THE ASTRONOMICAL JOURNAL, 115:2539–2550, 1998 June

THE EVOLUTION OF BLUE STRAGGLERS FORMED VIA STELLAR COLLISIONS

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Received 1997 November 6; revised 1998 February 25

ABSTRACT

We have used the results of recent smoothed particle hydrodynamic simulations of colliding stars to create models appropriate for input into a stellar evolution code. In evolving these models, we find that little or no surface convection occurs, precluding angular momentum loss via a magnetically driven stellar wind as a viable mechanism for slowing rapidly rotating blue stragglers that have been formed by collisions. Angular momentum transfer to either a circumstellar disk (possibly collisional ejecta) or a nearby companion are plausible mechanisms for explaining the observed low rotation velocities of blue stragglers. Under the assumption that the blue stragglers seen in NGC 6397 and 47 Tuc have been created solely by collisions, we find that the majority of blue stragglers cannot have been highly mixed by convection or meridional circulation currents at any time during their evolution. Also, on the basis of the agreement between the predictions of our nonrotating models and the observed blue straggler distribution, the evolution of blue stragglers is apparently not dominated by the effects of rotation.

Key words: blue stragglers — stars: evolution

1. INTRODUCTION

Blue stragglers lie roughly along an extension of a star cluster's main sequence (MS) and are generally bluer and brighter than the turnoff (TO) stars. While several possible mechanisms for their formation have been proposed (e.g., delayed star formation, binary mass transfer, binary coalescence, stellar collisions; see Livio 1993 and Stryker 1993 for reviews), the collision scenario, in which initially unbound stars come into contact and merge during a dynamical encounter, has received the most attention recently (e.g., Sandquist, Bolte, & Hernquist 1997; Ouellette & Pritchet 1996; Sills, Bailyn, & Demarque 1995). Blue stragglers formed by collisions are of particular interest, because their formation rate tells us about the stellar interaction rate in their cluster environment and the dynamical history of the cluster itself (Bailyn 1995).

There are at least two ways to investigate the formation rate of blue stragglers by collisions. One way is to perform N-body simulations, modeling the stellar environment and determining the collision rate directly. This requires knowledge of the stellar interaction cross section, local stellar density, mass function, and binary fraction. Another way is to model the structure of the merger remnants and to evolve them using a stellar evolution code. Just as the age of a cluster is found by comparing theoretical isochrones, based on standard stellar models, with an observed color-magnitude diagram (CMD), comparison of the evolution of the merger models with the observed numbers and distribution of blue stragglers in the CMD can tell us about the distribution of lifetimes, and the production rates, of these stars.

In this paper, we develop and evolve models of collisional merger remnants and compare the predictions of these models with the observed blue straggler distributions in NGC 6397 and 47 Tuc.

2. PREDICTIONS OF THE COLLISION SCENARIO: A REVIEW

There is direct and indirect evidence that blue stragglers are more massive than the TO stars in their parent clusters (spectroscopic measurements: Shara, Saffer, & Livio 1997; Rodgers & Roberts 1995; mass segregation: e.g., Sarajedini & Forrester 1995; Lauzeral et al. 1992; Mathieu & Latham 1986), suggesting a formation mechanism that involves combining the mass of at least two stars into one massive star (e.g., Livio 1993). Although there are a few possible mechanisms by which this can be achieved, this paper is concerned with only one of them—the collision scenario. This requires that two stars come into direct contact during a strong dynamical encounter and then merge, leaving behind a massive remnant. As well as being more massive, the collisionally merged star will have other properties that we will summarize here.

Hills & Day (1976) investigated the possibility that collisions could have occurred between single stars in globular clusters (GCs) within the lifetimes of the clusters. They found that, in the dense cores of some GCs, the timescale for single-star interactions is short enough that a significant number of collisions would have occurred. Also, because the impact parameter at the time of the collision is random, the collisions will, on average, occur off-axis. Angular momentum conservation then requires that the merged star be rapidly rotating. Hills & Day hypothesized that the excess energy from the collision would mix nuclear-processed material throughout the merger remnant, as well as mixing hydrogen into the core, essentially "resetting the nuclear clock" to the chemically homogeneous zero-age main-sequence (ZAMS) state.

Hoffer (1983) and Leonard (1989) extended the analysis of collision probabilities to include binary–single star and binary-binary interactions. Because of the increased physical cross section (essentially the semimajor axis of the binary) and the additional gravitational focussing caused by the larger mass of the bound pair, the probability that a collision will occur is greatly increased if one or more of the interacting objects is a binary. Since the rate at which binary interactions will occur in a population of stars will increase with the binary fraction, we expect to see a significant collision rate in clusters with a large binary fraction and in the cores of clusters, where most of the binaries are expected to be found as a result of mass segregation.

Leonard & Fahlman (1991) found that the rate of production of blue stragglers in NGC 5053 needed to be at least one blue straggler every $\sim 2.5 \times 10^8$ yr (assuming that the average lifetime of a blue straggler is $\sim 6 \times 10^9$ yr) to maintain the observed numbers. From their scattering experiments, a strong binary-binary interaction should occur every $\sim 2.2 \times 10^7$ yr, with only one in 20–40 such interactions actually resulting in a stellar collision [hence, one blue straggler produced every $\sim (4\text{–}8) \times 10^8$ yr]. According to Sigurdsson & Phinney (1993), the timescale for exchanges during strong interactions is significantly shorter than the timescale for collisions. Also, such exchanges tend to produce a net hardening of the binary, which will lengthen the timescale for collisions.

Leonard (1996) noted that the high frequency of binaries observed among blue stragglers (e.g., Kaluzny et al. 1997; Kaluzny 1997; Edmonds et al. 1996) could be accounted for by binary-binary interactions. A dynamical encounter that is strong enough to result in a direct collision between two of the component stars is also likely to result in a third star being captured into an eccentric orbit around the newly formed blue straggler; the fourth star is ejected from the system, removing the excess binding energy.

These dynamical studies treated the interacting stars as point sources, and assumed that a direct collision would take place and form a blue straggler if two of the binary components approached within some factor of their radii, ignoring the dynamics of the stellar collision itself. Benz & Hills (1987, 1992) studied stellar collisions by performing smoothed particle hydrodynamic (SPH) simulations of colliding $n = 3/2$ polytropes. They found that the resulting merger remnant's properties depended on the relative mass of the parent stars. During collisions involving equal-mass stars, Benz & Hills found that the material from the parent stars was distributed throughout the merger remnant, resulting in a star with a roughly homogeneous chemical composition: essentially a ZAMS star. Their collisions involving unequal-mass stars resulted in a merger remnant with the material from the less evolved, low-mass parent star in the core of the remnant; the material from the high-mass star was, in contrast, mixed throughout. In either case, the merger remnant roughly resembled a ZAMS star, because of the increased hydrogen content in the core.

Lombardi, Rasio, & Shapiro (1996, hereafter LRS) found that the high degree of mixing that Benz & Hills had observed in their merger remnants was largely an artifact of the low resolution of their simulations and their choice of $n = 3/2$ polytropes as approximations to GC stars. LRS noted that evolved GC stars would be more centrally condensed than $n = 3/2$ polytropes, and so chose to use $n = 3$ polytropes to approximate the structure of TO stars and $n = 3/2$ polytropes for lower mass stars. SPH simulations using these approximations demonstrated that there should be little or no hydrodynamical mixing during a stellar collision.

Benz & Hills (1987, 1992) and LRS found that significant mass loss could occur during grazing collisions; in the high impact velocity collisions of Benz & Hills, the two stars could be completely disrupted, leaving no single remnant behind. If the star was not disrupted during the collision, it was swollen to pre–main-sequence sizes by the injection of orbital kinetic energy into the stellar material. The greatest amount of mass loss for a given impact velocity occurred during off-axis collisions.

The high rotation velocity that the collision scenario predicts for blue stragglers is one of its potential weak points. In open clusters, such as M67 (Peterson, Carney, & Latham 1984; Mathys 1991; Pritchet & Glaspey 1991), blue stragglers are not observed to be rapidly rotating. The only reported observation of the rotation velocity of a blue straggler in a globular cluster (Shara et al. 1997), $V \sin i = 155 \pm 55$ km s^{-1}, is high, but not necessarily unusual for normal stars of the same spectral type (A7 V: $\bar{V}_{\rm rot} \sim$ 130–170 km s^{-1}, Lang 1992), nor is it rotating near to its break-up velocity of ~ 410 km s^{-1} (estimated using the mass and log g from Shara et al.).

Leonard & Livio (1995) suggested that the rotation velocity of a blue straggler could be lowered by angular momentum loss (AML) via a magnetically driven stellar wind, similar to the mechanism proposed for pre–main-sequence stars by Tout & Pringle (1992). For this mechanism to work, the blue straggler would need to be left in a largely convective, Hayashi-like phase after the collision.

In addition to providing an AML mechanism for blue stragglers, a highly convective phase would also mix the stars, homogenizing them despite the fact that they will be initially inhomogeneous (LRS). Bailyn & Pinsonneault (1995) and Sills et al. (1995) found that such a high degree of mixing is needed to explain the colors and luminosities of blue stragglers seen in some globular clusters. On the other hand, Ouellette & Pritchet (1996) found that blue stragglers tend to avoid the ZAMS and that a high degree of mixing during a pre–main-sequence phase is essentially ruled out for most blue stragglers by their distribution on the CMD. If significant surface convection occurs (though not necessarily enough to mix the star), abundance anomalies may be observable. In fact, many of the blue stragglers in M67 have been observed to be underabundant in lithium (Pritchet & Glaspey 1991) and have anomalous CNO abundances (Mathys 1991), possibly indicative of mixing.

In short, the properties of blue stragglers formed through collisions are that they should be more massive than the cluster TO stars, they *might* be rapidly rotating, they *might* have surface chemical anomalies, and they are likely to have a companion in an eccentric orbit.

3. STRUCTURE OF MERGER REMNANTS

Scattering experiments, such as those of Leonard (1989) and Sigurdsson & Phinney (1993), tell us at what rate we can expect stellar collisions to occur. However, in order to test the accuracy and viability of the collision scenario, we need to know the timescales over which collisional merger remnants will be observable as blue stragglers. This can be done by creating stellar models from the predictions of the SPH simulations of stellar collisions, and following their evolution with a stellar evolution code. Once these timescales are known, they can be combined with the results of scattering experiments to predict the number of blue stragglers and other remnants of such strong interactions (Bailyn 1995) that should be observable in a cluster. Also, calculation of the evolution of merger remnants allows us to make predictions concerning the distribution of blue stragglers in the CMD. Comparison of this distribution with observations allows us to infer whether collisional mergers are likely to have occurred in a population of blue stragglers, and even the dynamical history of the cluster itself.

As mentioned earlier, LRS have studied stellar collisions by performing SPH simulations of colliding polytropes.

Their results showed that little, if any, hydrodynamic mixing occurred during the collision. The composition profiles of their merger remnants can be understood as an "entropy stratification" of the stellar material during the collision: the gas from the parent stars settles in the merger remnant such that the final entropy profile increases from the core outward. This entropy stratification can result in some unusual chemical profiles (as shown by LRS), but also allows some prediction of the chemical profile of a collisional remnant—which we will use in this study.

The collisions studied by LRS were between equal-mass polytropes and unequal-mass polytropes, for a variety of different masses. We choose here to model only mergers between equal-mass stars and mergers between a TO star and a lower mass star. Throughout the rest of this paper, we refer to these merger events as "equal-mass mergers" and "unequal-mass mergers," respectively. Although there are any number of possible combinations of parent star masses that would result in a particular merger mass between $1M_{\rm TO}$ and $2M_{\rm TO}$, the mergers considered here represent the extremes of hydrogen content: equal-mass mergers will result in the highest possible hydrogen content for a particular merger mass, whereas unequal-mass mergers will result in the lowest possible hydrogen content.

3.1. *Predictions from Entropy Stratification*

In their simulations, LRS approximated MS TO stars by $n = 3$ polytropes, while lower mass stars were approximated by $n = 3/2$ polytropes or composite polytropes. Their initial polytropic models had entropy and density profiles similar to those shown in Figure 1. During a collision involving a $n = 3$ polytrope ($\sim 0.80\ M_\odot$, MS TO star) and an $n = 3/2$ polytrope ($M \lesssim 0.40\ M_\odot$, lower MS star), entropy stratification predicts that the material of the lower mass, and presumably less evolved, $n = 3/2$ polytrope will settle into the core of the merger remnant, bringing with it a fresh supply of hydrogen. The subsequent evolution of the merger remnant will be strongly affected by the amount of hydrogen brought into the core by the entropy stratification. This stratification of the material also provides a simple explanation of why no nuclear-processed material is brought to the surface of the merger remnant during a collision.

If the entropy of the stellar gas is not modified during a collision, the distribution of the parent stars' material throughout the merger remnant can be found using the entropy profiles in Figure 1—this leads directly to the merger remnant's chemical profile if those of the parent stars are also known. However, shock heating during the collision can modify the entropy of the gas depending on the dynamics of the collision and the form of viscous dissipation chosen for the SPH (LRS). During relatively gentle collisions, such as the head-on, parabolic collisions studied by LRS, little shock heating will occur, and so the entropy of the gas at the time of the collision can be used to determine the final merger profile (see § 3.1.2).

3.1.1. *Stellar Collisions versus Polytropic Collisions*

During the previous discussion we have stressed that the simulations of LRS and Benz & Hills (1987, 1992) describe *polytropic* collisions. The reason for this pointed emphasis is that stars, especially evolved ones in GCs, are not polytropes.

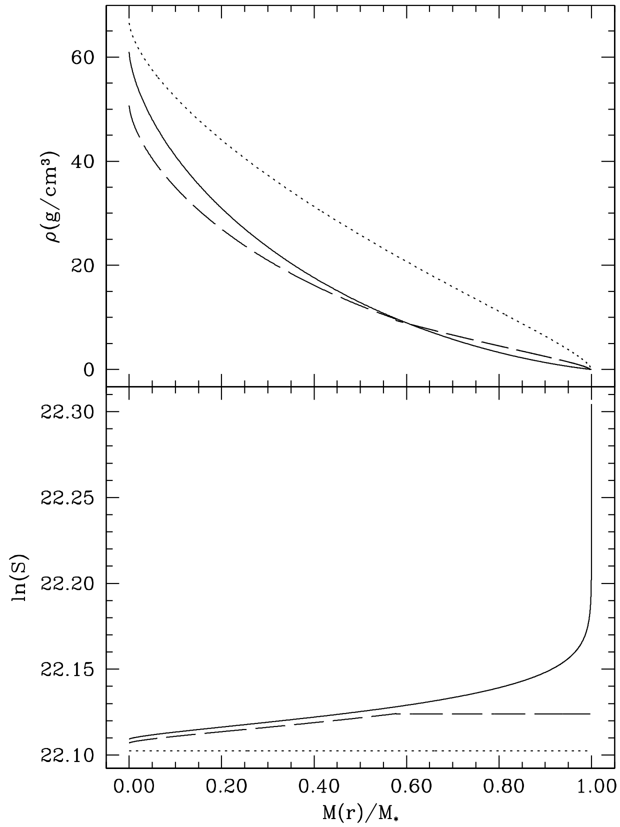

FIG. 1.—Density (ρ) and entropy (S) profiles for representative polytropes. Shown are profiles for 0.80 $M_\odot$ (*solid line*; $n = 3$, $R = 1.0\ R_\odot$), 0.60 $M_\odot$ (*dashed line*; $n = 3$, $R = 0.56\ R_\odot$), and 0.40 $M_\odot$ (*dotted line*; $n = 3/2$, $R = 0.37\ R_\odot$) polytropes, calculated using the radii and polytropic indices given. The 0.6 $M_\odot$ composite polytrope has an $n = 3$ core and an $n = 1.5$ envelope with the boundary between the two located at $R_{\rm bnd} = 0.29\ R_\odot$.

The structure of a polytrope is given by

$$P(r) = K\rho(r)^{1+1/n}\ , \quad (1)$$

where P is the pressure, ρ is the gas density, n is the polytropic index, and K is a constant. Using this relationship as a constraint, $\rho(r)$ and K can be found using only two of the equations of stellar structure: the equations of hydrostatic equilibrium and mass continuity. For a simple polytrope, the molecular weight μ is constant throughout, in which case both K and n are also constants. This allows the structure of the polytrope to be determined with no information about sources of opacity or nuclear processes.

The entropy profile of a polytrope can be found by assuming an ideal gas equation of state of the form

$$P = A\rho^{\Gamma_1}\ , \quad (2)$$

where Γ_1 is one of the adiabatic exponents and is defined by

$$\frac{dP}{P} + \Gamma_1 \frac{dV}{V} = 0\ . \quad (3)$$

If radiation pressure within the gas can be ignored, then $\Gamma_1 = \gamma = c_P/c_V = 5/3$ for an ideal, monatomic gas. Equating the pressure from the polytropic structure to the pressure from the equation of state yields A throughout the gas. The relationship between the entropy S and the constant A can be found by integrating the first law of thermodynamics, $T\,dS = dU + P\,dV$, using the definition of γ and equation (3). This yields

$$S = \frac{N_0 k}{\mu\gamma} \ln A + \text{const} , \quad (4)$$

where N_0 is Avagadro's number and k is Boltzmann's constant. From the form of the polytropic structure equation (eq. [1]) and the equation of state (eq. [2]), if $1 + 1/n$ is not equal to γ, then A, and hence the entropy S, will not be constant throughout the star. Polytropes with $n = 3/2$ will have $1 + 1/n = \gamma$, and so will have constant entropy.

Polytropes are reasonable approximations to ZAMS stars since they, like polytropes, are chemically homogeneous, or at least nearly so. As pointed out by LRS, $n = 3$ polytropes approximate the structure of radiative stars, while $n = 3/2$ polytropes have a structure similar to that of fully convective stars. However, although GC TO stars are radiative, they are far from being chemically homogeneous. Figure 2 shows the entropy and density profiles for a GC TO star and lower MS stars for a cluster like 47 Tuc. The differences between the entropy and density profiles of GC stars and the polytropes shown in Figure 1 are largely due to the chemical inhomogeneity of the GC stars. The increased molecular weight toward the center of an evolved star results in a lower pressure at a fixed density and temperature, requiring the star to adjust its internal structure to maintain hydrostatic equilibrium. Equation (4) shows that this increase in the molecular weight results in a decrease in the entropy of the gas.

The decrease of the entropy in the core of an evolved star will directly affect the final chemical profile of a merger

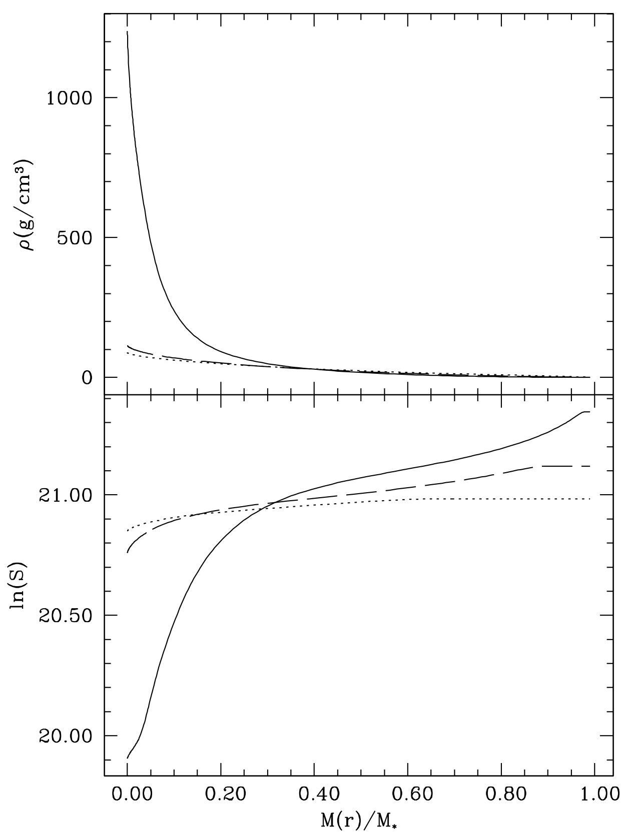

FIG. 2.—Density (ρ) and entropy (S) profiles for real GC stars. Shown are profiles for 0.864 $M_\odot$ (*solid line*), 0.60 $M_\odot$ (*dashed line*), and 0.40 $M_\odot$ (*dotted line*) stellar models at an age (14 Gyr) and metallicity ([Fe/H] = −0.71) appropriate for 47 Tuc.

found through entropy stratification. From Figure 2, it is obvious that no material from a low-mass star will be able to penetrate to the core of a MS TO star, leaving the merger remnant with a helium-rich core, unlike the collision between two equivalent polytropes. The evolution of two such merger remnants would be completely different if no mixing were to take place in a subsequent convective phase of evolution.

Comparison of Figures 1 and 2 shows that an $n = 3$ polytrope underestimates the central density of a TO star by more than a factor of 10. This in itself is enough to argue that no hydrogen-rich material from a lower mass star will penetrate to the core of a merger involving a TO star and a lower MS star (an unequal mass merger in our terminology). Entropy stratification of the material during such a collision will produce a merger remnant with a dense, helium-rich core.

A collision involving equal-mass stars or one involving equal-mass polytropes should produce a merger remnant with a composition profile nearly identical to those of the parent stars.

3.1.2. *Shock Heating*

As mentioned earlier, shock heating can modify the entropy of the gas during the collision and so affect the distribution of material throughout the merger remnant. If the amount of shock heating is significant, the structure of the merger remnant could differ considerably from what would be expected if the unmodified entropy profiles of the parent stars were used to estimate the final structure.

The amount of shock heating caused by the passage of a shock can be estimated from the pre-shock state of the gas, and the velocity of the shock. Making the simple assumptions that the sound velocity within the gas is equal to the adiabatic sound speed, $c_s^2 = \partial P/\partial\rho$, and that there is no change of state due to the passage of the shock (i.e., $\gamma_{\text{final}} = \gamma_{\text{initial}}$), then, taking the Mach number of the shock as $M_0(=V_S/c_s)$, the change in entropy after the passage of the shock is

$$\Delta S = S_1 - S_0 = \frac{N_0 k}{\mu\gamma} \ln\left[\left(\frac{P_1}{P_0}\right)\left(\frac{\rho_1}{\rho_0}\right)^{-\gamma}\right] \quad (5)$$

(e.g., Shore 1992), where

$$\frac{\rho_1}{\rho_0} = \frac{(\gamma+1)M_0^2}{(\gamma-1)M_0^2+2} ,$$

$$\frac{P_1}{P_0} = \frac{2\gamma M_0^2-(\gamma-1)}{\gamma+1} .$$

Here, the subscript 0 refers to the conditions on the immediate preshock side of the shock discontinuity, and the subscript 1 refers to those conditions immediately on the postshock side of the discontinuity. Figure 3 shows the effect of shock heating on the entropy profiles of the stars shown in Figure 2 under the assumption that the shock velocity remains constant throughout the star.[1] It is obvious that there is potentially significant shock heating near the surface of the stars and very little in the cores. In

[1] This also neglects the effects of viscous dissipation, which would affect both ΔS and the velocity of the shock. The initial shock velocity is taken to be the impact velocity (see § 4). Typically, the shock has a Mach number ~ 5 at $M(r) \sim 0.99 M_*$, and $\lesssim 2$ at the center.

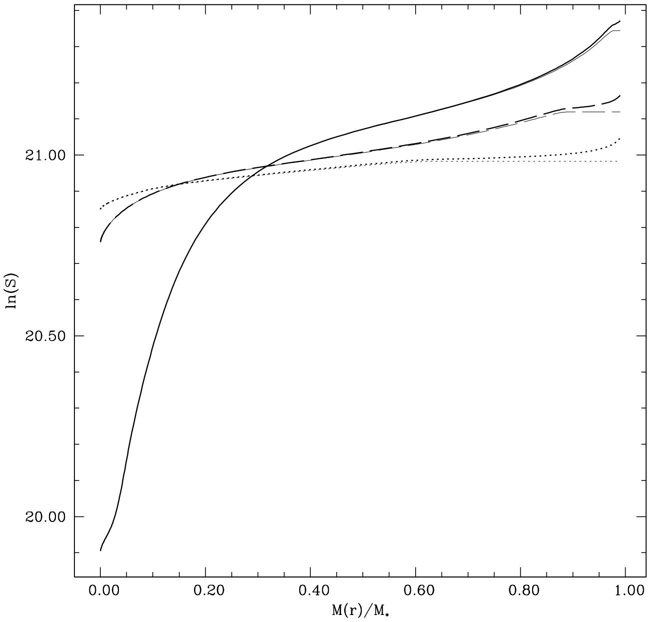

FIG. 3.—Comparison of preshock (*thin lines*) and postshock (*thick lines*) entropy profiles for the stars shown in Fig. 2. See § 3.1.2 for a description of the calculation of the amount of entropy production during shock heating

particular, entropy stratification of the parent stars' material using the "postshock" entropy profiles in Figure 3 would produce a very similar stratification to that found using the preshock profiles. The change in entropy near the surface of the star may result in some mixing of the surface layers; however, since these layers should contain no nuclear processed material, this would not significantly affect the subsequent evolution of the merger remnant.

The amount of shock heating calculated above is undoubtedly an underestimate: an actual collision would result in multiple shocks, especially during grazing collisions, and would have a much more complex geometry than that used in the above calculation. A more accurate estimate of the amount of shock heating could be found through simulations using SPH, which has been shown to model shocks well, depending on the form of artificial viscosity used (Monaghan 1992). Despite the rough nature of this estimate of the amount of shock heating, the *relative* amounts of shock heating should be correct: that is, the envelopes of the stars are more affected by the shocks than the interiors, and that the amount of shock heating is not enough to greatly affect the stratification of nuclear processed material when dealing with dissimilar stars.

3.2. *Physical Structure of Merger Remnants*

Leonard & Livio (1995) have suggested that a blue straggler formed through a stellar collision would resemble a pre–main-sequence star immediately after the collision. The SPH simulations of LRS show that the remnant of a collision involving two polytropes can be several times larger than either of the parent stars; however, unlike a pre–main-sequence star, the central density of the remnant will be much higher than that found on the Hayashi track. The decrease in central density of polytropic mergers relative to the parent stars is generally less than 50%, and is as little as $\sim 10\%$ in head-on mergers (from the simulations of LRS). Because of the size and dense core of a merger remnant, it will more closely resemble a red giant branch star than a star on the Hayashi track.

4. CONSTRUCTION OF INITIAL MODELS

Despite the facts that the SPH simulations of LRS use polytropes rather than evolved stars, and that the differences between the two species of objects can be extreme, we can use their results as the basis for producing models for input into a stellar evolution code. Entropy stratification of the material, which is a direct consequence of the dynamics of the fluid interaction and a requirement for the SPH fluid stability (see LRS for a discussion), can be used to predict the chemical profile of the merger remnant. Although there is no equivalent procedure for predicting the density profile of the remnant, the simulations of LRS show that the central density will not, in general, be much lower than that of the parent stars—this can be used as an additional constraint when constructing models.

Since head-on, parabolic collisions are relatively gentle, they are also the easiest to approximate for our purposes. We choose to ignore mass loss during the collision, the effects of rotation and the departure from spherical symmetry it produces. Because head-on collisions produce the least amount of mass loss and lowest rotation rates, these approximations are reasonably valid. We also choose to ignore shock heating during the collision, and use the precollision entropy profiles of our parent stars to define the chemical profile of the merger product using entropy stratification. The amount of shock heating during a head-on collision is small (§ 3.1.2 and LRS), and so should not strongly affect the end result.

The initial models we used to investigate blue straggler evolution were formed in three steps from a series of standard stellar models of the appropriate age and metallicity. First, for collisions between a TO star and a lower MS star, an appropriate TO model was scaled in mass so that its mass agreed with the total mass of the two parent stars; for equal-mass mergers, the mass of one of the parent stars was scaled. Next, the scaled model's composition profile was replaced with the chemical profile found by entropy stratification of the parent stars. This model was then expanded to simulate the expansion of the star during the collision.

The expansion of the model was performed by adding an additional term ϵ_x to the energy balance equation of stellar structure:

$$\frac{\partial L}{\partial M} = \epsilon_{\rm nuc} - \epsilon_\nu + \epsilon_{\rm grav} + \epsilon_x ,$$

where $\epsilon_{\rm nuc}$ is the nuclear energy generation rate, ϵ_ν is the energy loss due to neutrinos, and $\epsilon_{\rm grav}$ is the energy generation caused by contraction of the star ($= -T\,\partial S/\partial t$). If ϵ_x were held constant throughout the star, the central density would decrease rapidly as the star expanded and the star would end up on, or near, the Hayashi track (this is similar to the procedure that Sandquist et al. 1997 used to produce their Hayashi models). As mentioned earlier, the density of the core does not decrease by a large factor during the collision; to ensure this, we assumed that ϵ_x falls off as $1/\rho$ (see below). During the expansion of our models, the central density does not decrease by more than 20%, which, as explained below, results in our final models being virtually independent of the form of ϵ_x after they have relaxed to the MS.

One additional consideration is how much energy to inject into the star before allowing it to contract to the MS and evolve normally. We have chosen to use a criterion that takes into account the binding energy of the parent stars and the kinetic energy of the collision. The binding energy of a star can be expressed as

$$E_{\rm bind} = -q\,\frac{GM^2}{R}\,,$$

where M and R are the mass and radius of the star, and q is related to the degree of central concentration

$$q = \int_0^1 \frac{M(r/R)/M}{r/R}\,\frac{dM(r/R)}{M}$$

(Cox & Giuli 1968). In the center of mass frame of the two stars the orbital energy is

$$E_{\rm orb} = \frac{1}{2}\,\mu v^2 - \frac{GM\mu}{r}\,,$$

which, for a parabolic orbit, is conveniently equal to zero. [Here M is the total mass of the two stars, $M_1 + M_2$, and μ is the reduced mass of the system, $\mu = (M_1 M_2/M_1 + M_2)$.] Hence, the kinetic energy of the orbit is

$$K = \frac{GM\mu}{r}\,.$$

Assuming that one-half of the orbital kinetic energy goes into expanding the merger remnant, the final binding energy of the merger remnant is $E'_{\rm bind} = E_{\rm bind_1} + E_{\rm bind_2} + \frac{1}{2}K$. For the purposes of determining the kinetic energy at the time of the collision, we have set the separation of the two stars r equal to the average of their radii, $r = (R_1 + R_2/2)$. The expansion of the model is halted when its binding energy equals $E'_{\rm bind}$. For the same reasons that the form of ϵ_x is not critical, as long as the density constraint is obeyed (discussed below), the structure of the blue straggler after it has contracted to its MS is not highly dependent on the exact value of $E'_{\rm bind}$, although the duration of any convective zones during the pre–main-sequence evolution is affected.

One might be concerned about the ramifications for the models of assuming an energy injection term proportional to $1/\rho$. We have investigated this by adopting different energy injection schemes and comparing the resultant models. We find that, although the tracks that the models follow on the CMD as they expand and contract can differ considerably, the final models that satisfy the above criteria for energy and central density are very similar and evolve identically after they have contracted to the MS. This can be explained using the Vogt-Russell theorem, which states that the structure of a star in hydrostatic and thermal equilibrium is uniquely determined by the total mass and the run of chemical composition throughout the star (Vogt 1926; Russell 1927; see, e.g., Cox & Giuli 1968). Thus, if the amount of mixing that takes place is not significantly affected by the form of the energy injection term, the evolution of the merger remnant, at least after it has contracted to the MS, should also be unaffected by the choice. Our tests using different forms of ϵ_x show that this is the case as long as the central density does not decrease beyond what is indicated by the SPH simulations.

As pointed out by the referee, our energy injection scheme is similar to that used by Podsiadlowski (1996) in his investigation of the response of stars to heating by tidal effects. In his exploratory paper, Podsiadlowski investigated the effect of various forms of energy injection (e.g., centrally concentrated, uniform, surface) on a 0.8 $M_\odot$ GC ZAMS star—the different forms of energy injection were intended to approximate the zones in which tidally excited oscillations might dissipate their energy. Podsiadlowski found that the response of the star and its structure after a fixed amount of energy had been injected were strongly dependent upon the way in which energy was injected. Our initial experiments into the various forms of ϵ_x were quite similar to Podsiadlowski's and lend support to his findings; the differences in the final structures from our early experiments and those of Podsiadlowski's were simply a result of the fact that our initial models were evolved, whereas his were chemically homogeneous. Unlike Podsiadlowski's investigation, however, the form of ϵ_x for our models is constrained by the results of SPH: the central density of head-on mergers does not decrease dramatically from that of the parent stars'. From our experience, which is similar to what is reported by Podsiadlowski, forms of ϵ_x that are uniform throughout the star or are centrally concentrated result in a rapid decrease in the central density—by the time enough energy is injected to meet our energy constraint ($E'_{\rm bind}$, discussed earlier), the star's structure would resemble that of a star on the Hayashi track.

4.1. *Cluster Parameters*

We have chosen to compare our models of merger remnants with the blue stragglers observed in NGC 6397 and 47 Tuc. Both clusters have high central densities (see Table 1), a fact that makes the blue stragglers in these clusters excellent candidates for formation by collisions.

The stellar evolutionary code used to produce and evolve our merger models is a modified version of the VandenBerg et al. (1997) stellar evolution code and opacities. For each cluster, a grid of stellar models was produced, and isochrones were extracted using the method of Bergbusch & VandenBerg (1992). Ages and TO masses were found for each cluster by matching the TO luminosity with these isochrones; absolute TO luminosities were found by using the "best-fit" $M_V^{\rm HB}$-[Fe/H] relationship of Chaboyer, Demarque, & Sarajedini (1996) to obtain a distance modulus for each cluster. The ages derived in this manner agree with those derived by Chaboyer et al., and are shown in Table 1 along with the other derived cluster parameters. It should be noted that, although the ages and TO masses derived here are dependent upon the $M_V^{\rm HB}$-[Fe/H] relationship or distance calibration chosen, the final characteristics of our models (other than mass) are only weakly dependent upon the choice—in particular, the surface convection seen

TABLE 1

CLUSTER PARAMETERS

Parameter	NGC 6397	47 Tuc
Accepted:		
[Fe/H]	−1.91	−0.71
[α/Fe]	0.30	0.30
$E(B-V)$	0.18	0.04
V(HB)	12.90	14.09
c[a]	2.50	2.04
Derived:		
$M_V^{\rm HB}$	0.598	0.838
$(m-M)_V$	12.30	13.25
Age (Gyr)	19.0	14.0
$M_{\rm TO}$ ($M_\odot$)......	0.708	0.864

[a] Central concentration, defined as $c = \log (r_{\rm tidal}/r_{\rm core})$.

in the models presented here becomes less effective if the distance scale is increased.

5. RESULTS AND DISCUSSION

Blue straggler models were produced as described in the previous sections for both unequal-mass and equal-mass mergers, and for masses up to twice the cluster TO mass. Figures 4 and 5 show the evolution of these models on the CMD. As would be expected, because of the chemical inhomogeneity of the merger remnant caused by entropy stratification, the stars tend to avoid the ZAMS; only the low-mass, relatively unevolved, equal-mass mergers approach what would appear to be normal ZAMS stars. The differences between the tracks for the mergers of NGC 6397 and 47 Tuc are largely caused by the difference in cluster metallicity. These differences, including convection and a comparison with the observations, will be discussed in the next sections.

5.1. *Surface Convection*

Leonard & Livio (1995) and Sills et al. (1995) have suggested that blue stragglers must become largely convective during their postcollision, pre–main-sequence phase of evolution to explain both the observed rotation velocities and colors. While the blue straggler models of Sandquist et al. (1997) lend some support to this, our models do not, nor do the similar models of Sills et al. (1997).

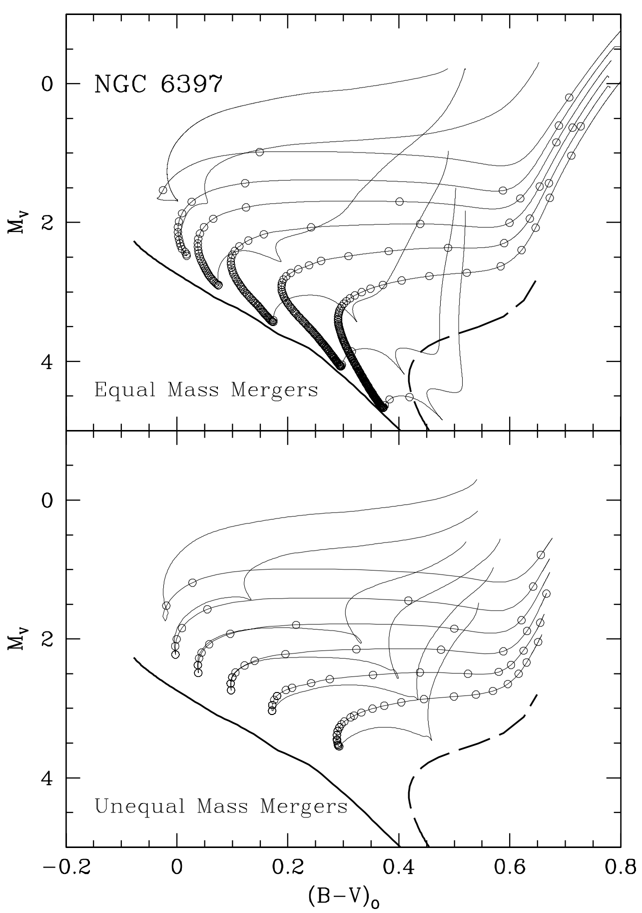

FIG. 4.—Evolutionary tracks (*thin solid lines*) for equal-mass (*top*) and unequal-mass (*bottom*) merger models for NGC 6397. Masses of the models range from 0.90 $M_\odot$ to 1.40 $M_\odot$ in steps of 0.10 $M_\odot$. Circles are placed along the tracks at equal intervals of 0.05 Gyr. Also shown are the theoretical cluster ZAMS (*thick solid line*) and a 19 Gyr isochrone (*dashed line*). Note the lack of a convective " hook " in all of the models, except for the most massive equal-mass merger model.

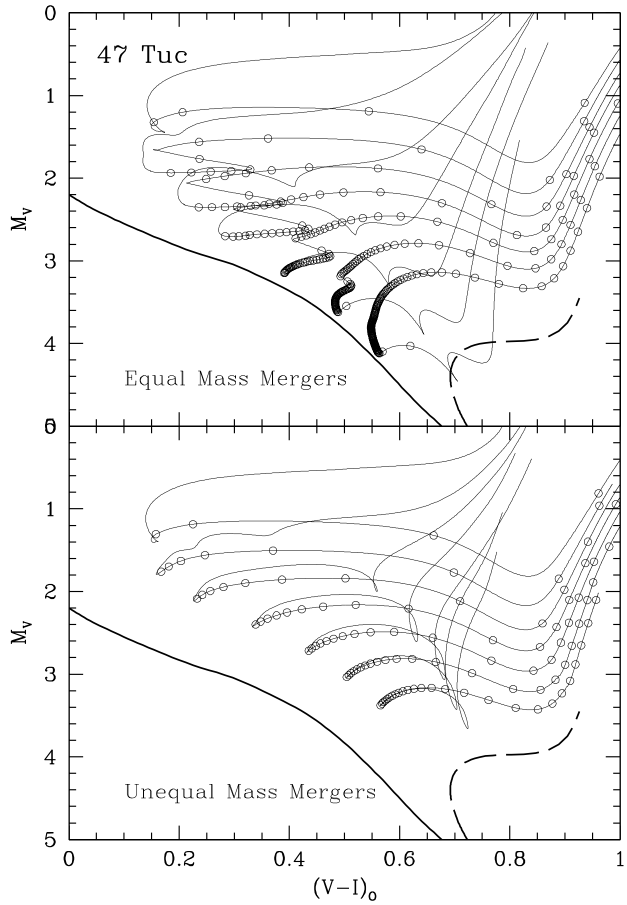

FIG. 5.—Similar to Fig. 4, except for 47 Tuc. The masses of the models range from 1.10 to 1.70 $M_\odot$ in steps of 0.10 $M_\odot$. Also shown is a 14 Gyr isochrone. Symbols and line styles have the same meanings as in Fig. 5.

Sandquist et al. (1997) performed SPH simulations of mergers of equal-mass polytropes and evolved the products of these mergers by imposing the resultant composition profile on to standard stellar models of the same mass as the mergers, and then forcing the stars to expand until they had reached the Hayashi track. Their models developed deep surface convection, enough to bring at least some helium to the surface of the stars. However, forcing the stars onto the Hayashi track would decrease the central density beyond what is observed to occur in SPH simulations (LRS)—thus requiring an additional energy source that would not be present in the actual collision. In addition, the deep convective envelopes seen by Sandquist et al. are a consequence of the fact that their models are forced onto the Hayashi track, at which point the low surface temperatures will require surface convection. Surface convection will persist until the opacity in the outer regions of the star decreases to the point where radiative transfer becomes efficient—either when the star has evolved to higher surface temperatures, or convection has brought a significant amount of helium to the surface.

Sills et al. (1997) created initial blue straggler models directly from the entropy and density profiles produced by SPH. None of their models developed any surface convection until they had evolved onto the red giant branch. The total lack of surface convection during the pre–main-sequence phase prevents any helium-rich material from being dredged up to the surface layers of the star, and makes spin-down of a rapidly rotating blue straggler by a magnetic wind mechanism (Leonard & Livio 1995) implausible. However, because the outer portions of SPH merger remnants are not necessarily in dynamical equilibrium, Sills et al. found it necessary to extrapolate the outer structure of their merger models from the purely radiative interior, forcing the envelope to be radiative, at least initially.

Surface convection does occur in some of our models during the pre–main-sequence phase, unlike the models of Sills et al., but not to the extent found by Sandquist et al. As in standard models evolving to the MS, the depth and duration of the surface convection zones in our models depends on the star's mass. The most massive merger remnants [$M \sim (1.8\text{–}2.0)M_{TO}$], whether formed by an unequal-mass or equal-mass merger, never develop surface convection zones during the phases prior to reaching the MS. The convection zones seen in our lower mass models are generally deeper and longer lived in the lowest mass stars, decreasing in depth and duration for mergers of higher mass. The convective zones typically contain $\lesssim 4\%$ of the star's mass and usually last for less than a few times 10^6 yr.

5.1.1. *Consequences of Surface Convection and Angular Momentum Loss*

The thin, short-lived surface convection seen in our models, and the absence of surface convection in the models of Sills et al. (1997), precludes any wind-driven AML mechanism for slowing down the rapidly rotating blue stragglers predicted by the collision scenario. However, blue stragglers are not observed to be rapidly rotating in open clusters, where, despite the comparatively low stellar density, it is possible for a fraction of blue stragglers to be formed by collisions (Leonard 1996; Leonard & Linnell 1992). To account for the low rotation velocities, there must be either an additional AML mechanism acting that is not dependent upon surface convection, or the blue stragglers in open clusters are being formed by a mechanism other than collisions that might produce more slowly rotating blue stragglers (e.g., binary coalescence, binary mass transfer), in which case the estimated numbers of collisionally generated blue stragglers in open clusters is incorrect. Even if the production of blue stragglers by collisions is ruled out in open clusters other formation mechanisms can be assisted, or accelerated, by strong dynamical interactions (Sigurdsson & Phinney 1993).

Little is known about the rotation of blue stragglers in GCs, although the stage has been set to rectify this problem by the observations of Shara et al. (1997). BSS 19 (Paresce et al. 1991) in 47 Tuc is estimated to have a mass of $1.70 \pm 0.40\ M_\odot$,[2] compared with the cluster TO mass of $\sim 0.86\ M_\odot$, and a high rotation velocity ($V \sin i = 155 \pm 55$ km s^{-1}). From our models, this star should not have had a convective envelope at any time during its "pre–blue straggler" phase, assuming it was created by a collisional merger, and so should contain the same angular momentum with which it was created, unless an AML mechanism other than a magnetically driven wind has acted upon it. During its contraction to the MS, its rotational velocity should have increased from its initial value by a factor of $\sim 5\text{–}6$, due to angular momentum conservation, implying an initial rotation velocity of ~ 30 km s^{-1}. From the results of LRS, this would have required a nearly head-on collision that, although not unlikely, is less probable than an off-axis collision. Hence, it is more probable that BSS-19, if created by a collision, is either inclined close to the line of sight (sin $i \lesssim \pi/4$), has experienced some AML, or was instead formed through some mechanism other than a collision, as suggested by Shara et al.

It is possible that AML from an initially rapidly rotating blue straggler could be achieved by angular momentum transfer to a circumstellar disk, possibly ejecta from the collision, or to a nearby companion, possibly captured during a binary interaction. Cameron, Campbell, & Quaintrell (1995) found that angular momentum transfer to a circumstellar disk is an extremely efficient mechanism for slowing the rotation of stars as they contract to the main sequence. This mechanism does require that the star have a convective envelope for the generation of a magnetic field, but the field strength required to shed a given amount of angular momentum is not necessarily as high as that needed for a wind-driven mechanism. Angular momentum transfer to a nearby companion has the advantage that a convective envelope is not required and demands that many blue stragglers be in binary systems, as is observed. During the distended contraction phase of the blue straggler's evolution, a nearby companion could exert a considerable torque on the star, forcing the stars to approach tidal lock. The angular momentum transfer to the companion will tend to force it into a larger orbit and reduce any initial eccentricity in the orbit, as well as slowing the rotation of the blue straggler.

5.1.2. *Surface Abundances*

Although it is questionable whether surface convection is sufficient to explain the moderate rotation rates of most blue stragglers, any amount of surface convection could act to alter the surface abundances of these stars. Since we find that surface convection does not occur in the most massive blue stragglers, it is possible that abundance anomalies might be a way in which to distinguish between the formation mechanisms, as suggested by Sills et al. (1997)—but only for the most massive stragglers. The convective zones in some of our lower mass models ($M \lesssim 1.40 M_{TO}$) can penetrate to depths where the temperature is high enough to destroy the more fragile elements, such as lithium (Pritchet & Glaspey 1991; Hobbs & Mathieu 1991; Glaspey, Pritchet, & Stetson 1994). However, although the amount of hydrodynamic mixing that occurs during a collision appears to be small, sufficient mixing should occur in the envelope during the merger (as a result of shock heating, for example; see § 3.1.2) that some chemical anomalies might be expected, even in the absence of convection. In addition, meridional currents, which should occur to some extent in rapidly rotating stars, will also mix the surface layers even if the core is not penetrated (Tassoul & Tassoul 1984).

5.2. *Core Convection*

In the normal main-sequence stars of NGC 6397 and 47 Tuc, core convection should persist for the entire main-sequence lifetime of stars with masses greater than ~ 1.40 and $\sim 1.20\ M_\odot$, respectively. Core convection does not

[2] Comparing the observations of effective temperature ($T_{eff} = 7630 \pm 300$ K) and surface gravity ($\log g = 4.09 \pm 0.1$) directly with our models yields a mass estimate of $1.55 \pm 0.10\ M_\odot$, independent of distance, in reasonable agreement with the determination of Shara et al.

occur at any time in our unequal-mass mergers, whereas core convection appears in most of our equal-mass models computed for 47 Tuc and for a short period of time in one equal-mass merger model computed for NGC 6397. No core convection occurs during the distended pre–main-sequence phase, but rather starts when the model has contracted close to its main sequence.

That no core convection occurs in our unequal-mass mergers is not surprising, given the high central density and helium abundance. This is in contrast to the results of Sills et al. (1997), who find that unequal-mass mergers typically do develop core convection. The simulations of Sills et al., however, are based on models produced using the end products of polytropic collisions, which, as shown earlier in the discussion on entropy stratification, will tend to have hydrogen-rich cores. The increased hydrogen abundance in the core results in a higher central opacity, making the central layers convectively unstable.

The differences in the amount of convection seen between the models for NGC 6397 and 47 Tuc are a consequence of the fact that NGC 6397 has a metallicity that is ~ 15 times lower than that of 47 Tuc, reducing the efficiency of the CNO cycle for masses less than $\sim 1.40\ M_{\odot}$. At that mass our equal-mass mergers for NGC 6397 ($M_{\rm TO} \sim 0.71\ M_{\odot}$) have sufficiently low central hydrogen content that convective transport is not necessary.

5.3. *Consequences of Calculated Mixing Scales*

According to the Vogt-Russell theorem, the structure of a star in hydrostatic and thermal equilibrium is determined by its mass and composition profile, which maps into a point on the H-R diagram. We would expect that a star perturbed slightly from its position of equilibrium on the H-R diagram would relax back to the same equilibrium position and structure. If the star is perturbed from its equilibrium state to a greater extent, such that no additional convection occurs that might change its chemical profile significantly, and such that no mass is lost during the perturbation, we would still expect it to relax back to the same equilibrium state on a timescale equal to the star's Kelvin-Helmholtz timescale. Similarly, two merger remnants produced in collisions involving identical sets of stars and having identical masses and chemical profiles, but which are initially at different points in the H-R diagram, should produce identical blue stragglers if no significant mixing occurs during their pre–main-sequence phase.

This same argument was used earlier to explain why the exact details of the mechanism used to expand our blue straggler models to their initial pre–main-sequence position are unimportant. However, here it has the additional implication that two theoretical merger remnant models that have identical masses and chemical profiles, but were produced using different assumptions about their structure, should produce identical blue stragglers if no significant convective mixing occurs during their pre–main-sequence phase. For this reason, we expect the evolution of the models of Sills & Lombardi (1997) and of Sills et al. (1997) to be similar to the evolution of our own unequal-mass merger models and equal-mass merger models, respectively.

5.4. *Comparison with Observations*

Shown on each of the tracks in Figures 4 and 5 are equally spaced intervals of 0.05 Gyr. Since the probability of seeing a star in any phase of its evolution is proportional to the amount of time it spends in that phase, a blue straggler that was created at some random time in the past has an equal probability of being seen in any of the marked intervals on the appropriate evolutionary track. This implies that the observed blue straggler distribution should cluster roughly where the marked intervals are closest *if* the blue stragglers have been created by processes similar to those represented by the models.

Figures 6 and 7 show the observed blue straggler distributions in NGC 6397 (Lauzeral et al. 1992) and 47 Tuc (Guhathakurta et al. 1992) compared with the *expected* distribution[3] of blue stragglers from our equal- and unequal-mass mergers. Ignoring, for the moment, those blue stragglers that lie blueward of the ZAMS, the blue straggler distributions in both clusters lie roughly in the region predicted by our unequal-mass mergers; the blue stragglers formed through equal-mass mergers tend to cluster too near to the ZAMS. For our unequal-mass mergers, the observed and expected distributions are similar at the 68% and 96% confidence levels for NGC 6397 and 47 Tuc, respectively; the same comparison for our equal-mass mergers yields 18% and 2% confidence levels (see Ouellette & Pritchet 1998 for details). The obvious interpretation of this is that most of the blue stragglers in these two clusters may have been created through unequal-mass mergers or a similar event that leaves the remnant in an apparently evolved state after formation (binary coalescence?).

There is, however, an additional, more critical observation to be made from the apparent agreement between the distribution of the blue stragglers and the predictions of our unequal-mass merger models. The collisional scenario for the formation of blue stragglers predicts that such mergers will, on average, be born rapidly rotating. This rapid rotation can affect the evolution of the star by providing nonthermal pressure support, which can extend the MS lifetime of a star, or by initiating meridional circulation currents, which could mix the star. Nonthermal pressure support would extend the lifetime of a star by requiring less energy to be liberated from the core in the form of nuclear reactions, thereby prolonging the hydrogen-burning lifetime of the star. Additionally, the star will appear cooler and less luminous than its nonrotating equivalent (Clement 1994). Meridional circulation currents are a consequence of the distortion of the star's gravitational potential by the rotation; as a result of this distortion, circulation currents will be initiated and mix the star. If hydrogen-rich material is brought into the core of the star by the circulation currents, the hydrogen-burning lifetime of the star, and so its lifetime as an observable blue straggler, will be greatly extended. Also, because of the increased central hydrogen abundance, the position of the blue straggler on the CMD should be closer to the ZAMS than it would be otherwise.

[3] The expected distribution was found by extracting stars from the tracks at random, using probability distributions created from the evolution timescales of the tracks themselves. The two distributions of blue stragglers, the observed and expected distributions, have the same distribution of apparent masses, excluding the extreme blue stragglers, as noted in the text. By this we mean that a comparison of the photometric positions of the blue stragglers and our tracks yields, for each blue straggler, a probability distribution for the mass. The fake blue stragglers have masses drawn from this probability distribution. The distribution of fake blue stragglers also takes into account the expected photometric errors as indicated on the figures. The probabilities quoted in the text are from Monte Carlo simulations that will be described in Ouellette & Pritchet (1998).

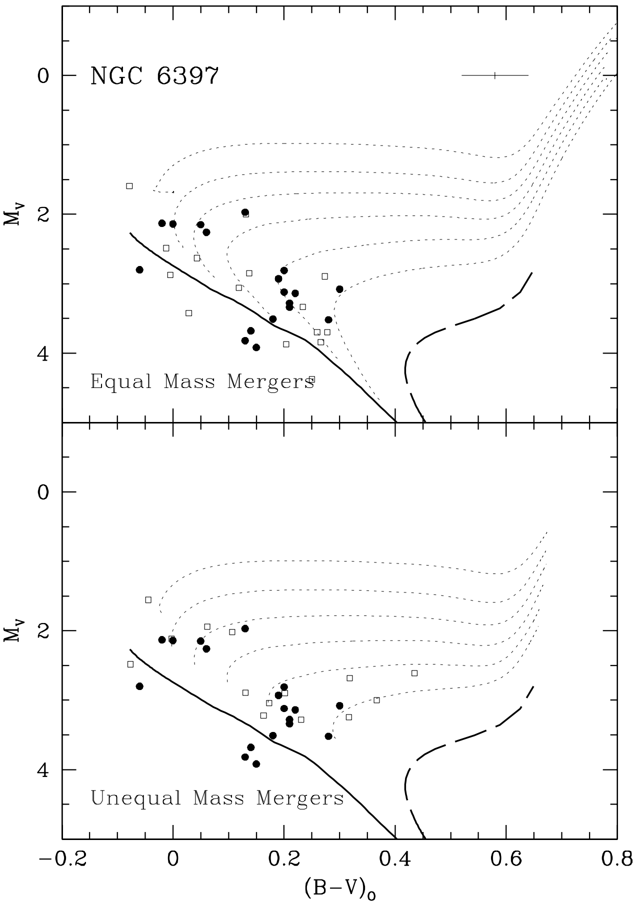

FIG. 6.—Observed distribution of blue stragglers in NGC 6397 (*circles*) compared with blue stragglers created from the tracks of our merger models (*squares*; see text). Also shown are the tracks for the appropriate mergers with the "pre–blue straggler" phase omitted for clarity (*dotted lines*). The error bar at the top of the figure corresponds to the random photometric errors appropriate for the observations. The theoretical cluster ZAMS (*solid line*) and a 19 Gyr isochrone (*dashed line*) are also shown.

If blue stragglers created by collisions are rotating rapidly enough for these effects to be important, the agreement between the predicted blue straggler distribution from our tracks, which do not include any effects of rotation, and the observed blue straggler distribution should be poor. The apparent agreement on the CMD between the prediction of the unequal-mass mergers and the observed blue stragglers places limits on the importance of the effects of rotation. First, although the hydrogen-burning lifetimes of the blue stragglers might in fact be different than what our models predict, the difference cannot be extreme (certainly much less than a factor of 2), else the observed blue straggler distribution would be significantly more clustered near the MS. Second, if meridional circulation were to mix the star, the observed blue stragglers would have significant scatter blueward toward the ZAMS. This does not exclude meridional circulation in the envelope (e.g., Tassoul & Tassoul 1984), but little or no helium-rich material will be brought to the surface in such a case.

5.4.1. *Blue Straggler Formation Rates*

A simple estimate of the formation rate of blue stragglers in these clusters can be made by finding the average amount of time our merger models will spend as observable blue stragglers. Assuming, as suggested by the comparison of the observed and expected blue straggler distributions, that the blue stragglers in these clusters are formed by unequal-mass mergers, the average formation rates of blue stragglers in NGC 6397 and 47 Tuc are one blue straggler every $\sim 5.0 \times 10^7$ and $\sim 3.6 \times 10^7$ yr, respectively. For comparison, if the blue stragglers in these clusters were formed solely by equal-mass mergers the rates would be 1 blue straggler every $\sim 2.3 \times 10^8$ and $\sim 1.2 \times 10^8$ yr. Assuming that the binary fraction in the core of each cluster is 100% (following Leonard & Fahlman 1991), and that the average binary semimajor axis is ~ 0.1 AU, equation (4.12) of Sigurdsson & Phinney (1993) yields an average time between collisions of $\sim 6 \times 10^7$ and $\sim 7 \times 10^5$ yr for NGC 6397 and 47 Tuc, respectively. These collision timescales are close to lower limits, since the binary fraction is most likely not 100%; for the timescale for 47 Tuc to agree with the model predictions, the binary fraction would have to be $\sim 10\%$, or the average binary separation would have to be smaller. (A more detailed analysis will be presented in Ouellette & Pritchet 1998.)

5.5. *Extreme Blue Stragglers*

In the above discussion and analysis we have pointedly ignored the blue stragglers that lie blueward of the ZAMS. While it is possible that some of these "extremely blue" blue stragglers are merely field objects[4] or photometric errors, many other studies of clusters with excellent photometric quality and low foreground object contamination (e.g., Bolte 1992) exhibit such extreme blue stragglers as well. This suggests that these blue stragglers are produced by a different process than that modeled here. In fact, as other studies have shown (Sandquist et al. 1997; Sills et al. 1997; Ouellette & Pritchet 1996), these stars lie roughly where we would expect fully mixed merger remnants to be. However, inspection of the "fake" blue straggler distributions in Figures 6 and 7 shows that photometric scatter may be responsible for some of these stars.

Earlier we suggested that meridional currents produced by rapid rotation are not necessarily important when modeling blue stragglers produced by collisions. It may, in fact, be that some rotational threshold exists above which this process will become important for the more rapidly rotating blue stragglers.

Mestel (1953) pointed out that, although the average rotation rate of early-type (A–F) stars is rapid enough that one would expect meridional circulation to be ongoing within these stars, the fact that we see such stars evolved to the giant phase implies that they have chemically inhomogeneous structures, which would not be the case if meridional circulation currents were present. Mestel (and Tassoul & Tassoul 1984) found that meridional currents were strongly inhibited by even a slight chemical gradient throughout the star. Since the stars in a GC are evolved and so not chemically homogeneous, and we expect chemical gradients in merger remnants, this may be enough to disrupt circulation currents in the majority of blue stragglers. Tassoul & Tassoul (1984) found that, for stars rotating well below rotational break-up, the velocity of the

[4] Guhathakurta et al. (1992) estimate that fewer than 3 of the blue stragglers they observed in 47 Tuc can be explained by background objects in the SMC.

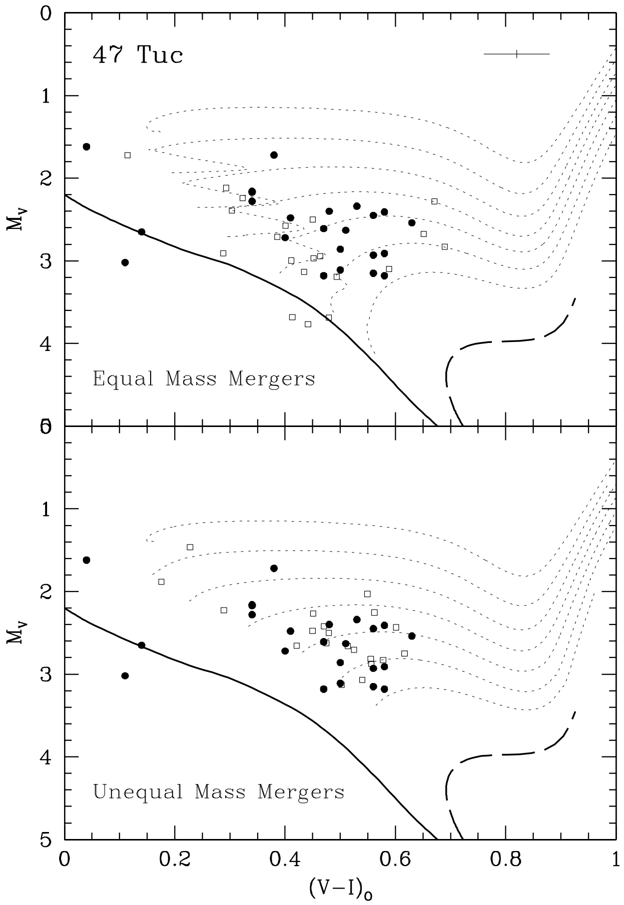

FIG. 7.—Similar to Fig. 6, except for 47 Tuc. Also shown is a 14 Gyr isochrone. Symbols and line styles have the same meanings as in Fig. 6.

circulation currents increased with increasing rotational velocity, but the depth of penetration of the currents was virtually unchanged—hence, it is very unlikely that meridional currents could be invoked to mix fresh hydrogen to the cores of these stars. The predicted circulation in the envelope may still be enough to affect observed chemical abundances, although it is questionable whether or not enough helium will be mixed to the surface to cause the star to appear as an extreme blue straggler. However, it may be possible that, if a star was rotating near to rotational breakup, the meridional currents might be able to penetrate further into the star than in Tassoul & Tassoul's more slowly rotating models—this would have to be confirmed through additional theoretical studies into rotation using two-dimensional stellar models, and is purely speculative at this point. If meridional currents are responsible for mixing helium to the surface of extreme blue stragglers, then these stars are most likely those that were created with the highest initial rotation velocities.

6. CONCLUSIONS

We have evolved stellar models that are appropriate for stars created during collisions between equal- and unequal-mass stars. In doing so we have used the results of SPH simulations of colliding polytropes, most importantly the predictions of entropy stratification. To produce these models, we have made several assumptions to facilitate the incorporation of the results of the SPH simulations of collisions between *polytropic* stars into models of real stars; these assumptions include neglecting mass loss, rotation, and shock heating during the collision. Because we have restricted our attention to head-on parabolic collisions, we believe these approximations to be reasonably valid; mass loss and rotational velocity are shown to be small in this case by the simulations of LRS; shock heating is shown to be small and, most importantly, should not affect the distribution of helium-rich gas in the merger remnant (§ 3.1.2), although it is likely that shock heating will affect the distribution of material near the surface of the star. The form of the energy injection term ϵ_x that we use to expand our merger remnant models is constrained by the results of SPH, but its form is nonetheless not crucial as long as significant mixing is not artificially introduced (via the Vogt-Russell theorem; Vogt 1926; Russell 1927). In comparing the predictions of these models with the blue stragglers seen in two dense clusters, NGC 6397 and 47 Tuc, we have also assumed that all of the blue stragglers in these clusters have been formed through collisions.

The apparent agreement between the predictions of our unequal-mass merger models and the observed blue straggler distributions NGC 6397 and 47 Tuc suggests the following:

1. Little or no mixing occurred during the formation of these blue stragglers, either of fresh hydrogen to the cores, or of helium to the surfaces. As explained earlier, such mixing would produce a significant blueward scatter in the distribution of blue stragglers in the CMD—as it is, our models with the best agreement with the observations, the unequal mass mergers, produce rather red stars, relative to the clusters' ZAMS. Regardless of the formation mechanism for the blue stragglers in these clusters, it is apparent from their location redward of the ZAMS that they are formed as *evolved* stars (Ouellette & Pritchet 1996).
2. There may be some form of efficient AML mechanism acting to slow the rotation of these stars. Blue stragglers that are formed by collisions are expected to be rapidly rotating: the effect of rapid rotation on the observed distribution of blue stragglers in the CMD should result in a poor agreement with our nonrotating models, and yet the agreement appears to be quite good. The thin, short-lived convective envelopes seen in our models precludes a magnetically driven stellar wind AML mechanism to explain the slow rotation rates and apparent lack of rotational effects; however, angular momentum transfer to a circumstellar disk or nearby companion are still plausible.

The lack of surface convection in our most massive models and the thin convective envelopes seen in our less massive models means that surface abundances of collisionally merged blue stragglers might not be altered from those of the parent stars, unless some amount of hydrodynamic mixing occurs during the collision. In addition, meridional circulation currents, which should be confined to the stellar envelope except in the most extreme cases, may act to mix the surface layers, regardless of the amount of convection.

The extreme blue stragglers observed in both NGC 6397 and 47 Tuc occupy the region of the CMD that should be populated by highly mixed stars. It is possible that these blue stragglers are the few that were born rotating rapidly enough that the meridional circulation currents are able to penetrate more deeply in to the star; if so, these stars should still be rapidly rotating. It is, however, possible that some fraction of these stars are merely photometric errors.

We would like to thank both Russell Robb and Don VandenBerg for providing useful comments on an earlier version of this manuscript. Additional thanks must go to the latter for allowing us to use his stellar evolution code, and for numerous insightful discussions on the topic of stellar evolution. Finally, we wish to thank the Natural Sciences and Engineering Research Council for financial support through a Research Grant to C. J. P.

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Note added in proof.—It has become apparent (Rasio & Lombardi, 1998, private communication) that our use of the thermodynamic entropy to obtain the stratification of the stellar material after a collision may be an incorrect extrapolation of the results of LRS. However, if the stratification is done with the "hydrodynamically correct", entropy-like quantity $A(=P/\rho^{5/3})$, the change in our models is small—a slight blueward shift of the location of our unequal-mass merger models (no change occurs in our equal-mass merger models) with no appreciable change in the evolutionary timescales. This change does not change our conclusions, nor the relevance of our results.

The justification for using the quantity A, rather than the entropy S, comes from the fact that the equation of state (eq. [2]) used in the SPH calculations is not a *global* relation, but rather is local to each particle: the SPH particles are adiabatically isolated from their surroundings, and so equation (2) holds even in the presence of changing composition.