PUBLICATIONS OF THE ASTRONOMICAL SOCIETY OF THE PACIFIC
98:914-921, October 1986

SPECTROSCOPIC OBSERVATIONS OF YELLOW SUPERGIANTS: I. RADIAL PULSATIONS OF RHO CASSIOPEIAE

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Received 1986 May 16, revised 1986 July 21

ABSTRACT

High-resolution Reticon spectra of the F8 Ia-0 supergiant ρ Cas providing lines of differing excitation potential are employed to construct radial-velocity curves which reveal a semiregular pulsation of the atmosphere. Analysis of line-doubling episodes supports the presence of a dominant radial pulsation mode with a period around 500 days, significantly longer than the radial fundamental mode predicted on the basis of Cepheid-like PLC relations for a star evolving to the red supergiant region. Predicted periods for fundamental radial pulsations in stars returning toward the blue supergiant region are comparable to the observed quasi-period.

Key words: supergiant star–radial pulsations–spectroscopic observations–stellar variability

I. Introduction

Yellow I*a*-0 supergiants such as ρ Cassiopeiae are visually among the most luminous of stars and spend their post-main-sequence lifetime near the upper boundary of the H-R diagram (HRD). The progenitor of ρ Cas was probably a main-sequence star between 40 and 50 $\mathfrak{M}_\odot$, which through continuous mass loss evolved to become a supergiant of a lower mass. Such a star is predicted to become a red supergiant and then to double back in the HRD to become a Wolf-Rayet star. Hence, very luminous yellow supergiants may be found in one of two different evolutionary intervals.

Observational studies of supergiants reveal complex changes in brightness and radial velocity, vigorous mass-loss episodes, and indications of complicated velocity fields (cf. de Jager 1984). Searches of periodicities in photometric and spectroscopic data have resulted in assignments of quasi-periods. But all studies to date suffer from various defects. Surveys differ with respect to spectral-type coverage, number of objects observed, and temporal resolution. Very few published surveys have provided simultaneous photometric and spectroscopic observations. Finally, the reported observations are generally of insufficient duration and completeness to follow adequately the long quasi-periods.

In this paper, we report on an intensive spectroscopic monitoring program that has provided high-resolution spectra of ρ Cas over more than four years, with periods of near-uniform sampling intervals of one month. This data base, the most detailed spectroscopic coverage of ρ Cas so far, has been examined for radial-velocity and line-profile variations. We report here on the inferred pulsational behavior of ρ Cas and attempt a consistent qualitative description of the atmospheric pulsations and the evolutionary status of this star. We suggest that its dominant pulsation is the primary radial mode with a period of about 520 days. Comparison of the observed and predicted periods suggests that ρ Cas is now evolving from the red-supergiant region toward the blue, and in about 7000 years may become a Wolf-Rayet star.

II. Observations

The spectra were obtained at the coudé focus of the 2.7-m telescope at McDonald Observatory. Our analysis covers selected intervals in the visible and near-infrared observed between September 1979 and February 1984, many of which have been secured on a regular basis in consecutive months. All spectra have been digitally recorded using Reticon detectors (Vogt, Tull, and Kelton 1978). The detector used up to June 1983 was a Reticon array of 1024 diodes, each 25 microns across. After June 1983, the replacement array had 1872 diodes, each 15 microns across. Typical dispersions were 0.1 Å diode^{-1} (3.5 to 5.1 km s^{-1} diode^{-1} at the observed wavelengths) throughout the first period, and 0.06 Å diode^{-1} (2.1 to 3.0 km s^{-1} diode^{-1}) later on, using normal gratings, and 0.01 Å to 0.03 Å diode^{-1} (0.7 to 1.2 km s^{-1} diode^{-1}) using an echelle grating. The normal entrance slit projected onto 2 to 7 (echelle) diodes of the 1024 array or 3 to 12 (echelle) diodes of the 1872 array. Since the stellar lines are greatly broadened by turbulence, the spectrum is essentially fully resolved by our observations. At the lower dispersion, an exposure covered roughly 100 Å at specific regions of interest: 6430 Å, 6568 Å, 8063 Å, 8667 Å, and 8786 Å. Echelle segments were 25 Å to 40 Å wide, centered on 6142 Å and 8690 Å, the latter overlapping a part of the 8667 Å lower-dispersion observation.

Estimates of the visual brightness have been taken from AAVSO circulars (see the Appendix).

III. Methods of Analysis

In order to explore different depths of the extended photosphere, one should examine both low- and high-excitation lines. Our sample of spectral regions provides an adequate sample with which to follow the radial-velocity behavior of lines formed at different depths. Furthermore, the linearity and high signal-to-noise ratio provided by the Reticon spectrophotometer make it possible to obtain accurate measurements of radial velocities for individual lines. In order to exploit these benefits, we used a software cross-correlation program (J. Tomkin and S. Sawyer, private communication) that operated directly on the recorded data. The program measures relative shifts between two spectra of different epochs by using any desired range of diodes for a certain spectral feature. This relative shift is automatically corrected for the Earth's orbital motion and compensated for different settings of the gratings by cross-correlating comparison iron-neon spectra obtained immediately following the stellar observations.

The stellar line profiles are often asymmetric and sometimes exhibit two or more components. The cross-correlation program enabled us to sample the entire profile (or a part of it) and thus to reduce to a minimum the departures caused by individual components. The relative radial-velocity curves, then, show the motion averaged over the entire observed hemisphere for each line, referred to an arbitrarily chosen date for which we measured the absolute velocity. No corrections have been made for projection or limb effects. We employed various tests to check the reduction and measurement procedures, and to derive the errors associated with our radial-velocity determinations (see Sheffer 1985). Summarizing those evaluations, our curves should be accurate to 1 km s^{-1} for relative velocities, an uncertainty much smaller than the actual velocity variations that we have detected.

IV. Results

A. *Radial Velocities*

Radial velocities were obtained and analyzed for several representative lines:

- the N I lines near 8700 Å. With a lower excitation potential (χ'') of 10.3 eV, these lines are formed deep in the photosphere.
- the H I Paschen line at 8750 Å. This line must also be sensitive to conditions in the deep photosphere but, thanks to the greater abundance of H relative to N, higher atmospheric layers are also sampled by this line.
- Fe II lines at 6417 Å and 6456 Å. These low-excitation lines (χ'' = 3.9 eV) should be formed in the upper to middle layers of the photosphere.
- Low-excitation lines of Fe I and other species. This sample is most sensitive to conditions in the upper photosphere.
- The Ca II 8662 Å and Ba II 6142 Å lines. As shown by Lambert, Hinkle, and Hall (1981, here LHH), the cores of these lines may contain a circumstellar component.

A quasi-periodic behavior is readily apparent in the radial-velocity curves of ρ Cas photospheric lines. The high-excitation (10.3 eV) N I line at 8703.3 Å formed in the deep photosphere exhibits a total amplitude of 14 km s^{-1} (see Fig. 1). Velocities from the independent high-resolution echelle observations centered on 8690 Å, and the lower-resolution normal-grating observations of the 8667 Å window are in good agreement. Our observations span 1000 days and seem to contain two quasi-periods of ρ Cas. One estimate of a representative quasi-period according to two consecutive radial-velocity maxima is 510 days long; another based on two consecutive minima is 530 days long.

The highest excitation line available to us is the 12.1 eV 8750.4 Å H I line with a radial-velocity behavior (Fig. 1) similar in form to that exhibited by the N I lines: the same cycle is present, but the H I line shows a larger amplitude (27 km s^{-1}). It is very likely that the velocity of this line is contaminated by components that are seen to vary in strength over its global profile. A lower limit on the

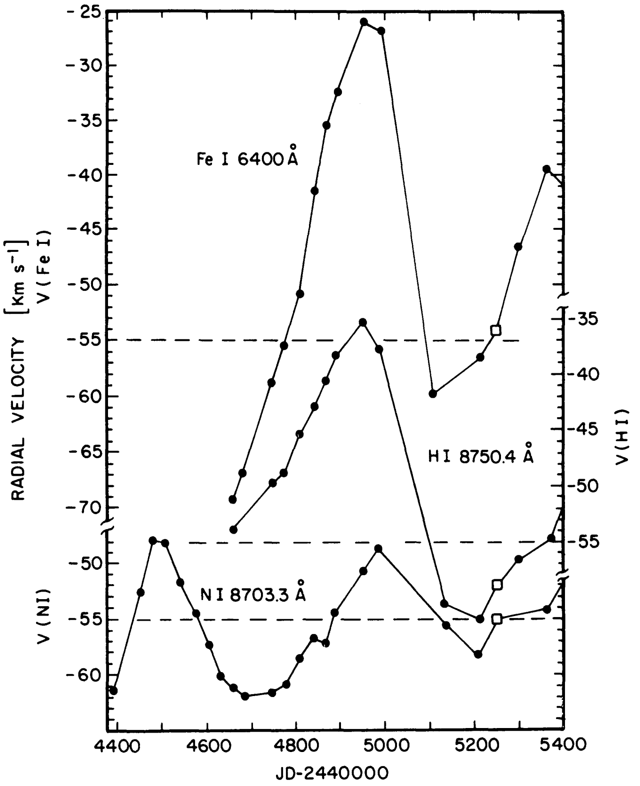

FIG. 1–Heliocentric radial-velocity curves for ρ Cas from a N I, H I, and Fe I line. The standard epochs (used for deriving relative velocities) are shown in special symbols. The broken lines indicate the approximate systemic velocity of −55 km s^{-1}.

quasi-period based on the minima of this line is 550 days.

Although the coverage is incomplete, the observations show that the two 3.9 eV Fe II lines have a total velocity amplitude of 18 to 20 km s^{-1}, a value slightly larger than that from the N I lines, but in phase with them.

A different behavior is exhibited by the low-excitation lines of neutral atoms and ions. Consecutive quasi-periods differ significantly in amplitude, both relative to the N I lines and even among themselves. The 0.3 eV La II line at 6390.5 Å shows a total amplitude of about 30 km s^{-1} mostly in phase with the N I line. The most extreme case is displayed by the 6400 Å Fe I blend of 3.6 and 0.9 eV excitation potentials (see Fig. 1). The total amplitude for this feature reaches a value of 43 km s^{-1}. The measured lines are representative of their class. During the entire interval between February 1981 and March 1983, this 6400 Å Fe I blend is observed to be a single line, and the entire profile below -25% intensity was cross correlated in order to find the radial velocity. In August 1983 there began a line-splitting episode that went on for a few months, during which the components were separated by up to 50 km s^{-1}. The line became single again in February 1984 with a velocity in accord with the evolution of the blue component.

Another pattern is shown by the Ca II and Ba II lines. For the 8662.1 Å Ca II line, we used only the deepest half of the profile in the cross correlations so as to reduce to a minimum the interference that might arise due to the broader high-excitation line of H I at 8665.0 Å. During an interval of 750 days, there is to be seen a monotonic rise in the radial velocity. The onset of the rise appears to be in phase with a rise shown by the N I, Fe I and II, and La II lines (compare Figs. 1 and 2), but, whereas the latter features show a quasi-periodic behavior with a velocity decline after about 330 days, the velocity of the Ca II line continues to increase for more than 440 days until an approximately constant velocity is maintained to the end of the observing period. The radial-velocity curve of the 6141.7 Å Ba II line that is very similar to that of the Ca II line (see Fig. 2), probably reflects the common formation of the lines in very high photospheric-chromospheric layers. The Ba II line shows a monotonic rise for at least 330 days following which it enters a state of roughly constant velocity for 380 days. The radial-velocity variation of the Ba II line is quite similar in shape to that of Ca II, although of lower amplitude, and shows a phase difference of -90 days. These Ba II observations cover an earlier period when there is no corresponding Ca II data, and they show one small cyclic change in velocity: rising and declining (only 3 km s^{-1} amplitude) just before the monotonic rise described above. This cyclic variation almost parallels a much larger amplitude cycle exhibited by the N I lines and described earlier. Over the common period of coverage the amplitude of the Ba II line is 9 km s^{-1} whereas that of the Ca II line is 13 km s^{-1}. Both the

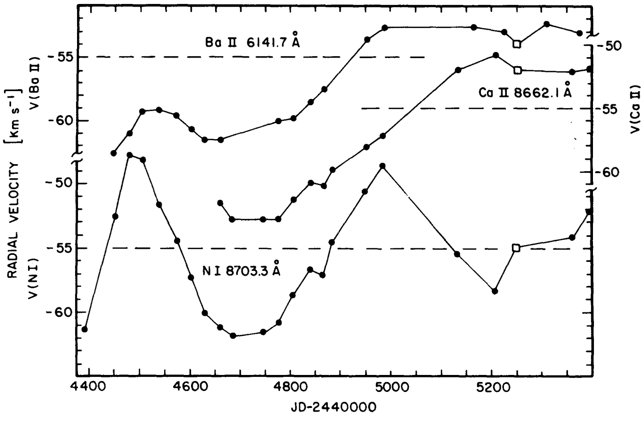

FIG. 2–Heliocentric radial-velocity curves for ρ Cas using a N I, a Ca II, and a Ba II line. The broken lines indicate the approximate systemic velocity of -55 km s^{-1}.

Ca II and Ba II lines are accompanied by a strong blueshifted circumstellar component in spectra illustrated by LHH. As we have used the entire profiles below the -50% and -25% intensity levels, various components that may be present in the profile are not distinguished. Obviously, decomposition of profiles and monitoring of individual components is a much-needed task that will necessitate a series of continuous and frequent observations of very high spectral resolution.

The periods derived for ρ Cas according to the cyclic radial velocity curves are 510 days between maxima occurring in September 1980 and in January 1982, and 530 days between minima in May 1981 and in August 1982. These periods based on the N I velocity curve are well determined, but, of course, we cannot exclude longer-term modulation of the velocities. Two other quasi-periods can be derived from the velocities of the low-excitation lines. This data set suffers from serious gaps in the observational coverage. The rough periods are 400 days to the following maximum in January 1983, and 300 days to the following maximum in November 1983. Possibly—if not probably—the former is not established well enough, so that a much longer interval of 700 days may exist between two maxima.

B. *Line-Doubling Episodes*

We have followed the evolution of many lines that, at certain phases, develop a double absorption core. Our sample is dominated by Fe I and Fe II lines and includes the two lines at 6147.7 Å and 6149.2 Å that were reported by LHH as showing the (then remarkable) splitting. In addition to these two Fe II lines, other prominent lines were 6430.9 Å and 8824.2 Å of Fe I and 6432.7 Å of Fe II. Our spectroscopic coverage of ρ Cas from 1979 through 1984 enables us to observe the evolution of the doubling in the lines (see Fig. 3): At first a component develops in the blue wing of the photospheric line. It grows deeper and deeper over several months until it rivals the intensity of the main (red) line. We take the time of rough

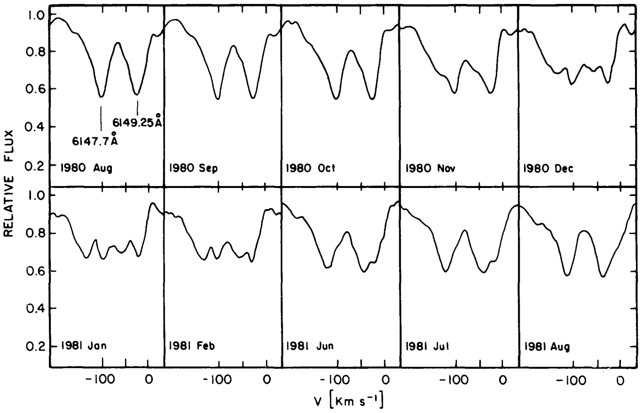

FIG. 3–A detailed view of the growth and decay of line splitting in the 6149.25 Å line of Fe II. Heliocentric velocity scale is in km s^{-1}. Also seen is a second Fe II line at 6147.7 Å (blended with Fe I).

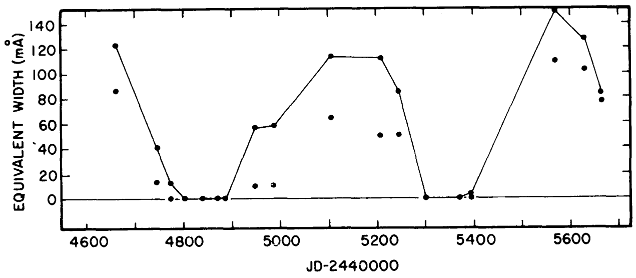

FIG. 4–Equivalent widths in mÅ for Fe I emission lines of ρ Cas. The curve shows the 8047 Å line, while the unconnected dots show the 8075 Å line.

equality of the two components as a reference point in timing the period. From that point on, the red component diminishes, and in the course of several more months, the line returns to a single profile. Line splitting of this type was discovered in W Virginis Population II stars (Sanford 1952) and has been interpreted as a signature of two absorbing layers, separated by a shock wave.

We now have four instances of doubling in ρ Cas that cover three periods which we determine to be 513 ± 45, 521 ± 45, and 525 ± 45 days in length. The uncertainties are primarily set by the sampling gaps in our observations, namely, of about one month. The shape and time of appearance of equally deep red and blue components depend on the type of line monitored, most notably varying with excitation potential. Such a variation with the excitation potential of a line is an interesting discovery which supports the interpretation of the pulsations as radial rather than nonradial modes, and we discuss this topic below.

C. *Emission Lines*

Two prominent emission lines in the 8063 Å window have been analyzed: these are the 8047.6 Å and 8075.2 Å lines of Fe I. The measured equivalent width of both lines varies on a time scale consistent with the previously mentioned quasi-periods according to duration and phase (Fig. 4). The total velocity change of the 8047.6 Å emission line is 10 km s^{-1}. This stands in marked contrast to the 8046.1 Å Fe I absorption line which exhibits changes of up to 40 km s^{-1}, similar to the 6400 Å and other Fe I lines. We interpret this as a consequence of the larger volume of material which contributes to the emission: these lines basically reflect a range of radial velocities of matter in more than one hemisphere around the star, with an average velocity that is not far from the systemic velocity. At the same time the absorption lines represent matter that occupies much less than a whole hemisphere around the star, hence clearly showing the variations along the line of sight. The equivalent widths of the emission lines and the AAVSO visual-magnitude estimates are correlated (see below).

The 8000 Å region is crossed by many CN absorption lines which also vary on a time scale equal to the photospheric quasi-period. Blending CN lines, which may contribute to the Fe I emission and absorption lines, appear to have a low-excitation temperature. The CN is presumed to reside in a region well above the photosphere. In a later paper, we shall analyze the CN lines in detail.

V. Discussion

Our radial-velocity curves show decisively that the yellow supergiant ρ Cas is undergoing photospheric pulsations on a semiregular basis. The line-forming layers experience different motions in the course of a pulsational cycle: we see regular amplitudes in high-excitation lines, bigger variations including line doubling in low-excitation lines, and marked departures from regularity are shown by the Ca II and Ba II lines to which blueshifted gas contributes. The period as provided by two cycles of observations appears to be about 520 days. We note that the radial-velocity curves are rather similar in shape to that of a Cepheid in that the ascending branch is of appreciably longer duration than the descending branch.

The photospheric activity of this supergiant may involve either radial or nonradial modes, or both. Although our span of observations is too short to permit definitive comments, we suggest that the amplitude of the radial-velocity variations may be more variable than the period. The fact that the pulsation is not purely regular may suggest the presence of a few modes with different periods. However, by using most of the line profile in the cross-correlation, the roles of individual components have been suppressed so that the averaged behavior for the visible hemisphere has been measured with a better signal-to-noise ratio. Individual components in the line profiles may arise as a result of surface nonradial motions. Such components (or strong convective motions) appear necessary to account for the large width of the absorption lines, an attribute customarily divided between microturbulence and macroturbulence.

To date, photometrists have provided most of the data on the variability of the supergiants. Unfortunately, the overlap between photometric and spectroscopic observations is poor. When we compare our radial-velocity data to the extensive AAVSO data, only one interval of apparent casual relationship is discerned (see Figs. 1 and 6), centered on JD2444800. It seems that this coincidence involved inflation of the photosphere, assuming a dominant radial mode. An expansion interval ended about JD2444875, while an interval of collapse began at that time and terminated around JD2445075. This is borne out by the accelerations computed from the velocity curve. Around JD2444875, ρ Cas reached maximum radial extent and was at or near a brightness minimum. Therefore, we suggest that it was also coolest at this phase. This is confirmed by colors: Arellano Ferro's (1985) obervations for the period JD2444860 to JD2444875 are redder than colors observed prior to the former date. In this sense ρ Cas is similar to all radially pulsating stars, being hotter and brighter when it is more compressed.

Arellano Ferro (1985) found that brightness and velocity variations were in phase during 1980–81, leading him to favor a nonradial mode for the pulsation. Our radial velocities and the AAVSO magnitudes, both of longer and denser coverages, are clearly not in phase. If fact, there seems to be a certain phase lag between brightness and size curves (based on radial motion) as is the rule for all radial pulsators.

Our observations of line doubling also support the suggestion that the dominant pulsation in the photosphere of ρ Cas is the fundamental radial mode. As described below, these episodes are attributed to a deep atmospheric pulsation which stimulates growth of a shock. The shock propagates outward and absorption lines are seen from gas ahead of and behind the shock. Line doubling is seen in Fe I (and other) lines, but not the N I high-excitation lines formed in the deepest photospheric layers. Understandably, these line splittings occur at times of velocity minima of the high-excitation lines of N I and H I; the deep photosphere is rising then at maximum expansion velocity past the equilibrium radius of the star. In several months, the layer which forms the iron lines and is collapsing onto the shock wave (as manifested by the red and blue components, respectively) will have been reduced to a layer at the top of the atmosphere and then will disappear. Averaging the three observed periods results in a mean piston period for ρ Cas 519 ± 45 days in length; this estimate is identical to our earlier estimates from other characteristics of the radial-velocity curves. Since the detected doublings occurred in September 1979, February 1981, July 1982, and February 1984 (± one month uncertainty at most), we can predict the next episode of ρ Cas line doubling to have occurred during August 1985, with intervals of two or three months preceding and following the interval of equality in which the line profiles changed. We have not inspected yet any spectra to check this prediction.

Examination of the evolution of the line splitting in eleven Fe I lines of differing excitation potential led to an interesting discovery (Fig. 5). The blue component appears earlier in the highest excitation lines (i.e., those lines formed deepest in the photosphere). The red component, whose disappearance marks the conclusion of the line-doubling phase, is last seen in the lines of lowest excitation potential. This evolution is readily interpreted if the pulsation is predominantly radial. A shock wave that reverses the velocity of photospheric layers develops first in the deepest layer and propagates outward and, hence, the line doubling occurs first in high-excitation lines and last in the low-excitation lines. An approximate linear fit to the data in Figure 5 gives a shock "velocity" of 1 eV per 65 ± 10 days. A nonlinear fit may prove to be better, especially in the higher photospheric layers.

Caution is urged in the interpretation of the double lines. There is a possibility that a part of the profile between the two apparent absorption lines may be an emission peak on an absorption line that is largely unchanging through the line-doubling episode. Two emission lines, both of low (0.9 eV) excitation potential, were described above. Others undoubtedly exist. Hence, the presence of emission in the cores of lines cannot be excluded completely as the source of the line doubling. However, since the profile of the (assumed) underlying single absorption line changes, a true doubling of an absorption line appears to be the more likely model. In any event, the evolution of a line doubled by superimposition of shock-produced emission on the central absorption core will follow the observed pattern represented by Figure 3 if the shock propagates outward driven by the

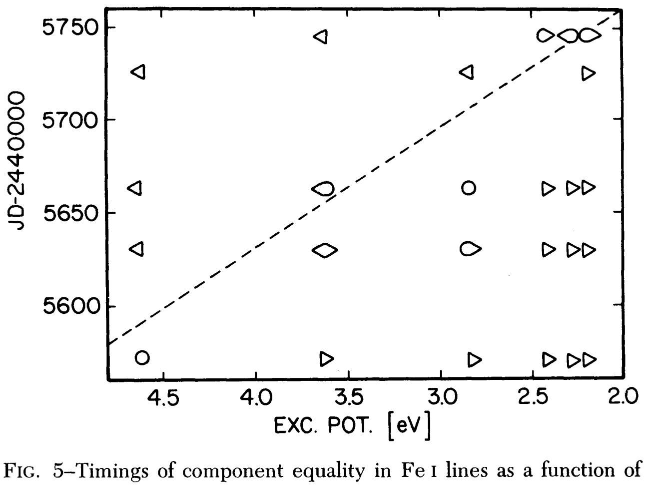

FIG. 5–Timings of component equality in Fe I lines as a function of lower excitation potential. Left (right) pointing arrowheads indicate dominance of blue (red) line components. Circles identify timings of equality. Combinations of symbols represent averaged results from two lines or more. An approximate linear fit is shown for illustrative purposes. It represents a "velocity" of 65 days per 1 eV.

radial pulsation, i.e., the precise interpretation of the nature of the line doubling is not critical to our assertion that the underlying pulsation is radial.

As described above, we have followed the behavior of Fe I emission lines in radial velocity and equivalent widths (see Fig. 4). We clearly see how the lines tend to fade to almost zero intensity at times of maximum radial acceleration of the N I lines, that is, at times of maximum expansion of the supergiant. It is then that the amount of excitation is reduced to a minimum. The emission reappears and intensifies during the collapse of the photosphere. Times of maximum emission correspond to intervals of maximum deceleration of the high-excitation lines, indeed, when the atmospheric layers of ρ Cas are fully compressed. No attempt has been made to correct for the underlying absorption component as the equivalent widths of the emission lines are much larger than those of their absorption companions at times of maximum excitation. These variations of the excitation of the emission lines correlate well with the AAVSO magnitudes (compare Figs. 4 and 6). Visual brightness and emission strength fade concurrently, then appear to rise together and reach a maximum before completing one cycle.

The Geneva photometric surveys of supergiants randomly sample their brightness, with no particular emphasis on individual objects (Maeder and Rufener 1972; Burki 1978; Maeder 1980). These surveys calibrate the period-luminosity lines in the HRD, but they surely cannot compete with the AAVSO data set as far as continuous temporal coverage is concerned for selected stars. We find from the latter that ρ Cas shows an average semi-amplitude of about 0.15 magnitude while the Geneva-calibrated HRD shows amplitudes of 0.05 to 0.09 magnitude for less-luminous F–G supergiants. This agrees well with Maeder's (1980) conclusion that amplitudes increase with luminosity.

A power spectrum analysis by Percy and Keith (1985) indicates the possible presence of a 275 ± 25 day period in the estimated brightness of ρ Cas as recorded by the British Astronomical Association. The AAVSO data do not show this feature (see also Percy, Fabro, and Keith 1985). This may serve as yet another warning that photometric and spectroscopic periods should not be blindly regarded

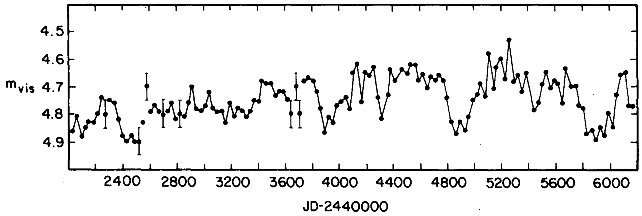

FIG. 6–A light curve for ρ Cas as constructed from visual-magnitude estimates published in the AAVSO circulars. Usual reporting is to two decimal places, except for the points shown with reporting error bars that were published to one decimal place only. Note the three big fadings near JD2440000 +3900, +4900, and +5900.

as identical. More importantly, there may be a 2 to 1 ratio between our shock-wave period and the estimated brightness period! Arellano Ferro (1985), by reviewing all photoelectric magnitudes of ρ Cas obtained between 1964 and 1981 by various sources, has proposed a period of 484 days. Perhaps his estimate should be given less weight than the above determinations due to inhomogeneity and incompleteness of the observations. However, it is consistent with our period of about 520 days.

When the observed periods (whether photometric or spectroscopic) of supergiants are compared to the radial fundamental mode of pulsation calculated theoretically, many stars are found to pulsate on longer time scales than predicted. Maeder (1980) favored a nonradial mode as the explanation for this discrepancy. But Lovy *et al.* (1984) developed new stellar evolutionary models that incorporated high rates of mass loss. When supergiants evolve back to the blue from the red supergiants region, their radial modes are likely to be longer than on the first crossing because they now have a dense interior and have lost mass. With this in mind, we can try to resolve the evolutionary status of ρ Cas: Is it traveling redward or blueward along the HRD upper boundary? Lovy *et al.* provide a fundamental period calibration of the HRD from which we can predict the radial fundamental period of ρ Cas. Their quasi-PLC relation for stars with $\log T_{\rm eff}$ less than 4.1 is $\log P = -0.275\ M_{\rm bol} - 3.918 \log T_{\rm eff} + 14.543$ for stars evolving to the red supergiant region. Assuming that the absolute magnitude is -9.4 (Humphreys 1978) and that $\log T_{\rm eff}$ is 3.79, then the location of ρ Cas corresponds to a predicted fundamental period of 190 days. It is readily seen that the observed period of 520 days for ρ Cas is longer by a factor of 2.7 than this prediction.

An estimate of the uncertainties in the data and in the relation itself shows that this difference in periods is significant. The predicted period increases with luminosity, but it is highly improbable that ρ Cas is intrinsically brighter than the value given above. Its membership in Cas OB5 (Humphreys 1978) already puts it among the visually most-luminous stars. We estimate the upper limit on the uncertainty in the formal value of $\log T_{\rm eff}$ to be ± 0.02; if this maximum uncertainty is applied to lower the adopted $T_{\rm eff}$, the predicted $\log P_0$ increases by 0.078. We should note in passing that ρ Cas exhibits somewhat larger temperature changes during pulsation and certainly major disturbances may occur during times when it irregularly shifts toward redder spectral types. During our reported interval of observations, there were no significant departures from the formal F8 type assigned to this supergiant. The relation for predicting radial fundamental periods given by Lovy *et al.* (1984) has a mean uncertainty of 0.053 in $\log P_0$ which they have determined from their models. These two errors in tandem may increase the predicted period by $\Delta \log P_0 = 0.131$, hence changing the 190-day value mentioned above into 260 days. We conclude that our observed 520 ± 45-day period is significantly longer than the predicted fundamental radial mode.

Rho Cas is not exceptional in this respect: Lovy *et al.* note that 60% of their photometrically observed supergiants have periods which are 1.5 to 4 times longer than the predicted fundamental radial periods. Sample models of a 60 $\mathfrak{M}_\odot$ star experiencing severe mass loss led to an increased predicted fundamental period for the post-red-supergiant star evolving to the blue. The increase by a factor of 1.7 applied to the above prediction of ρ Cas would give 320 days. We suppose that a model more appropriate to ρ Cas could be persuaded to pulsate in the fundamental radial mode with a period closer to the observed 520 days. An alternative explanation considers ρ Cas to still be evolving redward in the HRD, while pulsating nonradially over a period longer than the predicted fundamental mode, but the spectroscopic evidence compiled here points to a radial mode as dominant in the pulsations. We suggest, therefore, that ρ Cas is a supergiant in a very advanced evolutionary stage, crossing the top of the HRD toward the blue. Maeder's (1981) models (using a mass-loss rate $\log \dot{\mathfrak{M}}_{\rm WR} = -4.4$) lead us to speculate that ρ Cas is a 25 $\mathfrak{M}_\odot$ "remnant" of an original 45 $\mathfrak{M}_\odot$ ZAMS supergiant, born about 5 million years ago and which will explode in about 200,000 years.

To summarize, this study of the pulsational behavior of ρ Cas establishes the period to be about 520 days over the interval JD2444400 to JD2445800. The radial-velocity curves are somewhat variable and, hence, "quasi-period" may be a better description than period. We consider the dominant mode to be radial rather than nonradial on the following grounds:

1. The shape of the radial-velocity curves and the phase lag relative to maximum brightness find parallels in the Cepheids where a radial pulsation is obviously dominant.

2. Line-doubling episodes are quite readily interpreted as the effects of a radially propagating shock initiated by a radial pulsation.

We propose ρ Cas as the prototype of the I*a*-0 pulsators; we shall report elsewhere on similar pulsations in a second star HR 8752 = V509 Cassiopeiae (see also LHH). These pulsating stars of extreme Population I appear to be the high-mass, high-luminosity, and long-period cousins of the Population II RV Tauri variables. Common characteristics of these two groups include the light curves which show alternating deep and shallow minima (see Fig. 6), a smooth radial-velocity curve exhibited by the lines formed in the deep photosphere, and a line doubling present over a narrow interval in phase in lines formed in the higher layers—see Preston (1962, 1964) and Mozurkewich *et al.* (1986) for a discussion of R Scuti, an RV Tauri variable.

We thank Drs. V. V. Smith and J. Tomkin for obtaining some spectra, and Dr. C. Sneden for helpful discussions. The AAVSO and its members deserve an acknowledgment for the data we analyzed. This research has been supported in part by the National Science Foundation (grant AST 83-16635).

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APPENDIX
The AVVSO Light Curve

We have collected visual-magnitude estimates of ρ Cas from the American Association of Variable Stars Observers (AAVSO) circulars. These monthly-averaged values are given to a hundredth of a magnitude as determined by 4 to 21 observers (mostly 10 to 12). The data cover a time interval spanning more than a decade: the interval of 4140 days encompasses our spectroscopic observations (Fig. 6). There is, obviously, a rather uncertain error bar to be assigned for each individual observer and date combination, but when ten or more observers are averaged together, the most probable value for a consistent relative error is about 0.02 to 0.03 magnitude (see also a similar estimate by Percy and Keith 1985). An offset must be present relative to the standard photoelectric scale, but it should not affect the search for periodicities.

The star seems to be active for most of the time: brightenings and fadings of 0.15 magnitude are present relative to the averaged light level which may jump from one month to the next by a few hundredths of a magnitude (because of systematic errors?). So far, there is only one identified case of a coincidence between brightness and radial-velocity variations; this occurred around JD2444750 to JD2445150, and is discussed in the text. As noted in Section V, the variations of the emission-line strengths and brightness estimates are correlated over slightly more than two periods, roughly between JD2444700 and JD2445700.

The estimated light curve assembled out of this AAVSO data set exhibits three fadings of the star which are separated by roughly the same interval: around JD2440000 +3900, +4900, and +5900 dates. This "cyclic" behavior hints at a 1000-day-long photometric period: there is no similar fading at an earlier date (by 1000 days), but there is one seen at around JD2442500. This serves as yet another indication that, perhaps, we should not expect a complete correspondence between photometric and spectroscopic periods; the latter observed for ρ Cas are definitely shorter than 1000 days in length. Lower amplitude cycles with shorter quasi-periods of about 500 days seem to be present in the light curve (in part supported by correlation with the emission intensity), but the suspected amplitudes are close to the noise level.