*}
\ell=(2 / 3) e^{2} \gamma_{e}^{4} \Omega^{2} / c, \tag{13}
\end{equation*}
$$

is the total power radiated by an electron in curvature radiation. Substituting for all the quantities on the right-hand side of equation (10), we get finally the number of photons emitted per second from a single millisecond pulsar as

$$
\begin{equation*}
\dot{N}_{\gamma}=9.7 \times 10^{36} B_{8}^{5 / 4} P_{\mathrm{ms}}^{-9 / 4} R_{6}^{15 / 4} \int_{E_{1}}^{E_{2}} \frac{F(E)}{E} d E . \tag{14}
\end{equation*}
$$

For an estimate of the total contribution from all the millisecond pulsars present in the disk and the globular clusters, their space and period distribution has to be taken into consideration. It must be noted that we have not relied upon any particular fraction of the total spin-down energy as energy given out in $\gamma$-rays, as has been done in some modelings of the $\gamma$-rays from millisecond pulsars in the disk and globular clusters (Bhattacharya \& Srinivasan 1991; Tavani 1993). Indeed, the limited available data on $\gamma$-rays from pulsars give no clue at all to the ratio between the spin-down energy and the energy radiated as $\gamma$-rays.

The model for the space distribution of millisecond pulsars in the disk of the Galaxy suggests that their volume density $\phi_{d}(R, z)$ as a function of the Galactocentric radius $R$ and of the height from the plane of the Galaxy $z$ is given by (Kulkarni \& Narayanan 1988)

$$
\begin{equation*}
\phi_{d}(R, z)=\frac{N_{\text {total }}}{4 \pi R_{\mathrm{sf}}^{2} z_{\mathrm{sf}}} \exp \left(-\frac{R}{R_{\mathrm{sf}}}-\frac{|z|}{z_{\mathrm{sf}}}\right), \tag{15}
\end{equation*}
$$

where $R_{\mathrm{sf}}$ and $z_{\mathrm{sf}}$ are the scale factors having values 4 kpc and 300 pc , respectively. $N_{\text {total }}$ is the estimated number of millisecond pulsars present in the disk of the Galaxy. The period distribution of the disk component may be approx-
imated to (Bhattacharya \& Srinivasan 1991)

$$
\begin{equation*}
\rho_{\mathrm{Pd}}=2 P_{\mathrm{ms}} /\left(P_{\max }^{2}-P_{\min }^{2}\right), \tag{16}
\end{equation*}
$$

with $P_{\max }=10 \mathrm{~ms}$ and $P_{\min }=1.6 \mathrm{~ms}$.
The distribution of globular cluster pulsars is not known. As a working hypothesis it may be assumed that they follow the space distribution of the clusters themselves. Harris (1976) compiled the distance moduli of 111 globular clusters based on the assumption that $\left\langle M_{v}(\mathrm{RR})\right\rangle=\left\langle M_{v}(\mathrm{HB})\right\rangle=$ $0.6 \pm 0.3$ and corresponding to the distance between the Sun and the Galactic center, of 8.5 kpc . On the basis of these data Harris (1976) suggested that the space density distribution, $\phi_{\mathrm{gc}}$, of the globular clusters is spherical and followed the power law $\rho^{-3.5}$ where $\rho$ is radial distance from the center of the galaxy. Using the data of Harris (1976) and Madore (1980), we found the best power-law fit to the density distribution of globular clusters to be given by $13.56 \rho^{-3.347}$. The density distribution of millisecond pulsars in the globular clusters may then be written as

$$
\begin{equation*}
\phi_{\mathrm{gc}}=N_{\mathrm{gcp}}\left(13.56 \rho^{-3.347}\right), \tag{17}
\end{equation*}
$$

where $N_{\mathrm{gcp}}$ is the average number of pulsars in a globular cluster. As discussed above, we have adopted this number as $5 \times 10^{2}$.

The period distribution of the known globular cluster millisecond pulsars has been suggested by Chen (1991) on the basis of their observed periods. It is a power-law distribution:

$$
\begin{equation*}
\rho_{\mathrm{gcp}}(P)=(\alpha-1) P_{\min }^{\alpha-1} P_{\mathrm{ms}}^{-\alpha}, \tag{18}
\end{equation*}
$$

where the period $P$ is in milliseconds, $\alpha=1.4$ for $P_{\mathrm{ms}}>$ $P_{\min }$, and the minimum value of the period, $P_{\min }$, is 1.6 ms .

The $\gamma$-ray flux due to each population of millisecond pulsars, if the center of the Galaxy is the point from which the observation is made, can be estimated from the expression

$$
\begin{equation*}
F_{\gamma}=\iiint q \phi(R, z, \theta) d R d z d \theta, \tag{19}
\end{equation*}
$$

where $\theta$ is the azimuthal angle in the plane of the Galaxy (Fig. 1). In this equation $\phi$ could be either $\phi_{d}$ or $\phi_{\mathrm{gc}}$ (transformed to the cylindrical system), and $q$ stands for the
integral

$$
\begin{equation*}
q=\frac{1}{4 \pi} \int_{P_{\min }}^{P_{\max }} \dot{N}_{\gamma} \rho_{P}(P) d P \text { photons s}{ }^{-1} \mathrm{sr}^{-1} \tag{20}
\end{equation*}
$$

where $\rho_{P}$ would stand for $\rho_{\mathrm{Pd}}$ if $\phi$ in equation (19) is $\phi_{d}$, and would be $\rho_{\mathrm{gcp}}$ if $\phi$ is $\phi_{\mathrm{gc}}$. Substituting the value of $\dot{N}_{\gamma}$ from equation (14) we can rewrite the above expression for $q$ as

$$
\begin{equation*}
q=7.71 \times 10^{35} \int_{P_{\min }}^{P_{\max }} B_{8}^{5 / 4} P_{\mathrm{ms}}^{-9 / 4} \rho_{p}(P) d p \int_{E_{1}}^{E_{2}} \frac{F(E)}{E} d E \tag{21}
\end{equation*}
$$

where we have put $R_{6}=1$. All those millisecond pulsars whose magnetic fields have been measured seem to follow the relation $B_{8} \approx P_{\mathrm{ms}}^{7 / 6}$, which is in accordance with equation (1). We shall be using this relation for evaluating the integral in equation (21).

The observations are usually carried out in terms of the Galactic coordinate system with Earth as the center of the system. The transformation of equation (19) to this set of coordinates is effected by the following relations (Fig. 1):

$$
\begin{gather*}
r=s \cos b \\
z=s \sin b \\
R=\left(R_{\odot}^{2}+r^{2}-2 R_{\odot} r \cos l\right)^{1 / 2} . \tag{22}
\end{gather*}
$$

With the coordinate transformation carried out, the expression for the flux (eq. [19]) averaged over longitude range $\Delta l$ becomes

$$
\begin{equation*}
F_{\gamma}=\frac{1}{\Delta l} \iiint q \phi(s, l, b) \cos b d s d l d b \text { photons } \mathrm{s}^{-1} \mathrm{sr}^{-1} \tag{23}
\end{equation*}
$$

where $l$ is longitude and $b$ latitude, and $\Delta l$ is in radians. The flux due to both the populations of millisecond pulsars is calculated in this manner.

## 3. RESULTS AND DISCUSSION

The calculation of the $\gamma$-ray flux involves integration over energy range $E_{1}-E_{2}$. The choice of $E_{1}$ and $E_{2}$ has to be made depending on the energy range over which the background radiation of $\gamma$-ray flux is observed in the Galaxy. We choose this range to be $300-5000 \mathrm{MeV}$, close to the characteristic curvature photon energies of pulsars with periods between 1.6 and 10 ms . Integration of equation (21) involves double integration, which has been done numerically. We recall that $E$ is expressed in terms of $E_{c}$, which according to equation (7) is itself a function of the pulsar period. So for each $P_{\mathrm{ms}}$ from $P_{\min }$ to $P_{\max }$, the inner integral over $E$ is evaluated and then the outer integral is calculated. This value of $q$ is then used in equation (21).

To facilitate comparison with observations, we integrate equation (23) for various latitudes over all distances and longitude range $300^{\circ}<l<60^{\circ}$ excluding the range $350^{\circ}<l<10^{\circ}$ because the region close to the Galactic center deviates in behavior from the rest of the Galactic plane (Strong et al. 1988). Figure 2 shows our results superposed on the COS B observations (Strong et al. 1988). The curve labeled 1 shows the calculated $\gamma$-ray flux due to the disk component of $\sim 10^{4}$ millisecond pulsars. Curve 2 shows the flux due to the globular cluster millisecond pulsars on the assumption that their number is $5 \times 10^{2}$ per globular cluster. Curve 3 corresponds to the contribution of the two populations together. We note that the combined $\gamma$-ray flux due to the millisecond pulsars of both the disk and globular cluster populations falls short of the observations by a factor of $\sim 2$ at latitude zero and by a factor of $\sim 3$ at other latitudes. Since the flux of the disk millisecond pulsars falls sharply with latitude, it is obvious that if we had 3 times as many globular cluster pulsars as we have assumed, then the combined flux of the two components


Fig. 2.-Expected contribution from millisecond pulsars to the Galactic $\gamma$-ray intensity in the energy range $300-5000 \mathrm{MeV}$ and in the longitude interval $-60^{\circ}$ to $60^{\circ}$ is shown as a function of Galactic latitude. The observations of COS B (Strong et al. 1988) are also shown. Curve 1 : Intensity expected from the millisecond pulsars in the disk of the Galaxy (assuming $N_{\text {total }}=10^{4}$ ); Curve 2: Intensity expected from the millisecond pulsars in the globular clusters of the Galaxy (number per globular cluster assumed 500); Curve 3: Expected combined intensity from all millisecond pulsars in the Galaxy.


Fig. 3.-Same as Fig. 2, except that the average number of millisecond pulsars per globular cluster assumed is 1500
agrees with the observations (Fig. 3). The integrated $\gamma$-ray flux from the millisecond pulsars could then be proposed as a source of background $\gamma$-rays, as an alternative to the model of Strong et al. (1988), which is based on the interaction of the interstellar $\mathrm{H}_{\mathrm{I}}$ and $\mathrm{H}_{2}$ with cosmic rays and the inverse Compton emission.
To test whether a combination of disk and globular cluster components of millisecond pulsars can explain satisfactorily the background $\gamma$-ray flux in the Galaxy, we plot the longitude profile of the $\gamma$-rays from these sources in Figure 4. We find that this profile is quite different from the
one that is observed. The latitude profile in the outer Galaxy (longitudes $60^{\circ}-300^{\circ}$ ), too, is quite inconsistent with the observations (Fig. 5). The lack of agreement between the longitude profiles and the latitude profiles in the outer Galaxy is perhaps not unexpected, given the concentration of millisecond pulsars near the center of the Galaxy indicated by the distributions derived from equations (15) and (17).

It seems, therefore, that the integrated $\gamma$-ray emission from the millisecond pulsars cannot account for the Galactic $\gamma$-ray background. The model of Strong et al. (1988)


Fig. 4.-Expected $\gamma$-ray flux from the millisecond pulsars as a function of longitude. COS B observations are also shown.


Fig. 5.-Expected $\gamma$-ray flux from the millisecond pulsars in the outer Galaxy (longitude range $60^{\circ}-300^{\circ}$ ). $\operatorname{COS} B$ observations are also shown.
seems to offer a better explanation. With this in mind we can use our calculations to suggest constraints on the number of millisecond pulsars in the Galaxy. In order that the latitude distribution of $\gamma$-ray flux predicted by Strong et al. is not disturbed by the contribution of the integrated flux from the millisecond pulsars, the number of millisecond pulsars in the globular clusters should be much less than $5 \times 10^{2}$ per globular cluster. If we take into account the error bars in the $\operatorname{COS} B$ observations, the number of millisecond pulsars in the globular clusters can be further constrained to be $\sim 2 \times 10^{2}$ per cluster. The total number of millisecond pulsars in the globular clusters would then be close to the upper limit derived by Michelson et al. (1994) from the EGRET measurements. The population of disk millisecond pulsars that can be accommodated by the COS $B$ observations and the predictions of Strong et al., in case
the number of globular cluster millisecond pulsars is very small, is constrained to be $\lesssim 10^{4}$. However, if allowance is made for the contribution of $\gamma$-ray flux from $\lesssim 2 \times 10^{2}$ millisecond pulsars per globular cluster, then the number of millisecond pulsars of the disk population must be scaled down to $<5 \times 10^{3}$, which is within a factor of 2 of the bright pulsars like PSR $1855+09$ found by Johnston \& Bailes (1991). The flux from these pulsars at zero latitude would then be just sufficient to fill the gap between the observed flux and that predicted by Strong et al. (1988) without disturbing the latter curve at other latitudes (Fig. 6 ).
As we have discussed above, there is lot of uncertainty regarding the number of millisecond pulsars in the Galactic disk and in globular clusters. Different surveys have thrown up different numbers. In this state of uncertainty we believe


Fig. 6.-The expected $\gamma$-ray flux from the disk component of millisecond pulsars ( $10^{4}$ in number) (Curve 1 ), supplementing the predictions of Strong et al. (1988), is shown as Curve 2. COS B observations are also shown.
that the limits we derived would serve some useful purpose for surveys now in operation and those in the planning stage.

## 4. CONCLUSIONS

In this paper we have calculated the diffuse $\gamma$-ray flux expected from a superposition of $\gamma$-rays from millisecond pulsars in the disk of the Galaxy and in the globular clusters. Comparison with the $\operatorname{COS} B$ observations and the predictions of the model of Strong et al. (1988) allows us to
place constraints on the number of millisecond pulsars of both populations. We conclude that the number of disk millisecond pulsars must be $\lesssim 5 \times 10^{3}$ and the number of millisecond pulsars in the globular clusters must be less than $\sim 2 \times 10^{2}$ pulsars per globular cluster.

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