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ATMOSPHERIC MOTIONS IN RED SUPERGIANTS

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Radial velocities obtained from interferometric spectra of a α Ori, α Her, and α Sco have been analyzed to investigate expansion and other large-scale motions in their atmospheres. In α Ori evidence of such motions is found. They appear to be sporadic in time or position on the surface of the star. If these motions are an observational indication of mass loss, they lead to extremely high instantaneous loss rates. It has not been possible to infer an average rate of mass loss.

Key words: supergiant — radial velocity — mass loss — infrared — interferometer

I. Introduction

It has long been known (see, for example, Merrill and Greenstein 1956) that lines of neutral metals in long-period variables are displaced in a systematic way that depends on the excitation potential of the lower atomic level. This means that different levels in the atmosphere are moving at different velocities with respect to the sun. In variable stars this motion is most obviously attributable to pulsation of the atmosphere, and Merrill (1960) has looked unsuccessfully for the effect in late-type supergiants that are not long-period variables. However the presence of doubled resonance lines in both types of stars has been shown by Deutsch (1956) to be caused by a gas shell moving away from the star, and Weymann (1962) has deduced from a high-dispersion study of its shell spectrum that α Orionis (M1–2 I*ab*) is losing 4×10^{-6} solar masses per year. Deutsch (1960) could find no circumstellar absorption lines in normal giants earlier than M0, but Bopp and Edmonds (1970) found a velocity difference of -0.31 ± 0.25 km sec^{-1} between first-overtone CO vibration-rotation lines and neutral metal lines in α Bootis (K2 IIIp). This star shows no obvious pulsational characteristics. They interpreted the velocity difference as being caused by expansion of the upper atmosphere, leading to mass loss.

We have obtained high-resolution interferometric spectra of three of the M supergiants that have been prominent in the literature concerning circumstellar gas and mass loss: α Ori (M1–2 I*ab*), α Herculis (M5 II), and α Scorpii (M2 I*ab*). These three stars all show double resonance lines, variable radial velocities from the photospheric lines, and irregular variations in light. Our spectra are in the infrared, where most atomic transitions have lower levels above 4 eV, but where the blending is not as severe a problem as it is in the visual. The vibration-rotation lines of the molecules CO and OH are prominent in our spectra. These are generally formed at small continuum optical depths and are the counterpart of the low excitation atomic lines. With these new data we have reexamined the question of systematic motions in these supergiant atmospheres.

II. Observational Data

The interferograms were obtained with the JPL Mark III Planetary Interferometer (Beer, Norton, and Seaman 1971) at the 2.72-meter telescope of the McDonald Observatory. Details concerning them are presented in Table I.

III. Radial Velocities

Bopp and Edmonds (1970) measured the displacements of lines in α Boo relative to the same lines in the sun. Thus, their result that the radial velocity from the atomic lines is slightly larger than that from the CO lines assumes that there are no systematic shifts between solar atomic and molecular lines. This assumption is of questionable validity (see the appendix for details). In this study, we refer

TABLE I

OBSERVATIONAL DETAILS

Star	Run No.	Resolution (cm^{-1})	S/N (rms)	Wavenumber Range (cm^{-1})	Date (UT)	JD (2440000+)
α Ori	357	0.10	22	1800-3100	Jan. 24, 1972	1340.77
α Ori	374	0.10	46	5500-6600	Feb. 04, 1972	1351.66
α Ori	392	0.10	28	5500-6600	Mar. 29, 1972	1405.61
α Ori	400	0.10	37	5500-6600	Mar. 31, 1972	1407.63
α Ori	425	0.14	38	5500-6600	Apr. 06, 1972	1413.60
α Sco	440	0.15	56	4000-4800	Apr. 10, 1972	1417.94
α Her	399	0.10	34	4000-4800	Mar. 30, 1972	1406.95
α Boo	432	0.10	17	4000-4500	Mar. 08, 1972	1415.93

stellar line positions to rest wavenumbers obtained directly from laboratory spectra or calculated from energy levels.

Laboratory determinations of wavenumbers for atomic lines in the infrared are widely scattered through the literature and are, unfortunately, often unreliable. We have found that the recent Swedish measurements give consistent radial velocities, and we have used them when possible. We have also found that, in general, line positions from the older energy level data (Moore 1949, 1952) can lead to velocities with large internal errors. An exception is iron, for which Roth (1970) has revised the designations of some of the levels. Line positions obtained using the new designations are reliable enough for us to include iron lines in our list. We have excluded lines of titanium, vanadium, and scandium whose positions are only obtainable from energy levels, because they are apparently subject to uncertainties larger than a few hundredths of a wavenumber. The energy level data for silicon, sodium, and magnesium are reliable enough to give accurate positions for a few lines that have not been directly measured in the laboratory. In addition, there are at least 40 strong lines that we have been unable to identify. We have listed the numbers of lines and the sources for the adopted wavenumbers in Table II.

For the molecular lines, calculated or astronomical wavenumbers have been used. We have found that for CO the molecular constants of Rao and Mantz (1972) predict line positions that agree well with our spectra up to a rotational quantum number of about $J = 70$. The standard isotope shift formulation was used to predict the positions of the $^{13}C^{16}O$ lines which in the fundamental and first-overtone bands can be as strong as the $^{12}C^{16}O$ lines. The best available frequencies for the strong first-overtone P branch of OH are solar, from the sunspot data of Hall (1970). He has calibrated the wavenumber scale of his spectra using solar interferograms, but obviously the possibility of velocity shifts still exists. The first-overtone bands of SiO near 4 microns are clearly resolved in our spectrum of this region. These bands have not yet been observed in the laboratory, and molecular constants for the ground state derived from analyses of electronic transitions in the ultraviolet and are not accurate enough to predict reliable line positions for lines in the 4-micron band. Again, the best available data are provided by sunspot spectra (Hall 1973, unpublished). We have also used Hall's (1970) data for six (2,0) lines of CO with rotational quantum number greater than 70.

We calculated radial velocities for all unblended lines with satisfactory positions, and,

TABLE II

SOURCES FOR LINES USED

Species	Reference	Wavenumber region 1800–3300cm^{-1} no. of lines	4000–4800cm^{-1} no. of lines	5500–6600cm^{-1} no. of lines
$^{12}C^{16}O$	Rao and Mantz (1972)	22	60	63
	Hall (1970)		6	
$^{13}C^{16}O$	see note (1)	10	14	
SiO	Hall (unpublished data, 1973)	70		
OH	Hall (1970)			13
Al	Eriksson and Isberg (1962)		2	3
Ca	Risberg (1968)		5	
Fe	Moore (1952), Roth (1970)		7	6
Mg	Risberg (1964)		1	2
	Moore (1949)		2	1
Sc	Fisher, Knopf and Kinney (1959)		2	
Si	Litzen (1965)		7	7
	Moore (1967)		1	
Na	Johansson (1961)		2	
	Moore (1949)		2	
S	Jakobsson (1967)		1	1

NOTES TO TABLE II

(1) Wavenumbers calculated from the $^{12}C^{16}O$ predictions and standard relations for isotopic molecular constants.

as a check, included a large sample of sharp terrestrial lines. These were from transitions of N_2O and CO_2 using positions from the list of McClatchey et al. (1973), and of H_2O, for which accurate line positions and energy levels have recently become available (Flaud and Camy-Peyret 1973; Camy-Peyret and Flaud 1973; Camy-Peyret et al. 1973). This check revealed a small error in our frequency scale. Using over 200 lines of the three molecules above, and all the spectra used here, we have been able to define it quite precisely as mimicing a velocity shift of 1 km sec^{-1} (with very small differences between spectra) and to make the necessary corrections in the wavenumber scale. We do not yet know the cause of the shift.

As a further check we have measured the radial velocity of α Boo in exactly the same way as we did those of the supergiants. The result, -5.1 ± 0.3 km sec^{-1} from 74 lines, is in adequate agreement with the result of -5.50 ± 0.03 obtained by Griffin and Griffin (1973). Our quoted error is an internal probable error, and reflects the measurement uncertainty of both the stellar lines and the terrestrial lines used to evaluate the zero-point shift. The spectrum of α Boo covers a restricted wavenumber range (see Table I) and does not exhibit lines from enough atoms and molecules to be used in the analysis which follows.

IV. Results

In Table III we list the velocity obtained from each spectrum. The errors quoted are internal probable errors, and for the mean velocities from all lines they reflect both the uncertainty in the velocity shift and the uncertainty in the measurements of the stellar lines. We also list mean velocities from small groups of lines which will be explained and discussed below. In this case the uncertainty in the wavenumber shift is of no import since we are interested only in the difference between groups within the same spectrum. Hence these probable errors do not contain the uncertainty in the velocity shift. The most important contribution to these errors is purely random: we have read line positions to one-tenth of a resolution element, and the probable error per line is about the same, 0.013 cm^{-1}. Of course this is a larger error in velocity at 5 microns than it is at 1.6. Also identifiable is a scatter caused by the inhomogeneous frequency data. This is most noticeable for the atomic lines, but even there amounts to no more than an additional 0.01 cm^{-1} per line. As expected, the root-mean-square deviation decreases in spectra of high signal-to-noise ratio, and for the strongest lines.

At the bottom of Table III are listed mean photospheric velocities obtained from coudé plates, the range in velocity observed over several years of measurement, and mean circumstellar velocities, which are much less variable than photospheric ones. In all cases our mean velocities are well within the range observed in the photosphere in the blue. The velocity variation of α Ori has a period of about 8 years with apparently random short-term fluctuations imposed on it. The curve established by Adams' and Weymann's data, if extrapolated to the epoch of our observations, indicates that the radial velocity of the star should be close to the median value as we observe. Coudé scanner observations (D. L. Lambert and E. Luck, 1973, unpublished observations) confirm this.

To investigate expansion of the atmosphere we must ascribe a depth of formation to each line used. This is a complicated problem involving level populations, line saturation, and molecular dissociation. Line strength and excitation potential are both useful parameters in ordering lines by depth of formation. A strong line is formed at a smaller continuum optical depth than a weak one, and a line of high excitation potential is formed at larger depths than one of low excitation.

We have made two completely separate groupings of the lines using these ideas so that radial velocities of lines formed at different depths can be compared. The first grouping was by species. The metal lines were divided into two groups; those above and below 5.5 eV, and the molecular lines were grouped by molecule. We ordered these groups by level formation by calculating mean depths of formation for representative lines in each group using an α Ori model kindly supplied by Professor Hollis Johnson (see Faÿ and Johnson 1973). In Table III, the lower the entry for a group, the greater is the continuum optical depth at which that group is assumed to be formed. In the plots of these velocities in Figures 1 and 2, the further to the left the point in the diagram for each spectrum, the closer to the surface of the star the group is formed.

The criterion for the second grouping was the central intensity of the line. In any optically thick line, the central intensity is a measure of the source function in the region where the line is formed. Given the monotonic decrease in temperature outward from the center of the star, the deepest lines are formed closest to the surface of the star. The selected lines are all relatively strong and the assumption of optical thickness appears valid. Again the groups assumed to be formed closest to the top of the atmosphere appear highest in Table III and farthest to the left in the plot of each spectrum.

The spectrum of α Ori in the 3- to 5-micron region has not been treated as elaborately. All the observed SiO lines are much weaker than any of the CO lines in the sample, so a species grouping is also an intensity grouping. The lines of the fundamental CO band are formed at extremely small optical depths; the SiO lines

TABLE III

SUMMARY OF RADIAL VELOCITY RESULTS

	α Ori 357	α Ori 374	α Ori 392	α Ori 400	α Ori 425	α Her 399	α Sco 440
All lines	18.5 ± 0.5	18.9 ± 0.3	20.4 ± 0.3	20.1 ± 0.3	19.3 ± 0.3	−35.2 ± 0.4	−5.0 ± 0.3
n*	102	86	91	77	87	112	88
CO $\Delta v = 1$	21.8 ± 0.8						
n	32						
SiO $\Delta v = 2$	17.0 ± 0.5						
n	70						
$^{12}C^{16}O$ $\Delta v = 2$						−35.3 ± 0.2	−5.0 ± 0.2
n						66	54
OH $\Delta v = 2$		16.7 ± 0.4	19.5 ± 0.6	19.4 ± 0.6	16.8 ± 0.6		
n		12	13	9	11		
low exc. metals		17.8 ± 0.9	20.7 ± 0.5	19.1 ± 0.5	17.8 ± 0.6	−35.1 ± 0.4	−5.0 ± 0.6
n		5	5	4	5	19	18
$^{13}C^{16}O$ $\Delta v = 2$						−34.3 ± 0.4	−5.0 ± 0.7
n						14	7
$^{12}C^{16}O$ $\Delta v = 3$		19.2 ± 0.4	20.6 ± 0.3	20.0 ± 0.3	20.0 ± 0.3		
n		56	60	54	58		
high exc. metals		20.2 ± 0.7	20.3 ± 0.5	21.4 ± 0.7	18.2 ± 0.9	−35.8 ± 0.9	−5.3 ± 1.1
n		13	13	10	13	13	9
central intensity group 1						−35.2 ± 0.5	−5.5 ± 0.5
n						21	18
group 2		17.2 ± 0.5	20.4 ± 0.6	19.5 ± 0.5	17.3 ± 0.7	−35.4 ± 0.2	−4.8 ± 0.3
n		15	16	14	14	27	20
group 3		19.8 ± 0.5	20.8 ± 0.4	21.2 ± 0.5	19.7 ± 0.5	−34.9 ± 0.3	−4.2 ± 0.4
n		24	26	22	26	28	25
group 4		18.4 ± 0.4	20.2 ± 0.3	19.7 ± 0.5	20.2 ± 0.4	−34.8 ± 0.5	−5.6 ± 0.8
n		26	27	23	27	12	10
group 5		19.0 ± 0.7	20.0 ± 0.5	19.8 ± 0.6	19.1 ± 0.8	−35.5 ± 0.5	−4.8 ± 0.7
n		20	21	18	20	23	15
visual photospheric velocity					+21.0 (1)	−33.1 (1)	−3.2 (1)
observed range					7 (4)	5 (2)	9 (3)
velocity circumstellar					+10 (4)	−45 (2)	−18 (5)

NOTES TO TABLE III

* n indicates number of lines measured for each tabulated velocity.

(1) Wilson, 1953 (2) Deutsch, 1956 (3) Evans, 1961 (4) Adams, 1956 (5) Adams and McCormack, 1935.

are formed deeper in the atmosphere. The results for this spectrum appear in Table III but have not been plotted.

V. Discussion

Inspection of Figures 1 and 2 shows that the two methods for grouping the lines gives results which are similar and differ only in detail. The most obvious difference is caused by the 6-volt lines of neutral metals, which are stronger than predicted. In the species groupings they are assumed to be formed at comparatively

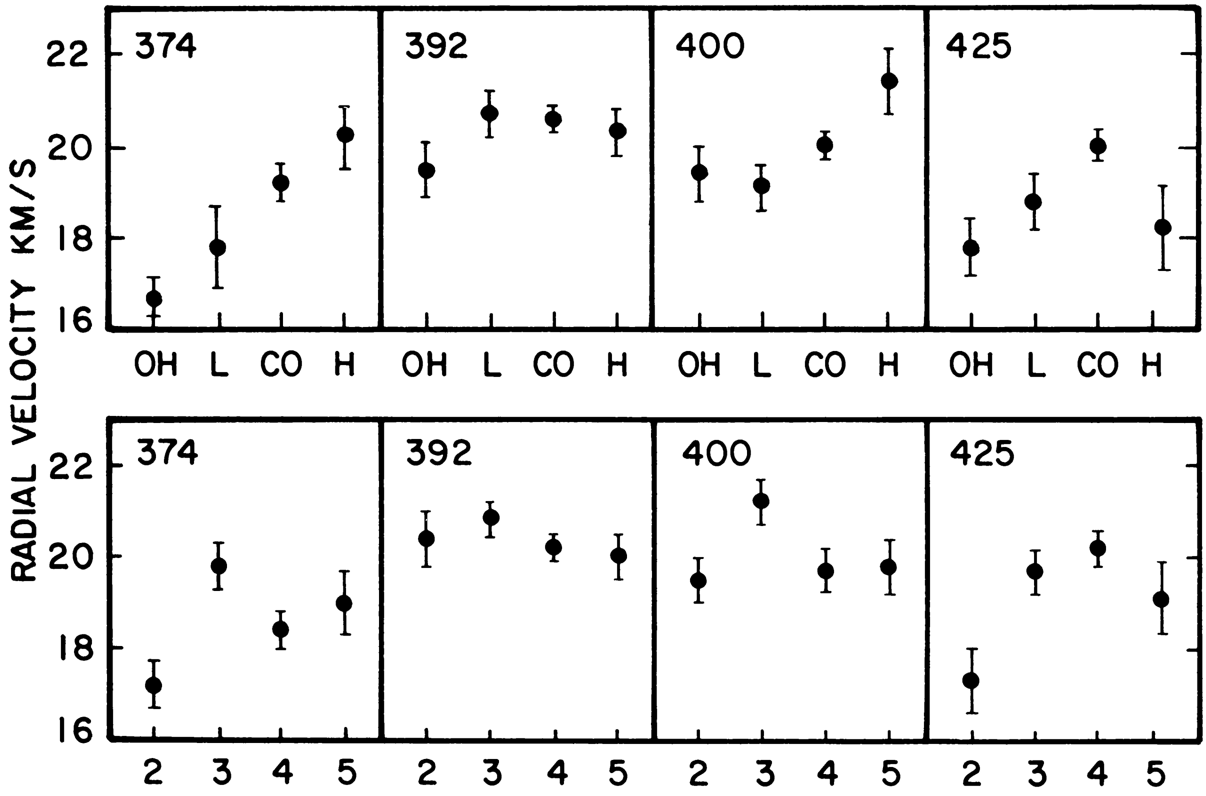

FIG. 1 — Mean radial velocities from lines in four spectra of α Ori. The spectra are identified by the run number at the top of each diagram. The top section shows results from grouping the lines by species; the bottom results from a grouping by line intensity. Depth of line formation increases to the right in each plot. Key: L = metal lines below 5.5 eV excitation, H = metal lines above 5.5 eV.

large depths (group H in Figs. 1 and 2), yet their intensity puts them closer to the surface (group 3 in Figs. 1 and 2). This may be an indication that these lines are affected by departures from local thermodynamic equilibrium. A comparison of the two plots for run 374 shows the effect clearly: the highest radial velocity is from the high-excitation metal lines, and they dominate group 3 in the plot of lines grouped by intensity.

The radial velocities for α Sco and α Her are independent of depth of formation to within the observational uncertainty of about 0.5 km sec^{-1}. The radial velocity results for run 374 of α Ori show evidence for a depth dependence. The 4-micron spectrum of α Ori also shows significant depth dependence, but this time in the sense that the atmosphere is contracting! This result is not necessarily in conflict with the ideas outlined below, but more spectra in this region and a reexamination of the molecular constants for SiO would clearly be of great interest.

In the analysis which follows we have placed the most reliance on the CO and OH results for the four spectra of α Ori in the 5500 to 6600 cm^{-1} region. The frequencies used for the molecular lines are more self-consistent than they are for the atomic lines, but, more important, the metal line strengths are likely to be affected by departures from local thermodynamic equilibrium. For the molecules the situation is better. Studies of the collision rates and of the source functions (Thompson, 1973; K. H. Hinkle and D. L. Lambert, in preparation) indicate that for vibration-rotation transitions LTE is a reasonable assumption in late-type atmospheres. This means that we have more confidence in the assigned depths of formation for OH and CO lines than we do for the metal lines.

We have used the statistical t test (Hoel 1971)

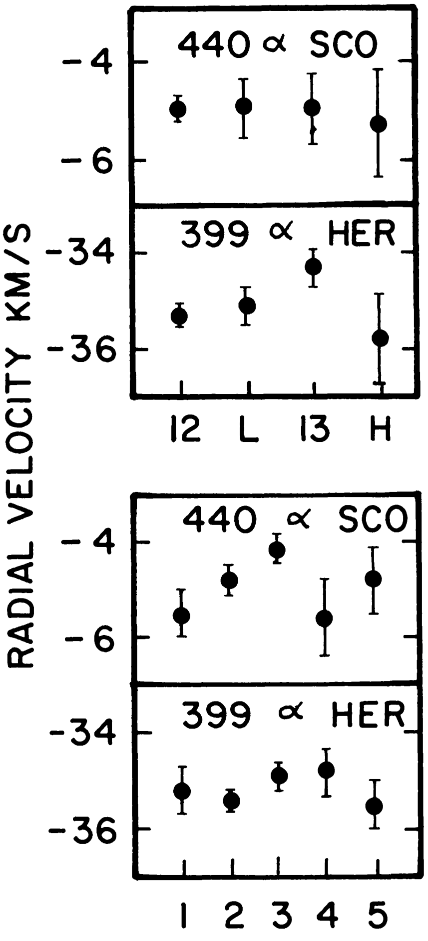

FIG. 2 — Mean radial velocities from lines in α Sco and α Her. The top section shows results from grouping lines by species; the bottom results from a grouping by line intensity. Depth of line formation increases to the right in each plot. Key: 12 = $^{12}C^{16}O$ $\Delta v = 2$ lines, L = metal lines below 5.5 eV excitation, 13 = $^{13}C^{16}O$ $\Delta v = 2$ lines, H = metal lines above 5.5 eV.

to check the reality of the differences between some of the OH and CO velocities. In runs 374 and 425 the OH and CO velocities are significantly different at the 99% confidence level, as are the OH velocities in runs 374 and 392. The difference between the OH and CO velocities in runs 392 and 400 is not significant. The OH velocities are based upon solar wavenumbers. Their systematic error is unlikely to exceed 0.2 to 0.3 km sec^{-1}. Such an error is, of course, common to all scans. The fluctuations in the OH-CO relative velocity are real according to our statistical tests and independent of this possible systematic error. The magnitude of the errors appears to to be too small to negate our conclusion that the OH layer is stationary or expanding relative to the CO layer in the present series of scans. A more extended series of scans is necessary before a relative contraction of the OH layer can be excluded.

Two effects which may contribute to this intermittent relative expansion are outlined below. The first may be labeled 'macroturbulence'. We suppose that the atmosphere consists of large elements (supergranules) moving in a random way and approximating the classical description of macroturbulence. We imagine that atmospheric conditions may vary from supergranule to supergranule. If one supergranule has the physical conditions favoring OH formation, the OH absorption line seen in integrated light will have a radial velocity weighted toward that of this exceptional supergranule. Then, if the supergranules are relatively few in number (say five to ten) a fluctuation in supergranule numbers, or in physical condition, could lead to a variation of radial velocity with species. The observed velocity variations are a small fraction of the total linewidth and this would suggest that several granules must contribute to the spectrum: two or three granules would result in severe asymmetries as one granule faded or came into prominence.

The OH and CO velocities, when they differ, do so in the sense that the atmosphere is expanding. Bopp and Edmonds (1970) used a model of continuous expansion of the atmosphere over its entire surface to investigate this effect. Their calculations contained an error and considerably underestimated the mass loss rate: their velocity difference of 0.31 km sec^{-1} gives a mass loss rate for **α** Boo of 2×10^{-3} solar masses per year. For **α** Ori, Weymann's (1962) mass loss rate of 4×10^{-6} $\mathfrak{M}_{\odot}$ yr^{-1} implies a line-of-sight velocity difference of 2×10^{-4} km sec^{-1} across the atmospheric region spanned by our line sample. Evidently any observable velocity differences lead to very high loss rates, so we reject this model. Our results indicate that expansion may occur, but that it is variable with time and/or position on the stellar photosphere. For this intermittent case it is not possible to infer a mass-loss rate. The results would also be consistent with pulsation of the photosphere.

Some of the velocity sequences in Figure 1 are suggestive of waves or shocks propagating through the atmosphere; see, for example, runs 400 and 425 for α Ori. A shock traveling at 10 km sec^{-1} (the sound speed is in the range 3 to 7 km sec^{-1} across the photosphere) would take about a week to propagate between the atmospheric levels spanning the region of formation of our atomic and molecular lines. An event such as this can readily contribute to our observed velocity differences, and to intermittent mass loss.

Two important factors prompt us to suggest that the mass loss rate for α Ori might be reexamined. First, accurate absolute oscillator strengths should now be available for all the low excitation lines seen in the shell spectrum. Second, Field (1974) has suggested that the considerable underabundances of some elements in the interstellar medium may be a result of their condensation into grains in stellar atmospheres or nebulae (i.e., circumstellar shells). The excess infrared emission is presumably attributable to dust grains in the circumstellar shell. Analyses of the interstellar line spectrum of ζ Ophiuchi compiled by Field show that Ti and Ca are underabundant by a factor of 10^3. He argues that these elements can condense out near stars. The mass-loss rate provided by Weymann is based on these and similar elements. He derived the column density (i.e., the total mass) on the assumption that the element abundance was solar. If these elements are largely condensed out into grains, the mass flow was underestimated. Assuming the factor for ζ Oph is applicable, the mass flow in α Ori must be increased from 4×10^{-6} to 4×10^{-3} $\mathfrak{M}_{\odot}$ yr^{-1}, which is a very large rate of loss. The larger flow would give a velocity differential of about 0.2 km sec^{-1} for our line groups which is close to the observed effect, but the mass-loss rate is uncomfortably large. Clearly, the condensation hypothesis must be scrutinized very carefully in this case.

The elements C, N, and O are thought by Field to condense out in the interstellar medium and not in material surrounding supergiant stars. Therefore the column densities of these elements may also be a factor of 10^3 higher than previously realized, and, in this case, circumstellar components of low excitation OH and CO lines are much more likely to perturb our photospheric velocities. A simple slab estimation shows the equivalent widths of the circumstellar components of the lines observed in the 5500-6600 wavenumber region to be extremely small, even with the larger column densities. For CO this is because of the very small f values, which were calculated from Chackerian (1970). For OH the $\nu = 0$, $N = 9$ level, the lowest we observe, has an excitation of 0.19 eV, and, more importantly, OH is seriously depleted by H_2O formation at temperatures less than 3000° K. For the fundamental CO lines the situation is quite different. Even Weymann's original column density for carbon implies completely saturated circumstellar lines for (1,0) transitions to at least $J =$ 25, assuming a shell excitation temperature of 500° K and that most of the carbon is in the CO molecule, as in the photosphere. These lines will be slightly weaker than the photospheric ones, but they will be superposed on the photospheric profiles. We have seen no evidence for such lines. The radial velocities derived from the (1,0) lines agree well with those from higher vibrational states (there are some (5,4) lines in our sample) and there are no systematic differences in equivalent widths. Further, the fundamental CO spectrum can be satisfactorily predicted using only a model photosphere (Lambert 1973, unpublished). We conclude that the CO column density in the shell is substantially less than the predicted carbon column density, either because CO has been dissociated or because it has condensed onto grains. This is to be contrasted with the situation in low-gravity stars with thick shells, such as VY Canis Majoris (Geballe, Wollman, and Rank 1973) in which low excitation lines at the shell velocity are the only features seen in the CO fundamental band. Use of the enhancement factor suggested by Field does not make the α Ori shell optically thick: for a hydrogen column density of 2×10^{25} cm^{-2} only the extreme ultraviolet suffers any measureable continuous absorption due to the shell.

The problems of atmospheric motions in supergiants obviously need further study. We plan to continue with simultaneous observation across the 4000 to 6600 cm^{-1} interval with good time coverage. We are also monitoring the radial velocity of α Ori using the Tull coudé scanner and hope to acquire biweekly or monthly observations throughout a year. Fine analysis of line profiles will be required to define the velocity field properly and to ascribe contributions to micro- and macroturbulence.

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APPENDIX

In their radial velocity study of interferometric spectra of Arcturus, Bopp and Edmonds (1970) measured stellar line displacements relative to the solar spectrum rather than using the frequency scale provided from the stellar interferogram. Their interpretation of the slight radial velocity shift between stellar metallic and CO lines is based on their implicit assumption that solar metallic and CO lines show no relative velocity displacement. This is probably an invalid assumption.

A simple and direct indication which casts doubt on their assumption is provided by measurements of the sunspot umbral velocity field. Observations of spots in which the umbral lines are measured relative to their photospheric counterparts show that the umbral motion is predominantly *downwards;* for example, Holmes (1963) reported $v \sim -0.1$ km sec^{-1}. An *upflow* must surely be present in order to feed the Evershed flow in the penumbra. The explanation would appear to be that the photospheric lines used in these measurements are themselves displaced to the blue by the complex photospheric velocity field (or other effects). This displacement has long been noted by observers searching for the gravitational redshift predicted by Einstein; see, for example, Pierce and Breckenridge (1973) who report on a recent reinvestigation of solar wavelengths. The typical photospheric line used in umbral studies has a blueward displacement equal to a substantial fraction of the gravitational redshift of 0.64 km sec^{-1}. Hence, umbral lines formed in a different type of velocity field may experience a redward displacement relative to the photospheric lines. Of course, their displacement relative to a laboratory spectrum would be to the blue as the flow is upward in the umbra.

Differential velocities for photospheric lines are well established; see, for example, Lambert and Mallia (1968) and Glebocki and Stawikowski (1971) for recent discussions. The velocity shift is a function of equivalent width and species. A detailed analysis would be required in order to predict the systematic shift between the CO lines and the metal-line sample used by Bopp and Edmonds. The effect is potentially of the correct magnitude and sign to explain the apparent expansion of the CO line relative to the metallic lines in Arcturus.

Note added in proof:

Boesgaard (1973) has reported that Fe II emission lines in α Ori are redshifted by 5 km sec^{-1} with respect to photospheric absorption lines. This is analogous to the situation in our 3- to 5-micron spectrum of α Ori. Presumably the fundamental CO lines are formed in the same part of the atmosphere as the ultraviolet Fe II lines. This redshift of lines formed at the top of the atmosphere is not inconsistent with the presence of an expanding gas shell around the star (see the discussion following Boesgaard's paper).

REFERENCE

Boesgaard, A. M. 1973, in *Stellar Chromospheres, SP-317,* S. D. Jordan and E. H. Arnett, eds. (Washington, D.C.: NASA), p. 153.