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Breit-Wheeler pair creation by finite laser pulses

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Abstract. Probability distributions of electron-positron pair creation in collisions of a laser beam and a nonlaser photon (Breit-Wheeler process) are calculated in the framework of strong-field quantum electrodynamics. The driving laser field is modeled as a finite pulse, similar to the formulation introduced in Phys. Rev. A **86**, 052104 (2012). The sensitivity of pair production to the driving pulse peak intensity and duration, in the context of spin effects, are investigated.

1. Introduction

With the rapid development of high-power laser technology, the laser radiation in the near-visible spectrum with intensities as high as 10^{22} W/cm² can be produced in the laboratory [1]. These new capabilities have led to a renaissance of theoretical interest in strong-field quantum electrodynamics (QED), in which electron dynamics in a powerful laser field must be treated relativistically and in which nonlinear QED processes become important. One of such processes is nonlinear electron-positron (e^-e^+) pair creation which occurs in collisions of an intense laser beam with a nonlaser photon. This process, known as the *nonlinear Breit-Wheeler process*, is the topic of our paper (for recent reviews, see, Refs. [2, 3, 4]).

In the original paper of Breit and Wheeler [5], a collision of two light photons which combined their energies to produce the e^-e^+ pair was considered. This idea was further developed for the case when one of the photons is replaced by a strong laser field [6, 7, 8]. In these studies, the laser field was treated as a monochromatic plane wave. The process in a weakly nonmonochromatic laser field was investigated by Narozhny and Fofanov [9]. It is crucial to realize that the process was realized in the series of experiments performed at SLAC [10, 11], which aimed at testing the hypothesis of light-light scattering.

Since the synthesis of laser pulses with well-controlled properties is currently feasible, it is important to investigate their impact on the nonlinear Breit-Wheeler process. This was realized in recent works where pair creation stimulated by a pulsed laser field was analyzed [12, 13, 14, 15, 16]. While different aspects were studied in these papers, including the dependence of probability distributions of created pairs on intensity, shape, duration, and carrier-envelope phase of the driving pulse, neither of them was analyzed in the context of spin effects. In this paper, we demonstrate that generation of lepton pairs in well-defined spin states can be effectively altered by varying the parameters of the driving pulse. This is in contrast to previous investigations of spin effects in strong-field QED processes which rely on a monochromatic plane wave field approximation (see, for instance, Refs. [17, 18, 19, 20, 21, 22, 23, 24, 25, 26]).

This paper is organized as follows. In Section 2, we present the theoretical formulation of the nonlinear Breit-Wheeler process which is along the lines presented in our previous paper [15].



In Section 3, we specify the laser pulse shape for which the actual numerical calculations are performed. The latter are presented in Section 4, showing a strong dependence of spin effects on parameters of a driving laser pulse such as pulse duration and peak intensity. A brief summary of our results is given in Section 5.

In the theoretical formulas below we keep $\hbar = 1$, however the numerical results are presented in relativistic units such that $\hbar = c = m_e = 1$ where m_e is the electron mass.

2. Theory

The probability amplitude for the Breit-Wheeler pair creation process, $\gamma_{\mathbf{K}\sigma} \rightarrow e_{\mathbf{p}_e-\lambda_{e-}}^- + e_{\mathbf{p}_e+\lambda_{e+}}^+$, with electron and positron momenta and spin polarizations $\mathbf{p}_e-\lambda_{e-}$ and $\mathbf{p}_e+\lambda_{e+}$, respectively, equals

$$\mathcal{A}(\gamma_{\mathbf{K}\sigma} \rightarrow e_{\mathbf{p}_e-\lambda_{e-}}^- + e_{\mathbf{p}_e+\lambda_{e+}}^+) = -ie \int d^4x j_{\mathbf{p}_e-\lambda_{e-}, \mathbf{p}_e+\lambda_{e+}}^{(+)}(x) \cdot A_{\mathbf{K}\sigma}^{(+)}(x), \quad (1)$$

where $\mathbf{K}\sigma$ denotes the initial nonlaser photon momentum and polarization. Here,

$$A_{\mathbf{K}\sigma}^{(+)}(x) = \sqrt{\frac{1}{2\varepsilon_0\omega_{\mathbf{K}}V}} \varepsilon_{\mathbf{K}\sigma} e^{-i\mathbf{K}\cdot x}, \quad (2)$$

where V is the quantization volume, ε_0 is the vacuum electric permittivity, $\omega_{\mathbf{K}} = cK^0 = c|\mathbf{K}|$ ($K \cdot K = 0$), and $\varepsilon_{\mathbf{K}\sigma} = (0, \boldsymbol{\varepsilon}_{\mathbf{K}\sigma})$ is the linear polarization four-vector satisfying the conditions,

$$K \cdot \varepsilon_{\mathbf{K}\sigma} = 0, \quad \varepsilon_{\mathbf{K}\sigma} \cdot \varepsilon_{\mathbf{K}\sigma'} = -\delta_{\sigma\sigma'}, \quad (3)$$

for $\sigma, \sigma' = 1, 2$. The matrix element of the pair current operator is defined as

$$[j_{\mathbf{p}_e-\lambda_{e-}, \mathbf{p}_e+\lambda_{e+}}^{(+)}(x)]^\nu = \bar{\psi}_{\mathbf{p}_e-\lambda_{e-}}^{(+)}(x) \gamma^\nu \psi_{\mathbf{p}_e+\lambda_{e+}}^{(-)}(x). \quad (4)$$

Moreover, $\psi_{\mathbf{p}\lambda}^{(\beta)}(x)$ (with $\beta = +1$ for electron and $\beta = -1$ for positron) is the Volkov solution of the Dirac equation coupled to the electromagnetic field [27, 28]

$$\psi_{\mathbf{p}\lambda}^{(\beta)}(x) = \sqrt{\frac{m_e c^2}{VE_{\mathbf{p}}}} \left(1 - \beta \frac{e}{2k \cdot p} A k\right) u_{\mathbf{p}\lambda}^{(\beta)} e^{-i\beta S_p^{(\beta)}(x)}, \quad (5)$$

with the phase:

$$S_p^{(\beta)}(x) = p \cdot x + \int_{-\infty}^{k \cdot x} \left[\beta \frac{e A(\phi) \cdot p}{k \cdot p} - \frac{e^2 A^2(\phi)}{2k \cdot p} \right] d\phi. \quad (6)$$

Here, $E_{\mathbf{p}} = cp^0 \geq m_e c^2$, $p = (p^0, \mathbf{p})$, $p \cdot p = (m_e c)^2$, and $u_{\mathbf{p}\lambda}^{(\beta)}$ are the free-electron (positron) bispinors normalized such that

$$\bar{u}_{\mathbf{p}\lambda}^{(\beta)} u_{\mathbf{p}\lambda'}^{(\beta')} = \beta \delta_{\beta\beta'} \delta_{\lambda\lambda'}. \quad (7)$$

The four-vector potential $A(k \cdot x)$ in Eq. (5) represents an external electromagnetic radiation generated by lasers such that $k \cdot A(k \cdot x) = 0$ and $k \cdot k = 0$.

We consider a linearly polarized laser pulse that propagates in the direction determined by the unit vector \mathbf{n} and that lasts for time T_p . Its fundamental frequency is $\omega = 2\pi/T_p$ whereas its wave four-vector is $k = k^0(1, \mathbf{n})$ with $k^0 = \omega/c$. The laser pulse is characterized by the following four-vector potential,

$$A(k \cdot x) = A_0 \varepsilon f(k \cdot x), \quad (8)$$

where A_0 relates to the peak laser field intensity, and where a shape function $f(k \cdot x)$ vanishes for $k \cdot x < 0$ and $k \cdot x > 2\pi$. Here, a linear polarization vector $\varepsilon = (0, \boldsymbol{\varepsilon})$ is such that $\varepsilon^2 = -\boldsymbol{\varepsilon}^2 = -1$

and $k \cdot \varepsilon = 0$. As explained in Ref. [15], such a choice of the polarization four-vector ε as a spacelike vector (the same concerns also $\varepsilon_{K\sigma}$) is justified by gauge invariance of the probability amplitude (1). Also in Ref. [15], we showed that this amplitude can be represented such that

$$\mathcal{A}(\gamma_{K\sigma} \rightarrow e_{\mathbf{p}_{e-}\lambda_{e-}}^- + e_{\mathbf{p}_{e+}\lambda_{e+}}^+) = i\sqrt{\frac{2\pi\alpha c(m_e c^2)^2}{E_{\mathbf{p}_{e-}}E_{\mathbf{p}_{e+}}\omega_K V^3}} \mathcal{A}_{\text{BW}}, \quad (9)$$

where $\alpha = e^2/(4\pi\varepsilon_0 c)$ is the fine-structure constant, V is the quantization volume, and

$$\mathcal{A}_{\text{BW}} = \sum_N (2\pi)^3 \delta^{(1)}(P_N^-) \delta^{(2)}(\mathbf{P}_N^\perp) D_N \frac{1 - e^{-2\pi i P_N^+/k^0}}{i P_N^+}. \quad (10)$$

Here, a four-vector

$$P_N = K - \bar{p}_{e-} - \bar{p}_{e+} + Nk, \quad (11)$$

is expressed in terms of the laser-dressed momenta of electron and positron,

$$\bar{p}_{e\mp} = p_{e\mp} \mp \mu m_e c \frac{\varepsilon \cdot p_{e\mp}}{k \cdot p_{e\mp}} \langle f \rangle k + \frac{1}{2} (\mu m_e c)^2 \frac{\langle f^2 \rangle}{k \cdot p_{e\mp}} k, \quad (12)$$

where the averaged values $\langle f^i \rangle$ for $i = 1, 2$ are defined as

$$\langle f^i \rangle = \frac{1}{2\pi} \int_0^{2\pi} d(k \cdot x) [f(k \cdot x)]^i. \quad (13)$$

In Eq. (10), the light cone coordinates have been used $P_N = (P_N^-, P_N^+, \mathbf{P}_N^\perp)$ in accordance with Ref. [29]. In addition, the Fourier coefficients, D_N , are defined as

$$\begin{aligned} D_N = & \frac{1}{2} \mu m_e c \left[\frac{2k^0}{Q^0} \left(\frac{\varepsilon \cdot p_{e+}}{k \cdot p_{e+}} - \frac{\varepsilon \cdot p_{e-}}{k \cdot p_{e-}} \right) \bar{u}_{\mathbf{p}_{e-}\lambda_{e-}}^{(+)} \not{\varepsilon}_{K\sigma} u_{\mathbf{p}_{e+}\lambda_{e+}}^{(-)} - \frac{1}{k \cdot p_{e+}} \bar{u}_{\mathbf{p}_{e-}\lambda_{e-}}^{(+)} \not{\varepsilon}_{K\sigma} \not{k} u_{\mathbf{p}_{e+}\lambda_{e+}}^{(-)} \right. \\ & - \left. \frac{1}{k \cdot p_{e-}} \bar{u}_{\mathbf{p}_{e-}\lambda_{e-}}^{(+)} \not{\varepsilon}_{K\sigma} \not{k} u_{\mathbf{p}_{e+}\lambda_{e+}}^{(-)} \right] G_N^{(1)} + \frac{(\mu m_e c)^2}{4(k \cdot p_{e+})(k \cdot p_{e-})} \left[\frac{2k^0}{Q^0} (k \cdot p_{e+} + k \cdot p_{e-}) \right. \\ & \times \left. \bar{u}_{\mathbf{p}_{e-}\lambda_{e-}}^{(+)} \not{\varepsilon}_{K\sigma} u_{\mathbf{p}_{e+}\lambda_{e+}}^{(-)} + \bar{u}_{\mathbf{p}_{e-}\lambda_{e-}}^{(+)} \not{\varepsilon}_{K\sigma} \not{k} u_{\mathbf{p}_{e+}\lambda_{e+}}^{(-)} \right] G_N^{(2)}, \end{aligned} \quad (14)$$

where $Q^0 = k^0 - p_{e-}^0 - p_{e+}^0 \neq 0$, and where we have introduced a relativistically invariant parameter, $\mu = |eA_0|/(m_e c)$. The functions $G_N^{(i)}$, with $i = 1, 2$, are expressed in terms of the generalized Bessel functions (for more details, see [15, 29]).

Now, it is possible to derive the probability distribution for positrons produced in the Breit-Wheeler process by a finite laser pulse [15]

$$\frac{d^3 \mathbf{P}^{(p)}}{dE_{\mathbf{p}_{e+}} d^2 \Omega_{\mathbf{p}_{e+}}} = \frac{\alpha (m_e c)^2 k^0 |\mathbf{p}_{e+}|}{(2\pi)^2 \omega_K (k \cdot p_{e-})} \left| \sum_N D_N \frac{1 - e^{-2\pi i P_N^0/k^0}}{P_N^0} \right|^2, \quad (15)$$

where the electron four-momentum p_{e-} equals

$$\mathbf{p}_{e-}^\perp = \mathbf{w}^\perp, \quad (16)$$

$$p_{e-}^\parallel = \frac{(m_e c)^2 + (\mathbf{w}^\perp)^2 - (w^-)^2}{2w^-}, \quad (17)$$

$$p_{e-}^0 = \frac{(m_e c)^2 + (\mathbf{w}^\perp)^2 + (w^-)^2}{2w^-}, \quad (18)$$

with $w = K - p_{e^+}$. The last formulas follow from solving the momentum conservation conditions, $P_N^- = 0$ and $\mathbf{P}_N^\perp = \mathbf{0}$, which are defined by the delta functions in Eq. (10). One can check that these solutions satisfy the on-mass shell relation, $p_{e^-} \cdot p_{e^-} = (m_e c)^2$. Note also that the corresponding probability distribution for electrons can be obtained from (15) by interchanging e^+ and e^- .

3. Laser pulse shape

For numerical illustrations, we choose the four-vector potential of the form

$$A(k \cdot x) = A_0 B \varepsilon f(k \cdot x), \quad (19)$$

with the following shape function:

$$f(k \cdot x) = N_A \sin^2\left(\frac{k \cdot x}{2}\right) \sin(N_{\text{osc}} k \cdot x). \quad (20)$$

Here, A_0 is related to the parameter μ whereas N_{osc} denotes the number of field oscillations. As one can understand, the pulse lasts for time $T_p = 2\pi N_{\text{osc}}/\omega$. In addition, the parameter B is introduced in Eq. (19). If we choose $B = N_{\text{osc}}$ the mean intensity contained in the pulse is fixed, irrespective of its duration. For this to happen, we have to keep

$$\frac{1}{2\pi} \int_0^{2\pi} d(k \cdot x) [f'(k \cdot x)]^2 = \frac{1}{2}, \quad (21)$$

where the derivative is with respect to $k \cdot x$ (for more details, see [30]). In the following, we assume that the laser pulse propagates in the z -direction ($\mathbf{n} = \mathbf{e}_z$) and its polarization vector is $\varepsilon = \mathbf{e}_x$. In the considered configuration, the nonlaser photon is counterpropagating ($\mathbf{n}_K = -\mathbf{e}_z$) and its polarization vector is either parallel, $\varepsilon_K = \mathbf{e}_x$, or perpendicular, $\varepsilon_K = -\mathbf{e}_y$, to the polarization direction of the laser pulse. For these two configurations, in the next section we compare the angle-fixed energy spectra of created positrons.

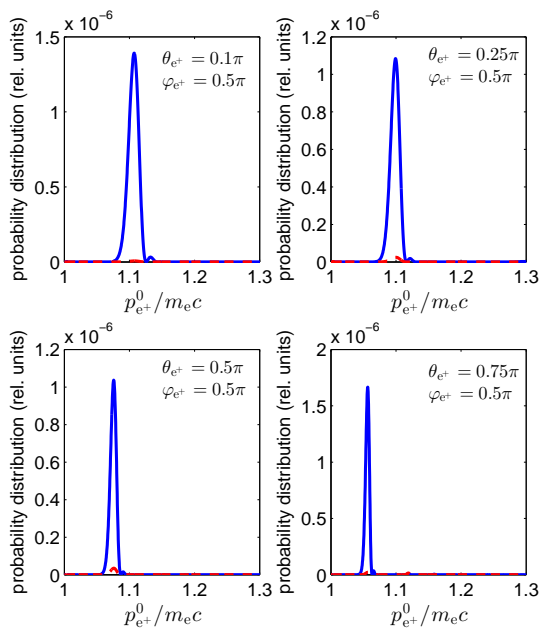


Figure 1. The angle-fixed energy spectra of positrons [defined by Eq. (15)] which are produced in a head-on collision of a laser pulse with a nonlaser photon. The parameters of the colliding pulse are such that $\mu = 1$, $\omega_L = 0.3m_e c^2$, and $N_{\text{osc}} = 32$, while for the nonlaser photon we have $\omega_K = m_e c^2$. Both polarization directions are parallel, i.e., $\varepsilon = \varepsilon_K = \mathbf{e}_x$. While the solid blue line corresponds to the creation of a positron and an electron with the same helicity ($\lambda_{e^+} \lambda_{e^-} = 1$), the dashed red line (which is hardly visible on the scale of this figure) corresponds to the opposite helicity configuration for the produced particles ($\lambda_{e^+} \lambda_{e^-} = -1$).

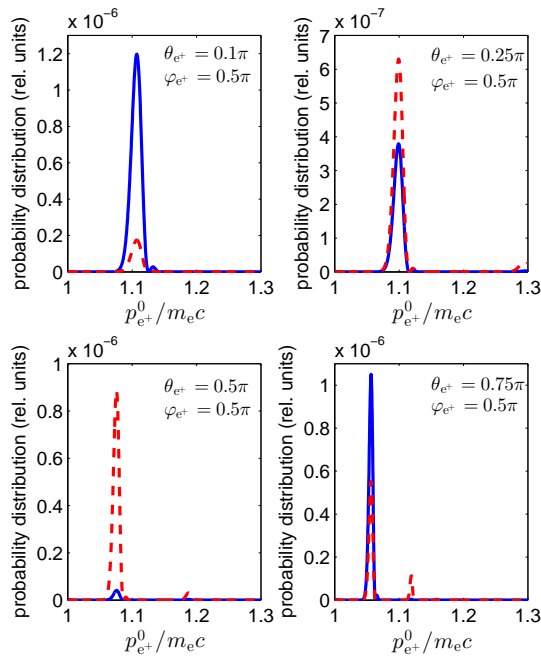


Figure 2. The same as in Fig. 1 but for $\varepsilon_{\mathbf{K}} = -\mathbf{e}_y$.

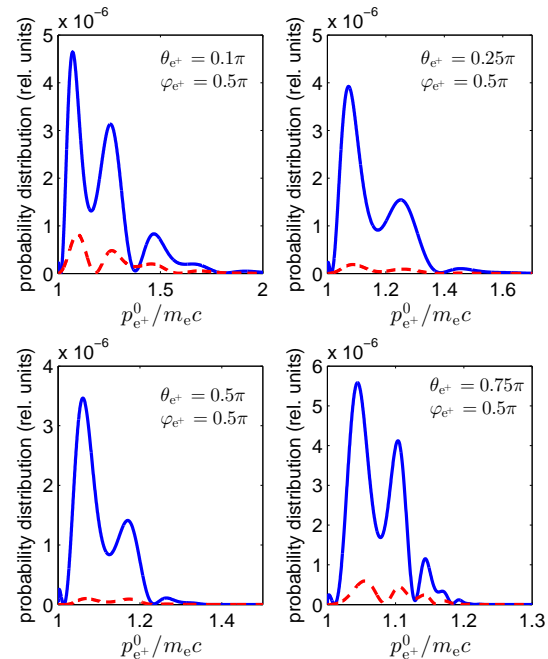


Figure 3. The same as in Fig. 1 but for $N_{\text{osc}} = 4$.

4. Numerical illustrations

The distribution (15) is defined for well-defined spin states of produced electrons and positrons. Based on this formula, we shall study now the spin effects in the nonlinear Breit-Wheeler process.

In Figs. 1 and 2 we show the positron spectra (15) calculated at fixed detection angles in the (yz) -plane, as denoted in each panel. We present the results for the case when linear polarizations of the laser pulse and the colliding photon are parallel (Fig. 1), $\varepsilon = \varepsilon_{\mathbf{K}} = \mathbf{e}_x$, or perpendicular (Fig. 2) to each other, $\varepsilon = \mathbf{e}_x$ and $\varepsilon_{\mathbf{K}} = -\mathbf{e}_y$. The remaining parameters are given in the caption to Fig. 1. Here, we focus on spin effects of the created pairs. Therefore, we compare the energy distributions for the case when the positron and electron are created with the same (solid lines) or the opposite (dashed lines) spin projections. As one can see in Fig. 1, the former configuration dominates. This tendency, however, can be manipulated by changing the polarization direction of the colliding photon or that of the laser pulse, as shown in Fig. 2.

The results presented in Figs. 1 and 2 relate to a rather long driving pulse, which contains 32 field oscillations. In this case, we observe separate multiphoton peaks in the energy spectra of created particles, with dominant peaks as shown in the figures and smaller peaks which are hardly visible on the scale of these figures (see, also Ref. [15]). The question arises whether the pulse duration has any observable impact on the spin effects. To answer this question we present the results for a much shorter pulse, which contains only four oscillations of the electromagnetic field, $N_{\text{osc}} = 4$. Both configurations, when the polarization vectors of the pulse and the nonlaser photon are parallel or perpendicular to each other, are considered (Fig. 3 and 4, respectively).

When comparing Figs. 1 and 3, we observe the same tendency which is that electrons and positrons are mostly created with the same spin projection. It turns out that for perpendicular polarizations, the creation of particles with the opposite spin projections is more significant for shorter pulses (compare Figs. 2 and 4). The latter depends also on the peak intensity of the driving pulse. This can be seen when comparing Figs. 4 and 5. Both figures relate to the same physical geometry, but the peak field strength is different in these two cases (it is either $\mu = 1$

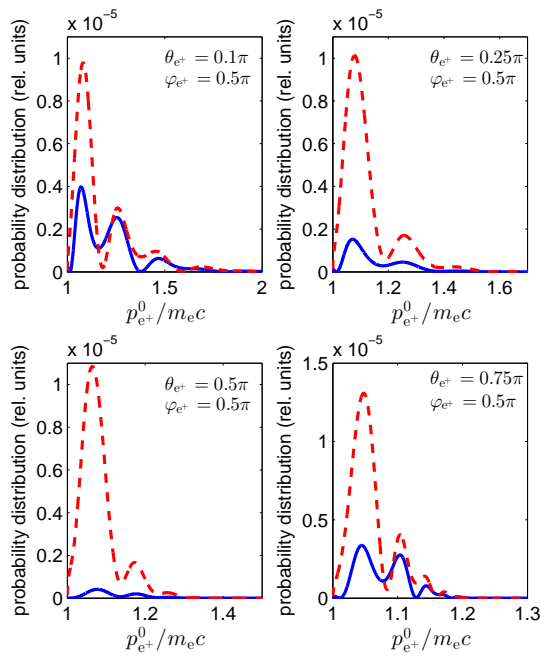


Figure 4. The same as in Fig. 2 but for $N_{\text{osc}} = 4$.

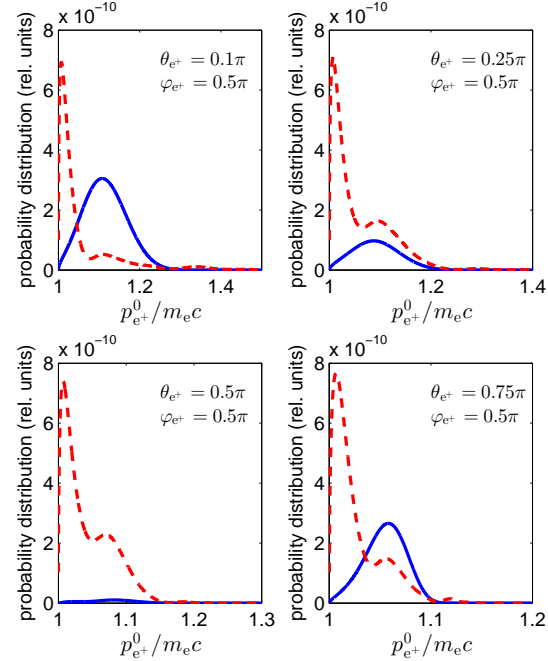


Figure 5. The same as in Fig. 4 but for $\mu = 0.1$.

in Fig. 4, or $\mu = 0.1$ in Fig. 5). Note that for a weaker field (Fig. 5), we observe quite broad regions of positron energy where both product particles are created in the same spin states [this specifically concerns the case when the particles are detected in the propagation direction of the pulse, i.e., for small θ_{e^+} (see, the upper left panel for $\theta_{e^+} = 0.1\pi$)]. However, if we increase the peak field strength (Fig. 4), this energy region significantly deteriorates.

Here, we have looked at the energy spectra of created positrons in the (yz) -plane, i.e., in the plane normal to the polarization direction of the pulse. While for the colinear polarizations of the pulse and the nonlaser photon the electron and the positron are predominantly created in the same spin states (Figs. 1 and 3), the situation is changed for the perpendicular polarizations of the pulse and the nonlaser photon (Figs. 2, 4, and 5). Although we do not present here the corresponding results we have also looked at the positron spectra in the (xz) -plane, i.e., in the plane defined by the polarization and the propagation direction of the driving field. In this case, we observe that the configuration with the opposite spin projections becomes dominant for a colinear configuration of $\boldsymbol{\varepsilon}$ and $\boldsymbol{\varepsilon}_{\mathbf{K}}$ unless the particles are detected in the propagation direction of the pulse (at small angles θ_{e^+}). The electrons and the positrons produced at small angles θ_{e^+} are in the same spin states. On the other hand, when the laser pulse and the nonlaser photon polarizations are perpendicular, the leptons which are created at small angles are in the opposite spin states. This stays true for different values of μ and N_{osc} .

In closing, we note that the stronger the peak field intensity the more effective is pair creation, as illustrated by Figs. 4 and 5. This explains also why the signal of pair creation becomes larger when, for a fixed μ , we decrease the incident pulse duration. Once we keep the average intensity contained in a pulse fixed (see, the discussion in Sec. 3), the maximum value of the electric and magnetic fields must be effectively increased when decreasing the pulse duration. Consequently, an enhancement of the probability of pair production induced by shorter laser pulses is observed. Specifically, if one compares Figs. 1 and 3 the spectra are enhanced roughly four times; if one compares Figs. 2 and 4 the spectra are enhanced by one order of magnitude.

5. Conclusions

In this paper, we have focused on spin effects in the electron-positron pair creation which happens via collisions of a strong laser pulse with a nonlaser photon; both being linearly polarized. We have compared two cases of the colinear and perpendicular polarizations of the laser pulse and the nonlaser photon. We have observed that while for the colinear polarizations, both leptons (when detected at small angles) tend to be in the same spin state thus for the perpendicular polarizations this tendency is reversed. This cannot be generalized for particles detected at bigger angles. We have demonstrated that the spin effects in the nonlinear Breit-Wheeler process are sensitive to both the driving pulse peak strength and its duration. Therefore, they can be used as control parameters when producing highly polarized positron (electron) beams.

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References

- [1] Yanovsky V, Chvykov V, Kalinchenko G, Rousseau P, Planchon T, Matsuoka T, Maksimchuk A, Nees J, Cheriaux G and Krushelnick K 2008 *Opt. Exp.* **16** 2109
- [2] Ehlotzky F, Krajewska K and Kamiński J Z 2009 *Rep. Prog. Phys.* **72** 046401
- [3] Ruffini R, Vereshchagin G and Xue S S 2010 *Phys. Rep.* **487** 1
- [4] Di Piazza A, Müller C, Hatsagortsyan K Z and Keitel C H 2012 *Rev. Mod. Phys.* **84** 1177
- [5] Breit G and Wheeler J A 1934 *Phys. Rev.* **46** 1087
- [6] Reiss H R 1962 *J. Math. Phys.* **3** 59
- [7] Nikishov A I and Ritus V I 1964 *Sov. Phys. JETP* **19** 529
- [8] Narozhny N B, Nikishov A I and Ritus V I 1965 *Sov. Phys. JETP* **20** 622
- [9] Narozhny N B and Fofanov M S 1997 *Laser Phys.* **7** 141
- [10] Bula C *et al* 1996 *Phys. Rev. Lett.* **76** 3116
- [11] Bamber C *et al* 1999 *Phys. Rev. D* **60** 092004
- [12] Heinzl T, Ilderton A and Marklund M 2010 *Phys. Lett. B* **692** 250
- [13] Titov A I, Takabe H, Kämpfer B and Hosaka A 2012 *Phys. Rev. Lett.* **108** 240406
- [14] Nousch T, Seipt D, Kämpfer B and Titov A I 2012 *Phys. Lett. B* **715** 246
- [15] Krajewska K and Kamiński J Z 2012 *Phys. Rev. A* **86** 052104
- [16] Titov A I, Kämpfer B, Takabe H and Hosaka A 2013 *Phys. Rev. A* **87** 042106
- [17] Bunkin F, Kazakov A and Fedorov M V 1973 *Sov. Phys. Usp.* **15** 416
- [18] Szymanowski C, Vénier V, Taïeb, Maquet A and Keitel C H 1997 *Phys. Rev. A* **56** 3846
- [19] Walser M V, Szymanowski C and Keitel C H 1999 *Europhys. Lett.* **48** 533
- [20] Panek P, Kamiński J Z and Ehlotzky F 2002 *Phys. Rev. A* **65** 033408
- [21] Panek P, Kamiński J Z and Ehlotzky F 2004 *Phys. Rev. A* **69** 013404
- [22] Di Piazza A, Milstein A I and Müller C 2010 *Phys. Rev. A* **82** 062110
- [23] Müller T O and Müller C 2011 *Phys. Lett. B* **696** 201
- [24] Müller T O and Müller C 2012 *Phys. Rev. A* **86** 022109
- [25] Ahrens S, Bauke H, Keitel C H and Müller C 2012 *Phys. Rev. Lett.* **109** 043601
- [26] Ahrens S, Müller T O, Villalba-Chavez S, Bauke H and Müller C 2013 *J. Phys.: Conf. Ser.* **414** 012012
- [27] Volkov D M 1935 *Z. Phys.* **94** 250
- [28] Krajewska K and Kamiński J Z 2010 *Phys. Rev. A* **82** 013420
- [29] Krajewska K and Kamiński J Z 2012 *Phys. Rev. A* **85** 062102
- [30] Krajewska K, Müller C and Kamiński J Z 2013 *Phys. Rev. A* **87** 062107